Booth School of Business, University of Chicago  
Business 41202, Spring Quarter 2009, Mr. Ruey S. Tsay

Midterm

GSB Honor Code:  
I pledge my honor that I have not violated the Honor Code during this examination.

Signature:  
Name:  
ID:

Notes:
• Open notes and books.
• For each question, write your answer in the blank space provided.
• Manage your time carefully and answer as many questions as you can.
• The exam has 7 pages and the R output has 8 pages. Please check to make sure that you have all the pages.
• For simplicity, ALL tests use the 5% significance level.
• Round your answer to 3 significant digits.

Problem A: (30 pts) Answer briefly the following questions. Each question has two points.

1. For Questions 1 to 5, consider the monthly simple returns of CRSP Decile 10 portfolio from January 1970 to December 2008. The basic statistics of the data are given in the attached R output. Is the mean return of Decile 10 portfolio significantly different from zero? Why?

2. What is the unconditional standard error of the simple monthly returns?

3. Are the monthly simple returns symmetric with respect to their mean value? Why?
4. Do the monthly simple returns have heavy tails? Why?

5. Are the monthly simple returns serially correlated? Why?

6. **For Questions 6 and 7**, consider the monthly simple returns of the Decile 9 portfolio. As shown in homework assignment, the series follows the model \( r_t = 0.0099 + a_t + 0.155a_{t-1} \), \( \sigma^2 = 0.00242 \). Based on the model, what is the expected mean return of the Decile 9 portfolio?

7. Again, based on the model, what is the unconditional variance of the monthly simple returns of Decile 9 portfolio?

8. What is the null hypothesis when one applies the Ljung-Box test \( Q(15) \) to a time series \( x_t \)?

9. **For Questions 9 and 10**, consider the quarterly earnings per share of Intel Corporation from the first quarter of 1992 to the first quarter of 2009. The series follows approximately the AR(1) model \( (1 - 0.81B)(r_t - 0.19) = a_t \), \( \sigma^2 = 0.0044 \). It is also known that the company earned $0.04 in the first quarter of 2009. Based on the model, obtain the forecast of the earnings per share for the second quarter of 2009.

10. What is the standard deviation of the associated forecast error for the Intel earnings prediction?

11. Both the EGARCH and GJR models can handle asymmetric responses of asset volatility to prior positive and negative innovations. Give a main difference between these two models.
12. **For Questions 12 to 13**, consider a the GARCH(1,1) model

\[
    r_t = 0.0075 + a_t, \quad a_t = \sigma_t \epsilon_t, \quad \epsilon_t \sim iid(0, 1)
\]

\[
    \sigma_t^2 = 0.001 + 0.853 \sigma_{t-1}^2 + 0.125 a_{t-1}^2.
\]

Suppose that at the forecast origin \( t = h \), \( \sigma_h = 0.053 \) and \( a_h = -0.012 \). Compute the volatility forecast for time \( t = h + 1 \).

13. Compute the volatility forecast for time \( t = h + 2 \) made at the forecast origin \( t = h \).

14. Describe a consequence of overlooking the serial correlations in a linear regression model.

15. Describe two differences between the empirical density function of an asset return series and the normal density that has the same mean and standard deviation.
Problem B. (37 pts) Consider the daily log returns, in percentages, of the 3M stock from January 1999 to December 2008. The relevant R output is attached. Answer the following questions.

1. (2 points) Is there any serial correlation in the log return series? Why?

2. (2 points) An MA model is used to handle the mean equation, which appears to be adequate. Write down the fitted MA model, including the residual variance.

3. (6 points) A GARCH(1,1) model with Gaussian distribution is used for the volatility equation. It turns out the MA coefficients become insignificant so that the mean equation is refined. Write down the fitted model, including the mean equation.

4. (2 points) Except for the normality, is the fitted GARCH(1,1) model adequate? Why?

5. (4 points) To check the risk premium of volatility, a GARCH(1,1)-M model is used. Is the risk premium statistically significant? Why?

6. (3 points) A GARCH(1,1) model with Student-\( t \) innovations is applied. Write down the volatility equation, including the degrees of freedom.

7. (3 points) Focus on an IGARCH(1,1) model, write down the fitted model.

