Midterm

**ChicagoBooth Honor Code:**
*I pledge my honor that I have not violated the Honor Code during this examination.*

**Signature:**

**Name:**

**ID:**

**Notes:**

- Open notes and books. Exam time: 180 minutes.
- You may use a calculator or a PC. However, turn off Internet connection and cell phones. Internet access and phone communication are strictly prohibited during the exam.
- The exam has 8 pages and the R output has 12 pages. Please check that you have all 20 pages.
- For each question, write your answer in the blank space provided.
- Manage your time carefully and answer as many questions as you can.
- For simplicity, if not specifically given, use 5% Type-I error in hypothesis testings.
- Round your answer to 3 significant digits.

**Problem A:** (30 pts) Answer briefly the following questions. Each question has two points.

1. Give two reasons by which the return series of an asset tend to contain outliers.

2. Describe two differences between an AR(1) model and an MA(1) model of a time series.
3. Give two characteristics of the return $r_t$ if it follows the model $r_t = 0.05 + a_t$, $a_t = \sigma_t \epsilon_t$, where $\epsilon_t$ are iid $N(0, 1)$ and $\sigma_t^2 = 0.02 + 0.4a_{t-1}^2$.

4. (Questions 4 to 6): Suppose that the asset return $r_t$ follows the model

$$
\begin{align*}
    r_t &= a_t \\
    a_t &= \sigma_t \epsilon_t, \quad \epsilon_t \sim \text{iid } t^*_6 \\
    \sigma_t^2 &= 0.09 + 0.145a_{t-1}^2 + 0.855\sigma_{t-1}^2.
\end{align*}
$$

Does the unconditional variance of $r_t$ exist? Why?

5. Suppose that $r_{100} = -0.05$ and $\sigma_{100} = 0.3$. Compute 1-step and 2-step ahead volatility forecasts at the forecast origin $t = 100$. (Note that it is volatility, not $\sigma^2$.)

6. Compute the 22-step ahead mean and volatility forecasts (one month ahead).

7. Give an advantage of Spearman’s $\rho$ over the Pearson correlation.

8. Give a feature that GARCH-M models have, but the GARCH models do not.

9. Suppose that $r_t$ follows the model

$$
    r_t = r_{t-1} + a_t - 0.9a_{t-1},
$$

and we have $r_{1001} = 1.2$ and $r_{1000}(1) = 1.0$, where $r_t(1)$ denotes the 1-step ahead prediction of $r_{t+1}$ at the forecast origin $t$. Compute $r_{1001}(1)$. 

2
10. Why is the usual $R^2$ measure not proper in time series analysis?


12. (Questions 12-13) Suppose that the daily simple returns of an asset in week 1 were -0.5%, 1.2%, 2.5%, -1.0%, and 0.6%. What are the corresponding daily log returns?

13. What is the weekly simple return of the asset?

14. (Questions 14-15): The summary statistics of daily simple returns of an asset are given below:

```r
> basicStats(rtn)

   rtn
   nobs 2515.000000
   Mean 0.000410
   SE Mean ?????????
   LCL Mean -0.000257
   UCL Mean 0.001077
   Stdev 0.017060
   Skewness 0.517184
   Kurtosis 6.661044
```

What is the standard deviation of the mean? Is the expected return of the asset significantly different from zero? Why?

15. Based on the summary statistics, are the returns normally distributed? Perform a statistical test to justify your conclusion.
Problem B. (23 points) Consider the monthly U.S. unemployment rates from January 1947 to March 2016. Due to strong serial dependence, we analyze the differenced series $x_t = r_t - r_{t-1}$, where $r_t$ is the seasonally adjusted unemployment rate. Answer the following questions, using the attached R output. Note: A fitted ARIMA model should include residual variance.

1. (2 points) The `auto.arima` command specifies an ARIMA(2,0,2) model for $x_t$. The fitted model is referred to as `m1` in the output. Write down the fitted model.

