

Lecture 11: Unit Root and Unit-Root Test
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1 Introduction

Unit-root problem is concerned with the existence of characteristic roots of a time series model on the unit circle. Recall that a random walk model is

$$z_t = z_{t-1} + a_t,$$

where $\{a_t\}$ is a white noise process. In general, $\{a_t\}$ can be a sequence of martingale differences, that is, $E(a_t|F_{t-1}) = 0$, $\text{Var}(a_t|F_{t-1})$ is finite, and $E(|a_t|^{2+\delta}|F_{t-1}) < \infty$ for some $\delta > 0$, where F_{t-1} is the σ -field generated by $\{a_{t-1}, a_{t-2}, \dots\}$. For simplicity, one often assumes that $Z_0 = 0$. It will be seen later that this assumption has no effect on the limiting distributions of unit-root test statistics. This simple model plays an important role in the unit-root literature.

The assumption that a_t is a martingale difference is the basic setup used in Chan and Wei (1988, Annals of Statistics) for their famous paper on limiting properties of unstable AR processes. However, this assumption can be relaxed without introducing much complexity. In what follows, we adopt the approach of Phillips (1987, Econometrica) with a single unit root and a_t is a stationary series with weak serial dependence.

The case of unit roots with multiplicity greater than 1 or other characteristic roots on the unit circle can be handled via the work of Chan and Wei (1988).

2 Basic theory of time series with a unit root

Consider the discrete-time process $\{z_t\}$ generated by

$$z_t = \pi z_{t-1} + y_t, \quad t = 1, 2, \dots \quad (1)$$

where $\pi = 1$, z_0 is a fixed real number, and y_t is a stationary time series to be defined shortly. It will become clear later that the starting value z_0 has no effect on the limiting distributions discussed in this chapter.

Define the partial sum of $\{y_t\}$ as

$$S_t = \sum_{i=1}^t y_i. \quad (2)$$

For simplicity, we define $S_0 = 0$. Then, $z_t = S_t + z_0$. A fundamental result to unit-root theory is the limiting behavior of S_t . To this end, one must properly standardize the partial sum S_T as $T \rightarrow \infty$. It is common in the literature to employ the average variance of S_T given by

$$\sigma^2 = \lim_{T \rightarrow \infty} E(T^{-1}S_T^2) \quad (3)$$

which is assumed to exist and positive. Define the function

$$X_T(r) = \frac{1}{\sqrt{T}\sigma} S_{[Tr]}, \quad 0 \leq r \leq 1, \quad (4)$$

where $[Tr]$ denotes the integer part of Tr . In particular, $X_T(1) = \frac{1}{\sqrt{T}\sigma} S_T$. Under certain conditions, $X_T(r)$ is shown to converge weakly to the well known standard Brownian motion or the Wiener process. This is commonly referred to as the *functional central limit theorem*.

Basic assumption A: Assume that $\{y_t\}$ is a stationary time series such that (a) $E(y_t) = 0$ for all t , (b) $\sup_t E(|y_t|^\beta) < \infty$ for some $\beta > 2$, (c) the average variance σ^2 of Eq. (3) exists and is positive, and (d) y_t is strong mixing with mixing coefficients α_m that satisfy

$$\sum_{m=1}^{\infty} \alpha_m^{1-2/\beta} < \infty.$$

Strong mixing is a measure of serial dependence of a time series $\{y_t\}$. Let $F_{-\infty}^q$ and F_r^∞ be the σ -field generated by $\{y_q, y_{q-1}, \dots\}$ and $\{y_r, y_{r+1}, \dots\}$, respectively. That is, $F_{-\infty}^q = F\{y_q, y_{q-1}, \dots\}$ and $F_r^\infty = F\{y_r, y_{r+1}, \dots\}$. We say that y_t satisfies a strong mixing condition if there exists a positive function α_m satisfying $\alpha_m \rightarrow 0$ as $m \rightarrow \infty$ so that

$$|P(A \cap B) - P(A)P(B)| < \alpha_{r-q}, \quad A \in F_{-\infty}^q, \quad B \in F_r^\infty.$$

If y_t is strong mixing, then the serial dependence between y_t and y_{t-h} approaches zero as h increases.

The following two theorems are widely used in unit root study. See Herrndorf (1983) and Billingsley (1968) for more details.