8. (2 points) Why are the volatility forecasts of the IGARCH(1,1) model not constant for all forecast horizons?
9. (3 points) A GJR model is also fitted. Write down the fitted volatility equation.

10. (2 points) Based on the GJR model, is the leverage effect significant? Why?

11. (4 points) An EGARCH(1,1) model is also fitted. Write down the fitted volatility equation. You may ignore the insignificant parameter, if any.

12. (4 points) Based on the EGARCH model, calculate the ratio \( \frac{\sigma_t^2(\epsilon_{t-1}=-2)}{\sigma_t^2(\epsilon_{t-1}=2)} \) to see the leverage effect.
Problem C. (13 pts) Consider the monthly unemployment rate for the State of Illinois from 1976 to March 2009. The data are obtained from the Federal Reserve Bank at St Louis and are seasonally adjusted. Let \( x_t \) be the monthly unemployment rate.

1. (5 points) Write down the final fitted model for \( x_t \), including the residual variance.

2. (2 points) Is the fitted model adequate? Why?

3. (2 points) Based on the fitted model, are there business cycles in the Illinois economy? Why?

4. (2 points) Calculate the average length of the business cycles implied by the fitted model.

5. (2 points) Use the data and fitted model, obtain a 95% interval forecast for the May unemployment rate of Illinois.
**Problem D.** (20 pts) As mentioned in class, VIX of CBOE is widely used as a volatility index for the U.S. stock market. This problem is concerned with the relationship between the VIX index and the volatility series estimated from the daily log returns of the S&P 500 index. The sample period is from January 2, 2004 to April 16, 2009 for 1331 observations. Note that I used a Gaussian AR(2)-GARCH(1,1) model to estimate the volatility. In addition, the GARCH daily volatility is annualized (multiplied by $\sqrt{252}$) to match with VIX measure. Let $Y_t = \ln(VIX_t)$ and $X_t = \ln(V_t)$, where $V_t$ is the volatility obtained from the GARCH(1,1) model.

1. (2 points) Is the fitted GARCH(1,1) model for the daily S&P 500 index highly persistent? Why?

2. (3 points) Write down the linear regression model between $Y_t$ and $X_t$.

3. (3 points) Let $y_t$ and $x_t$ be the first differenced series of $Y_t$ and $X_t$, respectively. Is the correlation between $y_t$ and $x_t$ significant at the 5% level? Why?

4. (6 points) Write down the final fitted model between $y_t$ and $x_t$.

5. (2 points) Is there any serial correlation in the residuals of the final fitted model? Why?

6. (4 points) Does the VIX index depend on the GARCH volatility? Why?
```r
> library(fBasics)
> source("garchoxfit_R.txt")

*** Problem A ***
> da=read.table("m-deciles08.txt",header=T)
> d10=da[,5]
> basicStats(d10)

   d10
nobs 468.000000
Minimum -0.199203
Maximum  0.176525
1. Quartile -0.015977
3. Quartile  0.035473
Mean      0.008265
Median    0.010325
Sum       3.868040
SE Mean   0.001993
LCL Mean  0.004350
UCL Mean  0.012181
Variance  0.001858
Stdev     0.043106
Skewness  -0.420791
Kurtosis  2.229552

> Box.test(d10,12,type='Ljung')

Box-Ljung test

data: d10
X-squared = 10.6872, df = 12, p-value = 0.5559

*** Problem B *****
> da=read.table("d-3msp9908.txt",header=T)
> da[1,]

date   mmm     sp
1 19990104 0.049209 -0.000919

> mmm=log(da[,2]+1)*100  % Percentage log returns
> Box.test(mmm,10,type='Ljung')

Box-Ljung test

data: mmm
X-squared = 28.733, df = 10, p-value = 0.001376

> acf(mmm)  % Compute ACF

> m1=arima(mmm,order=c(0,0,2))
Coefficients:
     ma1     ma2 intercept
```
\[
\begin{bmatrix}
-0.0218 & -0.0773 & 0.0279 \\
0.0200 & 0.0205 & 0.0293
\end{bmatrix}
\]

\(\text{s.e.} 0.0200 0.0205 0.0293\)