2. (3 points) Model checking shows two large outliers. An ARIMA(2,0,2) model with two outliers are then specified, `m3`. Write down the fitted model.

3. (3 points) Model checking shows some serial correlations at lags 12 and 24. A seasonal model is then employed and called `m4`. Write down the fitted model.

4. (3 points) The outliers remain in the seasonal model. Therefore, a refined model is used and called `m5`. Write down the fitted model.

5. (2 points) Based on the model checking statistics provided, are there serial correlations in the residuals of model `m5`? Why?

6. (2 points) Among models `m1`, `m3`, `m4` and `m5`, which model is preferred under the in-sample fit? Why?

7. (2 points) If root mean squares of forecast errors are used in out-of-sample prediction, which model is preferred? Why?
8. (2 points) If mean absolute forecast errors are used in out-of-sample comparison, which model is selected?

9. (2 points) Consider models m1 and m3. State the impact of outliers on in-sample fitting.

10. (2 points) Again, consider models m1 and m3. State the impact of outliers on out-of-sample predictions.

Problem C. (27 points) Consider the daily log returns of Amazon (AMZN) stock obtained via quantmod. Statistical analysis is included in the attached R output. Answer the following questions. Note, a model should include both mean and volatility equations and the innovation distribution used.

1. (2 points) Are there serial correlations in the daily log returns? Why? Write down the proper null hypothesis for testing.

2. (3 points) A standard GARCH(1,1) model is fitted. Write down the fitted model.

3. (3 points) Model checking shows the normality is rejected. A skew standardized Student-t distribution is used. Write down the fitted model. Model m3.
4. (2 points) Based on the fitted model \( m3 \). Does the model support that the innovation is skewed? Perform a test to support your conclusion.

5. (2 points) Compute the 95% interval forecasts for 1-step and 2-step ahead predictions using model \( m3 \).

6. (2 points) An IGARCH(1,1) model is also entertained. Write down the fitted model. Model \( m4 \).

7. (2 points) Why are the 1-step to 5-step ahead volatility forecasts of the IGARCH(1,1) model not constant?

8. (2 points) An EGARCH model is also entertained. Write down the fitted model? Model \( m5 \).

9. (2 points) Based on the fitted EGARCH model, is the leverage effect significant? Why?

10. (3 points) The lag-1 VIX index is used as an explanatory variable for volatility. Write down the fitted model. Model \( m6 \)
11. (2 points) Based on the fitted model, does the lag-1 VIX index affect significantly the AMZN volatility? Why?

12. (2 points) Among all volatility models entertained, which model provides best in-sample fit? Why?

**Problem D.** (10 points) Consider the monthly log returns of Procter and Gamble stock from January 1960 to March 2015. Use the R output to answer the following questions.

1. (2 points) An IGARCH(1,1) model is entertained. Write down the fitted model.

2. (2 points) Based on the statistics provided, is the model adequate? Why?

3. (4 points) Based on the fitted IGARCH(1,1) model, compute the 1-step and 2-step ahead forecasts for mean and volatility of the log returns.

4. (2 points) A GARCM-M model is entertained. Based on the fitted model, is the risk premium statistically significant? Perform a test to justify your answer.
Problem E. (10 points) Consider the monthly log returns of value-weighted index and the S&P composite index from January 1960 to March 2015. Our goal is to study the relationship between the volatility of the two market indexes. Based on the output provided, answer the following questions:

1. (1 points) A GARCH(1,1) model with skew standardized Student-\(t\) innovations is employed for the S&P index returns. Does the fitted model support the use of skew innovations? Why?

2. (2 points) A similar GARCH(1,1) model is also employed for the value-weighted index returns. Let the resulting volatility be \(\text{volvw}_t\). Let \(\text{volsp}_t\) be the corresponding volatility of the S&P index return. Write down the fitted simple linear regression model for the dependent variable \(\text{volsp}_t\). Is this simple linear regression model adequate? Why?

