Solutions to Homework Assignment #4

1. Consider the daily log returns of Caterpillar stock.
   (a) Are there any serial correlations in the log return series \( r_t \)? Why?
   A: No, the Ljung-Box statistics give \( Q(10) = 16.23 \) with \( p \)-value 0.093.
   (b) Are there any ARCH effects in the log return series \( r_t \)? Why?
   A: Yes, the Ljung-Box statistics of squared returns give \( Q(10) = 917.6 \) with \( p \)-value close to zero.
   (c) Fit a Gaussian ARMA-GARCH model to the \( r_t \) series.
   A: The QQ-plot is in Figure 1. The model is not adequate as the normality assumption is rejected. The fitted model is
   \[
   r_t = 4.93 \times 10^{-4} + a_t \\
   a_t = \sigma_t \epsilon_t, \quad \epsilon_t \sim iid \ N(0,1) \\
   \sigma_t^2 = 4.42 \times 10^{-6} + 0.049a_{t-1}^2 + 0.939\sigma_{t-1}^2.
   \]
   (d) Build a GARCH model with standardized Student-\( t \) innovations for the \( r_t \) series.
   A: QQ-plot is shown in Figure 2. The model seems to be adequate.
   (e) Write down the fitted model.
   A: The model is
   \[
   r_t = 5.9 \times 10^{-4} + a_t \\
   a_t = \sigma_t \epsilon_t, \quad \epsilon_t \sim iid \ t^*_{5.11} \\
   \sigma_t^2 = 4.21 \times 10^{-6} + 0.072a_{t-1}^2 + 0.92\sigma_{t-1}^2.
   \]
   (f) Obtain 1-step to 5-step ahead mean and volatility forecasts using the fitted ARMA-GARCH model with standardized Student-\( t \) innovations.
   A: The predictions are
   \[
   \begin{array}{llll}
   & \text{meanForecast} & \text{meanError} & \text{standardDeviation} \\
   1 & 0.0005900709 & 0.01557845 & 0.01557845 \\
   2 & 0.0005900709 & 0.01565642 & 0.01565642 \\
   3 & 0.0005900709 & 0.01573343 & 0.01573343 \\
   4 & 0.0005900709 & 0.01580951 & 0.01580951 \\
   5 & 0.0005900709 & 0.01588467 & 0.01588467 \\
   \end{array}
   \]
(g) Compute the 95% 1-step to 5-step interval predictions of the log return series using standardized student-\(t\) innovations.

A: The 95% interval predictions are

\[
\begin{align*}
> CI \\
[1,] & -0.03041105 0.03159119 \\
[2,] & -0.03056620 0.03174635 \\
[3,] & -0.03071946 0.03189960 \\
[4,] & -0.03087085 0.03205100 \\
[5,] & -0.03102042 0.03220056
\end{align*}
\]

2. Consider again the daily log returns of the CAT stock of Problem 1.

(a) Igarch model:

\[
a_t = \sigma_t \varepsilon_t, \quad \varepsilon_t \sim_{iid} N(0,1) \\
\sigma_t^2 = 1.92 \times 10^{-6} + (1 - 0.948)a_{t-1}^2 + 0.948\sigma_{t-1}^2.
\]

(b) See Figure 3.

(c) No, the Ljung-Box statistics of the standardized residuals give \(Q(10) = 14.26\) with \(p\)-value 0.16.

(d) No, the Ljung-Box statistics of the squared standardized residuals show \(Q(10) = 1.15\) with \(p\)-values close to 1.

(e) Except for the normality, the Igarch(1,1) model seems adequate. The 1-step to 4-step ahead volatility forecasts are 0.01567, 0.01573, 0.01579, and 0.01585.

3. Consider the monthly returns of Coke (KO) stock from January 1951 to December 2016.

(a) Yes, the expected return is different from zero. The t-test shows 4.99 with \(p\)-value close to zero. No, there are no serial correlations in the returns. The Ljung-Box statistics show \(Q(12) = 17.06\) with \(p\)-value 0.15. Yes, there are ARCH effects in the returns, because the Ljung-Box statistics of the squared returns (mean-adjusted) show \(Q(12) = 256\) with \(p\)-value close to zero.

