Graduate School of Business
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Bus 41910, Univariate Time Series Analysis, Mr. R. Tsay

Homework Assignment #1, Due in one week.

1. For the second-order difference equation: $Y_t = \phi_1 Y_{t-1} + \phi_2 Y_{t-2}$, determine the stable regime on the (ϕ_1, ϕ_2)-plane. (This is the triangular regime you will see in many time series textbooks, e.g. Box, Jenkins and Reinsel, 1976.)

2. Compute the solutions to the difference equations:

a. $Y_t = 1.2Y_{t-1} - .5Y_{t-2}$, $Y_0 = 8$ and $Y_1 = 8.54$.

b. $Y_t - .5Y_{t-1} = 1 + .4X_t$, $Y_0 = 1$ and X_t satisfies $X_t = -.5X_{t-1}$, $X_0 = 2$.

3. Prove that the moment generating function of a stationary linear process $X_t = \psi(B)a_t = \sum_{i=0}^{\infty} \psi_i a_{t-i}$ is given by

$$\Gamma(z) = \sigma^2 \psi(z)\psi(z^{-1}),$$

where $\sigma^2 = \text{var}(a_t)$, $E(a_t) = 0$, and $\psi_0 = 1$.

4. Simulation is informative in studying time series. Use any package available to you to generate the following series.

a. a Gaussian white noise series with 200 observations.

b. an MA(2) process with 300 observations., say $X_t = 0.4 + (1 - 1.1B + 0.4B^2)a_t$.

c. an AR(1) process with 200 observations, say $(1 - 0.8B)X_t = 1.0 + a_t$.

d. an ARMA(1,1) process with 500 observations, say $(1 - 0.9B)X_t = 1.0 + (1 - 0.4B)a_t$.

For each simulated series, show the sample mean, standard error, and the first 5 lags of sample autocorrelation function.

Experiment with the parameters of the series, plot the data and comment on the relation between autocorrelation and the behavior of the sample path.

To simulate a time series in **SCA**, use the command "utsm" (stands for univariate time series model) to setup the model and the command SIMULATE to generate the data. You may use "help utsm" and "help simu" in SCA for on-line help.

For example, in SCA, use the following commands to specify an ARMA(1,1) model.

```
utsm m1. model (1-p1*b)x=c+(1-t1*b)noise. simulate.
p1 = 0.9
t1 = 0.4
c = 1.0
```

Then, use the following command to generate 300 data points.

```
simu model m1. noise n(0,1). nobs 300.
```

The data is stored in x.
To compute ACF of x, use the command
acf x. maxlag 5.
In **S-Plus**, you may use
x=arima.sim(model=list(ar=c(.9),ma=c(-.4)),n=300)
acfx = acf(x,lag.max=5)
If you use **R**, you can use the package "tseries" with some simple "programming". Below is an illustration, where % denotes explanation.

```
n=301      % I used 301 because an ARMA(1,1) model needs a starting value
at=rnorm(n)   % generate 501 normal N(0,1) random variates
x=double(n)   % set workspace for the time series
x[1]=0        % initial value of the first data point
for (i in 2:n)  % A do-loop to generate ARMA(1,1) model.
{
x[i]=0.9*x[i-1]+at[i]-0.4*at[i-1]
}
y=ts(x[2:301])  % Skip the initial value and store the results in ‘‘y’’.

To compute ACF first 10 ACFs, use the command

acf(y,lag.max=10)

To plot the series,

ts.plot(y)
```