3. (2 points) A refined model is employed. Write down the fitted linear regression model with time series errors.

4. (3 points) Alternatively, one can use \(\text{volvw}_t\) as an explanatory variable in volatility modeling of the S&P index return. Write down the fitted volatility model.

5. (2 points) Does \(\text{volvw}_t\) significantly contribute to the volatility modeling of the S&P index returns? Why?
## Problem B

```r
rate <- as.numeric(UNRATE[,1])
xt <- diff(rate)  # Differenced series
require(forecast)
auto.arima(xt)
Series: xt
ARIMA(2,0,2) with zero mean
m1 <- arima(xt,order=c(2,0,2),include.mean=F)
m1
Call: arima(x = xt, order = c(2, 0, 2), include.mean = F)
Coefficients:
     ar1   ar2    ma1  ma2
  1.6546 -0.7753  1.6288  0.8440
s.e.  0.0427  0.0468  0.0420  0.0477
sigma^2 estimated as 0.03838: log likelihood = 172.36, aic = -334.71
which.min(m1$residuals)
[1] 22
i22[22]=1; i22 <- rep(0,818)
m2 <- arima(xt,order=c(2,0,2),xreg=i22,include.mean=F)
m2
Coefficients:
     ar1   ar2    ma1  ma2   i22
  1.6953 -0.7965  1.6286  0.8164 -1.5038
s.e.  0.0454  0.0477  0.0484  0.0509  0.1837
sigma^2 estimated as 0.03545: log likelihood = 204.92, aic = -397.84
which.max(m2$residuals)
[1] 21
i21 <- rep(0,818)
i21[21]=1
out <- cbind(i22,i21)
m3 <- arima(xt,order=c(2,0,2),xreg=out,include.mean=F)
m3
Call: arima(x = xt, order = c(2, 0, 2), xreg = out, include.mean = F)
Coefficients:
     ar1   ar2    ma1  ma2   i22   i21
  1.6901 -0.7909  1.6128  0.8014 -1.5302  1.1472
s.e.  0.0466  0.0504  0.0534  0.0592  0.1755  0.1757
sigma^2 estimated as 0.03368: log likelihood = 225.86, aic = -437.72
Box.test(m3$residuals,lag=12,type='Ljung')
Box-Ljung test
data: m3$residuals
X-squared = 31.83, df = 12, p-value = 0.00147
m4 <- arima(xt,order=c(2,0,2),seasonal=list(order=c(1,0,1),period=12),
```
include.mean=F)
> m4
Call:arima(x = xt, order=c(2, 0, 2), seasonal=list(order=c(1, 0, 1), period=12),
  include.mean = F)
Coefficients:
    ar1    ar2    ma1    ma2    sar1    sma1
  1.2357 -0.3608 -1.2354  0.5151  0.5542 -0.8220
s.e.  0.2413  0.2221  0.2241  0.1702  0.0662  0.0473

sigma^2 estimated as 0.03538:  log likelihood = 204.21, aic = -394.43

> m5 <- arima(xt, order=c(2, 0, 2), seasonal=list(order=c(1, 0, 1), period=12),
  include.mean=F, xreg=out)
> m5
Call:arima(x = xt, order = c(2, 0, 2), seasonal = list(order = c(1, 0, 1), period = 12),
  xreg = out, include.mean = F)
Coefficients:
    ar1    ar2    ma1    ma2    sar1    sma1    i22    i21
  1.5743 -0.6591 -1.4869  0.6720  0.5488 -0.8208 -1.4762  1.1441
s.e.  0.1159  0.1110  0.1111  0.0913  0.0659  0.0448  0.1620  0.1616

sigma^2 estimated as 0.03062:  log likelihood = 263.2, aic = -508.4
> Box.test(m5$residuals, lag = 24, type = 'Ljung')
  Box-Ljung test
data: m5$residuals
X-squared = 27.826, df = 24, p-value = 0.2674