\(\sigma^2\) estimated as 2.661: log likelihood = -4799.51, aic = 9607.02

> Box.test(m1$residuals,10,type='Ljung')

Box-Ljung test
data: m1$residuals
X-squared = 11.7898, df = 10, p-value = 0.2994

> Box.test(m1$residuals^2,10,type='Ljung')

Box-Ljung test
data: m1$residuals^2
X-squared = 427.5722, df = 10, p-value < 2.2e-16

> m2=garchOxFit(formula.mean=~arma(0,2),formula.var=~garch(1,1),series=mmm)

Maximum Likelihood Estimation (Std.Errors based on Second derivatives)

<table>
<thead>
<tr>
<th>Coefficient</th>
<th>Std.Error</th>
<th>t-value</th>
<th>t-prob</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cst(M)</td>
<td>0.045629</td>
<td>0.026143</td>
<td>1.745</td>
</tr>
<tr>
<td>MA(1)</td>
<td>-0.011194</td>
<td>0.022342</td>
<td>-0.5010</td>
</tr>
<tr>
<td>MA(2)</td>
<td>-0.036400</td>
<td>0.022313</td>
<td>-1.631</td>
</tr>
<tr>
<td>Cst(V)</td>
<td>0.112481</td>
<td>0.024814</td>
<td>4.533</td>
</tr>
<tr>
<td>ARCH(Alpha1)</td>
<td>0.090184</td>
<td>0.015217</td>
<td>5.926</td>
</tr>
<tr>
<td>GARCH(Beta1)</td>
<td>0.866898</td>
<td>0.021976</td>
<td>39.45</td>
</tr>
</tbody>
</table>

> m2=garchOxFit(formula.mean=~arma(0,0),formula.var=~garch(1,1),series=mmm)

** SPECIFICATIONS **

Dependent variable : X
Mean Equation : ARMA (0, 0) model. No regressor in the mean
Variance Equation : GARCH (1, 1) model. No regressor in the variance
The distribution is a Gauss distribution.

Maximum Likelihood Estimation (Std.Errors based on Second derivatives)

<table>
<thead>
<tr>
<th>Coefficient</th>
<th>Std.Error</th>
<th>t-value</th>
<th>t-prob</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cst(M)</td>
<td>0.045660</td>
<td>0.027350</td>
<td>1.669</td>
</tr>
<tr>
<td>Cst(V)</td>
<td>0.109661</td>
<td>0.023986</td>
<td>4.572</td>
</tr>
<tr>
<td>ARCH(Alpha1)</td>
<td>0.089748</td>
<td>0.015082</td>
<td>5.950</td>
</tr>
<tr>
<td>GARCH(Beta1)</td>
<td>0.868582</td>
<td>0.021504</td>
<td>40.39</td>
</tr>
</tbody>
</table>

***************
** FORECASTS **
***************
Number of Forecasts: 15

<table>
<thead>
<tr>
<th>Horizon</th>
<th>Mean</th>
<th>Variance</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.04566</td>
<td>4.005</td>
</tr>
<tr>
<td>2</td>
<td>0.04566</td>
<td>3.947</td>
</tr>
<tr>
<td>3</td>
<td>0.04566</td>
<td>3.891</td>
</tr>
<tr>
<td>4</td>
<td>0.04566</td>
<td>3.838</td>
</tr>
<tr>
<td>......</td>
<td></td>
<td></td>
</tr>
<tr>
<td>15</td>
<td>0.04566</td>
<td>3.38</td>
</tr>
</tbody>
</table>

***********

** TESTS **

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Q-Statistics on Standardized Residuals
Q(10) = 11.4192 [0.3258064]
Q(20) = 24.8356 [0.2077950]

H0 : No serial correlation ==> Accept H0 when prob. is High [Q < Chisq(lag)]

-------------

Q-Statistics on Squared Standardized Residuals
--> P-values adjusted by 2 degree(s) of freedom
Q(10) = 4.91159 [0.7669820]
Q(20) = 8.18053 [0.9758263]

H0 : No serial correlation ==> Accept H0 when prob. is High [Q < Chisq(lag)]

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> m3=garchOxFit(formula.mean=~arma(0,0),formula.var=~garch(1,1),
series=mmm,arch.in.mean=T)

********************

** SPECIFICATIONS **

Dependent variable : X
Mean Equation : ARMA (0, 0) model. No regressor in the mean
Variance Equation : GARCH (1, 1) model. in-mean
No regressor in the variance
The distribution is a Gauss distribution.

