Homework Assignment #2

Due Date: before class
- Campus class: April 23, 2010
- Weekend class: April 24, 2010

Notes:
- **Data files**: Datasets may be downloaded from the course web site.
- Use 5% level in all tests.
- The notation $\rho_i$ is the lag-$i$ autocorrelation coefficient.
- In some of the problems, I provide guidances to specify a time series model. This is to help you gain experience in empirical data analysis. You can try your own models to gain further experience.

Assignment:

1. Consider daily price of Apple stock from January 3, 2003 to April 6, 2010. The data are obtained from Yahoo Finance and have 9 columns, namely (Month, Day, Year, Open, High, Low, Close, Volume, Adjclose). We focus on the adjusted closing price in Column 9. You can use the command `rev` to reverse the time series.

   (a) Compute the daily log returns of Apple stock. Is there any serial correlation in the daily log returns? To answer this question, perform the test $H_0 : \rho_1 = \cdots = \rho_{10} = 0$ versus $H_a : \rho_i \neq 0$ for some $1 \leq i \leq 10$. Draw your conclusion.

   (b) Use the command `ar` with maximum likelihood method to select an AR model for the log return series. What is the specified order? Perform model fitting and write down the fitted model. Are all AR coefficients significant at the 5% level? Why?

   (c) Focus on the AR model specified in Part (b). Simplify the AR model by fixing the insignificant coefficients to zero. Write down the final model with all AR coefficient significant at the 5% level.

   (d) Consider the log price series of Apple stock. Is the log price series unit-root nonstationary? Perform a unit-root test to answer the question. Draw your conclusion. [Hint: you may use the order specified in Part (b) in the test.]
2. Consider the VIX index of CBOE from January 3, 1990 to October 9, 2009. Focus on the series of the daily closing index. Compute the log return \( r_t \) of the daily VIX index.

(a) Are there serial correlations in the daily \( r_t \) series? To answer the question, perform the test \( H_0 : \rho_1 = \cdots = \rho_{20} = 0 \) versus \( H_a : \rho_i \neq 0 \) for some \( 1 \leq i \leq 20 \). Draw your conclusion.

(b) Use the command `ar` to identify an AR model for \( r_t \). Perform estimation and write down the fitted model, including residual standard deviation.

(c) Perform model checking using \( Q(20) \) of the residuals. Are there serial correlations in the residuals? Draw your conclusion.

3. Consider the monthly unemployment rate of the State of Illinois. The data obtained from the Bureau of Labor Statistics are from January 1976 to February 2010 with the last observation being preliminary. The sample size is 410. Let \( u_t \) be the monthly unemployment rate.

(a) If you compute the ACF of \( u_t \), you see that the sample ACFs are large, indicating high serial dependence in the unemployment rate. In empirical modeling, this leads to consider the increment series \( y_t = u_t - u_{t-1} \). [In R, the command is \( y = \text{diff}(u) \).] Perform \( Q(24) \) test of the \( y_t \) series. Draw your conclusion.

(b) Use the command `ar` with maximum likelihood method to specify an AR model for \( y_t \). Denote the specified order by \( p \). This mean that \( u_t \) is an ARIMA(\( p,1,0 \)) process. Fit the model for \( u_t \). Perform model checking using \( Q(24) \) of the residuals. Is the fitted model adequate?

(c) Fit an ARMA(2,1,4) model to \( u_t \). Perform model checking using \( Q(24) \) of the residuals. Write down the fitted model.

(d) Compare the models in Parts (b) and (c). Which model is preferred? Why?

(e) Re-fit the two models in Parts (b) and (c). Which model is preferred? Why?

4. Consider the daily price data of the Apple stock of Problem 1. Compute the daily log price range as \( r_t = \ln(H_t) - \ln(L_t) \), where \( H_t \) and \( L_t \) denote daily high and low price of the stock, respectively. Daily range can be used to measure volatility of the stock log price. Answer the following questions:

(a) Compute the first 10 lags of ACF of \( r_t \). Test the null hypothesis that the first 10 lags of ACF are zero. Draw your conclusion. [Note: 10 lags correspond to the number of trading days in two weeks.]

(b) Compute the first 20 lags of PACF of \( r_t \).
(c) Specify an AR model for the range series \( r_t \). Perform model checking to justify your choice of model.

5. Consider the monthly simple return of CRSP Decile 10 portfolio from January 1967 to December 2009. [Column 5 in the file m-dec12910.txt.] This is the portfolio consists of the smallest 10 percent of the stocks in NYSE, AMEX, and NASDAQ.

(a) Test \( H_0 : \rho_{12} = 0 \) versus \( H_a : \rho_{12} \neq 0 \). Draw your conclusion.

(b) Test \( H_0 : \rho_{24} = 0 \) versus \( H_a : \rho_{24} \neq 0 \). Draw your conclusion.

(c) Compute the first 24 lags of ACF. Based on the ACF plot, it seems that only lags 1 and 12 have significant serial correlations. This suggests that the model

\[
r_t = \mu + (1 - \theta_1 B - \theta_{12} B^{12}) \alpha_t,
\]

is adequate for the return series. Fit the model and write down the fitted model. Are the two MA coefficients significant? Why?

**Reading assignments:** Chapter 2 of the textbook.