> source("backtest.R")
> backtest(m1, xt, 750, include.mean=F)
[1] "RMSE of out-of-sample forecasts"
[1] 0.1621524
[1] "Mean absolute error of out-of-sample forecasts"
[1] 0.1242145
> backtest(m3, xt, 750, include.mean=F, xre=out)
[1] "RMSE of out-of-sample forecasts"
[1] 0.1625846
[1] "Mean absolute error of out-of-sample forecasts"
[1] 0.1236525
> backtest(m4, xt, 750, include.mean=F)
[1] "RMSE of out-of-sample forecasts"
[1] 0.1499355
[1] "Mean absolute error of out-of-sample forecasts"
[1] 0.1164277
> backtest(m5, xt, 750, include.mean=F, xre=out)
[1] "RMSE of out-of-sample forecasts"
[1] 0.1492887
Mean absolute error of out-of-sample forecasts
0.1162959

Problem C

```r
getSymbols("AMZN")

getSymbols("^VIX") ## to be used later.

vix <- as.numeric(VIX[,6])
vixm1 <- vix[-1]
amzn <- diff(log(as.numeric(AMZN[,6])))

Box.test(amzn,lag=10,type='Ljung')

require(rugarch)

spec1 <- ugarchspec(variance.model=list(model="sGARCH"),
                      mean.model=list(armaOrder=c(0,0)))
m1 <- ugarchfit(data=amzn,spec=spec1)
m1
```

```
*---------------------------------*
| GARCH Model Fit             *
*---------------------------------*

Conditional Variance Dynamics

GARCH Model : sGARCH(1,1)
Mean Model : ARFIMA(0,0,0)
Distribution : norm

Optimal Parameters

|             | Estimate | Std. Error | t value | Pr(>|t|) |
|-------------|----------|------------|---------|----------|
| mu          | 0.001364 | 0.000496   | 2.7513  | 0.005936 |
| omega       | 0.000002 | 0.000001   | 2.9754  | 0.002927 |
| alpha1      | 0.008162 | 0.0000591  | 13.8111 | 0.000000 |
| beta1       | 0.988780 | 0.000329   | 3004.2634 | 0.000000 |

Information Criteria

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Weighted Ljung-Box Test on Standardized Residuals
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<td>5.3300</td>
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<td>H0 : No serial correlation</td>
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Weighted Ljung-Box Test on Standardized Squared Residuals

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<td>Lag[1]</td>
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<td>9.439</td>
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```r
> spec2 <- ugarchspec(variance.model=list(model="sGARCH"),
 mean.model=list(armaOrder=c(0,0)),distribution.model="std")
> m2 <- ugarchfit(data=amzn,spec=spec2)
> m2
```

Conditional Variance Dynamics

GARCH Model : sGARCH(1,1)
Mean Model : ARFIMA(0,0,0)
Distribution : std

Optimal Parameters

|                | Estimate | Std. Error | t value | Pr(>|t|) |
|----------------|----------|------------|---------|----------|
| mu             | 0.000865 | 0.000384   | 2.2551  | 0.024124 |
| omega          | 0.000004 | 0.000003   | 1.3177  | 0.187617 |
| alpha1         | 0.021701 | 0.003228   | 6.7223  | 0.000000 |
| beta1          | 0.972435 | 0.006398   | 151.9797| 0.000000 |
| shape          | 3.601122 | 0.252180   | 14.2800 | 0.000000 |

