

Solutions to Homework Assignment #1

Assignment:

1. Consider the daily stock return of the Apple Inc. (tick symbol AAPL) and the Standard and Poor's 500 Composite index from January 2000 to December 2007. The data are simple returns and in the file d-aaplsp0007.txt (three columns with date, AAPL, SP).

- (a) Express the simple returns of Apple stock in percentages. Compute the sample mean, standard deviation, skewness, excess kurtosis, minimum, and maximum of the percentage simple returns.

Answer: See Row 1 of Table 1.

- (b) Transform the simple returns of AAPL to log returns and express the log returns in percentages. Compute the sample mean, standard deviation, skewness, excess kurtosis, minimum, and maximum of the percentage log returns.

Answer: See Row 2 of Table1.

Table 1: Summary statistics of daily Apple stock returns.

return	Mean	St.D.	Skew.	Ex.Kur.	Min	Max
Simple	0.158	3.26	-1.88	33.64	-51.87	13.69
Log	0.102	3.45	-4.78	101.7	-73.12	12.83

- (c) Is the sample mean of AAPL log returns statistically different from zero?

Answer: No, test statistic of the one-sample t-test assumes the value 1.321 with p-value 0.1866 so that we cannot reject the null hypothesis of zero mean at the 5% significance level. [Use the command t.test in R.]

- (d) Consider the simple returns of AAPL and S&P 500 index. Is there any difference between the mean returns of the two series?

Answer: Let x_t = simple return of AAPL – simple return of S&P 500 index. Test the null hypothesis that the mean of x_t is zero. The test statistic is 2.36 with p-value 0.018 so that we conclude that the two mean returns are significantly different at the 5% level. [The result suggests that Apple stock outperform the S&P 500 index during the sample period.]

2. Consider the monthly simple returns for the International Business Machines Corp. stock (tick symbol IBM) and the S&P 500 composite index from January 1977 to December 2007. The returns include dividend distributions, and the data file is m-ibmsp7707.txt. Transform the simple returns to log returns and express the log returns in percentages. Compute the sample mean, standard deviation, skewness, excess kurtosis, minimum, and maximum of each percentage log-return series.

Answer:

Table 2: Summary statistics of monthly log returns of IBM and S&P 500 index

return	Mean	St.D.	Skew.	Ex.Kur.	Min	Max
IBM	0.716	7.54	-0.08	1.67	-30.37	30.29
S&P 500	0.703	4.24	-0.84	3.48	-24.54	12.38

3. Consider the monthly 3-month Treasury Bill rates on the secondary market from February 1, 1977 to January 1, 2008. The data file consists of four columns (namely, year, month, day, and rate). The rates are in percentages. Answer the following questions:

- (a) Compute the mean, standard deviation, skewness, and kurtosis of the interest rate series.

Answer: See Row 1 of Table 3. The interest rate series seems a bit skew to the right, but the excess kurtosis is not big.

- (b) Compute the change series of interest rate, i.e. $x_t = y_t - y_{t-1}$ with y_i being the i th observation of the interest rate. Compute the mean, standard deviation, skewness, and kurtosis of the change series.

Answer: See Row 2 of Table 3. The change series is skew to the left with heavy tails.

Table 3: Summary statistics of monthly 3-m Treasury Bill rates.

Series	Size	Mean	St.D.	Skew.	Ex.Kur.	Min	Max
3mTB	372	5.95	3.09	0.88	16.3	0.87	0.90
Change	371	0.002	0.029	-0.50	0.14	-2.45	48.60

- (c) Obtain an empirical density function of the x_t series. [Show a density plot, which can be obtained using the command *density* in R.] Is the density function symmetric with respect to its mean?

Answer: For density function, see Figure 1. For the skewness test, the test statistic is $t = \frac{-2.45}{\sqrt{6/371}} = -19.30$, which is highly significant compared with $N(0, 1)$ distribution. Thus, the density function is not symmetric with respect to its mean.

4. Consider the daily log returns of Apple stock from January 2000 to December 2007 as in Problem 1. Conduct the following tests by (a) state the null and alternative hypotheses, (b) perform the test, and (c) draw your conclusions:

- (a) Test the null hypothesis that the skewness measure of the returns is zero;

- (b) Test the null hypothesis that the excess kurtosis of the returns is zero;

- (c) Test the null hypothesis that the log returns is normally distributed. You may use the Jarque-Bera test.

Answer:

- (a) $H_o : m_3 = 0$ versus $H_a : m_3 \neq 0$, where m_3 denotes the skewness measure. The test statistic is $S^* = \frac{\hat{S}}{\sqrt{6/2010}} = -87.45$, which is highly significant. Conclusion: The skewness of the daily log returns of Apple stock is not zero, indicating that the return distribution is not symmetric. It is skew to the left.