Functional Central Limit Theorem (FCLT): If $\{y_t\}$ satisfies the basic assumption A, then $X_T(r) \Rightarrow W(r)$, where $W(r)$ is a standard Brownian motion for $r \in [0, 1]$ and \Rightarrow denotes weak convergence, i.e., convergence in distribution.

Continuous Mapping Theorem: If $X_T(r) \Rightarrow W(r)$ and $h(\cdot)$ is a continuous functional on $D[0,1]$, the space of all real valued functions on $[0,1]$ that are right continuous at each point on $[0,1]$ and have finite left limits, then $h(X_T(r)) \Rightarrow h(W(r))$ as $T \rightarrow \infty$.

The ordinary least squares estimate of π in Eq.(1) is

$$\hat{\pi} = \frac{\sum_{t=1}^T z_{t-1} z_t}{\sum_{t=1}^T z_{t-1}^2},$$

and its variance is estimated by

$$\text{var}(\hat{\pi}) = \frac{s^2}{\sum_{t=1}^T z_{t-1}^2},$$

where s^2 is the residual variance given by

$$s^2 = \frac{1}{T-1} \sum_{t=1}^T (z_t - \hat{\pi} z_{t-1})^2. \quad (5)$$

The usual t -ratio for testing the null hypothesis $H_o : \pi = 1$ versus $H_a : \pi < 1$ is given by

$$t_\pi = \left(\sum_{t=1}^T z_{t-1}^2 \right)^{1/2} \frac{\hat{\pi} - 1}{s} = \frac{\sum_{t=1}^T z_{t-1} y_t}{s \sqrt{\sum_{t=1}^T z_{t-1}^2}}. \quad (6)$$

We have the following basic results of a unit-root process z_t .

Theorem: Suppose that $\{y_t\}$ satisfies the basic Assmption A and $\sup_t E|y_t|^{\beta+\eta} < \infty$, where $\beta > 2$ and $\eta > 0$, and $\pi = 1$, then as $T \rightarrow \infty$, we have

- (a) $T^{-2} \sum_{t=1}^T z_{t-1}^2 \Rightarrow \sigma^2 \int_0^1 W(r)^2 dr$,
- (b) $T^{-1} \sum_{t=1}^T z_{t-1} y_t \Rightarrow \frac{\sigma^2}{2} (W(1)^2 - \frac{\sigma_y^2}{\sigma^2})$,
- (c) $T(\hat{\pi} - 1) \Rightarrow \frac{(1/2)(W(1)^2 - (\sigma_y^2/\sigma^2))}{\int_0^1 W(r)^2 dr}$,
- (d) $\hat{\pi} \rightarrow_p 1$, where \rightarrow_p denotes convergence in probability,
- (e) $t_\pi \Rightarrow \frac{(\sigma/(2\sigma_y))[W(1)^2 - (\sigma_y^2/\sigma^2)]}{[\int_0^1 W(r)^2 dr]^{1/2}}$,

where σ^2 and σ_y^2 are defined as

$$\sigma^2 = \lim_{T \rightarrow \infty} E(T^{-1} S_T^2), \quad \sigma_y^2 = \lim_{T \rightarrow \infty} T^{-1} \sum_{t=1}^T E(y_t^2).$$

Proof. For part (a), we have

$$\begin{aligned} T^{-2} \sum_{t=1}^T z_{t-1}^2 &= T^{-2} \sum_{t=1}^T (S_{t-1} + z_0)^2 \\ &= T^{-2} \sum_{t=1}^T (S_{t-1}^2 + 2z_0 S_{t-1} + z_0^2) \\ &= \sigma^2 \sum_{t=1}^T \left(\frac{1}{\sigma\sqrt{T}} S_{t-1} \right)^2 \frac{1}{T} + 2z_0 \sigma T^{-1/2} \sum_{t=1}^T \left(\frac{1}{\sigma\sqrt{T}} S_{t-1} \right) \frac{1}{T} + T^{-1} z_0^2 \\ &= \sigma^2 \sum_{t=1}^T \int_{(t-1)/T}^{t/T} \left(\frac{1}{\sigma\sqrt{T}} S_{[Tr]} \right)^2 dr + 2z_0 \sigma T^{-1/2} \sum_{t=1}^T \int_{(t-1)/T}^{t/T} \frac{1}{\sigma\sqrt{T}} S_{[Tr]} dr + T^{-1} z_0^2 \\ &= \sigma^2 \int_0^1 X_T^2(r) dr + 2z_0 \sigma T^{-1/2} \int_0^1 X_T(r) dr + T^{-1} z_0^2 \\ &\Rightarrow \sigma^2 \int_0^1 W(r)^2 dr, \quad T \rightarrow \infty. \end{aligned}$$