Information Criteria

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</tr>
<tr>
<td>Hannan-Quinn</td>
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```r
> spec3 <- ugarchspec(variance.model=list(model="sGARCH"),mean.model=
list(armaOrder=c(0,0)),distribution.model="sstd")
> m3 <- ugarchfit(data=amzn,spec=spec3)
> m3
```

Conditional Variance Dynamics

GARCH Model : sGARCH(1,1)
Mean Model : ARFIMA(0,0,0)
Distribution : sstd

Optimal Parameters
------------------------------------

| Parameter | Estimate  | Std. Error | t value | Pr(>|t|) |
|-----------|-----------|------------|---------|---------|
| mu        | 0.001461  | 0.000453   | 3.2223  | 0.001272|
| omega     | 0.000004  | 0.000003   | 1.5644  | 0.117724|
| alpha1    | 0.022732  | 0.003547   | 6.4097  | 0.000000|
| beta1     | 0.971489  | 0.004124   | 235.5422| 0.000000|
| skew      | 1.076577  | 0.031540   | 34.1333 | 0.000000|
| shape     | 3.573507  | 0.275739   | 12.9597 | 0.000000|

Information Criteria
------------------------------------

Akaike -4.8078
Bayes -4.7931
Shibata -4.8078
Hannan-Quinn -4.8024

Weighted Ljung-Box Test on Standardized Residuals
------------------------------------

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<th>Lag</th>
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<th>p-value</th>
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<td>Lag[1]</td>
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<td>H0 : No serial correlation</td>
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Weighted Ljung-Box Test on Standardized Squared Residuals
------------------------------------

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> ugarchforecast(m3,n.ahead=5)

*------------------------------------*
| GARCH Model Forecast |
*------------------------------------*

Model: sGARCH
Horizon: 5
0-roll forecast [T0=1976-06-05 19:00:00]:

Series Sigma
T+1 0.001461 0.02494
T+2 0.001461 0.02495
T+3 0.001461 0.02496
T+4 0.001461 0.02497
T+5 0.001461 0.02498
spec4 <- ugarchspec(variance.model=list(model="iGARCH"),
                     mean.model=list(armaOrder=c(0,0)),distribution.model="sstd")
>m4 <- ugarchfit(data=amzn,spec=spec4)
>m4

Conditional Variance Dynamics
-----------------------------------
GARCH Model : iGARCH(1,1)
Mean Model : ARFIMA(0,0,0)
Distribution : sstd

Optimal Parameters
------------------------------------

|        | Estimate | Std. Error | t value | Pr(>|t|) |
|--------|----------|------------|---------|----------|
| mu     | 0.001528 | 0.000462   | 3.3095  | 0.000935 |
| omega  | 0.000002 | 0.000002   | 1.5149  | 0.129803 |
| alpha1 | 0.024331 | 0.004152   | 5.8598  | 0.000000 |
| beta1  | 0.975669 | NA         | NA      | NA       |
| skew   | 1.079980 | 0.031788   | 33.9747 | 0.000000 |
| shape  | 3.280276 | 0.137902   | 23.7871 | 0.000000 |

Information Criteria
------------------------------------
Akaike   -4.8073
Bayes   -4.7950
Shibata -4.8073
Hannan-Quinn -4.8028

Weighted Ljung-Box Test on Standardized Residuals
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<td>H0 : No serial correlation</td>
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Weighted Ljung-Box Test on Standardized Squared Residuals
------------------------------------

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> ugarchforecast(m4,n.ahead=5)

