The University of Chicago, Booth School of Business
Business 41202, Spring Quarter 2009, Mr. Ruey S. Tsay

Solutions to Final Exam

Problem A: (42 pts) Answer briefly the following questions.

1. **Questions 1 to 4.** Consider the daily log returns of Motorola stock from January 2001 to December 2008. The summary statistics and some preliminary analysis of the returns are given in the attached output. Based on the results provided, does the distribution of the log returns have heavy tails? Perform a proper test and draw your conclusion.

   **A:** Yes, it has heavy tails. The kurtosis test is $t = \frac{9.245}{\sqrt{24/2011}} = 84.627$ with $p$-values close to zero. Thus, the null hypothesis of zero excess kurtosis is rejected.

2. The Ljung-Box statistic $Q(10)$ indicates that the log returns are serially correlated. To gain further insight, perform the following hypothesis tests: (a) $H_0 : \rho_1 = 0$ vs $H_a : \rho_1 \neq 0$ and (b) $H'_0 : \rho_4 = 0$ vs $H'_a : \rho_4 \neq 0$, where $\rho_i$ is the lag-$i$ ACF of the log returns. Draw your conclusions based on the two tests.

   **A:** (a) $t = \frac{0.01}{\sqrt{12011}} = 0.448$ with $p$-value 0.65. (b) $t = \frac{-0.0772}{\sqrt{12011}} = -3.462$ with $p$-value 0.00054. Based on the two tests, the returns have no lag-1 serial correlation, but have lag-4 serial correlation.

3. An MA(4) model is entertained for the log return series. Write down the fitted MA(4) model, including residual variance.

   **A:** $r_t = -7 \times 10^{-4} + (1 + 0.0088B - 0.055B^2 + 0.0158B^3 - 0.0784B^4)a_t$, where $\sigma^2_a = 9.573 \times 10^{-4}$.

4. Based on the output, is there any evidence that the log returns have conditional heteroscedasticity? Why?

   **A:** Yes, the Q-statistics of the squared residuals of the MA(4) model give $Q(10) = 263.95$, which is highly significant. On the other hand, the residuals show not serial correlation.

5. State two characteristics of high-frequency financial data.

   **A:** Any two of (a) heavy tails, (b) irregular time intervals, (c) large sample size.

6. Consider the price change from transaction to transaction of Stock A in a given time period. Let $A_i$ be the indicator variable of a price change for the $i$-th transaction, i.e. $A_i = 1$ if and only if the $i$-th transaction results in a price change. In addition, let $p_i = P(A_i = 1)$. Employing a logistic regression model, one obtains the fit logit($p_i$) = $-1.10 + 1.05A_{i-1}$. What is the probability of two consecutive price changes?

   **A:** $p(A_i = 1|A_{i-1} = 1) = \exp(-1.1 + 1.05)/(1 + \exp(-1.1 + 1.05)) = 0.488$. 

7. Suppose that the price $P_t$ of a stock follows the stochastic diffusion model

$$\frac{dP_t}{P_t} = \mu dt + \sigma P_t^{-0.5} dw_t,$$

where $\mu$ and $\sigma$ are constant and $w_t$ is the standard Brownian motion. What is the distribution of the log return of the stock from time $t$ to $T$?

A: Let $G_t = \ln(P_t)$. Using Ito’s Lemma, we have

$$dG_t = (\mu - \frac{\sigma^2}{2} P_t) dt + \sigma P_t^{-0.5} dw_t.$$

There the log return from $t$ to $T$ is

$$\ln(P_T) - \ln(P_t) \sim N((\mu - \frac{\sigma^2}{2} P_t)(T-t), \frac{\sigma^2(T-t)}{P_t}).$$

8. Consider a nondividend-paying stock. If the current price of the stock is $40.00 and the risk-free interest rate is 2% per annum. What is the price of a European put option contingent on the stock with time-to-expiration 1 month and strike price $39.00? (Annualized volatility is 20%)

A: Based on the Black-Scholes formula, $h_+ = [\ln(40/39) + (0.02 - (0.2 * 0.2/2)) * (1/12)]/(0.2*\sqrt{1/12}) = 0.496$ and $h_- = 0.439$. Therefore, $p_t = 39 \exp[-0.02/12]\Phi(-h_-) - 40\Phi(-h_+) = 0.474$.

