

THE UNIVERSITY OF CHICAGO
Graduate School of Business
 Business 41202, Spring Quarter 2008, Mr. Ruey S. Tsay

Solutions to Homework Assignment #6

1. Consider the Qualcomm stock.

- Calculate the VaR of your position for the next trading day using the RiskMetrics method at the time point $T = 2514$. (You may estimate the IGARCH(1,1) model without constant to obtain the parameter values needed in the calculation.)

Answer: The IGARCH(1,1) estimation gives $\alpha = 0.97$. Thus, $\sigma_{2515}^2 = 0.03a_{2514}^2 + 0.97\sigma_{2514}^2 = 0.03(0.311) + 0.97(3.32) = 3.23$. Consequently, $\text{VaR} = 2.33\sqrt{3.23}/100 * 1000000 = \41875 .

- Build a GARCH(1,1) model for the log return series with Gaussian innovations. What is the VaR based on the fitted model?

Answer: The fitted Gaussian GARCH(1,1) model is

$$r_t = 0.11 + a_t, \quad a_t = \sigma_t \epsilon_t, \quad \epsilon \sim N(0, 1).$$

$$\sigma_t^2 = 0.051 + 0.061a_{t-1}^2 + 0.936\sigma_{t-1}^2.$$

From the model, $r_{2514}(1) = 0.11$ and $\sigma_{2514}^2(1) = 0.051 + 0.061(-0.6679)^2 + 0.936(3.6198) = 3.466$. $\text{VaR} = |0.11 - 2.33\sqrt{3.466}|/100 * 1000000 = \42278 .

- Build a GARCH(1,1) model with t -innovations for the log return series. What is the VaR based on the fitted model?

Answer: The fitted model is

$$r_t = 0.056 + a_t, \quad a_t = \sigma_t \epsilon_t, \quad \epsilon_t \sim t_{6.37}^*.$$

$$\sigma_t^2 = 0.023 + 0.041a_{t-1}^2 + 0.957\sigma_{t-1}^2.$$

From the model, $r_{2514}(1) = 0.056$ and $\sigma_{2514}^2(1) = 0.023 + 0.041(-0.614)^2 + 0.957(3.64) = 3.52$. The 1% quantile of $t_{6.37}$ is -3.082 . Consequently, $\text{VaR} = |0.056 + (-3.082)\sqrt{3.52}/\sqrt{6.37/4.37}|/100 * 1000000 = \47354 .

2. Again, consider the daily log returns of Qualcomm stock. Using blocks of size 21, fit a generalized extreme value distribution to the negative return series. Write down the estimates and their standard errors. Compute the 1% VaR of your financial position based on the fitted parameters. What is the 1% VaR of your financial position for the next 10 trading days?

Answer: The estimates are $\xi = 0.241$, $\sigma = 2.223$ and $\mu = 3.852$. Using the program `evtVaR`, the VaR is $\$80476$. For 10 days, $\text{VaR} = 10^{0.241} * 80476 = \140174 .

3. Again, consider the negative log returns of the Qualcomm stock. Fit a generalized Pareto distribution to the return series with threshold 4.0%. Based on the fitted model, what is the 1% VaR of your position? What is the associated expected shortfall? Repeat the analysis using threshold 5.0%. Are the results sensitive to the choice of thresholds?

Answer: For threshold 4%, the estimates are $\xi = 0.016$ and $\beta = 2.591$. The estimate of ξ is not statistically significant, but the calculation continues to work. The VaR is \$96970 and the expected shortfall is \$124255.

For the threshold 5%, the estimates are $\xi = -0.053$ and $\beta = 2.970$. The VaR is \$98730 and expected shortfall is \$123810. Since the estimates of ξ are close to zero, the GPD estimation does not indicate a heavy left tail for the distribution. The VaR and expected shortfall are not sensitive to the choices of threshold.

4. Consider now the log returns of McDonald's stock. Calculate the VaR using Risk-Metrics method. Also, what is the VaR for the combined position of Qualcomm and McDonald's stocks?

Answer: For MCD stock, the estimates of the special IGARCH(1,1) model gives $\alpha = 0.964$. The *garch11v* program shows $a_{2514}^2 = 0.993$ and $\sigma_{2514}^2 = 1.717$. Consequently, the VaR = $2.33\sqrt{0.036(0.993) + 0.964(1.717)}/100 \times 1000000 = \29867 .

The sample correlation between QCOM and MCD is 0.178. The combined VaR is $\sqrt{41875^2 + 29867^2 + 2 * 0.178 * 41875 * 29867} = \55595 .

5. Again, consider the daily log returns of McDonald's stock. Using blocks of size 63, fit a generalized extreme value distribution to the negative returns. Write down the estimates and their standard errors. Compute the 1% VaR of your financial position based on the fitted parameters. Repeat the calculation if block sizes of 21 are used. Are the results sensitive to the choice of block sizes?

Answer: With block size 63, the estimates are $\xi = 0.385(0.185)$, $\sigma = 1.257(0.213)$, $\mu = 3.222(0.240)$, where the number in parentheses is standard error. The VaR is \$38501. For block size 21, the estimates are $\xi = 0.322$, $\sigma = 1.045$, and $\mu = 2.293$. The VaR is \$44032, which is slightly higher. In this particular case, the VaR seems sensitive to the choice of block size.