THE UNIVERSITY OF CHICAGO
Graduate School of Business
Business 41914, Spring Quarter 2007, Mr. Ruey S. Tsay

Solutions to Homework Assignment 2

The SCA output of my analysis is in a separate file. You may compare it with yours. Keep in mind that there is no "true" model so that multiple models can fit the same data well. In what follows, I use the notation $\boldsymbol{z}_t$ to denote a vector time series.

1. Based on the sequential Chi-square test and AIC, I identify a VAR(2) model for the data. After removing the insignificant parameters, the final model is

$$\boldsymbol{z}_t - \begin{bmatrix} .55 & .36 \\ 0 & 0 \end{bmatrix} \boldsymbol{z}_{t-1} - \begin{bmatrix} -.36 & .52 \\ 0 & .46 \end{bmatrix} \boldsymbol{z}_{t-2} = \begin{bmatrix} 2.44 \\ 6.10 \end{bmatrix} + \boldsymbol{a}_t, \quad \boldsymbol{\Sigma}_a = \begin{bmatrix} 1.57 & 1.59 \\ 1.59 & 19.2 \end{bmatrix}.$$

   Model checking indicates that the fitted model is adequate. The multivariate Ljung-Box statistics all fail to reject the hypothesis of no cross-correlations in the residuals.

   This model indicates there is a unidirectional relationship between the components series with z_{2t} as input and z_{1t} as output.

2. Again, the sequential Chi-square statistics and AIC suggest a VAR(1) model. The fitted model is

$$\boldsymbol{z}_t - \begin{bmatrix} .73 & -.26 \\ -.78 & 0 \end{bmatrix} \boldsymbol{z}_{t-1} = \begin{bmatrix} 11.0 \\ 37.6 \end{bmatrix} + \boldsymbol{a}_t, \quad \boldsymbol{\Sigma}_a = \begin{bmatrix} 1.61 & 1.31 \\ 1.31 & 18.2 \end{bmatrix}.$$

   The Ljung-Box statistics of the residuals fail to reject the hypothesis that the fitted model is adequate. Based on the fitted model, there are feedback relationship between the two series.

3. The sample cross-correlation matrices suggest a VMA(1) model for the data. The fitted model is

$$\boldsymbol{z}_t = \begin{bmatrix} 17.0 \\ 24.8 \end{bmatrix} + \boldsymbol{a}_t - \begin{bmatrix} .45 & 0 \\ 1.89 & 0.76 \end{bmatrix} \boldsymbol{a}_{t-1}, \quad \boldsymbol{\Sigma}_a = \begin{bmatrix} 1.59 & 1.47 \\ 1.47 & 18.9 \end{bmatrix}.$$

   The fitted model is adequate based on the Ljung-Box statistics of the residuals. Based on the fitted model, there is a unidirectional relationship between the two components with z_{1t} being the input variable.

4. Make use of Eq. (10) of Lecture note 2 on page 20. Define $\boldsymbol{y}_0 = \boldsymbol{0}$, $\boldsymbol{y}_1 = \boldsymbol{z}_1$, and $\boldsymbol{y}_t = \boldsymbol{z}_t + \sum_{i=1}^{t-1} \boldsymbol{\theta}^i \boldsymbol{z}_{t-i}$, and $\boldsymbol{x}_0 = \boldsymbol{0}$, and $\boldsymbol{x}_t = -\boldsymbol{\theta}^t$. Then, we have the system of equations as

$$\begin{aligned} \boldsymbol{y}_0 &= \boldsymbol{x}_0 \boldsymbol{a}_0 + \boldsymbol{a}_0 \\ \boldsymbol{y}_1 &= \boldsymbol{x}_1 \boldsymbol{a}_0 + \boldsymbol{a}_1 \\ \vdots &= \vdots \\ \boldsymbol{y}_n &= \boldsymbol{x}_n \boldsymbol{a}_0 + \boldsymbol{a}_n. \end{aligned}$$

Let $\boldsymbol{\Sigma}_a^{1/2}$ as the square-root matrix of $\boldsymbol{\Sigma}$. Pre-multiplying the above equations by $\boldsymbol{\Sigma}^{-1/2}$, we obtain a multivariate linear regression setup. From which we can estimate the parameter $\boldsymbol{a}_0$. The exact likelihood function of the data can then be obtained by integrating out $\boldsymbol{a}_0$ using

$$\boldsymbol{a}_t = (\boldsymbol{y}_t - \boldsymbol{x}_t \hat{\boldsymbol{a}}_0) + \boldsymbol{x}_t(\boldsymbol{a}_0 - \hat{\boldsymbol{a}}_0)$$

with the first term on the right side of the equation being independent of $\boldsymbol{a}_0$. Details are similar to that of the univariate MA(1) case and is omitted.

You may also use the method in the paper by Nicholls and Hall (1979) mentioned in class.