THE UNIVERSITY OF CHICAGO
Graduate School of Business
Business 41202, Spring Quarter 2008, Mr. Ruey S. Tsay

Solutions to Homework Assignment #2

Assignment:

1. Monthly simple returns of Decile 1 portfolio.

   (a) The ACF and PACF are shown in Figure 1. The actual values are given below:

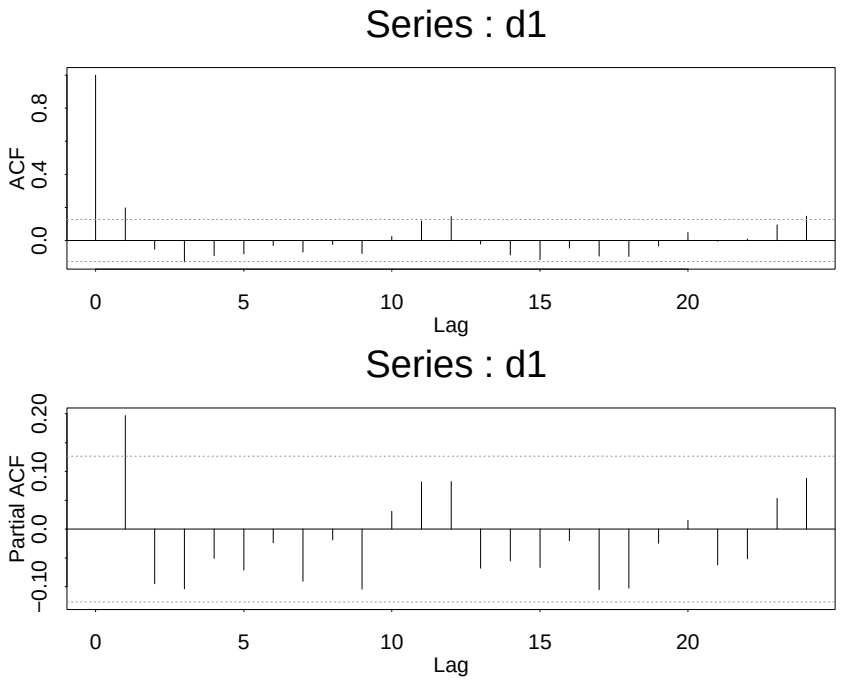
```
 Lag    ACF   PACF
 [1,]  0.197  0.20
 [2,] -0.052 -0.09
 [3,] -0.129 -0.10
 [4,] -0.091 -0.05
 [5,] -0.080 -0.07
 [6,] -0.030 -0.02
 [7,] -0.071 -0.09
 [8,] -0.023 -0.02
 [9,] -0.079 -0.10
[10,]  0.025  0.03
[11,]  0.118  0.08
[12,]  0.145  0.08
[13,] -0.022 -0.07
[14,] -0.087 -0.06
[15,] -0.115 -0.07
[16,] -0.047 -0.02
[17,] -0.095 -0.11
[18,] -0.097 -0.10
[19,] -0.032 -0.02
[20,]  0.048  0.01
[21,] -0.004 -0.06
[22,]  0.008 -0.05
[23,]  0.095  0.05
[24,]  0.146  0.09
```

   (b) The Ljung-Box statistic is $Q(12) = 30.01$ with p-value 0.003, which is less than 0.05 so that we reject the null hypothesis. In other words, there are serial correlations in the returns.

   (c) Lag 12 ACF: The t-ratio is $t = 0.145/\sqrt{1/240} = 2.26 > 1.96$ so that the lag-12 ACF is significantly different from zero.

   (d) The t-ratio if $t = 0.2/\sqrt{1/240} = 3.10$ so that the lag-1 PACF is significantly different from zero.

Figure 1: ACF and PACF of the monthly simple returns of Decile 1 portfolio



2. Monthly simple returns of Decile 10. in Problem 1.

   (a) The Ljung-Box statistic is $Q(12) = 6.97$ with p-value 0.86 so that there is no serial correlations for the first 12 lags of the returns.

   (b) For the absolute returns, $Q(12) = 31.75$ with p-value 0.002 so that there exists serial correlations in the absolute returns.

3. Apple stock daily log price range.

   (a) The ACF are

```
 Lag  ACF
 [1,] 0.50
 [2,] 0.44
 [3,] 0.41
 [4,] 0.41
 [5,] 0.40
 [6,] 0.39
 [7,] 0.38
 [8,] 0.37
 [9,] 0.37
[10,] 0.35
```

The Ljung-Box statistic is $Q(10) = 3662.5$ with p-value 0. Therefore, there is serial correlations in the daily range series.

