Lecture 10: Intervention Analysis

Bus 41910, Time Series Analysis, Mr. R. Tsay

Intervention analysis or event study is used to assess the impact of a special event on the time series of interest. The main focus is to estimate the dynamic effect on the mean level of the series, but other effects can also be considered. The main reference of intervention analysis is Box and Tiao (1975, JASA).

We start with an example. Figure 1 shows the time plot of weekly differences in market shares of Crest and Colgate toothpaste before and after the approval of Crest by ADA (American Dental Association). The approval is seen to have some impact on the market shares. How can we assess the impact? Is the impact transitory or permanent? This is a typical example of *intervention analysis* or *event study*. Here the time of intervension is known. In our particular instance, it is $t_0 = 136$. The goal is to assess the impact (permanent or transitory? magnitude?) of the event on market share.

The difference between intervention analysis and outlier detection is that the time point of intervention is known. There is no need to "identify" the time point, which is a critical issue in outlier detection. Because the time point is known, statistical inference is easier in intervention analysis than in outlier detection.

A simple way to study intervention analysis is to consider some simple dynamic models. To this end, we consider two types of *input* series. They are (a) the pulse function and (b) the step function. A pulse function indicates that the intervention only occurs in the single time index t_0 whereas a step function shows that the intervention continues to exist starting with the time index t_0. Consider the weekly sales of a product. If a discount promotion is in effect only for a given week, then we use a pulse function to signify the existence of the promotion. On the other hand, if a key characteristic of the product is permanent changed, then the intervention becomes a step function. Mathematically, these two input functions are

$$P_t^{(t_0)} = \begin{cases} 0 & \text{if } t \neq t_0 \\ 1 & \text{if } t = t_0 \end{cases}$$

$$S_t^{(t_0)} = \begin{cases} 0 & \text{if } t < t_0 \\ 1 & \text{if } t \geq t_0 \end{cases}$$

With a given input, the effect of the intervention can be summarized as

$$f_t = \frac{\omega(B)}{\delta(B)} I_t^{(t_0)},$$

where $I_t^{(t_0)} = P_t^{(t_0)}$ or $S_t^{(t_0)}$ and $\omega(B) = \omega_0 + \omega_1 B + \cdots + \omega_s B^s$ and $\delta(B) = 1 - \delta_1 B - \cdots - \delta_r B^r$ with r and s are non-negative integers. All zeros of $\delta(B)$ are assumed to be on or outside the unit circle and $\omega(B)$ and $\delta(B)$ have no common factors.

The intervention model can then be written as

$$Y_t = Z_t + \frac{\omega(B)}{\delta(B)} I_t^{(t_0)}, \qquad (1)$$

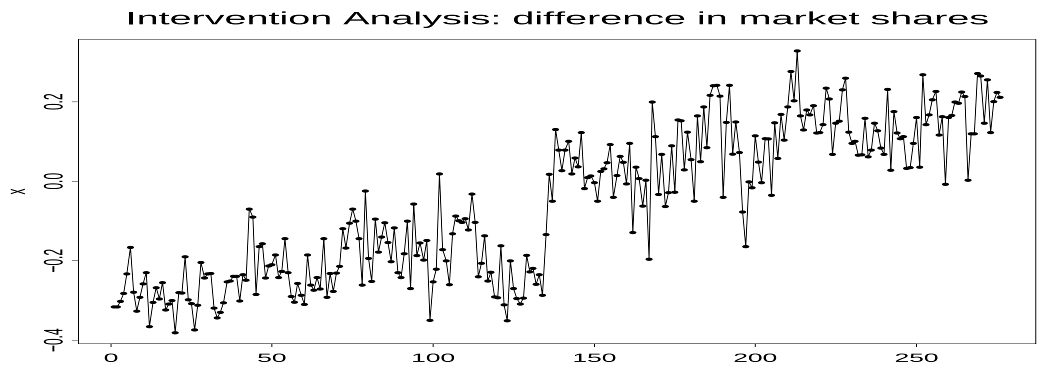

Figure 1: Time plot of the difference in marke shares of two types of toothpaste

where Y_t is the observed series and Z_t is the underlying intervention-free series. Intervention analysis is to specify the model for Z_t and the polynomial $\omega(B)$ and $\delta(B)$ so that the impact of the intervention can be estimated.
To aid the specification of $\omega(B)$ and $\delta(B)$, it is helpful to consider some special cases.

1. $\frac{\omega(B)}{\delta(B)} = \omega_0$:

2. $\frac{\omega(B)}{\delta(B)} = \omega_1 B$:

3. $\frac{\omega(B)}{\delta(B)} = \omega_0 + \omega_1 B$:

4. $\frac{\omega(B)}{\delta(B)} = \frac{\omega_0}{1-B}$:

5. $\frac{\omega(B)}{\delta(B)} = \frac{\omega_0}{1-\delta_1 B}$:

In practice, linear combinations of the foregoing functional forms can be used.

Modeling procedure

A general procedure for intervention analysis is as follows:

- Specify the model for Z_t using $\{Y_1, \ldots, Y_{t_0-1}\}$, i.e., using data before intervention.
- Use the model built for Z_t to predict Z_t for $t \geq t_0$. Let the prediction be $\hat{Z}_t$ for $t \geq t_0$.
- Examine $Y_t - \hat{Z}_t$ for $t \geq t_0$ to specify $\omega(B)$ and $\delta(B)$.
- Peform a joint estimation using all the data.
- Check the entertained model for model inadequency.

In applications, multiple interventions may exist. One can analyze the interventions sequentially before perform a final joint estimation.

Example: Toothpaste example.