For part (b), we have

$$\begin{aligned}
T^{-1} \sum_{t=1}^T z_{t-1} y_t &= T^{-1} \sum_{t=1}^T (S_{t-1} + z_0) y_t \\
&= T^{-1} \sum_{t=1}^T S_{t-1} y_t + z_0 \bar{y} \\
&= T^{-1} \sum_{t=1}^T \frac{1}{2} (S_t^2 - S_{t-1}^2 - y_t^2) + z_0 \bar{y} \\
&= (2T)^{-1} S_T^2 - (2T)^{-1} \sum_{t=1}^T y_t^2 + z_0 \bar{y} \\
&= \frac{\sigma^2}{2} X_T(1)^2 - \frac{1}{2} T^{-1} \sum_{t=1}^T y_t^2 + z_0 \bar{y} \\
&\Rightarrow \frac{\sigma^2}{2} \left(W(1)^2 - \frac{\sigma_y^2}{\sigma^2} \right),
\end{aligned}$$

because $\bar{y} \rightarrow 0$ and $T^{-1} \sum_{t=1}^T y_t^2 \rightarrow \sigma_y^2$ almost surely as $T \rightarrow \infty$.

Part (c) follows parts (a) and (b) and the continuous mapping theorem. Part (d) follows part (c). In particular, $\hat{\phi}$ converges to 1 at the rate of T^{-1} , not the usual rate $T^{-1/2}$. This is referred to as the super consistency in the unit-root literature.

Using the fast convergence rate of $\hat{\pi}$, parts (a) and (b), and Eq. (5), we have

$$\begin{aligned}
s^2 &= \frac{1}{T-1} \sum_{t=1}^T (z_t - \hat{\pi} z_{t-1})^2 \\
&= \frac{1}{T-1} \sum_{t=1}^T [(z_t - z_{t-1}) + (1 - \hat{\pi}) z_{t-1}]^2 \\
&= \frac{1}{T-1} \sum_{t=1}^T y_t^2 + 2(1 - \hat{\pi})(T-1)^{-1} \sum_{t=1}^T z_{t-1} y_t + (T-1)^{-1} (1 - \hat{\pi})^2 \sum_{t=1}^T z_{t-1}^2 \\
&\rightarrow_p \sigma_y^2,
\end{aligned}$$

because the last two terms vanish as $T \rightarrow \infty$. Thus, the t -ratio can be written as

$$\begin{aligned}
t_\pi &= \frac{\sum_{t=1}^T z_{t-1} y_t}{\sigma_y (\sum_{t=1}^T z_{t-1}^2)^{0.5}}, \\
&= \frac{\sigma^{-2} T^{-1} \sum_{t=1}^T z_{t-1} a_t}{\sigma_y [(\sigma^{-2} T^{-1})^2 \sum_{t=1}^T z_{t-1}^2]^{1/2}}.
\end{aligned}$$

Using the results of parts (a) and (b), and continuous mapping theorem, we have

$$t_\pi \rightarrow_d \frac{\frac{\sigma}{2\sigma_y} [W(1)^2 - \frac{\sigma_y^2}{\sigma^2}]}{[\int_0^1 W(r)^2 dr]^{1/2}}.$$

In what follows, we consider the unit-root theory for some special cases. Since y_t is stationary with zero mean, we have

$$E(S_T^2) = T\gamma_0 + 2 \sum_{i=1}^{T-1} (T-i)\gamma_i \quad (7)$$

where γ_i is the lag- i autocovariance of y_t . As $T \rightarrow \infty$, we have

$$\lim_{T \rightarrow \infty} E(T^{-1}S_T^2) = \gamma_0(1 + 2\rho_1 + 2\rho_2 + 2\rho_3 + \dots) = \gamma_0 \left[2 \left(\sum_{i=0}^{\infty} \rho_i \right) - 1 \right].$$

