Bus41202: Analysis of Financial Time Series, Spring 2008
Solutions to Self-Study HW Assignments

1. Yes, r_t^o is serially correlated. Based on the result of Section 5.1, the first three lags of ACF are -9.95×10^{-4}, -5.97×10^{-4}, and -3.58×10^{-4}, respectively.

2. (a) Yes, the diurnal pattern exists. The ACF of the n_t series show a periodic pattern. See Figure 1. (b) Ignoring the time intervals between trades, we simply consider the series $r_i = \ln(P_i) - \ln(P_{i-1})$, where P_i is the ith transaction price as observed. This r_i series has a negative lag-1 serial correlation of -0.28, which is highly significant. Thus, there is bid-ask bounce. Alternatively, one can employ the Decomposition model of Section 5.4.2 to verify the bid-ask bounce. (c) The frequency table is as follows

	Negative						Positive				
Size	5	4	3	2	1	0	1	2	3	4	5
Freq.	30	45	137	800	3577	12163	3574	776	151	52	41

3. The result of serial correlation test in R is given below:

```
> x=read.table("taq-ge-dec5-5m.txt")
> dim(x)
[1] 385   1
> Box.test(x,lag=10,type='Ljung')

        Box-Ljung test

data:  x
X-squared = 24.326, df = 10, p-value = 0.00678
```

The $Q(10) = 24.32$, with a p-value smaller that 0.01. Therefore, we reject the null hypothesis of no serial correlation.

4. The price is $P_t = \exp(p_t)$. Taking partial derivatives, we have

$$\frac{\partial P_t}{\partial t} = 0, \quad \frac{\partial P_t}{\partial p_t} = \exp(p_t) = P_t, \quad \frac{\partial P_t^2}{\partial p_t^2} = P_t.$$

By Ito's lemma,

$$dP_t = (P_t \gamma + \frac{1}{2} P_t \sigma^2) dt + P_t \sigma dw_t.$$

That is,

$$dP_t = (\gamma + \frac{\sigma^2}{2}) P_t dt + \sigma P_t dw_t.$$

5. Apply the Black-Scholes pricing formula. (a) Price of a call option with strike price \$125 is $c_t = \$10.71$. (b) Price of a put option with strike price \$118 is $p_t = \$9.82$. (c) If the volatility is increased to 80%, then the call option becomes $c_t = \$17.86$ and the put option becomes $p_t = \$16.75$.

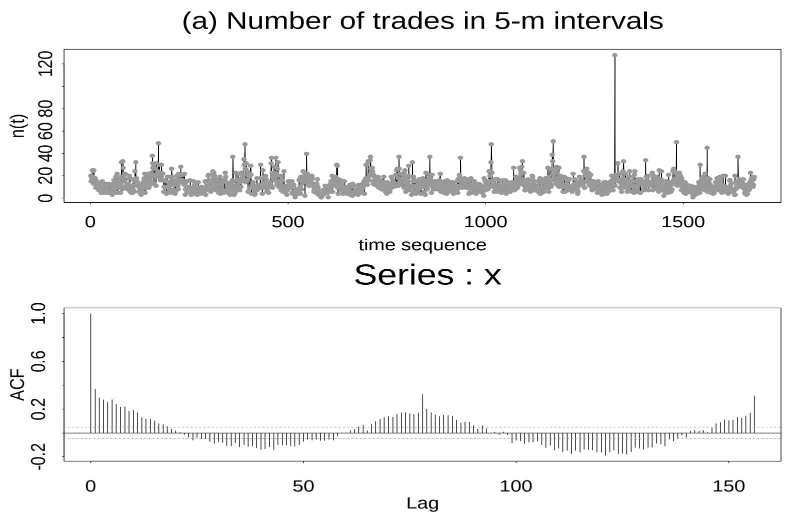

Figure 1: (a) The number of trades in 5-minute intervals for 3M stock in December 1999. (b) The sample ACF of the n_t series in part (a).