Maximum Likelihood Estimation (Std.Errors based on Second derivatives)

<table>
<thead>
<tr>
<th>Coefficient</th>
<th>Std.Error</th>
<th>t-value</th>
<th>t-prob</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cst(M)</td>
<td>-0.012853</td>
<td>0.055702</td>
<td>-0.2308</td>
</tr>
<tr>
<td>Cst(V)</td>
<td>0.110108</td>
<td>0.023951</td>
<td>4.597</td>
</tr>
<tr>
<td>ARCH(Alpha1)</td>
<td>0.089994</td>
<td>0.015045</td>
<td>5.982</td>
</tr>
<tr>
<td>GARCH(Beta1)</td>
<td>0.868067</td>
<td>0.021463</td>
<td>40.44</td>
</tr>
<tr>
<td>ARCH-in-mean(var)</td>
<td>0.029423</td>
<td>0.024376</td>
<td>1.207</td>
</tr>
</tbody>
</table>

> m4=garchOxFit(formula.mean=~arma(0,0),formula.var=~garch(1,1),series=mmm,cond.dist="t")

********************
** SPECIFICATIONS **

Dependent variable : X
Mean Equation : ARMA (0, 0) model. No regressor in the mean
Variance Equation : GARCH (1, 1) model. No regressor in the variance
The distribution is a Student distribution, with 4.67985 degrees of freedom.

Maximum Likelihood Estimation (Std.Errors based on Second derivatives)

<table>
<thead>
<tr>
<th>Coefficient</th>
<th>Std.Error</th>
<th>t-value</th>
<th>t-prob</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cst(M)</td>
<td>0.025556</td>
<td>0.023673</td>
<td>1.080</td>
</tr>
<tr>
<td>Cst(V)</td>
<td>0.006889</td>
<td>0.006981</td>
<td>0.9868</td>
</tr>
<tr>
<td>ARCH(Alpha1)</td>
<td>0.037292</td>
<td>0.009582</td>
<td>3.892</td>
</tr>
<tr>
<td>GARCH(Beta1)</td>
<td>0.961873</td>
<td>0.010957</td>
<td>87.78</td>
</tr>
<tr>
<td>Student(DF)</td>
<td>4.679848</td>
<td>0.41824</td>
<td>11.19</td>
</tr>
</tbody>
</table>

> m5=garchOxFit(formula.mean=~arma(0,0),formula.var=~igarch(1,1),series=mmm)

** SPECIFICATIONS **

Dependent variable : X
Mean Equation : ARMA (0, 0) model. No regressor in the mean
Variance Equation : IGARCH (1, 1) model. No regressor in the variance
The distribution is a Gauss distribution.

Maximum Likelihood Estimation (Std.Errors based on Second derivatives)

<table>
<thead>
<tr>
<th>Coefficient</th>
<th>Std.Error</th>
<th>t-value</th>
<th>t-prob</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cst(M)</td>
<td>0.045923</td>
<td>0.027442</td>
<td>1.673</td>
</tr>
<tr>
<td>Cst(V)</td>
<td>0.058513</td>
<td>0.015052</td>
<td>3.887</td>
</tr>
<tr>
<td>ARCH(Alpha1)</td>
<td>0.117026</td>
<td>0.020937</td>
<td>5.589</td>
</tr>
<tr>
<td>GARCH(Beta1)</td>
<td>0.883174</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

** FORECASTS **

Number of Forecasts: 15

<table>
<thead>
<tr>
<th>Horizon</th>
<th>Mean</th>
<th>Variance</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.04592</td>
<td>5.609</td>
</tr>
<tr>
<td>2</td>
<td>0.04592</td>
<td>5.668</td>
</tr>
<tr>
<td>3</td>
<td>0.04592</td>
<td>5.726</td>
</tr>
<tr>
<td>4</td>
<td>0.04592</td>
<td>5.785</td>
</tr>
<tr>
<td>.........</td>
<td></td>
<td></td>
</tr>
<tr>
<td>15</td>
<td>0.04592</td>
<td>6.428</td>
</tr>
</tbody>
</table>
Q-Statistics on Standardized Residuals
\[ Q(10) = 9.81177 \ [0.4571597] \]
\[ Q(20) = 21.6296 \ [0.3609447] \]
H0: No serial correlation ==> Accept H0 when prob. is High [Q < Chisq(lag)]