2.1 AR(1) case

Consider the simple AR(1) model $z_t = \pi z_{t-1} + a_t$, i.e., $y_t = a_t$ being an *iid* sequence of random variables with mean zero and variance $\sigma_a^2 > 0$. In this case, it is easy to see that, from Eq. (7),

$$\sigma^2 = \lim_{T \rightarrow \infty} E(T^{-1}S_T^2) = \sigma_a^2, \quad \sigma_y^2 = \sigma_a^2.$$

Consequently, for the random-walk model $z_t = z_{t-1} + a_t$, we have

1. $T^{-2} \sum_{t=1}^T z_{t-1}^2 \Rightarrow \sigma_a^2 \int_0^1 W^2(r) dr$,
2. $T^{-1} \sum_{t=1}^T z_{t-1}(z_t - z_{t-1}) \Rightarrow \frac{\sigma_a^2}{2} [W^2(1) - 1]$,
3. $T(\hat{\pi} - 1) \Rightarrow \frac{0.5[W^2(1)-1]}{\int_0^1 W^2(r) dr}$,
4. $t_\pi \rightarrow_d \frac{0.5[W^2(1)-1]}{[\int_0^1 W^2(r) dr]^{1/2}}$.

The critical values of t_π has been tabulated by several authors. See, for instance, Fuller (1976, Table 8.5.2). Some critical values are given in the next section.

2.2 AR(p) case

We start with the AR(2) case in which $(1 - B)(1 - \phi B)z_t = a_t$, where $|\phi| < 1$. The model can be written as

$$z_t = z_{t-1} + y_t, \quad y_t = \phi y_{t-1} + a_t.$$

For the stationary AR(1) process y_t , $\sigma_y^2 = \sigma_a^2/(1 - \phi^2)$ and $\gamma_i = \phi^i \gamma_0$. Thus, by Eq. (7), $\sigma^2 = \sigma_y^2(1 + \phi)/(1 - \phi) = \sigma_a^2/(1 - \phi)^2$. Consequently, the limiting distributions discussed depend on the AR(1) coefficient ϕ . For instance, the t -ratio of $\hat{\pi}$ becomes

$$t_\pi \Rightarrow \frac{\frac{1}{2} \sqrt{\frac{1+\phi}{1-\phi}} [W(1)^2 - \frac{1-\phi}{1+\phi}]}{[\int_0^1 W(r)^2 dr]^{1/2}}.$$

Such a dependence makes it difficult to use t_π in unit-root testing, because the asymptotic critical values depend on the nuisance parameter ϕ . This dependence continues to hold for the general AR(p) process y_t . We shall discuss test statistics that can overcome this difficulty in the next section. It is the *augmented Dickey-Fuller* test statistic of Said and Dickey (1984).

2.3 MA(1) case

Next, assume that $z_t = z_{t-1} + y_t$ and $y_t = a_t - \theta a_{t-1}$ with $|\theta| < 1$. In this case, we have $\gamma_0 = (1 + \theta^2)\sigma_a^2$, $\gamma_1 = -\theta\sigma_a^2$, and $\gamma_i = 0$ for $i > 1$. Consequently, $\sigma_y^2 = (1 + \theta^2)\sigma_a^2$ and, by Eq. (7), $\sigma^2 = (1 - \theta)^2\sigma_a^2$. The limiting distributions of unit-root statistics become

1. $T^{-2} \sum_{t=1}^T z_{t-1}^2 \Rightarrow (1 - \theta)^2 \sigma_a^2 \int_0^1 W^2(r) dr,$
2. $T^{-1} \sum_{t=1}^T z_{t-1} (z_t - z_{t-1}) \Rightarrow \frac{(1-\theta)^2 \sigma_a^2}{2} [W^2(1) - \frac{1+\theta^2}{(1-\theta)^2}],$
3. $T(\hat{\pi} - 1) \Rightarrow \frac{\frac{1}{2}[W^2(1) - \frac{1+\theta^2}{(1-\theta)^2}]}{\int_0^1 W^2(r) dr},$
4. $t_\pi \rightarrow_d \frac{\frac{1-\theta}{2\sqrt{1+\theta^2}} [W^2(1) - \frac{1+\theta^2}{(1-\theta)^2}]}{[\int_0^1 W^2(r) dr]^{1/2}}.$

From the results, it is clear that when θ is close to 1 the asymptotic behavior of t_π is rather different from that of the case when y_t is a white noise series. This is not surprising because when θ approaches 1, the z_t process is close to being a white noise series. This might explain the severe size distortions of Phillips-Perron unit-root test statistics seen in Table 1 of Phillips and Perron (1988).

