Graduate School of Business, University of Chicago
Business 41914, Spring Quarter 2005, Mr. Ruey S. Tsay

Midterm

GSB Honor Code:
I pledge my honor that I have not violated the Honor Code during this examination.

Signature: **Name**: **UC. ID**:

Notes:

- Open notes and books.
- Write your answer in the blank space provided for each question.
- **Manage** your time carefully and answer as many questions as you can.
- For simplicity, **ALL** tests use the 5% significance level.

Problem A: (30 points; 2 points per question) Answer briefly the following questions.

1. (For problems 1 to 5) Consider a 2-dimensional, stationary time series $\boldsymbol{z}_t$. Define the lag-ℓ auto-covariance matrix $\boldsymbol{\Gamma}_\ell$ of $\boldsymbol{z}_t$.

2. Show that $\boldsymbol{\Gamma}_\ell = \boldsymbol{\Gamma}'_{-\ell}$, where $\ell \geq 0$.

3. Define the lag-ℓ auto-correlation matrix of $\boldsymbol{z}_t$.

4. Suppose that $\boldsymbol{z}_t = (z_{1t}, z_{2t})'$. Give a sufficient condition that z_{1t} does not depend on any past values of z_{2t}, but z_{2t} depends on some past values of z_{1t}.

5. If $\boldsymbol{z}_t$ follows a VARMA(1,1) model $\boldsymbol{z}_t - \boldsymbol{\phi}\boldsymbol{z}_{t-1} = \boldsymbol{\phi}_0 + \boldsymbol{a}_t - \boldsymbol{\theta}\boldsymbol{a}_{t-1}$, what is the marginal model for z_{1t}?

6. (For problems 6 to 7) Suppose that the k-dimensional process $\boldsymbol{z}_t$ follows the model

$$\boldsymbol{\phi}(B)\boldsymbol{z}_t = \boldsymbol{\theta}(B)\boldsymbol{a}_t,$$

where $\boldsymbol{\phi}(B) = \boldsymbol{I} - \sum_{i=1}^{p} \boldsymbol{\phi}_i B^i$ and $\boldsymbol{\theta}(B) = \boldsymbol{I} - \sum_{j=1}^{q} \boldsymbol{\theta}_j B^j$. State sufficient conditions that the VARMA model is identifiable.

7. Write down a sufficient condition for the weak stationarity of $\boldsymbol{z}_t$.

8. Consider the transfer function model $y_t = v(B)x_t + N_t$, where x_t is the input variable, $v(B) = v_0 + v_1 B + v_2 B^2 + \cdots$ and N_t is independent of x_t, but may be serially correlated. Suppose further that x_t follows a univariate ARMA(1,1) model

$$(1 - 0.8B)x_t = (1 + 0.4B)b_t,$$

where $\{b_t\}$ is a univariate white noise series with mean zero and variance σ_b^2. Describe a method to provide consistent estimates of the impulse response weights v_i's.

9. Consider the random walk model $z_t = z_{t-1} + a_t$, where a_t is a sequence of independent and identically distributed random variables with mean zero, variance σ^2 and $E(|a_t|^{2+\delta}) < \infty$ for some $\delta > 0$. Assume also that $z_0 = 0$. What is the limiting distribution of $\frac{1}{T}\sum_{t=1}^{T} z_{t-1}a_t$ as $T \to \infty$? Briefly outline the proof.

10. Define co-integration of a k-dimensional time series.

11. Suppose that $\boldsymbol{z}_t$ follows the model

$$\boldsymbol{z}_t = \boldsymbol{\mu} + \boldsymbol{a}_t - \boldsymbol{\theta}\boldsymbol{a}_{t-1},$$

where $\{\boldsymbol{a}_t\}$ are Gaussian white noise with mean zero and covariance matrix $\boldsymbol{\Sigma} > 0$. Derive the moment equations of $\boldsymbol{z}_t$.

12. (For Problems 12 to 15) Suppose that$\boldsymbol{z}_t$ follows the model

$$\boldsymbol{z}_t - \begin{bmatrix} .6 & .2 \\ .4 & .8 \end{bmatrix} \boldsymbol{z}_{t-1} = \boldsymbol{a}_t - \begin{bmatrix} -.2 & .7 \\ -0.5 & 1 \end{bmatrix} \boldsymbol{a}_{t-1},$$

where $\{\boldsymbol{a}_t\}$ is a Gaussian white noise series with mean zero and $\mathrm{Cov}(\boldsymbol{a}_t) = \boldsymbol{I}$, the identity matrix. Is the model invertible? Why?

13. Show that z_{1t} and z_{2t} are unit-root nonstationary.

14. Write down an Error-Correction Model for the system.

15. Write down the co-integrating vector for the system in which the first element is 1.

Problem B. (20 points) Consider a 2-dimensional time series $\boldsymbol{z}_t = (z_{1t}, z_{2t})'$ with 125 observations. The SCA analysis of the data is attached in which z_{1t} and z_{2t} are called "x" and "y", respectively. Use the output and answer the following questions.

1. (4 points) Why a VAR(3) model is specified for the series?

2. (7 points) Write down the final fitted model, including the residual covariance matrix.

3. (6 points) Based on the fitted model, discuss the relationship between z_{1t} and z_{2t}.

4. (3 points) Why are the forecasts of z_{1t} a constant?

Problem C. (15 points) Consider the monthly JP-US exchange rate and US-UK exchange rate series from January 1971 to April 2005. Let $\boldsymbol{z}_t = (z_{1t}, z_{2t})'$ where z_{1t} is the log Japan-US exchange rate (Yens per U.S. Dollar) and z_{2t} is the log US-UK exchange rate (Dollars per U.K. Pound). A VAR(2) model and a VARMA(1,1) model are identified and estimated for the series.

1. (4 points) Write down the fitted final VAR(2) model.

2. (4 points) Write down the fitted final VARMA(1,1) model.

3. (4 points) Are the two exchange rate series co-integrated? Why?

4. (3 points) Is there any relationship between the two log exchange rate series?

Problem D. (15 points) Suppose that $\boldsymbol{z}_t$ is a linear stationary process with Kronecker indices $\{k_1 = 2, k_2 = 0, k_3 = 1\}$. Write down the implied VARMA model for $\boldsymbol{z}_t$. How many parameters in the AR and MA polynomials that require estimation?

Problem E. (20 points) Suppose that $\boldsymbol{z}_t$ is a linear stationary process with three linear independent scalar components $y_{1t} \sim$ SCM(0,1), $y_{2t} \sim$ SCM(1,0), and $y_{3t} \sim$ SCM(2,1).

1. (12 points) Write down a specified VARMA model for the $\boldsymbol{y}_t = (y_{1t}, y_{2t}, y_{3t})'$ series.

2. (4 points) Is there any redundant parameter implied by the SCM models? If yes, how many?

3. (4 points) How many parameters in the AR and MA matrix polynomials of Part (1) that require estimation?