Graduate School of Business
University of Chicago
Bus 41910, Time Series Analysis, Mr. R. Tsay

Solutions to Homework Assignment #3

1. Direct calculation gives the answers below:

- What is the 1-step ahead forecast $X_{100}(1)$? Answer: $X_{100}(1) = X_{100} + X_{97} - X_{96} - 0.5a_{100} - 0.6a_{97} + 0.3a_{96} = 126.201$. The variance of forecast error is $\text{Var}(a_{101}) = 1.0$.
- What is the 2-step ahead forecast $X_{100}(2)$? What is the variance of the associated forecast error?

  Answer: $X_{100}(2) = X_{100}(1) + X_{98} - X_{97} - 0.6a_{98} + 0.3a_{97} = 123.069$. The variance of forecast error $= (1 + \psi_1^2)\text{Var}(a_t) = 1.25$.
- What is the 3-step ahead forecast $X_{100}(3)$?

  Answer: $X_{100}(3) = X_{100}(2) + X_{99} - X_{98} - 0.6a_{99} + 0.3a_{98} = 126.196$.
- What is the 4-step ahead forecast $X_{100}(4)$?

  Answer: $X_{100}(4) = X_{100}(3) + X_{100} - X_{99} - 0.6a_{100} + 0.3a_{99} = 133.832$.

2. Using the aggregation results, we can derive the result below:
Answer: The least common multiple of $(1-B^2)$ and $(1+B+B^2+B^3)$ is $(1-B^4)$. Therefore, applying $(1 - B^4)$ to X_t, we have

$$\begin{aligned}(1 - B^4)X_t &= (1 + B^2)\epsilon_{1t} + (1 - B)\epsilon_{2t} + (1 - B^4)\epsilon_{0t} \\ &= (\epsilon_{0t} + \epsilon_{1t} + \epsilon_{3t}) - \epsilon_{2,t-1} + \epsilon_{1,t-2} - \epsilon_{0,t-4}.\end{aligned}$$

Thus, X_t follows an ARMA(4,4) process with AR polynomial $(1 - B^4)$ and MA polynomial $\theta(B) = 1 - \theta_1 B - \theta_2 B^2 - \theta_4 B^4$. [No lag-3 in MA.]

3. (Apply the updating formula.) $X_{101}(1) = (1 - \delta)X_{101} + \delta X_{100}(1) = 0.1 \times 55 + 0.9 \times 50 = 50.5$.

4. Identify possible ARIMA models for the data sets given below: (a) dat4a, (b) dat4b, (c) dat4c, (d) dat4d, and dat4e.
Answer: For simplicity, I do not show any computer output. (a) Based on PACF and EACF, an AR(2) model is identified. (b) The ACF and PACF identify the model as either an AR(2) or MA(2). (c) ACF indicates unit root nonstationarity. ACF and PACF of the first differenced series show an AR(1) model. Thus, the original series is an ARIMA(1,1,0)

series. The EACF identifies an ARMA(2,1) model for the data. (d) The PACF shows an AR(3) whereas EACF shows an ARMA(1,1) model. (e) The ACF shows an alternating decay pattern at lags 4, 8, 12, ... This is a seasonal time series. The PACF shows significant results at lags 4 and 8. Thus, an AR(8) with polynomial $(1-\phi_4 B^4-\phi_8 B^8)$ or an ARMA(4,4) with model $(1-\phi_4 B^4)x_t = c + (1-\theta_4 B^4)a_t$.

5. I used two programs “r-backtest.txt” and “r-foreden.txt” to answer this problem. These two R programs are posted on the course web.

- The MSFE for AR(7) and ARMA(1,1) are 0.320 and 0.307, respectively.
- Based on qq-plots, the transformed forecast errors are close to, but still have certain discrepancies with, uniform distribution. [The qq-plots are not shown.]
- The AR(7) model is slightly preferred, but the two models are close. Both seem to fit the data well.