Graduate School of Business
University of Chicago
Bus 41910, Time Series Analysis, Mr. R. Tsay

Homework Assignment #3, Due in one week

1. Consider the models

$$(1 - B)(1 - B^4)X_t = (1 - 0.5B)(1 - 0.6B^4)a_t,$$

where $\{a_t\}$ is a Gaussian white noise series with mean zero and variance 1. Assume also that $(X_{96}, X_{97}, \cdots, X_{100}) = (118.78,118.73,116.54,118.62,126.22)$ and $(a_{96}, \cdots, a_{100}) = (0.44, 0.56, 1.85, -0.82, -0.47)$. Use the minimum mean squared error criterion to compute forecasts at the forecast origin $t = 100$.

- What is the 1-step ahead forecast $X_{100}(1)$? What is the variance of the associated forecast error?
- What is the 2-step ahead forecast $X_{100}(2)$? What is the variance of the forecast error of the 2
- What is the 3-step ahead forecast $X_{100}(3)$?
- What is the 4-step ahead forecast $X_{100}(4)$?

2. Consider the structural model

$$X_t = T_t + S_t + \epsilon_{0t},$$

where T_t and S_t satisfy

$$(1 - B^2)T_t = \epsilon_{1t}, \quad (1 + B + B^2 + B^3)S_t = \epsilon_{2t},$$

where $\{\epsilon_{it}\}$ $(i = 0, 1, 2)$ are independent Gaussian white noise series with mean zero and variance 1. What is the ARMA model for X_t?

3. Consider the simple exponential smoothing method for forecasting. Suppose that the discount rate is 0.9 and the 1-step ahead forecast at origin $t = 100$ is $X_{100}(1) = 50$. If $X_{101} = 55$, then what is the 1-step ahead forecast at $t = 101$?

4. Identify possible ARIMA models for the data sets given below: (a) dat4a, (b) dat4b, (c) dat4c, (d) dat4d, and dat4e.

5. Consider the time series "bjsera.txt" of Box and Jenkins (1976). There are 197 observations. Suppose that we use the first 150 observations to perform model estimation and the last 47 observations for forecasting evaluation. Two models are used. The first model is an ARMA(1,1) and the second model an AR(7).

- Compute the mean squares of forecast errors of 1-step ahead out-of-sample forecasts for the two models. [You should re-estimate the model at each forecast origin, starting with $t = 150$.]

- Assume the errors are Gaussian. Transform the forecast errors using the Gaussian cumulative distribution function. Are the transformed forecast errors close to uniform random variates for each model? Justify your answer.

- Based on the analysis you have performed, which model is more adequate? Why?