THE UNIVERSITY OF CHICAGO
Graduate School of Business
Business 41914, Spring Quarter 2007, Mr. Ruey S. Tsay

Homework Assignment 3

Note:

- You may discuss problems with other students, but must hand in your OWN solutions.
- You may use any software to do the empirical analysis even though I use SCA in the demonstration.
- The assignment is due in one week once assigned.

1. Assume that $\{a_t\}$ is a sequence of martingale difference such that $E(a_t|F_{t-1}) = 0$, $E(a_t^2|F_{t-1}) = \sigma_a^2$, and $E(|a_t|^\delta|F_{t-1}) < \infty$ for some $\delta > 2$, where F_{t-1} is the σ-field generated by $\{a_{t-1}, a_{t-2}, \ldots\}$. Further, assume that $z_t = z_{t-1} + a_t$ with z_0 being a finite real number. In addition, let T be the sample size, $w(u)$ be a standard Brownian motion on the unit interval [0,1], and $\rightarrow_d$ denotes convergence in distribution. Derive the following results as $T \rightarrow \infty$.
   - $(\sigma_a T)^{-2} \sum_{t=1}^{T} z_{t-1}^2 \rightarrow_d \int_0^1 w(u)^2 du$.
   - $(\sigma T)^{-1} \sum_{t=1}^{T} z_{t-1} a_t \rightarrow_d \frac{1}{2}[w(1)^2 - 1]$.
   - Let $\hat{\phi} = \sum_{t=1}^{T} z_t z_{t-1} / \sum_{t=1}^{T} z_{t-1}^2$ be the ordinary least square estimate of AR(1) coefficient, and t_ϕ be the t-ratio for testing the null hypothesis $H_o : \phi = 1$. Show that $t_\phi \rightarrow_d \frac{(1/2)[w(1)^2-1]}{[\int_0^1 w(u)^2 du]^{1/2}}$.
2. For simplicity, assume that $z_t = z_{t-1} + a_t - \theta a_{t-1}$, where $\{a_t\}$ is an iid sequence of $N(0, \sigma_a^2)$, $|\theta| < 1$ and $\sigma_a^2 > 0$. Derive the distribution of $T^{-2} \sum_{t=1}^{T} z_{t-1}^2$ as $T \rightarrow \infty$.
3. Solve the same problem as Problem 2, but assuming that z_t follow the ARIMA(1,1,0) model $(1 - B)(1 - \phi B)z_t = a_t$, where $|\phi| < 1$.
4. Consider the U.S. monthly series of 30-year fixed mortgage rate from June 1976 to March 2007. Take the log transformation. Is there a unit root in the series? Test the hypothesis using 5% significance level.
5. Consider the prior mortgage rate series and the U.S. monthly series of treasury interest rate (constant maturity 2 years) of the same period. Again, take the log transformation of both series. Is there a co-integration between them? If yes, what is the co-integrating vector?