*------------------------------------*
| GARCH Model Forecast |
*------------------------------------*
Model: iGARCH
Horizon: 5

0-roll forecast [T0=1976-06-05 19:00:00]:

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<td>T+2</td>
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<td>T+3</td>
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<td>T+4</td>
<td>0.001528</td>
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<tr>
<td>T+5</td>
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<td>0.02642</td>
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> spec5 <- ugarchspec(variance.model=list(model="eGARCH"),
mean.model=list(armaOrder=c(0,0)))
> m5 <- ugarchfit(data=amzn,spec=spec5)
> m5

Conditional Variance Dynamics
-----------------------------------
GARCH Model : eGARCH(1,1)
Mean Model : ARFIMA(0,0,0)
Distribution : norm

Optimal Parameters
------------------------------------

| Estimate | Std. Error | t value | Pr(>|t|) |
|----------|------------|---------|---------|
| mu       | 0.001807   | 0.000386 | 4.6873  | 0.000003 |
| omega    | -0.456107  | 0.021264 | -21.4494| 0.000000 |
| alpha1   | -0.049813  | 0.013608 | -3.6605 | 0.000252 |
| beta1    | 0.936172   | 0.003462 | 270.4202| 0.000000 |
| gamma1   | 0.127728   | 0.022494 | 5.6783  | 0.000000 |

Information Criteria
------------------------------------

Akaike    -4.5318
Bayes     -4.5195
Shibata   -4.5318
Hannan-Quinn -4.5273

Weighted Ljung-Box Test on Standardized Residuals
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Weighted Ljung-Box Test on Standardized Squared Residuals
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<table>
<thead>
<tr>
<th>statistic</th>
<th>p-value</th>
</tr>
</thead>
</table>
Conditional Variance Dynamics
-----------------------------------
GARCH Model : sGARCH(1,1)
Mean Model : ARFIMA(0,0,0)
Distribution : sstd

Optimal Parameters
------------------------------------
       Estimate Std. Error  t value  Pr(>|t|)
mu     0.001905   0.000455  4.1847   0.000029
omega  0.000000   0.000010  0.0000   1.000000
alpha1 0.089867   0.031530  2.8502   0.004369
beta1  0.000173   0.133352  0.0013   0.998963
vxreg1 0.000026   0.000004  6.0994   0.000000
skew   1.118515   0.033620 33.2692   0.000000
shape  3.958441   0.320506 12.3506   0.000000

Information Criteria
------------------------------------
Akaike    -4.8371
Bayes     -4.8200
Shibata   -4.8372
Hannan-Quinn -4.8309

Weighted Ljung-Box Test on Standardized Residuals
------------------------------------

<table>
<thead>
<tr>
<th>statistic</th>
<th>p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lag[1]</td>
<td>1.457</td>
</tr>
<tr>
<td>Lag[2*(p+q)+(p+q)-1][2]</td>
<td>5.013</td>
</tr>
<tr>
<td>Lag[4*(p+q)+(p+q)-1][5]</td>
<td>7.953</td>
</tr>
<tr>
<td>d.o.f=0</td>
<td></td>
</tr>
<tr>
<td>H0 : No serial correlation</td>
<td></td>
</tr>
</tbody>
</table>

Weighted Ljung-Box Test on Standardized Squared Residuals
------------------------------------

<table>
<thead>
<tr>
<th>statistic</th>
<th>p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lag[1]</td>
<td>3.139e-05</td>
</tr>
<tr>
<td>Lag[4*(p+q)+(p+q)-1][9]</td>
<td>1.149e+00</td>
</tr>
<tr>
<td>d.o.f=2</td>
<td></td>
</tr>
</tbody>
</table>
#### Problem D ####