3 Unit-root testing

The results of the previous section can be used to test for a unit root, especially the t -ratio statistics. Assume that the true model is $z_t = z_{t-1} + y_t$ and, for simplicity, we start with the model

$$z_t = \pi z_{t-1} + e_t. \tag{8}$$

The null hypothesis is $H_o : \pi = 1$ and the alternative is $H_a : \pi < 1$. From the basic theorem, the limiting distribution of the t -ratio of the least squares estimate $\hat{\pi}$ depends on some parameters of y_t series. These parameters of y_t become nuisance parameters and we shall consider ways to overcome them. To this end, we shall focus on the case that y_t is driven by some martingale-difference sequence.

Later we shall consider the model

$$z_t = \alpha_0 + \pi z_{t-1} + e_t. \tag{9}$$

Table 1: Selected Critical Values of Unit-Root Test Statistics

sample size	Probability			
T	0.01	0.025	0.05	0.10
	Model without constant			
100	-2.60	-2.24	-1.95	-1.61
250	-2.58	-2.23	-1.95	-1.62
500	-2.58	-2.23	-1.95	-1.62
∞	-2.58	-2.23	-1.95	-1.62
	Model with constant			
100	-3.51	-3.17	-2.89	-2.58
250	-3.46	-3.14	-2.88	-2.57
500	-3.44	-3.13	-2.87	-2.57
∞	-3.43	-3.12	-2.86	-2.57
	Model with time trend			
100	-4.04	-3.73	-3.45	-3.15
250	-3.99	-3.69	-3.43	-3.13
500	-3.98	-3.68	-3.42	-3.13
∞	-3.96	-3.66	-3.41	-3.12

It turns out that including a constant term in the fitted model results in a different limiting distribution and, hence, different critical values. The limiting distribution of the t -ratio for testing $H_o : \pi = 1$ can be obtained by the same techniques shown in the previous section. The limiting distribution is different from those of the basic theorem because the inclusion of the constant term alters the design matrix. Finally, we also consider the model

$$z_t = \alpha_0 + \alpha_1 t + \pi z_{t-1} + e_t, \tag{10}$$

for which we have yet another set of critical values.

3.1 Critical values

When $y_t = a_t$, the martingale difference sequence, there is no insurance parameter involved and the critical values of the t -ratio of the least squares estimate $\hat{\pi}$ for testing a unit root can be tabulated. These values are obtained by simulation. More critical values are given in Table 8.5.2 of Fuller (1976).

3.2 Augmented Dickey-Fuller test

Consider in this subsection a high order AR(p) process, $\phi(B)z_t = a_t$. We focus on the case that $\phi(B) = \phi^*(B)(1 - B)$ where $p^*(B)$ is a stationary AR polynomial. Let $\phi^*(B) =$

$1 - \sum_{i=1}^{p-1} \phi_i^* B^i$. The model becomes $\phi(B)z_t = \phi^*(B)(1-B)z_t = (1-B)z_t - \sum_{i=1}^{p-1} \phi_i^*(1-B)z_{t-i} = a_t$. Testing for a unit root in $\phi(B)$ is equivalent to testing $\pi = 1$ in the model

$$z_t = \pi z_{t-1} + \sum_{j=1}^{p-1} \phi_j^* (z_{t-j} - z_{t-j-1}) + a_t.$$

Or equivalently, the same as testing for $\rho - 1 = 0$ in the model

$$\Delta z_t = (\pi - 1)z_{t-1} + \sum_{j=1}^{p-1} \phi_j^* \Delta z_{t-j} + a_t,$$

where $\Delta z_t = z_t - z_{t-1}$. The above model is the univariate version of error-correction form. It is easy to verify that (a) $\pi - 1 = -\phi(1) = \sum_{i=1}^p \phi_i - 1$ and $\phi_j^* = -\sum_{i=j+1}^p \phi_i$. In practice, the linear model

$$\Delta z_t = \beta z_{t-1} + \sum_{j=1}^{p-1} \phi_j^* \Delta z_{t-j} + a_t, \quad (11)$$

