Graduate School of Business
University of Chicago
Bus 41910, Time Series Analysis, Mr. R. Tsay

Homework Assignment #6, Due in one week (November 20)

1. Consider the ARMA(2,1) model

$$(1-1.3B+0.4B^2)z_t = (1+0.5B)a_t, \quad a_t \sim N(0,2).$$

Write down three state-space models for the time series z_t using the three approaches by Akaike, Aoki and Harvey discussed in the lecture.

2. The random vectors $\boldsymbol{X}$ and $\boldsymbol{Y}$ are jointly normal as

$$\begin{bmatrix} \boldsymbol{X} \\ \boldsymbol{Y} \end{bmatrix} \sim N\left(\begin{bmatrix} \boldsymbol{\mu}_x \\ \boldsymbol{\mu}_y \end{bmatrix}, \begin{bmatrix} \boldsymbol{\Sigma}_{xx} & \boldsymbol{\Sigma}_{xy} \\ \boldsymbol{\Sigma}_{yx} & \boldsymbol{\Sigma}_{yy} \end{bmatrix} \right),$$

where $\boldsymbol{\Sigma}_{xx}$ and $\boldsymbol{\Sigma}_{yy}$ are non-singular covariance matrices. Derive the conditional distribution of $\boldsymbol{X}$ given $\boldsymbol{Y} = \boldsymbol{y}$.

3. The random vectors $\boldsymbol{X}$, $\boldsymbol{Y}$ and $\boldsymbol{Z}$ are jointly normal as

$$\begin{bmatrix} \boldsymbol{X} \\ \boldsymbol{Y} \\ \boldsymbol{Z} \end{bmatrix} \sim N\left(\begin{bmatrix} \boldsymbol{\mu}_x \\ \boldsymbol{\mu}_y \\ \boldsymbol{\mu}_z \end{bmatrix}, \begin{bmatrix} \boldsymbol{\Sigma}_{xx} & \boldsymbol{\Sigma}_{xy} & \boldsymbol{\Sigma}_{xz} \\ \boldsymbol{\Sigma}_{yx} & \boldsymbol{\Sigma}_{yy} & \boldsymbol{0} \\ \boldsymbol{\Sigma}_{zx} & \boldsymbol{0} & \boldsymbol{\Sigma}_{zz} \end{bmatrix} \right),$$

where $\boldsymbol{\Sigma}_{xx}$, $\boldsymbol{\Sigma}_{yy}$ and $\boldsymbol{\Sigma}_{zz}$ are non-singular covariance matrices. Derive the conditional distribution of $\boldsymbol{X}$ given $\boldsymbol{Y} = \boldsymbol{y}$ and $\boldsymbol{Z} = \boldsymbol{z}$.

4. Consider the State-Space model of Lecture 12. That is, Eq. (1) and (2) of the handout. Suppose that the covariance matrix between e_t and ϵ_t is $\text{Cov}(e_t, \epsilon_t) = \boldsymbol{W}$, which is non-zero. Derive the corresponding Kalman Filter algorithm for the model, i.e. Eq. (6)-(12) of the handout.

5. Consider the linear regression model with time-series errors

$$y_t = \beta x_t + z_t, \quad (1-0.5B-0.24B^2)z_t = a_t,$$

where $\{a_t\}$ is a sequence of independent and identically distributed random variables with mean zero and variance 1. Write down a State-Space form for the model.