Graduate School of Business
University of Chicago
Bus 41910, Time Series Analysis, Mr. R. Tsay

Homework Assignment #3, Due in one week

1. Consider the models

$$(1-B)(1-B^4)Z_t = (1-0.5B)(1-0.6B^4)a_t,$$

where $\{a_t\}$ is a Gaussian white noise series with mean zero and variance 1. Assume also that $(Z_{96}, Z_{97}, \cdots, Z_{100}) = (119.0,118.7,116.5,118.6,126.2)$ and $(a_{96}, \cdots, a_{100}) = (0.4, 0.6, 1.9, -0.8, -0.5)$. Use the minimum mean squared error criterion to compute forecasts at the forecast origin $t = 100$.

- What is the 1-step ahead forecast $Z_{100}(1)$? What is the variance of the associated forecast error?
- What is the 2-step ahead forecast $Z_{100}(2)$? What is the variance of the forecast error of the 2
- What is the 3-step ahead forecast $Z_{100}(3)$?
- What is the 4-step ahead forecast $Z_{100}(4)$?

2. Suppose that Z_t follows the model

$$(1-0.7B)Z_t = a_t,$$

where $\{a_t\}$ is a Gaussian white noise series with mean zero and variance 1. Consider the non-overlapping 3 aggregates, i.e. $Y_\ell = Z_{3\ell} + Z_{3\ell-1} + Z_{3\ell-2}$. What is the order of the ARMA model for Y_ℓ? Can you write down the specific model for Y_ℓ?

3. Consider the structural model

$$Z_t = T_t + S_t + \epsilon_{0t},$$

where T_t and S_t satisfy

$$(1-B^2)T_t = \epsilon_{1t}, \quad (1+B+B^2+B^3)S_t = \epsilon_{2t},$$

where $\{\epsilon_{it}\}$ $(i = 0, 1, 2)$ are independent Gaussian white noise series with mean zero and variance 1. What is the ARMA model for Z_t?

4. Derive the autocorrelation function of the following models, where $\{a_t\}$ is a Gaussian white noise series with mean zero and variance 1 and $s > 1$ is a positive integer.

- $Z_t = (1 - \theta B)(1 - \Theta B^s)a_t$.
- $Z_t = (1 - \theta B - \Theta B^s)a_t$.

5. Consider the simple exponential smoothing method for forecasting. Suppose that the discount rate is 0.94 and the 1-step ahead forecast at origin $t = 100$ is $Z_{100}(1) = 50$. If $Z_{101} = 55$, then what is the 1-step ahead forecast at $t = 101$?