Q-Statistics on Squared Standardized Residuals
--> P-values adjusted by 2 degree(s) of freedom
\[ Q(10) = 6.48574 \ [0.5929897] \]
\[ Q(20) = 10.7767 \ [0.9035926] \]
H0: No serial correlation ==> Accept H0 when prob. is High [Q < Chisq(lag)]

\[ \text{m6=garchOxFit(formula.mean=~arma(0,0),formula.var=~gjr(1,1),series=mmm)} \]

** SPECIFICATIONS **

Dependent variable: X
Mean Equation: ARMA (0, 0) model. No regressor in the mean
Variance Equation: GJR (1, 1) model. No regressor in the variance
The distribution is a Gauss distribution.

Maximum Likelihood Estimation (Std.Errors based on Second derivatives)

<table>
<thead>
<tr>
<th>Coefficient</th>
<th>Std.Error</th>
<th>t-value</th>
<th>t-prob</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cst(M)</td>
<td>0.027421</td>
<td>0.275765</td>
<td>0.9948</td>
</tr>
<tr>
<td>Cst(V)</td>
<td>0.102334</td>
<td>0.021451</td>
<td>4.771</td>
</tr>
<tr>
<td>ARCH(Alpha1)</td>
<td>0.053279</td>
<td>0.013139</td>
<td>4.055</td>
</tr>
<tr>
<td>GARCH(Beta1)</td>
<td>0.873023</td>
<td>0.019740</td>
<td>44.23</td>
</tr>
<tr>
<td>GJR(Gamma1)</td>
<td>0.073447</td>
<td>0.019879</td>
<td>3.695</td>
</tr>
</tbody>
</table>

\[ \text{m7=garchOxFit(formula.mean=~arma(0,0),formula.var=~egarch(1,1),series=mmm)} \]

** SPECIFICATIONS **

Dependent variable: X
Mean Equation: ARMA (0, 0) model. No regressor in the mean
Variance Equation: EGARCH (1, 1) model. No regressor in the variance
The distribution is a Gauss distribution.

Maximum Likelihood Estimation (Std.Errors based on Second derivatives)

<table>
<thead>
<tr>
<th>Coefficient</th>
<th>Std.Error</th>
<th>t-value</th>
<th>t-prob</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cst(M)</td>
<td>0.015527</td>
<td>0.027089</td>
<td>0.5732</td>
</tr>
</tbody>
</table>
Cst(V)  1.142883  0.10703  10.68  0.0000
ARCH(Alpha1) -0.070013  0.24788 -0.2824  0.7776
GARCH(Beta1)  0.971283  0.0075914 127.9  0.0000
EGARCH(Theta1) -0.048788  0.013879 -3.515  0.0004
EGARCH(Theta2)  0.145827  0.044171  3.301  0.0010

***************
** FORECASTS **
***************
Number of Forecasts: 15

<table>
<thead>
<tr>
<th>Horizon</th>
<th>Mean</th>
<th>Variance</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.01553</td>
<td>4.432</td>
</tr>
<tr>
<td>2</td>
<td>0.01553</td>
<td>4.415</td>
</tr>
<tr>
<td>3</td>
<td>0.01553</td>
<td>4.372</td>
</tr>
<tr>
<td>4</td>
<td>0.01553</td>
<td>4.33</td>
</tr>
<tr>
<td>....</td>
<td>....</td>
<td>....</td>
</tr>
<tr>
<td>15</td>
<td>0.01553</td>
<td>3.964</td>
</tr>
</tbody>
</table>

** TESTS **

************

Q-Statistics on Standardized Residuals
Q( 10) = 10.2103 [0.4222412]
Q( 20) = 22.8594 [0.2957503]
H0 : No serial correlation ==> Accept H0 when prob. is High [Q < Chisq(lag)]