9. Obtain all non-zero serial correlations of the model $r_t = 0.1 + (1 - 0.4B)(1 - 0.5B^4)a_t$, where $\{a_t\}$ is a sequence of independently and identically distributed random variates with mean zero and variance 3.0.

A: The non-zero ACFs are (a) $\rho_0 = 1$, $\rho_1 = \frac{-4}{1+3^2} = -0.345$, $\rho_r = \frac{-0.5}{1+0.5^2} = -0.4$, and $\rho_3 = \rho_5 = 0.138$.

10. Give two univariate volatility models that can handle the leverage effect in volatility modeling.

A: Any two of (a) EGARCH models, (b) GJR models, (c) stochastic volatility model.

11. Pairs trading belongs to statistical arbitrage. One of the important assumptions used is that a linear combination of the prices (or log prices) is stationary. Why is this assumption important?

A: Stationarity implies mean-reverting, which is needed for pairs trading.

12. Consider a 2-2-1 feed-forward neural network with skip layer. Suppose that the output variable is continuous. Write down the econometric model of this network.

A: $o_t = \alpha_0 + \alpha_1 h_1 + \alpha_2 h_2 + \alpha_3 x_1 + \alpha_4 x_2$, where $h_i = \frac{\exp(\beta_0 + \beta_1 x_1 + \beta_2 x_2)}{1+\exp(\beta_0 + \beta_1 x_1 + \beta_2 x_2)}$ for $i = 1, 2$. 


13. Describe two methods that can be used to compare different econometric models in analyzing financial time series.
   A: Backtesting (Out of sample forecasts) and information criteria.

14. The quarterly U.S. unemployment rates from 1948 to 2004 follow approximately the model \( r_t = 1.69r_{t-1} - 0.88r_{t-2} + 0.14r_{t-3} + a_t \), where \( a_t \) is a white noise series with mean zero and variance 0.085. Describe a method that can be used to test the unit-root nonstationarity of the series, including the hypotheses involved.
   A: Rewrite the model in an error-correction form as
   \[
   \Delta r_t = \pi r_{t-1} + \phi_1^* \Delta r_{t-1} + \phi_2^* \Delta r_{t-2} + a_t,
   \]
   where \( \phi_2^* = -\phi_3 \) and \( \phi_1^* = -\phi_3 - \phi_2 \) and \( \pi = \phi_3 + \phi_2 + \phi_1 - 1 \), where \( \phi_i \) are the AR coefficients. The hypothesis testing is \( H_0 : \pi = 0 \) versus the \( H_a : \pi < 0 \). [You may use the actual number for the AR coefficients.]

15. We discussed two classes of nonlinear time series models, namely the threshold autoregressive model and Markov switching model. Give two differences between these two classes of models.
   A: Any two of the following: (a) The way regimes are determined (TAR is deterministic, MS model is stochastic), (b) TAR is easier in estimation, and (c) the states in MS model are not certain.

16. Describe two weaknesses of using realized volatility in financial applications.
   A: Any two of (a) overlooking over-night volatility, (b) subject to effects of microstructure noise, (c) it might not be the needed volatility.

17. Suppose that the log return \( r_t \) of an asset follows the special GARCH(1,1) model: \( r_t = 0.01 + \sigma_t \epsilon_t \), where \( \sigma_t^2 = 0.2 + 0.95\sigma_{t-1}^2 \) and \( \epsilon_t \sim N(0,1) \). Does the distribution of the log return \( r_t \) have heavy tails? Why?
   A: No, the excess kurtosis of \( r_t \) is zero.

18. Describe two methods that can be used to handle the diurnal pattern of intradaily stock returns.
   A: Any two of (a) fitting exponential functions, (b) using sample variances of intervals across trading days, (c) using seasonal models.

19. Suppose the log return \( r_t \) of an asset follows the model
   \[
   \begin{align*}
   r_t & = 0.01 + a_t, \quad a_t = \sigma_t \epsilon_t, \quad \epsilon_t \sim t_6^* \\
   \sigma_t^2 & = 0.01 + 0.07\sigma_{t-1}^2 + 0.90\sigma_{t-1}^2,
   \end{align*}
   \]
   where \( t_6^* \) is the standardized Student-\( t \) distribution with 6 degrees of freedom. Suppose also that \( r_{100} = -0.025 \) and \( \sigma_{100} = 0.40 \). What is the VaR for the log return \( r_{101} \), assuming holding a long position on the stock?
A: Based on the model, the 1-step ahead prediction for the return is 0.01 and for the conditional variance is $0.01 + 0.07(-0.025) + 0.9(0.4^2) = .154$ so that the volatility forecast is $\sqrt{0.154} = .393$. The 1% quantile is $0.01 - 3.143/\sqrt{6/4} \times 0.393 = -.998$. Therefore, we have VaR = 0.998.

