Assignment:

1. Climate change is a big concern for many. In this problem, we analyze the monthly global temperature anomalies from January 1880 to December 2015 for the Northern Hemisphere. The data consist of measurements from land and sea surface and are in 0.01°C. The original data are from Goddard Institute for Space Studies. See http://data.giss.nasa.gov/gistemp. The data are available in the file m-globaltemp.txt. To obtain the series, do the following in R:

```r
da <- read.table('m-globaltemp.txt',header=T)
dd <- da[,2:13]
xt <- c(t(dd))
zt <- diff(xt)
```

(a) Plot both the monthly temperature and its first differenced series on the same page. [You can use the command `par(mfcol=c(2,1))` in R.]

(b) Is there a unit root in the temperature series? Why?

(c) Let $z_t = x_t - x_{t-1}$ with $x_t$ being the global temperature. Test $H_0 : E(z_t) = 0$ versus $H_a : E(z_t) \neq 0$. Draw your conclusion.

(d) Compute ACF and PACF of the $z_t$ series. Plot them on the same page.

(e) Consider the $z_t$ series. Test $H_0 : \rho_1 = \cdots = \rho_{12} = 0$ versus $H_a : \rho_i \neq 0$ for some $1 \leq i \leq 12$. Draw the conclusion.

2. Again, consider the first-differenced series $z_t$ of Problem 1.
(a) Use the `ar` command with subcommand `method='mle'` to identify an AR model for the $z_t$ series. Fit the specified AR model, perform model checking, and write down the fitted model. [You may include the subcommand `include.mean=F` to remove the mean in light of the test result in part (b) of Problem 1.]

(b) Next, return to the global temperature series $x_t$. Based on the AR model in part (a), fit a AR model for $x_t$. Use the fitted model to compute 1-step to 12-step ahead point forecasts of the global temperature at the forecast origin December 2015. For your information, the actual data for January and February 2016 are 153 and 190, respectively.

(c) Compute the 1-step to 2-step ahead 95% interval forecasts for $x_t$. Are the actual values in these intervals?

3. Consider, again, the global temperature series $x_t$ of Problem 2.

(a) Use the command `auto.arima` in the package `forecast` to identify an ARIMA model for $x_t$.

(b) Is the model adequate? Why? Which residual ACF are significantly different from zero, if any?

(c) A refined model can be obtained. But one needs to use the first-differenced series $z_t$.

```r
mm = arima(zt,order=c(1,0,2),seasonal=list(order=c(1,0,0),period=12))
```
Perform model checking [use subcommand `gof=24`]. Is the model adequate? Why? Refer to this model as `mm`.

(d) Compare the `mm` model with the AR model built in Problem 2 for $z_t$. In terms of in-sample fitting, which model is preferred? Why?

(e) Use `backtest` to compare the AR and `mm` models. You may use the initial forecast origin at $t = 1600$ and forecast horizon $h = 1$. Which model is preferred? Why?

4. Consider the daily adjusted closing prices of two stocks from 2011-08-04 to 2014-02-28. The companies are Billiton Limited (BHP) and Vale S.A. (VALE). The data can be obtained from Yahoo Finance via the package `quantmod`. Once the data are downloaded, do the following:

```r
bhp <- as.numeric(BHP[,6])
vale <- as.numeric(VALE[,6])
```
These two commands transform BHP and VALE from an .xts object to an ordinary time series object in R.

(a) Does the bhp series have a unit root? Why?

(b) Let $r_t = bhp_t - 0.9749vale_t$. Build an AR model for the $r_t$ series. Is the model adequate? Why?

(c) Write down the fitted AR model for $r_t$. Compute the solutions of the characteristic function of the fitted AR model.

(d) Fit an ARMA(2,1) model to the $r_t$ series. Is the model adequate? Why? Write down the fitted ARMA model. [In the `arima` command of R, the order is c(2,0,1).]
(e) Fit an ARIMA(1,1,1) model to the $r_t$ series. Is the model adequate? Why? Write down the fitted model.

(f) Compare the AR, ARMA, and ARIMA models for $r_t$. Which model is preferred? Why?

(g) Use backtest with forecast origin $t = 600$ and forecast horizon $h=1$, to compare the AR, ARMA and ARIMA models. Draw your conclusion.

5. (Commodity prices). Consider the monthly crude oil prices from January 1986 to February 2016. The original data are from FRED and also available in m-COILWTICO.txt.

(a) Obtain the time plot of the oil prices and its first differenced series.

(b) Based on the plots, is the first differenced series weakly stationary? Why?

(c) Let $r_t$ be the first differenced price series. Test $H_0: \rho_1 = \cdots = \rho_{12} = 0$ versus $H_a: \rho_i \neq 0$ for some $1 \leq i \leq 12$. Draw your conclusion.

(d) Build an AR model for $r_t$, including model checking. Refine the model by excluding all estimates with $t$-ratio less than 1.645. Write down the fitted model.

(e) Build an ARIMA model for $r_t$, including model checking. Write down the fitted model.

(f) Use the fitted AR model to compute 1-step to 4-step ahead forecasts of $r_t$ at the forecast origin February, 2016. Also, compute the corresponding 95% interval forecasts.

Reading assignments: Chapter 2 of the textbook.