1. Monthly conventional 30-year mortgage rates. Using PACF or the ar command in R identifies an AR(3) model. Using the ar command with “mle” identifies an AR(4) model. Either model should be fine as the fitted AR-4 coefficient is relatively small in magnitude. My answer is based on the AR(4) model. The fitted model is

\[(1 - 1.59B + .99B^2 - .51B^3 + .12B^4)(m_t - 8.19) = a_t, \quad \sigma_a^2 = 0.0661.\]

Model checking via tsdiag command looks fine. For Ljung-Box statistics, we have \(Q(12) = 13.91\) with p-value 0.31. The 1-step to 4-step ahead forecasts are 4.88, 4.89, 4.95 and 4.98, respectively. The standard errors of the predictions are 0.26, 0.48, 0.62, and 0.72, respectively.

2. Use of monthly 3-month Treasury Bill rate. The ordinary linear regression model shows

\[\text{mort}_t = 4.29 + 0.83\text{tb3m}_t + e_t.\]

The residuals of this model have high serial correlations. Thus, we take the first differences of the mortgage and Treasury Bill rate. That is,

\[y_t = (1 - B)\text{mort}_t, \quad x_t = (1 - B)\text{tb3m}_t.\]

The regression model is

\[y_t = 0.28x_t + e_t, \quad \sigma_e = 0.266.\]

However, there are serial correlations in the residual series \(e_t\). Using PACF or the ar command with “mle” identifies an AR(2) model for the residuals. The model then becomes

\[(1 - 0.47B + 0.28B^2)(y_t - 0.217x_t) = a_t, \quad \sigma_a^2 = 0.0573.\]

The tsdiag command shows that the model is adequate with \(Q(12) = 14.07\) with p-value 0.30 for the residuals.

The regression coefficient 0.217 is highly significant because its standard error is only 0.025. Thus, the mortgage rate indeed depends on the interest rate.
3. Decile 1 returns. The ordinary linear regression is
\[ d_t = 0.0028 + 0.1253 \text{Jan}_t + e_t, \]
where \( d_t \) is the monthly simple return of Decile 1 portfolio. The coefficient of January dummy is highly significant, but the residuals of the fitted model show some serial correlations. In particular, lag-1 ACF is significant. [There is some minor serial correlation at lag 12.] The refined model is
\[ d_t = 0.0034 + 0.1186 \text{Jan}_t + a_t + 0.1918 a_{t-1}, \quad \sigma^2_a = 0.0046. \]
The tsdiag command of R indicates the fitted model is adequate. The residuals show \( Q(24) = 20.99 \) with \( p \)-value 0.64.

For prediction, we have
\[
> \text{predict(m2,newxreg=newjan,12)}
\]
\[
\begin{align*}
\text{pred} \\
\text{Start} & = 469 \\
\text{End} & = 480 \\
\text{Frequency} & = 1 \\
[1] & 0.1311 0.0034 0.0034 0.0034 0.0034 0.0034 \\
[7] & 0.0034 0.0034 0.0034 0.0034 0.0034 0.0034 \\
\end{align*}
\]
\[
\begin{align*}
\text{se} \\
\text{Start} & = 469 \\
\text{End} & = 480 \\
\text{Frequency} & = 1 \\
[1] & 0.0677 0.0689 0.0689 0.0689 0.0689 0.0689 \\
[7] & 0.0677 0.0689 0.0689 0.0689 0.0689 0.0689 \\
\end{align*}
\]

4. Sesonal model. The fitted model is
\[
(1 - .988B^{12})(d_t - 0.0119) = (1 + .1896B)(1 - 0.9135B^{12}) a_t, \quad \sigma^2_a = 0.0047. \]
Again, the tsdiag command fails to detect any model inadequacy. Indeed, \( Q(24) = 17.97 \) with \( p \)-value 0.80 for the residuals.

The predictions are
\[
> \text{predict(m3,12)}
\]
\[
\begin{align*}
\text{pred} \\
\text{Time Series:}
\end{align*}
\]
The two models are similar to each other. The fitted results are close between the two models.

5. First, consider the two sub-series. The regression models obtained are

\[ d_t = 0.0019 + 0.154 \text{jan}_t + a_t + 0.205a_{t-1}, \quad \sigma_a^2 = 0.0057 \]

for the first 240 observations, i.e., from 1970 to 1989, and

\[ d_t = 0.0049 + 0.081 \text{jan}_t + a_t + 0.215a_{t-1}, \quad \sigma_a^2 = 0.0032, \]

for the second part of the data from 1990 to 2008. From the two models, the effect of January decreases from 0.154 to 0.081. Given that the standard errors of the two coefficients are small, the difference seems significant.

Second, we use two dummy variables. The second dummy variable is for January from 1990 to 2008 whereas the first dummy variable is for January of the whole sample. The fitted model is

\[ d_t = 0.0029 + 0.158 \text{jan}_t - 0.071 \text{Jan1}_t + a_t + 0.205a_{t-1}, \quad \sigma_a^2 = 0.0045. \]

The negative coefficient, which is significantly different from zero, confirms that the January effect has decreased significantly in recent years.