where $\beta = \pi - 1$, is used. The least squares estimate of β can then be used in unit-root testing. Specifically, testing $H_o : \pi = 1$ versus $H_a : \pi < 1$ is equivalent to testing $H_o : \beta = 0$ versus $H_a : \beta < 0$. It can be shown that the t -ratio of $\hat{\beta}$ (against 0) has the same limiting distribution as t_π in the random-walk case. In other words, for an AR(p) model with $p > 1$, by including the lagged variables of Δz_t in the linear regression of Eq. (11), one can remove the nuisance parameters in unit-root testing. This is the well-known augmented Dickey-Fuller unit-root test. Furthermore, the limiting distribution of the LS estimates $\hat{\phi}_i^*$ in Eq. (11) is the same as that of fitting an AR($p - 1$) model to Δz_t . In other words, limiting properties of the estimates for the stationary part remain unchanged when we treat the unit-root as known a priori.

4 Example

We consider some examples of applying unit-root test.

4.1 Program note

In R, you may use the package **FinTS**, which automatically loads the package `fUnitRoots`, to perform unit-root test. The command is

```
adfTest(x,lags,type=c("nc","c","ct"))
```

where "x" is the time series, "lags" denotes the number of lags used, and "type" corresponds to the three fitted models discussed before.

You can also obtain critical values of Table 8.5.2 of Fuller (1976) from the program. The command is

```
adfTable(trend=c("nc","c","ct"),statistic=c("t","n")).
```

For instance, `adfTable(trend=c("c"),statistic=c("t"))` will provide the table of critical values for unit-root test with a constant in the model.

4.2 Demonstration

Consider the VIX of CBOE from 2004 to 2008. The data are obtained from the CBOE web site. From the plot, there appears to have a unit root. The unit-root test confirms it.

```
> da=read.table("vix08.txt",header=T)
> dim(da)
[1] 1080    7
> da[1,]
  mm day year VIXOpen VIXHigh VIXLow VIXClose
1  1  2 2004   17.96   18.68  17.54   18.22

> y=da[,7]

> m1=ar(y)    <== Find the AR order of the original series
> m1$order
[1] 11
```

```
> library(FinTS)
```

```
> m2=adfTest(y,lags=10,type=c("c"))
> m2
```

Title:

Augmented Dickey-Fuller Test

Test Results:

PARAMETER:

Lag Order: 10

STATISTIC:

Dickey-Fuller: -2.2726

P VALUE:

0.2112

```
> m2=adfTest(y,lags=10,type=c("ct"))
> m2
```

Title:

Augmented Dickey-Fuller Test

Test Results:

PARAMETER:

```

    Lag Order: 10
  STATISTIC:
    Dickey-Fuller: -2.7422
  P VALUE:
    0.2642

> m2=adfTest(y,lags=10)    <== Default: no constant term.
> m2

```

```

Title:
  Augmented Dickey-Fuller Test

```

```

Test Results:
  PARAMETER:
    Lag Order: 10
  STATISTIC:
    Dickey-Fuller: -0.3701
  P VALUE:
    0.4984

```

```

> adfTable(trend=c("c"),statistic=c("t"))

```

```
$x
```

```
[1] 25 50 100 250 500 Inf
```

```
$y
```

```
[1] 0.010 0.025 0.050 0.100 0.900 0.950 0.975 0.990
```

```
$z
```

```

    0.010 0.025 0.050 0.100 0.900 0.950 0.975 0.990
  25 -3.75 -3.33 -3.00 -2.63 -0.37 0.00 0.34 0.72
  50 -3.58 -3.22 -2.93 -2.60 -0.40 -0.03 0.29 0.66
 100 -3.51 -3.17 -2.89 -2.58 -0.42 -0.05 0.26 0.63
 250 -3.46 -3.14 -2.88 -2.57 -0.42 -0.06 0.24 0.62
 500 -3.44 -3.13 -2.87 -2.57 -0.43 -0.07 0.24 0.61
 Inf -3.43 -3.12 -2.86 -2.57 -0.44 -0.07 0.23 0.60

```

```
attr("class")
```

```
[1] "gridData"
```

```
attr("control")
```

```

  table      trend statistic
  "adf"      "c"      "t"

```

```
>
```