(a) Empirical density of change series of TB3M: 1977.2–2008.1

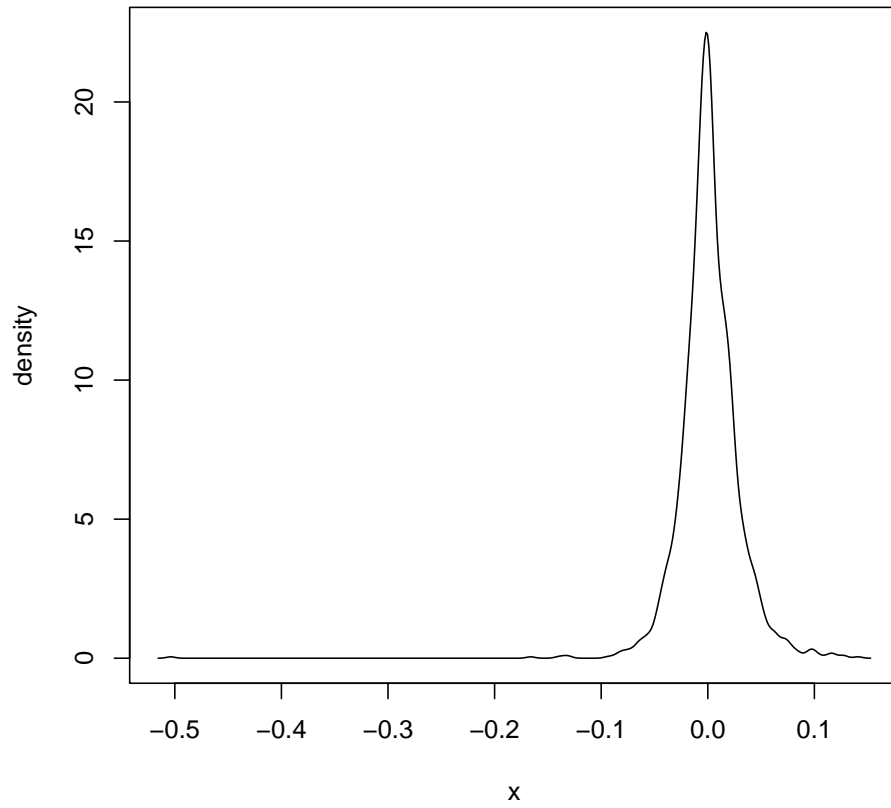


Figure 1: Empirical density

- (b) $H_o : m_4 = 0$ versus $H_a : m_4 \neq 0$, where m_4 denotes the excess kurtosis. The test statistic is $K^* = \frac{\hat{m}_4}{\sqrt{24/2010}} = 930.73$, which is extremely significant. Conclusion: the excess kurtosis of the daily log returns of Apple stock is not zero, indicating that the distribution of the log returns has heavy tails.
- (c) $H_o : r_t$ is normal versus $H_a : r_t$ is not normal. The test statistic is $JB = 875690.7$ with p-value close to zero. Conclusion: the daily log returns of Apple stock is not normally distributed.
5. Again, consider the monthly simple returns of IBM stock and the S&P 500 composite index from 1977 to 2007 and the 3-month Treasury Bill rates from February 1, 1977 to January 1, 2008. Since the Treasury Bill rates are annualized, we can approximate the monthly rates by dividing the rates by 12. Compute the monthly simple excess returns of the IBM stock and the S&P 500 index. Answer the following questions:
- (a) Are the mean excess returns of IBM stock and S&P 500 index significantly different from zero? Why?
Answer: Perform the one-sample test for mean return being zero. For the IBM excess returns, the test statistic is 1.29 with p-value 0.20. For the S&P 500 index excess returns, the test statistic is 1.37 with p-value 0.17. Thus, the means of excess returns are not significantly different from zero.
- (b) Is the mean excess return of the S&P 500 index positive? [In R, use the command `t.test(rt, alternative=c("greater"))`, where `rt` denotes the excess return.] What is the p-value of the test? If the significance level is 10%, what is your conclusion?
Answer: This is a one-sided test, namely $H_o : \mu \leq 0$ versus $H_a : \mu > 0$, where μ is the mean excess return. The test statistic is 1.37 with p-value 0.086. Therefore, we cannot reject the hypothesis that the mean excess return is not greater than zero at the 5% level, but we can reject the null hypothesis at the 10% level. In other words, at the 10% level, the mean of excess returns of the S&P 500 index is greater than zero.
- (c) Are the monthly excess returns of IBM stock symmetric with respect to the mean return?
Answer: Perform the skewness test. The test statistic is $S^* = 2.84$, which is greater than 1.96. Thus, the distribution of the monthly IBM excess returns is not symmetric.
- (d) Do the monthly simple excess returns of IBM stock have heavy-tails? Why?
Answer: Perform the kurtosis test. The test statistic is 7.28, which is greater than the critical value 1.96. Thus, the distribution of monthly IBM excess returns has heavy tails.

R script for the HW assignment

```
>setwd('C:/teaching/bs41202')
>library(fBasics)
# Problem 1
> x=read.table('d-aaplsp0007.txt')
> aapl = x[,2]*100
> basicStats(aapl)
```

```

> rt=log(x[,2]+1)*100
> basicStats(rt)
> t.test(rt)
> xt=x[,2]-x[,3]
> t.test(xt)
# Problem 2
> x=read.table('m-ibmsp7707.txt')
> ibm=log(x[,2]+1)*100
> basicStats(ibm)
> sp5=log(x[,3]+1)*100
> basicStats(sp5)
# Problem 3
> x=read.table('m-tb3ms7708.txt')
> int=x[,4]
> basicStats(int)
> xt=diff(int)
> basicStats(xt)
> d1=density(xt)
> plot(d1$x,d1$y,type='l',xlab='x',ylab='density')
> ss =skewness(xt)/sqrt(6/length(xt))
> ss
# Problem 4
> x=read.table('m-ibmsp7707.txt')
> y=read.table('m-tb3ms7708.txt')
> ibm=x[,2]-y[,4]/(100*12)
> sp5=x[,3]-y[,4]/(100*12)
> t.test(ibm)
> t.test(sp5)
> t.test(sp5,alternative=c('greater'))
> ss=skewness(ibm)/sqrt(6/length(ibm))
> ss
> kk = kurtosis(ibm)/sqrt(24/lenght(ibm))
> kk

```

Note: The command **diff** in R calculates the change series.