Q-Statistics on Squared Standardized Residuals
---> P-values adjusted by 2 degree(s) of freedom
Q(10) = 6.20758 [0.6239927]
Q(20) = 8.79235 [0.9643605]
H0 : No serial correlation ==> Accept H0 when prob. is High [Q < Chisq(lag)]

**** Problem C ****
> da=read.table("unem-il.txt")
> da[1,]
   V1 V2 V3  V4
1 1976 1 1 6.4
> il=da[,4]
> m1=ar(il,method="mle")
> m1$order
[1] 5
> m2=arima(il,order=c(5,0,0))
> m2
arima(x = il, order = c(5, 0, 0))
Coefficients:

<table>
<thead>
<tr>
<th></th>
<th>ar1</th>
<th>ar2</th>
<th>ar3</th>
<th>ar4</th>
<th>ar5</th>
<th>intercept</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1.2622</td>
<td>-0.1333</td>
<td>0.0133</td>
<td>-0.0535</td>
<td>-0.1002</td>
<td>7.0085</td>
</tr>
<tr>
<td>s.e.</td>
<td>0.0497</td>
<td>0.0807</td>
<td>0.0809</td>
<td>0.0808</td>
<td>0.0509</td>
<td>0.7136</td>
</tr>
</tbody>
</table>

$sigma^2$ estimated as 0.02950: log likelihood = 134.13, aic = -254.27

> m2=arima(il,order=c(5,0,0),fixed=c(NA,NA,0,0,NA,NA))
> m2

arima(x = il, order = c(5, 0, 0), fixed = c(NA, NA, 0, 0, NA, NA))

Coefficients:

<table>
<thead>
<tr>
<th></th>
<th>ar1</th>
<th>ar2</th>
<th>ar3</th>
<th>ar4</th>
<th>ar5</th>
<th>intercept</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1.2592</td>
<td>-0.1380</td>
<td>0</td>
<td>0</td>
<td>-0.1328</td>
<td>7.0297</td>
</tr>
<tr>
<td>s.e.</td>
<td>0.0496</td>
<td>0.0629</td>
<td>0</td>
<td>0</td>
<td>0.0228</td>
<td>0.7084</td>
</tr>
</tbody>
</table>

$sigma^2$ estimated as 0.02953: log likelihood = 133.89, aic = -257.77

> p1=c(1,-m2$coef[1:5])
> mm=polyroot(p1)
> mm

[1] 1.046594+0.000000i -1.979174+0.000000i -0.105095-1.780519i
[4] 1.142769-0.000000i -0.105095+1.780519i

> Mod(mm)

[1] 1.046594 1.979174 1.783618 1.142769 1.783618

> k=2*pi/acos(-.105095/1.783618)
> k

[1] 3.855299

> tsdiag(m2,gof=24)
> Box.test(m2$residuals,24,type='Ljung')

Box-Ljung test
data: m2$residuals
X-squared = 22.3796, df = 24, p-value = 0.5566

> predict(m2,4)

$pred

Time Series: Start = 400, End = 403
Frequency = 1


$se

Start = 400, End = 403
Frequency = 1
***** Problem D *****
> da=read.table("vix0409.txt",header=T)
> da[1,]
Mon Day year VIXOpen VIXHigh VIXLow VIXClose
1 1 2 2004 17.96 18.68 17.54 18.22
> vix=log(da[,7]) % Take log-transformation of the VIX index
> da=read.table("d-sp50409.txt",header=T)
> da[1,]
Mon Day Year Open High Low Close Volume AdjClose
1 4 16 2009 854.54 870.35 847.04 865.3 6598670000 865.3
> sp5=log(da[,9])
> sp=sp5
> length(sp)
[1] 1332
> for (i in 1:1332){ % Reverse the ordering
+ sp[i]=sp5[1332-i+1]
+ }
> sp5=diff(sp)*100 % Percentage log returns.

> m1=garchOxFit(formula.mean="arma(2,0),formula.var="garch(1,1),series=sp5)

********************
** SPECIFICATIONS **
********************
Dependent variable : X
Mean Equation : ARMA (2, 0) model. No regressor in the mean
Variance Equation : GARCH (1, 1) model. No regressor in the variance
The distribution is a Gauss distribution.