```r
> da = read.table("m-pg3dx-6015.txt", header=T)
> head(da)

PERMNO  date    RET    vwretd  ewretd  sprtrn
1 18163 19600129 -0.081667 -0.066244 -0.039202 -0.071464

> pg <- log(da[,3]+1)
> source("Igarch.R")
> m3 <- Igarch(pg)
Estimates: 0.9164492
Maximized log-likelihood: -967.2926

Coefficient(s):

| Estimate | Std. Error | t value | Pr(>|t|) |
|----------|------------|---------|----------|
| beta     | 0.9164492  | 0.0163915 | 55.9099 < 2.22e-16 *** |

> names(m3)
[1] "par" "volatility"

> r3 <- pg/m3$volatility
> Box.test(r3, lag=12, type='Ljung')

Box-Ljung test
data: r3
X-squared = 10.684, df = 12, p-value = 0.5562

> Box.test(r3^2, lag=12, type='Ljung')

Box-Ljung test
data: r3^2
X-squared = 5.7124, df = 12, p-value = 0.9299

> length(pg)
[1] 663

> pg[663]
[1] -0.03819212

> m3$volatility[663]
[1] 0.03961325

> source("garchM.R")
> m4 <- garchM(pg, type=1)
Maximized log-likelihood: 991.3017

Coefficient(s):

| Estimate     | Std. Error    | t value     | Pr(>|t|)          |
|--------------|---------------|-------------|-------------------|
| mu           | 0.007111978   | 0.004706627 | 1.51106 0.13077411 |
| gamma        | 0.707355559   | 1.574949256 | 0.44913 0.65333852 |
| omega        | 0.000416370   | 0.000223498 | 1.86297 0.06246653 |
| alpha        | 0.165418629   | 0.046037973 | 3.59309 0.00032678 *** |
| beta         | 0.709835860   | 0.101923405 | 6.96440 3.298e-12 *** |
```
### Problem E

```r
> da=read.table("m-pg3dx-6015.txt",header=T)
> head(da)

```

<table>
<thead>
<tr>
<th>PERMNO</th>
<th>date</th>
<th>RET</th>
<th>vwretd</th>
<th>ewretd</th>
<th>sprtrn</th>
</tr>
</thead>
<tbody>
<tr>
<td>18163</td>
<td>19600129</td>
<td>-0.081667</td>
<td>-0.066244</td>
<td>-0.039202</td>
<td>-0.071464</td>
</tr>
</tbody>
</table>

```r
> sp <- log(da[,6]+1)
> vw <- log(da[,4]+1)

```

```r
> m1 <- garchFit(~garch(1,1),data=sp,trace=F,cond.dist="sstd")
> summary(m1)
```

**Title:** GARCH Modelling  
**Call:**  
\texttt{garchFit(formula = \neg garch(1, 1), data = sp, cond.dist = "sstd", trace = F)}

**Mean and Variance Equation:**  
\texttt{data \sim garch(1, 1)[data = sp]}

**Error Analysis:**

| Estimate  | Std. Error | t value | Pr(>|t|) |
|-----------|------------|---------|----------|
| mu 6.092e-03 | 1.444e-03  | 4.218   | 2.46e-05 *** |
| omega 9.338e-05 | 4.132e-05  | 2.260   | 0.023822 *   |
| alpha1 1.360e-01 | 3.564e-02  | 4.031   | 5.56e-05 ***   |
| beta1 8.244e-01 | 3.764e-02  | 21.899  | < 2e-16 ***   |
| skew 7.699e-01 | 4.394e-02  | 17.267  | < 2e-16 ***   |
| shape 7.901e+00 | 2.212e+00  | 3.571   | 0.000355 ***   |

**Standardised Residuals Tests:**

<table>
<thead>
<tr>
<th>Test</th>
<th>Statistic</th>
<th>p-Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ljung-Box Test R Q(10)</td>
<td>7.547206</td>
<td>0.6729704</td>
</tr>
<tr>
<td>Ljung-Box Test R Q(20)</td>
<td>12.2038</td>
<td>0.9088825</td>
</tr>
<tr>
<td>Ljung-Box Test R^2 Q(10)</td>
<td>5.408172</td>
<td>0.8622992</td>
</tr>
<tr>
<td>Ljung-Box Test R^2 Q(20)</td>
<td>8.436177</td>
<td>0.9886569</td>
</tr>
</tbody>
</table>

```r
> volsp <- volatility(m1)

```

```r
> n1 <- garchFit(~garch(1,1),data=vw,trace=F,cond.dist="sstd")
> summary(n1)
```

**Title:** GARCH Modelling  
**Call:**  
\texttt{garchFit(formula = \neg garch(1, 1), data = vw, cond.dist = "sstd", trace = F)}

**Mean and Variance Equation:**  
\texttt{data \sim garch(1, 1) [data = vw]}

