

**Graduate School of Business
University of Chicago**

Bus 41910, Time Series Analysis for Forecasting & Model Building, Mr. R. Tsay
Homework Assignment #1, Due in one week.

1. Consider the quarterly GDP series mentioned in class. Construct the growth rate series $X_t = \ln(GDP_t) - \ln(GDP_{t-1})$. Compute the first 12 lags of sample autocorrelations of X_t .
2. Compute the solutions to the difference equations:
 - a. $Y_t = 1.2Y_{t-1} - .5Y_{t-2}$, $Y_0 = 5$ and $Y_1 = 6.74$.
 - b. $Y_t - .5Y_{t-1} = 1 + .4X_t$, $Y_0 = 1$ and X_t satisfies $X_t = -.5X_{t-1}$, $X_0 = 2$.

3. Prove that the moment generating function of a stationary linear process $X_t = \psi(B)a_t = \sum_{i=0}^{\infty} \psi_i a_{t-i}$ is given by

$$\Gamma(z) = \sigma^2 \psi(z) \psi(z^{-1}),$$

where $\sigma^2 = \text{var}(a_t)$, $E(a_t) = 0$, and $\psi_0 = 1$.

4. For the second-order difference equation: $Y_t = \phi_1 Y_{t-1} + \phi_2 Y_{t-2}$, determine the stable regime on the (ϕ_1, ϕ_2) -plane. That is, find the regime such that the solutions to $1 - \phi_1 B - \phi_2 B^2 = 0$ outside the unit circle. [You may find this regime in some textbook, e.g. Figure 3.2 of Box and Jenkins (1976).]
5. Simulation is informative in studying time series. Use any package available to you to generate the following series, where the innovations $\{a_t\}$ are independent and identically distributed $N(0, 1)$ random variables.
 - a. a Gaussian white noise series with 300 observations.
 - b. an MA(2) process with 400 observations., say $X_t = 0.4 + (1 - 1.1B + 0.4B^2)a_t$.
 - c. an AR(1) process with 300 observations, say $(1 - 0.8B)X_t = 1.0 + a_t$.
 - d. an ARMA(1,1) process with 600 observations, say $(1 - 0.9B)X_t = 1.0 + (1 - 0.4B)a_t$.

For each simulated series, show the sample mean, standard error, and the first 5 lags of sample autocorrelation function.

To simulate a time series in **SCA**, use the command “utsm” (stands for univariate time series model) to setup the model and the command SIMULATE to generate the data. You may use “help utsm” and “help simu” in SCA for on-line help.

For example, in SCA, use the following commands to specify an ARMA(1,1) model.

```
utsm m1. model (1-p1*b)x=c+(1-t1*b)noise. simulate.
```

```
p1 = 0.9
```

```
t1 = -0.4
```

```
c = 1.0
```

Then, use the following command to generate 600 data points.

```
simu model m1. noise n(0,1). nobs 600.
```

The data is stored in x.

To compute ACF of x, use the command

```
acf x. maxlag 5.
```

If you use **R**, you can use the package “tseries” with some simple “programs”. Below is an illustration, where % denotes explanation.

```
n=501      % I used 501 because there is a starting value for ARMA(1,1) model
at=rnorm(n) % generate 501 normal N(0,1) random variates
x=double(n) % set space for the time series
x[1]=0      % initial value of the first data point
for (i in 2:n) % A do-loop to generate ARMA(1,1) model.
{
x[i]=0.9*x[i-1]+at[i]-0.4*at[i-1]
}
y=ts(x[2:501]) % Skip the initial value and store the results in ‘y’.
```

To compute ACF,

```
acf(y,lag.max=10)
```

To plot the series,

```
ts.plot(y)
```

You can also use the command `{\bf arima.sim}` directly.

```
y=arima.sim(n=500,list(ar=c(0.9),ma=c(-0.4)),sd=1)
```