Maximum Likelihood Estimation (Std.Errors based on Second derivatives)

<table>
<thead>
<tr>
<th></th>
<th>Coefficient</th>
<th>Std.Error</th>
<th>t-value</th>
<th>t-prob</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cst(M)</td>
<td>0.029475</td>
<td>0.018514</td>
<td>1.592</td>
<td>0.1116</td>
</tr>
<tr>
<td>AR(1)</td>
<td>-0.089956</td>
<td>0.029151</td>
<td>-3.086</td>
<td>0.0021</td>
</tr>
<tr>
<td>AR(2)</td>
<td>-0.063881</td>
<td>0.028851</td>
<td>-2.214</td>
<td>0.0270</td>
</tr>
<tr>
<td>Cst(V)</td>
<td>0.010816</td>
<td>0.0036103</td>
<td>2.996</td>
<td>0.0028</td>
</tr>
<tr>
<td>ARCH(Alpha1)</td>
<td>0.076304</td>
<td>0.012076</td>
<td>6.319</td>
<td>0.0000</td>
</tr>
<tr>
<td>GARCH(Beta1)</td>
<td>0.915426</td>
<td>0.013389</td>
<td>68.37</td>
<td>0.0000</td>
</tr>
</tbody>
</table>

> garch=sqrt(m1$condvars*252) % Annualize the fitted GARCH volatility
> garch=log(garch) % Take the log-transformation of the fitted volatility
> plot(garch,vix) % The plot strong linear relation
> m2=lm(vix~garch)
> summary(m2)
Call:
\texttt{lm(formula = vix \sim garch)}

Coefficients:
\[ \begin{array}{lrrrr}
\text{Estimate} & \text{Std. Error} & \text{t value} & \text{Pr(>|t|)} \\
(Intercept) & 0.527727 & 0.018572 & 28.42 & <2e-16 *** \\
garch & 0.859372 & 0.006737 & 127.56 & <2e-16 *** \\
\end{array} \]

---

\texttt{> acf(m2$residuals) \% Show strong serial correlations in the residuals}
\texttt{> dvix=diff(vix) \% Use 1st differenced series}
\texttt{> dgarch=diff(garch) \% use 1st differenced series}
\texttt{> m3=lm(dvix \sim dgarch)}
\texttt{> summary(m3)}

Call:
\texttt{lm(formula = dvix \sim dgarch)}

Coefficients:
\[ \begin{array}{lrrrr}
\text{Estimate} & \text{Std. Error} & \text{t value} & \text{Pr(>|t|)} \\
(Intercept) & 0.0005378 & 0.0017987 & 0.299 & 0.76500 \\
dgarch & -0.0879575 & 0.0326944 & -2.690 & 0.00723 ** \\
\end{array} \]

---

Residual standard error: 0.0656 on 1328 degrees of freedom
Multiple R-squared: 0.005421, Adjusted R-squared: 0.004672

\texttt{> acf(m3$residuals) \% show some serial correlations at lags 1, 2, and 10.}

\texttt{> m4=arima(dvix,xreg=dgarch,include.mean=F,order=c(0,0,2),seasonal=list(order=c(0,0,1))}
\texttt{> m4}

Call:
\texttt{arima(x = dvix, order = c(0, 0, 2), seasonal = list(order = c(0, 0, 1), period = 10),}
\texttt{xreg = dgarch, include.mean = F)}

Coefficients:
\[ \begin{array}{lrrrr}
\text{ma1} & \text{ma2} & \text{sma1} & \text{dgarch} \\
-0.1656 & -0.0938 & 0.0887 & -0.0108 \\
\text{s.e.} & 0.0298 & 0.0291 & 0.0274 & 0.0351 \\
\end{array} \]

\texttt{sigma^2 estimated as 0.004132: log likelihood = 1762.99, aic = -3515.97}

\texttt{> tsdiag(m4,gof=20) \% The plot not shown.}
\texttt{> Box.test(m4$residuals,20,type='Ljung')}

Box-Ljung test
data: m4$residuals
X-squared = 14.9595, df = 20, p-value = 0.7787