Solutions to Homework Assignment #6

This assignment is concerned with VaR. Assume that the probability of interest is 1% and the financial position is long with $1 million for each of the two stocks involved. Two data sets are used. The first data set consists of the daily simple returns of the Apple stock from January 1999 to December 2008 for 2515 observations. The second data set contains the daily simple returns of the Wal-Mart stock for the same period. The data are in files d-aapl9908.txt and d-wmt9908.txt. Each file has two columns, namely date and sample returns. Transform the simple returns into log returns, in percentages.

1. Let $r_t$ be the daily log returns of Apple stock, in percentages.
   - The fitted IGARCH(1,1) model is
     \[ r_t = 0.229 + a_t, \quad a_t = \sigma_t \epsilon_t, \quad \epsilon_t \sim N(0,1) \]
     \[ \sigma_t^2 = 0.06673a_{t-1}^2 + 0.93347\sigma_{t-1}^2. \]
     Ignoring the constant term in the mean equation (due to program constraint), the 1-step ahead volatility prediction is $\sqrt{1.281} = 3.58$. The VaR for $r_t$ is $2.326 \times 3.58 = 8.3271$. Therefore, 1% VaR for Apple position is $83271.00.
   - The GARCH(1,1) model is
     \[ r_t = 0.212 + a_t, \quad a_t = \sigma_t \epsilon_t, \quad \epsilon_t \sim N(0,1) \]
     \[ \sigma_t^2 = 0.529 + 0.153a_{t-1}^2 + 0.818\sigma_{t-1}^2. \]
     The 1-step ahead prediction gives volatility $\sqrt{7.287} = 2.6994$ so that VaR for $r_t$ is $0.212 - 2.326 \times 2.6992 = -6.0668$ (the negative sign exists because of left tail). The VaR for the Apple position is $60668.00.
   - The fitted GARCH(1,1) model with $t$-innovation is
     \[ r_t = 0.155 + a_t, \quad a_t = \sigma_t \epsilon_t, \quad \epsilon_t \sim t^*_{5.59} \]
     \[ \sigma_t = 0.052 + 0.024a_{t-1}^2 + 0.971\sigma_{t-1}^2. \]
     The 1-step ahead volatility forecast is $\sqrt{16.7} = 4.0866$. The 1% VaR for the Apple return is $0.155 - (3.2214/\sqrt{(5.59/3.59)}) \times 4.0866 \approx -10.3947$ with negative sign indicating loss. The VaR for the financial position is $103947.00$.
2. Again, consider the daily log returns of Apple stock. The estimated parameters (standard errors) are 0.267 (.063), 1.964(.162), 4.138(.198), respectively, for xi, sigma, and mu. Based on the “evtVaR” program, the 1% VaR for the daily log return is $7.9355 so that the VaR for the Apple position is $79355. The 10-day VaR is $ 79355 \times 10^{2.67} = $146749.0.

3. Threshold is 4%: VaR and expected shortfall are $84466 and $124993, respectively. Threshold is 5%: VaR and expected shortfall are $84179 and $124704, respectively. The results are not sensitive to the choices of threshold.

4. Consider now the log returns of Wal-Mart stock. For the Wal-Mart long position, the VaR based on RiskMetrics is $ 1000000 \times 2.326 \times \sqrt{6.336/100} = $58549.

Since the correlation between AAPL and WMT is 0.224, the VaR of the combined position is $\sqrt{58549^2 + 83271^2 + 2 \times 0.224 \times 58549 \times 83271} = $112010.

5. For RiskMetrics, the assumption of normality and zero mean returns leads to the same VaR for short and long position. However, the correlation between the two assets becomes negative. The combined VaR is $\sqrt{58549^2 + 83271^2 - 2 \times 0.224 \times 58549 \times 83271} = $90431. On the other hand, the VaR of Wal-Mart stock changes when it is a short position (to the right tail) for GARCH(1,1) model. Specifically, for the short position, the VaR is $ 1000000 \times [0.012 + 2.326 \sqrt{5.65}] / 100 = $55408. The VaR for the combined position is $ $\sqrt{55408^2 + 60668^2 - 2 \times 0.224 \times 55408 \times 60668} = $72420.