Graduate School of Business
University of Chicago
Bus 41910, Time Series Analysis, Mr. R. Tsay

Homework Assignment #2 (due October 13, **Before** class)

1. Assume that $\{a_t\}$ is a Gaussian white noise series with mean zero and variance σ_a^2. Compute the mean and first 3 lags of autocorrelation function (ACF) of model (a) to (d). For model (e), compute the mean and the first 4 lags of ACF.

   a. $Z_t - 0.7Z_{t-1} = 3 + a_t$

   b. $(1 - .33B - .13B^2)Z_t = .044 + a_t$

   c. $Z_t = 2 + a_t - 1.3a_{t-1} + .4a_{t-2}$.

   d. $(1 - 0.8B)Z_t = 1.0 + (1 + .4B)a_t$

   e. $Z_t = 1.0 + a_t - 0.7a_{t-4}$.

2. Use SCA or S-plus or any other package to simulate the models.

   a. $(1 - B)Z_t = a_t$

   b. $(1 - B)Z_t = 3.0 + a_t$

   c. $(1 - 0.7B^4)Z_t = a_t$

   d. $(1 - B + B^2)Z_t = a_t$

   e. $(1 - 0.95B)Z_t = (1 - 0.90B)a_t$

For parts (a) to (d), use 200 observations. For part (e), use 1000 observations. Plot the generated time series, and compute the ACF of the series. Describe the feature of the plot and ACF of each series. [You don't need to turn in the time series plot.]

3. Obtain the first 3 π-weights and ψ-weights of the models:

   a. $(1 - \phi_1 B - \phi_2 B^2)Z_t = (1 - \theta B)a_t$

   b. $(1 - \sqrt{3}B + B^2)Z_t = a_t$

4. Consider the time series $Z_t = T_t + \epsilon_t$, where $\{\epsilon_t\}$ is a sequence of independent and identically distributed (iid) random variables with mean zero and variance 1, and $\{T_t\}$ and $\{\epsilon_t\}$ are independent. Assume also that T_t follows the model

$$T_t = 0.5T_{t-1} + a_t,$$

where $\{a_t\}$ is a sequence of iid random variables with mean zero and variance 2. Derive an ARMA model for Z_t, including the coefficients and variance of the innovations.

5. The data file "hous.dat" contains the monthly housing starts and house sold in the U.S. You may take the log transformation to stablize the variability.

- Compute the sample ACF of the series.
- Compute the sample ACF of $(1 - B)X_t$.
- Compute the sample ACF of $(1 - B^{12})X_t$.
- Compute the sample ACF of $(1 - B)(1 - B^{12})X_t$.

To use SCA to solve this problem, do the following.

1. Download the data file into your unix machine, e.g. GSBPHD or Galton.
2. Enter SCA and then use the following commands:
    - input x,y. file 'hous.dat'.
    - x = ln(x)
    - acf x.
    - acf x. dfor 1.
    - acf x. dfor 12.
    - acf x. dfor 1,12.
    - (do the same for y.)
    - stop

To load the data into S-Plus or R, use the following commands:

- x = read.table('hous.dat')
- x1 = log(x[1,])
- x2 = log(x[2,])
- dx1=diff(x1)
- d12x1=diff(x1,12)