```r
```

```r
```
Conditional Distribution: sstd

Error Analysis:

|        | Estimate | Std. Error | t value | Pr(>|t|) |
|--------|----------|------------|---------|----------|
| mu     | 8.787e-03 | 1.485e-03  | 5.918   | 3.27e-09 *** |
| omega  | 9.953e-05 | 4.376e-05  | 2.274   | 0.022940 *  |
| alpha1 | 1.271e-01 | 3.203e-02  | 3.967   | 7.27e-05 *** |
| beta1  | 8.274e-01 | 3.841e-02  | 21.542  | < 2e-16 *** |
| skew   | 7.383e+00 | 4.391e-02  | 16.811  | < 2e-16 *** |
| shape  | 7.428e+00 | 1.919e+00  | 3.871   | 0.000108 *** |

---

> volvw <- volatility(n1)
> k1 <- lm(volsp~volvw)
> summary(k1)

Coefficients:

|                        | Estimate | Std. Error | t value | Pr(>|t|) |
|------------------------|----------|------------|---------|----------|
| (Intercept)            | 0.0007918| 0.0002818  | 2.81    | 0.0051 ** |
| volvw                  | 0.9487851| 0.0062352  | 152.17  | <2e-16 *** |

---

Residual standard error: 0.001891 on 661 degrees of freedom
Multiple R-squared: 0.9722, Adjusted R-squared: 0.9722
F-statistic: 2.315e+04 on 1 and 661 DF, p-value: < 2.2e-16

> k2 <- ar(k1$residuals)
> k2$order
[1] 1
> k3 <- arima(volsp,order=c(1,0,0),xreg=volvw)
> k3

Call: arima(x = volsp, order = c(1, 0, 0), xreg = volvw)
Coefficients:

<table>
<thead>
<tr>
<th></th>
<th>ar1</th>
<th>intercept</th>
<th>volvw</th>
</tr>
</thead>
<tbody>
<tr>
<td>s.e.</td>
<td>0.0178</td>
<td>0.0004</td>
<td>0.0073</td>
</tr>
</tbody>
</table>

sigma^2 estimated as 7.603e-07: log likelihood = 3729.18, aic = -7450.36

> tsdiag(k3)
> Box.test(k3$residuals,lag=12,type='Ljung')

Box-Ljung test
data: k3$residuals
X-square = 11.216, df = 12, p-value = 0.5105

> spec1 <- ugarchspec(variance.model=list(model="sGARCH",external.regressors=as.matrix(volvw)),mean.model=list(armaOrder=c(0,0)),distribution.model="sstd")
> n4 <- ugarchfit(data=sp,spec=spec1)
> n4
Conditional Variance Dynamics
-----------------------------------
GARCH Model : sGARCH(1,1)
Mean Model : ARFIMA(0,0,0)
Distribution : sstd

Optimal Parameters
-----------------------------------

|        | Estimate | Std. Error | t value | Pr(>|t|) |
|--------|----------|------------|---------|---------|
| mu     | 0.005915 | 0.001475   | 4.00923 | 0.000061|
| omega  | 0.000000 | 0.000027   | 0.00000 | 0.999998|
| alpha1 | 0.115915 | 0.034151   | 3.39418 | 0.000688|
| beta1  | 0.771342 | 0.067762   | 11.38303| 0.000000|
| vxreg1 | 0.004918 | 0.002628   | 1.87140 | 0.061289|
| skew   | 0.769959 | 0.044977   | 17.11882| 0.000000|
| shape  | 7.983262 | 2.245018   | 3.55599 | 0.000377|

Weighted Ljung-Box Test on Standardized Residuals
-----------------------------------

<table>
<thead>
<tr>
<th></th>
<th>statistic</th>
<th>p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lag[1]</td>
<td>0.4769</td>
<td>0.4898</td>
</tr>
<tr>
<td>Lag[4*(p+q)+(p+q)-1][5]</td>
<td>2.1466</td>
<td>0.5842</td>
</tr>
</tbody>
</table>

d.o.f=0
HO : No serial correlation