Homework Assignment 3

Note:

- You may discuss problems with other students, but must hand in your OWN solutions.
- You may use any software to do the empirical analysis even though I use R in the demonstration, especially the MTS package.
- Due on May 11, 2017 (before class).

1. The data file hw3p1.txt contains 200 observations generated from the VMA(1) model

   \[ z_t = a_t - \begin{bmatrix} 0.3 & 0.2 \\ -0.5 & 1.1 \end{bmatrix} a_{t-1}, \quad \Sigma_a = \begin{bmatrix} 2 & 0.5 \\ 0.5 & 1 \end{bmatrix}, \]

   where \( \{a_t\} \) is a sequence of iid bivariate \( N(0, \Sigma_a) \) random vectors.

   - Show that the data support the specification of a VMA(1) model.
   - Fit a VMA(1) model. Perform model checking and write down the fitted model.
   - Use fitted model to perform 1-step to 2-step ahead forecasts at the forecast origin \( t = 200 \).
   - Fit a VAR model to the data. Perform model checking and write down the fitted model.
   - Compare the VMA(1) and the VAR model you specified. Draw your conclusions.

2. Again, consider the data set in Problem 1. Perform estimation via the conditional maximum likelihood method first. Then, perform estimation via the exact maximum likelihood method. Compare the two fitted models and draw your conclusions.

3. Consider the monthly log returns of CRSP decile portfolios 2, 3, and 8 from January 1961 to December 2016. Multiple the returns by 100, i.e. in percentage returns.

   - Estimate a VMA(1) model to the series via the conditional maximum likelihood method. Refine the model if necessary by removing insignificant estimates with \( t \)-ratio 1.645. Is the model adequate? Write down the fitted model.
   - Use the fitted model to obtain forecasts of the three returns for January and February of 2017.

4. Consider the quarterly U.S. Federal government debt held by (a) foreign and international investors (FDHBFIN.csv) and (b) by the Federal Reserve Banks (FDHBFBRBN.csv), in billions of dollars, from 1970.I to 2016.IV. Let \( z_t \) be the quarterly growth rate series of the debt, i.e., the first difference of the log debt.
• Build a VMA model for the growth rate series, including model checking, and write down the fitted model.
• Use the VMA model to obtain 1-step and 3-step ahead forecasts of the growth rate series at the forecast origin 2016.IV.

5. Again, consider the quarterly growth rates of U.S. Federal debt of Problem 4.
• Build a VAR model for the growth rate series, including model checking, and write down the fitted model.
• Consider the VAR model and the VMA model of Problem 4. Which model is preferred? Why?

**Reading assignment:** Chapter 3 of the textbook.