Solutions to Homework Assignment #5


<table>
<thead>
<tr>
<th>Type</th>
<th>Mean</th>
<th>Median</th>
<th>Minimum</th>
<th>Maximum</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma_{0,t}$</td>
<td>14.30</td>
<td>8.57</td>
<td>0</td>
<td>161.92</td>
</tr>
<tr>
<td>$\sigma_{1,t}$</td>
<td>18.38</td>
<td>11.80</td>
<td>0.00</td>
<td>193.48</td>
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<tr>
<td>$\sigma_{2,t}$</td>
<td>14.54</td>
<td>10.30</td>
<td>1.72</td>
<td>111.26</td>
</tr>
<tr>
<td>$\sigma_{3,t}$</td>
<td>25.85</td>
<td>18.24</td>
<td>3.01</td>
<td>194.81</td>
</tr>
<tr>
<td>$\sigma_{5,t}$</td>
<td>14.70</td>
<td>10.44</td>
<td>1.83</td>
<td>124.43</td>
</tr>
<tr>
<td>$\sigma_{6,t}$</td>
<td>26.77</td>
<td>19.03</td>
<td>3.33</td>
<td>228.92</td>
</tr>
</tbody>
</table>

2. The time plot is given in Figure 1. Let $x_t$ be the log volatility, the model selected is

$$(1 - 0.93B)(1 - B)x_t = (1 - 0.86B)a_t,$$

where $\sigma^2_a = 4.67 \times 10^{-4}$. The model fits the series well based on the model checking statistics, but there are a few large outliers. The 1-step to 5-step ahead forecasts are $-0.912, -0.912, -0.912, -0.912, -0.912$.

3. Amazon and S&P 500 index.

(a) Yes, they can. Because the coefficient of $A_{t-1}$ is statistically significant.

(b) The fitted model is

$$h_{1t} = \begin{cases} \frac{\exp(2.11 - 0.30A_{t-1} - 0.14A_{t-2} + 1.79M_{t-1} + 3.62M_{t-2})}{1 + \exp(2.11 - 0.30A_{t-1} - 0.14A_{t-2} + 1.79M_{t-1} + 3.62M_{t-2})} \\ 1 \\ \frac{\exp(0.82 - 2.63A_{t-1} - 0.10A_{t-2} - 0.67M_{t-1} + 0.32M_{t-2})}{1 + \exp(0.82 - 2.63A_{t-1} - 0.10A_{t-2} - 0.67M_{t-1} + 0.32M_{t-2})} \\ \frac{\exp(0.48 - 2.35A_{t-1} + 1.89A_{t-2} + 0.76M_{t-1} + 1.20M_{t-2})}{1 + \exp(0.48 - 2.35A_{t-1} + 1.89A_{t-2} + 0.76M_{t-1} + 1.20M_{t-2})} \end{cases}$$

$$h_{3t} = \begin{cases} \frac{\exp(0.48 - 2.35A_{t-1} + 1.89A_{t-2} + 0.76M_{t-1} + 1.20M_{t-2})}{1 + \exp(0.48 - 2.35A_{t-1} + 1.89A_{t-2} + 0.76M_{t-1} + 1.20M_{t-2})} \\ 0 \end{cases}$$

$$P(A_t = 1) = \begin{cases} 1 & \text{if } \ell_t > 0 \\ 0 & \text{if } \ell_t \leq 0, \text{ where} \\ \ell_t = 0.84 - 2.88h_{1t} + 1.66h_{2t} + 1.52h_{3t} - 0.24A_{t-1} - 0.54A_{t-2} + 0.11M_{t-1} - 0.13M_{t-2} \end{cases}$$

(c) In this particular instance, the neural network outperforms the logistic regression based on the root mean square error criterion.
Figure 1: Time plot of volatility via Yang-Zhang method

4. Walgreens

- The 5-minute intraday log returns are shown in Figure 2.
- No, we have $Q(10) = 11.32$ with $p$-value 0.33.
- The realized volatilities are given below:

  \[
  \begin{array}{cccccccc}
  [1] & 0.007308644 & 0.009063069 & 0.006214600 & 0.008271188 & 0.008469543 & 0.007716857 \\
  [7] & 0.006497201 & 0.005492192 & 0.009369673 & 0.007286883 \\
  \end{array}
  \]

- For 1-m returns, the realized volatilities are

  \[
  \begin{array}{cccccccc}
  [1] & 0.007887359 & 0.009203825 & 0.007482885 & 0.008118223 & 0.008179217 & 0.008297711 \\
  [7] & 0.009541117 & 0.005376000 & 0.009016982 & 0.006878587 \\
  \end{array}
  \]

5. Again, consider the tick-by-tick trade data of Walgreens stock from February 6 to 17, 2017.

- See Figure 3.
- Yes, there exists a diurnal pattern in the series.
Figure 2: Time plot of of intraday 5-m log returns Walgreens stock from February 6 to 17, 2017.

Figure 3: Trading intensity of Walgreens stock from February 6 to 17, 2017.