(b) The fitted model is

\[
r_t = 0.011 + a_t, \\
a_t = \sigma_t \varepsilon_t, \quad \varepsilon_t \sim_{iid} N(0,1) \\
\sigma_t^2 = 1.85 \times 10^{-4} + 0.095a_{t-1}^2 + 0.848\sigma_{t-1}^2.
\]

The normality assumption is clearly rejected. The model is not adequate.
(c) The fitted model is
\[
\begin{align*}
  r_t &= 0.011 + a_t, \\
  a_t &= \sigma_t \epsilon_t, \quad \epsilon_t \sim iid \ t^*_{7.49} \\
  \sigma_t^2 &= 1.75 \times 10^{-4} + 0.096a_{t-1}^2 + 0.85\sigma_{t-1}^2.
\end{align*}
\]

The model seems to be adequate.

(d) The fitted model is
\[
\begin{align*}
  r_t &= 0.011 + a_t, \\
  a_t &= \sigma_t \epsilon_t, \quad \epsilon_t \sim iid \ t^*_{7.62}(0.967) \\
  \sigma_t^2 &= 1.75 \times 10^{-4} + 0.096a_{t-1}^2 + 0.85\sigma_{t-1}^2.
\end{align*}
\]

The skewness test is \((0.967 - 1)/0.05 = -0.66\), which is less than 1.96 in absolute value. Thus, the assumption of symmetry in distribution cannot be rejected.

(e) The fitted model is
\[
\begin{align*}
  r_t &= 0.0094 + 0.284\sigma_t^2 + a_t, \\
  a_t &= \sigma_t \epsilon_t, \quad \epsilon_t \sim iid \ N(0, 1) \\
  \sigma_t^2 &= 1.86 \times 10^{-4} + 0.095a_{t-1}^2 + 0.848\sigma_{t-1}^2.
\end{align*}
\]

The \(t\)-ratio of risk premium is 0.195 with \(p\)-value 0.85 so that the risk premium is not statistically significant.

(f) The fitted model is
\[
\begin{align*}
  r_t &= 0.011 + a_t \\
  a_t &= \sigma_t \epsilon_t, \quad \epsilon_t \sim iid \ t^*_{7.52} \\
  \sigma_t^2 &= 0.00018 + 0.095(|a_{t-1}| - 0.052a_{t-1})^2 + 0.848\sigma_{t-1}^2
\end{align*}
\]

The leverage parameter has a \(t\)-ratio 0.475 with \(p\)-value 0.63. Thus, the leverage effect is not statistically significant.

4. Consider the monthly returns of the CRSP value-weighted index, including dividends, from 1961 to 2016.

(a) The fitted model is
\[
\begin{align*}
  r_t &= 0.0092 + a_t \\
  a_t &= \sigma_t \epsilon_t, \quad \epsilon_t \sim iid \ t^*_{8.19}(0.741) \\
  \sigma_t^2 &= 1.02 \times 10^{-4} + 0.118a_{t-1}^2 + 0.829\sigma_{t-1}^2.
\end{align*}
\]

(b) The predictions are
> predict(n2,5)

<table>
<thead>
<tr>
<th>meanForecast</th>
<th>meanError</th>
<th>standardDeviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.009156345</td>
<td>0.03273115</td>
</tr>
<tr>
<td>2</td>
<td>0.009156345</td>
<td>0.03342065</td>
</tr>
<tr>
<td>3</td>
<td>0.009156345</td>
<td>0.03406080</td>
</tr>
<tr>
<td>4</td>
<td>0.009156345</td>
<td>0.03465617</td>
</tr>
<tr>
<td>5</td>
<td>0.009156345</td>
<td>0.03521076</td>
</tr>
</tbody>
</table>

(c) The fitted model is

\[ r_t = 0.0089 + \alpha_t \]
\[ \alpha_t = \sigma_t \epsilon_t, \quad \epsilon_t \sim iid \const{\mathcal{N}}(0, 1) \]
\[ \sigma_t^2 = 3.19 \times 10^{-4} + (0 + 0.245N_{t-1})\alpha_{t-1}^2 + 0.696\sigma_{t-1}^2 \]

where \( t \)-ratio for the leverage is 1.02 with \( p \)-value 0.31. Thus, the leverage effect is not statistically significant.

5. CAT daily returns and VIX index.

A: \( t \)-ratio of \( \gamma \) is 1.39 with \( p \)-value 0.16. Thus, \( H_0 \) cannot be rejected. The 1-step ahead predictions for the return and its volatility are 0.00035 and 0.015, respectively.
Figure 2: QQ-plot of standardized residuals (t-innovations)

Figure 3: Fitted volatility series
Figure 4: QQ-plot of standardized residuals.