Assignment:

1. Daily gold price:
   (a) Time plots are shown in Figure 1.
   (b) The sample ACF is shown in Figure 2. All correlations are close to one, indicating strong serial dependence. Thus, the series has a unit root.
   (c) The Ljung-Box statistics give $Q(12) = 44.71$ with $p$-value $1.16 \times 10^{-5}$. The null hypothesis is rejected at the 5% level. That is, there are serial correlations in the return series.
   (d) An AR(14) is identified.
   (e) The fitted model is
   \[ r_t = -0.043r_{t-1} + 0.039r_{t-5} - 0.046r_{t-7} - 0.022r_{t-10} + 0.023r_{t-14} + a_t \]
   where $\sigma_a^2 = 1.16$. The model fits the data well, see model checking in Figure 3.
   (f) The point predictions (standard errors) are $-0.057(1.08), 0.021(1.08), 0.002(1.08), -0.031(1.08)$, respectively. The 95% interval forecasts are

\[
\begin{array}{ccc}
\text{Start} & \text{End} & \text{Frequency} = 1 \\
5622 & 5625 & 1 \\
\hline
\text{lcl} & \text{ucl} \\
5622 & -2.165231 & 2.051790 \\
5623 & -2.089822 & 2.131119 \\
5624 & -2.108014 & 2.112934 \\
5625 & -2.141583 & 2.079366 \\
\end{array}
\]

2. Returns of gold price:
   (a) The fitted model is
   \[ r_t = a_t - 0.041a_{t-1} + 0.038a_{t-5} - 0.042a_{t-7} \]
   where $\sigma_a^2 = 1.16$. Model checking looks fine; see Figure 4.
   (b) The Ljung-Box statistics give $Q(10) = 8.04$ with $p$-value 0.33 (after adjusting the degrees of freedom). The null hypothesis cannot be rejected. That is, there are no significant serial correlations in the residuals.
   (c) The AR model is preferred as it has a lower AIC (16784.8 versus 16788.9).
(d) Again the AR model is preferred as it has a slightly smaller MSE (0.6811 versus 0.6814).

3. Volatility modeling.

(a) Let $r_t = \text{vix}_t - 19.6$. The fitted model is

$$r_t = 0.88r_{t-1} + 0.033r_{t-3} + 0.028r_{t-5} + 0.035r_{t-8} + 0.066r_{t-10}$$

$$- 0.079r_{t-11} + 0.023r_{t-12} + a_t$$

where $\sigma_a^2 = 2.23$. The model is adequate; see Figure 5.

(b) The predictions are

12.44, 12.56, 12.62, 12.69, 12.71, 12.71, 12.70, 12.77, 12.84, 12.95

and the associated standard errors are

1.49, 1.99, 2.31, 2.54, 2.73, 2.89, 3.03, 3.16, 3.28, 3.39.

4. GDP growth rate.

(a) See Figure 6. The variability is lower in the 2nd part of the series.

(b) An AR(3) model is identified and it fits the data well. See Figure 7. The model is

$$r_t = 0.345r_{t-1} + 0.128r_{t-2} - 0.095r_{t-3},$$

where $r_t = \text{gdp}_t - 3.21$ and $\sigma_a^2 = 12.86$.

(c) Yes, the characteristic equation has a pair of complex roots.

(d) The average length is 11.06 quarters.

(e) The point predictions are 3.04, 2.98, 3.21, and 3.20, respectively. The 95% interval forecasts are

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<th>Start = 280</th>
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<th>Frequency = 1</th>
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5. Unemployment rates

(a) The fitted model is

$$(1 - 1.63B + 0.40B^2 + 0.23B^3)(1 - 0.54B^{12})(r_t - 5.69) = (1 - 0.63B)(1 - 0.80B^{12})a_t,$$

where $\sigma_a^2 = 0.035$. The model is adequate. See model checking in Figure 8.
Figure 1: Time plots of daily log gold price and its returns.

(b) The fitted model is
\[
(1 - 0.99B - 0.16B^2 + 0.16B^6 + 0.09B^{10} - 0.10B^{11})(r_t - 5.65) = a_t,
\]
where \( \sigma_a^2 = 0.038 \).

(c) The seasonal model is preferred as it has a smaller RMSE (0.138 versus 0.151).
Figure 2: Sample ACF of the daily log gold prices.

Figure 3: Model checking for AR model.
Figure 4: Model checking of MA(7) model for daily returns of gold price.

Figure 5: Model checking of an AR model for VIX index.
Figure 6: Time plot of U.S. quarterly GDP growth rates.

Figure 7: Model checking of an AR(3) model for US quarterly GDP growth rates.
Figure 8: Model checking of a seasonal model for US monthly unemployment rates.