(b) The first 25 PACFs are

```
 Lag    PACF
 [1,]  0.496
 [2,]  0.264
 [3,]  0.165
 [4,]  0.161
 [5,]  0.120
 [6,]  0.094
 [7,]  0.072
 [8,]  0.066
 [9,]  0.067
[10,]  0.027
[11,]  0.056
[12,]  0.029
[13,]  0.060
[14,]  0.071
[15,]  0.041
[16,]  0.063
[17,]  0.013
[18,]  0.005
[19,]  0.039
[20,] -0.001
[21,] -0.015
[22,]  0.014
[23,] -0.010
[24,]  0.018
[25,]  0.002
```

The standard error of the PACF is $1/\sqrt{2235} = 0.021$. The selected AR order is 16 because the PACF cuts off at lag 16 based on 2 standard-error limit.

4. U.S. quarterly unemployment rate.

(a) See Figure 2

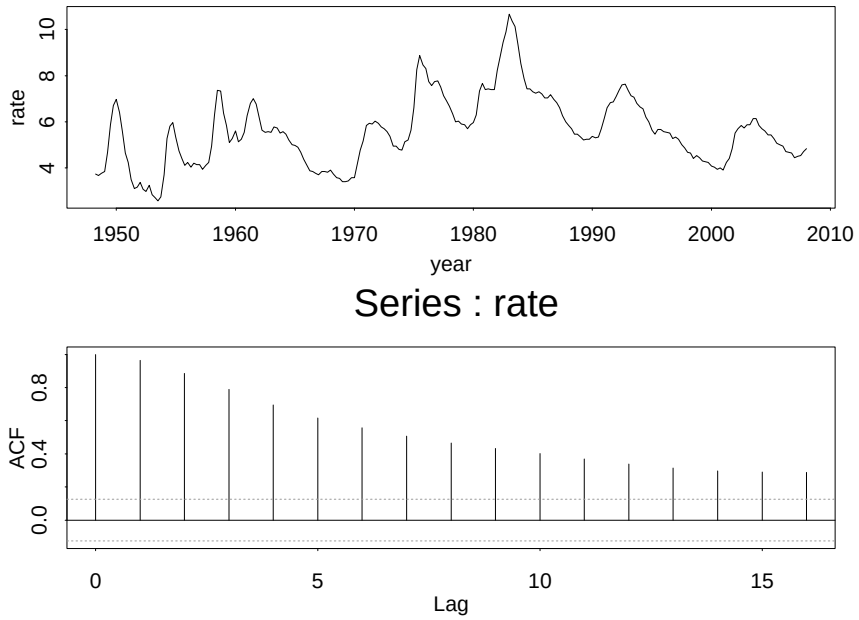
(b) Using the **ar** command, i.e. AIC, one identifies an AR(5) model for the series. The fitted model is

$$y_t = 1.66y_{t-1} - 0.78y_{t-2} + 0.04y_{t-3} - 0.05y_{t-4} + 0.09y_{t-5} + a_t,$$

where $y_t = r_t - 5.49$ and $\sigma_a^2 = 0.080$. Residual checking indicates that there are some minor serial correlations at higher lags in the residuals.

(c) For the fitted model, the characteristic function has five roots that include two pairs of complex conjugates. Thus, there are business cycles in the series. The average lengths of the business cycles are 10.96 quarters and 2.85 quarters. These match nicely with the

Figure 2: Quarterly unemployment rate and its ACF



common belief that the average period of expansion in the U.S. is about three years and the average period of U.S. recession is about 3 quarters.

(d) The 1-step to 4-step ahead forecasts are

```
    fore$pred fore$se
241       4.9    0.28
242       5.0    0.55
243       5.0    0.78
244       5.1    0.97
```

Note that the forecasts are close to what we saw in the recent publication. For example, last month unemploymnet rate was 5.1.

5. Decile 1 portfolio returns.

(a) The fitted model is

$$r_t = 0.015 + a_t + 0.22a_{t-1}, \quad \sigma_a^{@} = 0.0035.$$

(b) The forecasts are

```
    fore$pred fore$se
241     0.013   0.059
242     0.015   0.060
```

```
243     0.015   0.060
244     0.015   0.060
```

The 2-step to 4-step forecasts are the same because the MA(1) model has only one lag in memory.