20. Give a statistical distribution that can be used with GARCH-type models to analyze asset returns that have heavy tails and show negative skewness.
A: Any one of (a) skewed Student-$t$ distribution, (b) skewed generalized error distribution.

21. Consider a bivariate return series $r_t = (r_{1t}, r_{2t})'$. To test that $r_t$ has no serial correlations, one considers the hypothesis $H_0: \rho_1 = \cdots = \rho_{10} = 0$ versus $H_a: \rho_i \neq 0$ for some $1 \leq i \leq 10$, where $\rho_i$ denotes the lag-$i$ cross-correlation matrix of $r_t$. For a particular return series, we have $Q(10) = 45.53$ for the multivariate Ljung-Box statistic. Can one reject the null hypothesis? Why? [In R, one may use $1-pchisq(x,df)$ to calculate the $p$-value, where df denotes the degrees of freedom and $x$ is a positive real number.]
A: The $p$-value is 0.253 because the degrees of freedom is 40. No, one cannot reject the null hypothesis.

Problem B. (18 points) Again, consider the daily log returns of Motorola stock from January 2001 to December 2008. The sample size is 2011 and the tick symbol is MOT. Answer the following questions based on the attached output.

1. A Gaussian AR(4)-GARCH(1,1) model is entertained. Based on the $t$-ratios of the estimated AR coefficients, is there any significant AR coefficient? Why?
A: No, all AR coefficients are insignificant at the 5% value because the $p$-values are all greater than 0.05.

2. The above Gaussian model is refined. Write down the refined model, including both mean and volatility equations.
A: The model is $r_t = 0.000357 + a_t$, $a_t = \sigma_t \epsilon_t$, $\epsilon_t \sim N(0,1)$, $\sigma_t^2 = .104 \times 10^{-4} + 0.0727a_{t-1}^2 + 0.918\sigma_{t-1}^2$.

3. Is the refined model adequate? Why?
A: Yes, the Q statistics for the standardized residuals and the squared standardized residuals all fail to reject the null hypothesis.

4. The last values of the residual $a_t$ and the fitted volatility $\sigma_t^2$ are given. What are the 1-step ahead forecasts for the log return and the associated volatility?
A: $r_T(1) = 0.000357$. $\sigma_T^2(1) = .104 \times 10^{-4} + 0.0727(.0625)^2 + .918(.0426)^2 = .00196$ so that $\sigma_T(1) = 0.0443$.

5. To study the Market model for Motorola stock, we also consider the daily log return of the S&P composite index. Linear regression is used to estimate the Market model. Write down the fitted Market model. Is the model adequate? Why?
A: $r_t = -0.000394 + 1.352r_{m,t} + e_t$, where the residual standard error is 0.0251. No, the model is not adequate because the Q-statistics show that the residuals have serial correlations.

6. A regression model with time-series errors is entertained to refine the Market model. Write down the refined model. Is the model adequate? Why?
A: The model is $(1 - 0.0345B + 0.0148B^2 + 0.0143B^3 + 0.0684B^4)(r_t + 0.0004 - 1.345r_{m,t}) = a_t$, where $\sigma_a^2 = 0.000627$. The model is adequate as the Q-statistics fail to detect serial correlations in the residuals.

**Problem C.** (16 pts) Consider the quarterly earnings per share of MacDonald’s Corp. from 1993 to the first quarter of 2009 for 65 observations. Log-transformation of the earnings is taken to stabilize the variability.

1. Write down the fitted model for the log earning series, including residual variance.
A: The model is $(1 - B)(1 - B^4)x_t = (1 - 0.146B)(1 - 0.794B^4)a_t$, where $\sigma_a^2 = 0.0151$ and $x_t$ denotes the logarithm of quarterly earnings per share.

2. Is the fitted model adequate? Why?
A: Yes, the model is adequate. The Q-statistics show that (a) there are no serial correlations and (b) there are no ARCH effects in the residuals.

