Solutions to Homework Assignment #4

Note that Student-t distributions are standardized in volatility equations.

   - Yes, because $Q(10) = 29.90$ with $p$-value 0.0008 for the series. The AR(2) model $(1 - 0.004B + 0.096B^2)(r_t - 0.7 \times 10^{-4}) = a_t$ fits the dell well, where $r_t$ is the daily log returns of Dell stock. Note that only the AR(2) coefficient is statistically significant.
   - Yes, $Q(10) = 420.45$ for the squared residuals of the mean equation.
   - The fitted Gaussian GARCH(1,1) model is
     \[
     r_t = -0.0002 + 0.0159r_{t-1} + 0.0746r_{t-2} + a_t, \quad a_t = \sigma_t \epsilon_t
     \]
     \[
     \sigma_t^2 = 0.297 \times 10^{-4} + 0.038a_{t-1}^2 + 0.958\sigma_{t-1}^2.
     \]
     This model fits the data well as the $Q$-statistics of the standard residuals and the squares of standardized residuals all have high $p$-values.
   - The fitted model with Student-$t$ innovations is
     \[
     r_t = -0.0001 + 0.0014r_{t-1} + 0.0409r_{t-2} + a_t, \quad a_t = \sigma_t \epsilon_t, \quad \epsilon \sim t_{5.22}
     \]
     \[
     \sigma_t^2 = 0.003 \times 10^{-4} + 0.026a_{t-1}^2 + 0.974\sigma_{t-1}^2.
     \]
   - The fitted model is
     \[
     r_t = -0.0002 + 0.016r_{t-1} + 0.075r_{t-2} + a_t, \quad a_t = \sigma_t \epsilon_t
     \]
     \[
     \sigma_t^2 = 0.019 \times 10^{-4} + 0.0386a_{t-1}^2 + 0.9614\sigma_{t-1}^2.
     \]
   - Based on the AIC, the AR(2)-GARCH(1,1) model with Student $t$ innovations is selected for the Dell log returns.

2. Daily log returns of the VW index, denoted by $r_t$.
   - Yes, there are serial correlations in the log returns as shown by $Q(10) = 41.54$ with $p$-value $9.0 \times 10^{-6}$. Based on ACF, an MA(2) model is specified. The residuals of the MA(2) model have not significant serial correlations based on $Q(10) = 15.58$ with $p$-value 0.11.
• Yes, there are ARCH effects as the squared residuals of the fitted MA(2) model show \(Q(10) = 2434.1\) with \(p\)-value close to zero.

• The fitted IAGARCH(1,1) model is

\[
\begin{align*}
    r_t & = 0.00054 + a_t - 0.027a_{t-1} - 0.055a_{t-2}, \quad a_t = \sigma_t \epsilon_t, \quad \epsilon_t \sim t_{10.55} \\
    \sigma_t^2 & = 0.007 \times 10^{-4} + 0.085\sigma_{t-1}^2 + 0.915\sigma_{t-1}^2.
\end{align*}
\]

• Model checking indicates the fitted IGARCH(1,1) model is adequate at the 5% level. The 1-step to 5-step volatility forecasts are all around 2.75%.

3. Daily log returns of Dell stock denoted by \(r_t\).

• The fitted GARCH(1,1)-M model

\[
\begin{align*}
    r_t & = 3.4 \times 10^{-5} - 0.481\sigma_t^2 - 0.0012r_{t-1} - 0.041r_{t-2} + a_t, \quad a_t = \sigma_t \epsilon_t, \quad \epsilon_t \sim t_{5.21} \\
    \sigma_t^2 & = 3.09 \times 10^{-7} + 0.0259a_{t-1}^2 + 0.974\sigma_{t-1}^2.
\end{align*}
\]

Model checking indicates that the fitted model is adequate.

• No, it is not statistically significant. In addition, the negative sign seems counterintuitive.

• The fitted GJR(1,1) model is

\[
\begin{align*}
    r_t & = -2.97 \times 10^{-4} - 0.015r_{t-1} - 0.067r_{t-2} + a_t, \quad a_t = \sigma_t \epsilon_t, \quad \epsilon_t \sim N(0,1) \\
    \sigma_t^2 & = 1.7 \times 10^{-6} + (0 + 0.044N_{t-1})a_{t-1}^2 + 0.975\sigma_{t-1}^2,
\end{align*}
\]

where \(N_{t-1} = 1\) if \(a_{t-1} < 0\) and \(N_{t-1} = 0\), otherwise.

• Yes, the leverage parameter is significant at the 5% level. Indeed, it is highly significant with \(t\)-ratio \(5.45\).

4. Monthly log returns for 3M stock, denoted by \(r_t\) and measured in percentages.

• Yes, there are serial correlations in the daily log returns of 3M stock. The \(Q(12)\) statistic of the log return series assumes the value 27.69 with \(p\)-value 0.006. Based on PACF, an AR(3) model is specified.

• The fitted model is

\[
\begin{align*}
    r_t & = 1.014 - 0.054a_{t-1} - 0.047r_{t-2} - 0.099r_{t-3} + a_t, \quad a_t = \sigma_t \epsilon_t, \quad \epsilon \sim \text{ged}(1.557). \\
    \sigma_t^2 & = 4.43 + 0.066a_{t-1} + 0.822\sigma_{t-1}^2.
\end{align*}
\]

Model checking shows that the model is adequate.

• The 1-step to 5-step ahead forecasts for the return and volatility are (a) mean return: 2.41%, 1.32%, 2.53%, 0.78%, and 0.95%; (b) volatility: 7.26, 7.16, 7.07, 6.99, 6.91, respectively.
5. Daily log returns of Dollar-Euro exchange rate. Denote it by $r_t$.

- Yes, $Q(10) = 24.67$ with $p$-value 0.006 for the $r_t$. An MA(4) model with lag-4 coefficient only seems adequate in removing the serial correlations.

- Yes, we have $Q(10) = 323.06$ with $p$-value close to zero for the squared residuals of the MA(4) model.

- A simple Gaussian GARCH(1,1) model appears to fit the data adequately. The model is

$$
\begin{align*}
    r_t &= 1.74 \times 10^{-4} + a_t + 0.013a_{t-1} + 0.0002a_{t-2} - 0.01a_{t-3} + 0.053a_{t-4}, \\
    a_t &= \sigma_t \epsilon_t, \quad \epsilon_t \sim N(0, 1), \\
    \sigma_t^2 &= 6.31 \times 10^{-8} + 0.027a_{t-1}^2 + 0.972\sigma_{t-1}^2.
\end{align*}
$$

- The 1-step to 4-step ahead forecasts for the log return and its volatility are as follows: (a) log return: $1.98 \times 10^{-4}$, $9.82 \times 10^{-5}$, $3.07 \times 10^{-4}$ and $10.73 \times 10^{-4}$, and (b) volatility: $0.01095$, $0.01094$, $0.01094$, and $0.01093$, respectively.