Solutions to Homework Assignment #2

R commands used: The R commands and output I used is also posted. In Particular, in Problem 4, I demonstrate ways to fix non-significant parameter to zero in the ARIMA command of R.

1. Decile returns.

(a) The first 24 lags of ACF and PACF are given below:

ACF:
  0 1 2 3 4 5 6 7 8 9 10 11 12
  1.00 0.18 -0.03 -0.10 -0.06 -0.05 -0.03 -0.05 -0.07 -0.09 0.02 0.07 0.30
  13 14 15 16 17 18 19 20 21 22 23 24
  0.03 -0.10 -0.07 -0.02 -0.02 -0.04 -0.07 -0.06 -0.09 -0.05 0.05 0.19

PACF:
  1 2 3 4 5 6 7 8 9 10 11 12 13
  0.18 -0.06 -0.09 -0.03 -0.04 -0.03 -0.05 -0.07 -0.08 0.03 0.04 0.27 -0.08
  14 15 16 17 18 19 20 21 22 23 24
  -0.08 0.00 -0.02 -0.04 -0.04 -0.02 -0.06 -0.08 0.02 0.09

(b) The Ljung-Box statistic is $Q(12) = 76.66$ with p-value $1.79 \times 10^{-11}$. Thus, we reject the null hypothesis that the first 12 lags of ACF are all zero.

(c) The $t$-ratio of ACF at lag 12 is 6.414, which is highly significant with p-value close to zero. Thus, $\rho_{12}$ is not zero.

(d) The $t$-ratio of the lag-12 PACF is $\sqrt{468}\hat{\phi}_{12,12} = 5.83$, which is highly significant. Reject the null hypothesis, i.e. $\phi_{12,12} \neq 0$.

2. Decile 9 and Decile 10.

(a) For decile 9 returns, $\hat{\rho}_{12} = 0.035$ with $t$-ratio 0.75, which has a p-value 0.45. Thus, the null hypothesis of $\rho_{12} = 0$ cannot be rejected at the 5% level.

(b) Decile 10. The Ljung-Box statistic shows $Q(12) = 37.95$ with p-value 0.00015 so that there are serial correlations in the absolute simple returns of Decile 10.


(a) 10 lags of ACF:

```r
rt=log(da$ASKHI)-log(da$BIDLO)
acf2=acf(rt,lag=10)
print(acf2$acf,digits=2)
[1,] 1.00
```
The Ljung-Box statistic is $Q(10) = 5544.5$, which is highly significant. There are serial correlations in the daily range of Boeing stock.

(b) 20 lags of PACF

```r
> pacf2 = pacf(rt, lag = 20)
> print(pacf2, digits = 2)
```

Partial autocorrelations of series, by lag

```
         1       2       3       4       5       6       7       8       9      10      11      12      13
 0.63 0.29 0.19 0.12 0.12 0.13 0.07 0.08 0.08 0.04  0.01  -0.04  0.01
 14 15 16 17 18 19 20
0.04 0.03 0.00 0.02 0.02 0.00 0.02
```

The sample size is 2011 so that $2/\sqrt{2011} = 0.045$. Based on the PACF, one would specify an AR(9) or AR(10) model for the range series.


(a) See Figure 1.

(b) The `ar` command identifies an AR(11) model. The fitted model is

$$(1 - .99B - .24B^2 + .08B^3 + .07B^4 - .04B^5 + .13B^6 + .04B^7 - .06B^8 + .02B^9 + .13B^{10} - .12B^{11})(r_t - 5.65) = \alpha_t,$$

where the residual variance is 0.039.

(c) Yes, there are complex roots in the resulting characteristic equation.

(d) Predictions:

```r
> predict(m4, 4)
```

$pred$

```
<table>
<thead>
<tr>
<th></th>
<th>Apr</th>
<th>May</th>
<th>Jun</th>
<th>Jul</th>
</tr>
</thead>
</table>
```

$se$

```
<table>
<thead>
<tr>
<th></th>
<th>Apr</th>
<th>May</th>
<th>Jun</th>
<th>Jul</th>
</tr>
</thead>
<tbody>
<tr>
<td>2009</td>
<td>0.1966402</td>
<td>0.2764734</td>
<td>0.3656208</td>
<td>0.4532837</td>
</tr>
</tbody>
</table>
```
5. Decile 9 portfolio returns.

(a) The fitted model is

\[ r_t = 0.0099 + a_t + 0.16a_{t-1}, \quad \sigma_a^2 = 0.0024. \]

(b) Predictions:

```r
> predict(m5,4)

$pred
Time Series:
Start = 469
End = 472
Frequency = 1
[1] 0.019103745 0.009865929 0.009865929 0.009865929

$se
[1] 0.04917506 0.04976540 0.04976540 0.04976540
```

Because MA(1) model has a memory of 1 lag, the forecasts become sample mean after 1-step ahead.