3. Based on the fitted model, calculate the 95% interval forecast for the log earnings of the third quarter of 2009, i.e. 2-step ahead prediction.
A: The interval forecasts is $0.0294 \pm 1.96 \times 0.162$, i.e. (-0.288,0.347).

4. If you like to improve the fitted model, what is the first-step you would take? Why?
A: The lag-1 MA coefficient is not significant at the 5% level. One can simplify the model by removing that MA term.

**Problem D.** (24 points) Consider the daily log returns of the stocks of Motorola and Citigroup from January 2001 to December 2008. The tick symbols are MOT and C, respectively. Suppose that Manager A holds a long position of $1 million dollars on each of the two stocks. Use the attached output to answer the following questions.

1. Manager A decides to use RiskMetrics to calculate VaR of her financial position. To this end, the special Gaussian IGARCH(1,1) model is fitted to the two log-return series. The $\alpha$ parameter of the IGARCH model is 0.96 and 0.94, respectively, for MOT and C. A recursive method is used to calculate the fitted conditional variances of the two stocks. Values of the last returns and conditional variances of the two stocks are given in the output. Calculate the VaR for each stock for the next trading day.
A: The conditional variance for the next trading day is as follows: For MOT stock, $\sigma_T^2(1) = 0.04 \times 0.0629^2 + 0.96 \times 0.00303 = 0.00307$ and for Citi stock, $\sigma_T^2(1) = 0.06 \times (-0.0133)^2 + 0.94 \times 0.0103 = 0.00969$. Therefore, the VaRs are (a) for MOT, $\$1,000,000 \times (2.326\sqrt{0.00307}) = \$128,878$, (b) for C, $\$1,000,000 \times (2.326\sqrt{0.00969}) = \$228,966$. 

5
2. The correlation between the two log returns is 0.411. What is the VaR of Manager A’s financial position for the next 10 trading days?
   A: \[
   \text{VaR} = \sqrt{10(\sqrt{128878^2 + 228966^2} + 2 \times 0.411 \times 128878 \times 228966)} = $965873.
   \]

3. Manager A also adopts the Peaks over Threshold (POT) approach to calculate VaR for her position. After some considerations, a threshold of 3% is used. For MOT stock, what are the fitted parameters? Are the estimates significantly different from zero? Why?
   A: The estimated parameters are 0.193 and 0.0191 with standard errors 0.0724 and 0.00184, respectively. The estimates are all significant because their \( t \)-ratios are greater than 2.

4. For Manager A, what is the VaR for holding the MOT stock until the next trading day if the POT approach is used? What is the associated expected shortfall?
   A: From the output, \( \text{VaR} = $1000000 \times 0.0893417 = $89342 \). The expected shortfall is \( \$1000000 \times 0.127124 = $127124 \).

5. What is the VaR facing Manager A for the next trading day based on the POT approach?
   A: The VaR for Citi stock is \( $1000000 \times 0.089537 = $89537 \). The VaR for Manager A is \( \sqrt{89342^2 + 89537^2 + 2 \times 0.411 \times 89342 \times 89537} = $285177 \).

6. Manager A has equal weights for the two stocks so that we can obtain the simple return of her portfolio from the two individual returns. The simple portfolio return is transformed to log return, which is then used to perform POT analysis with threshold 3%. What is the VaR of the portfolio for the next trading day? Comment on the comparison between the current VaR and that obtained in the prior question.
   A: The VaR is \( $2000000 \times 0.076419 = $152838 \). This is much smaller than that obtained by individual VaR.

7. Manager B holds a short position of $1 million dollars each on the two stocks and decides to apply the traditional extreme value theory with block size 21 to calculate VaR. What is the VaR of each stock facing Manager B for the next trading day? What is the overall VaR of Manager B for the next trading day?
   A: For individual stocks, (a) VaR for MOT is $73090 and (b) VaR for C is $55004. The joint VaR is \( \sqrt{73090^2 + 55004^2 + 2 \times 0.411 \times 73090 \times 55005} = $108038 \).

8. Manager C holds a short position of $1 million dollars on Citigroup stock, but a long position of $1 million dollars on Motorola stock. Based on the POT approach with threshold 3%, calculate the VaR of Manager C for the next trading day.
   A: The VaR for C stock is \( 1000000 \times 0.081300 = $81300 \). The portfolio VaR for Manager C for the next trading day is \( \text{VaR} = \sqrt{81300^2 + 89342^2 - 2 \times 0.411 \times 81300 \times 89342} = $92850 \).