The Unemployment Volatility Puzzle and the Role of Wage Determination

Outline

I. Overview of Shimer (2005 AER)
II. Detailed Look at Hall (2005 AER)
III. Detailed Look at Hall and Milgrom (AER 2008)
IV. Why Study Wage Stickiness?
I. The Cyclical Behavior of Equilibrium Unemployment and Vacancies

Robert Shimer

American Economic Review,
March 2005
1. The Environment

• Builds on basic MP search and matching model (Pissarides 1985 and 2000, chapter 1)
• Recall: The only nontrivial decision in the model is how many vacancies employers create.
• Extend the model to encompass stochastic shocks to:
  – Labor productivity (common to all matches)
  – Rate at which workers separate from employers, equivalent in the model to rate at which jobs blow up
• Consider shocks to bargaining power as well
• Use Nash wage bargaining for the main analysis, but consider some implications of sticky wages
2. Main Point of the Paper

- **Unemployment Volatility Puzzle:** Given productivity shocks of plausible magnitude, the standard MP model cannot explain the observed cyclical behavior of unemployment, vacancies and the job-finding rate of unemployed workers.
  - Fluctuations in the model are *much* smaller than in the data
  - Related, the standard MP model has a weak internal propagation mechanism
Why So Little Amplification and Propagation?

Short answer: Under Nash wage bargaining, the equilibrium wage largely absorbs shocks to labor productivity.

As a result, productivity shocks have minor effects on the incentive to create vacancies → small equilibrium response of vacancies, job-finding rate and unemployment
A Bit More Detail

• Positive shock to productivity increases the incentive to create vacancies → more vacancies
• Higher v-u ratio raises job-finding rate and pulls unemployment down and economy moves southeast along the Beveridge Curve
• But more hires → shorter unemployment spells, raising worker threat point in wage bargains, and raising the PDV of wages in new matches
• Higher wages (i.e., higher PDV of wages in new matches) absorb most of the productivity gain under Nash bargaining, largely neutralizing the impact on vacancy creation incentives
• → small equilibrium responses of vacancies, v-u ratio, job-finding rate, unemployment rate
3. Additional Points

- Separation shocks cause unemployment and vacancies to covary positively in the model, at odds with the predominant pattern in the data.
  - For given productivity, a higher separation rate raises the inflow to unemployment and shortens employment durations.
  - Unemployment rises, but vacancies rise nearly in proportion, with only a modest decline in the v-u ratio.
• Assuming no aggregate shocks (to productivity or the separations rate), Shimer derives an expression for the elasticity of the v-u ratio w.r.t. the separation rate, s, in a steady-state equilibrium:

\[-s\]
\[\frac{(r + s)(1 - \eta(\theta_{p,s})) + \beta f(\theta_{p,s})}{(r + s)(1 - \eta(\theta_{p,s})) + \beta f(\theta_{p,s})}\]

where \(\eta(.)\) in [0,1) is the elasticity of the job-finding rate, \(f(.)\) w.r.t. to market tightness, and \(\beta\) is worker bargaining power (i.e., worker share of surplus)

• For his preferred calibration, this elasticity is about -0.1., implying that vacancies rise nearly as much as unemployment in response to a positive s shock.
Figure 4. Quarterly U.S. Beveridge Curve, 1951–2003

Notes: Unemployment is constructed by the BLS from the CPS. The help-wanted advertising index is constructed by the Conference Board. Both are quarterly averages of seasonally adjusted monthly series and are expressed as deviations from an HP filter with smoothing parameter $10^5$. 
• But separation/job destruction shocks are an important aspect of cyclical unemployment fluctuations. (Or shocks to deeper fundamentals cause recessionary spikes in separations and job destruction.)

• See the next slide for a time-series plot of several job loss measures in the U.S. from 1990 to 2011.

• We also saw evidence on the importance of cyclical movements in separations and unemployment inflows in the previous lecture, repeated next.
Job Loss Indicators, Quarterly, % of Employment

Job Destruction (BED)
Unemployment Inflows (CPS)
Layoffs (JOLTS)

Initial Claims for UI Benefits (Right Axis)

Reproduced from Figure 1 in Davis and von Wachter (2011)
Unemployment Flows with 2 States

• Steady-state approximation:

Unemp. Rate \equiv u_t \approx \frac{s_t}{s_t + f_t} \implies

d \log u_t \approx (1 - u_t)\left[d \log s_t - d \log f_t\right]

where \(s = E \rightarrow U\) hazard, and \(f = U \rightarrow E\) hazard

• Following Elsby, Michaels & Solon (AEJ-Macro, 2009), plot \(s\) and \(f\) contributions to log change in unemployment rate in each postwar recession.
Decomposition of log unemployment rate rises in postwar U.S. recessions

Courtesy of Mike Elsby.

With corrections for time aggregation.
Decomposition of log unemployment rate
rises in postwar U.S. recessions

Updated chart from Elsby et al. (2009)
Additional Points, Continued

• Separation shocks of plausible magnitude also generate small equilibrium response of unemployment under Nash wage bargaining, and for a similar reason

• Small fluctuations in bargaining power (i.e., the sharing parameter in a generalized Nash bargain) generate realistic movements in the v-u ratio, while inducing only a moderately countercyclical real wage.
  – The intuition for this result will become clear when we discuss Hall (2005 AER)
II. Employment Fluctuations with Equilibrium Wage Stickiness

Robert E. Hall

American Economic Review,
March 2005
1. Overview of the Environment

• Very similar to Shimer (AER, 2005):
  – Builds on basic MP search and matching model (Pissarides 1985 and 2000, chapter 1)
  – The only nontrivial private decision is how many vacancies employers create.
    – Hall limits attention to stochastic labor productivity shocks (common to all matches) – no shocks to separation/destruction rate or bargaining power

• Hall considers several wage-setting rules or regimes and explores their implications for shock amplification.
2. Main Point of the Paper

Wage Stickiness Can Resolve the Unemployment Volatility Puzzle in a Manner Consistent with Individual Rationality

– **Amplification**: Introducing wage stickiness into the MP framework greatly increases the sensitivity of unemployment and vacancies to exogenous driving forces

– **Individual Rationality**: Wage stickiness is, within bounds, fully consistent with individual rationality.
  
  • IR boils down to joint wealth-maximizing behavior by the parties to a match in this model.
  • Hall’s analysis overcomes a well-known critique of wage stickiness in earlier work. See Barro (1977) for an influential statement of the critique.
3. The Model

- Vacancies ($v$) and unemployment ($u$)

  \[(1) \quad \phi(x) = \omega x^\alpha \quad \text{Cobb-Douglas CRS Matching Technology}\]

- $x = v / u \quad \text{Tightness ratio}$

- $\phi(x) = \quad \text{Job-finding probability per period}$

- $\rho(x) = \phi(x) / x \quad \text{Vacancy filling probability per period}$

- $\delta = \text{Per-period probability that worker separates (constant)}$

- $\lambda = \text{Unemployment compensation + income value of leisure}$

- $z_s = \text{Productivity of labor in random state } s$

- $\pi_{s,s'} = \text{probability of transition from } s \text{ to } s'$

- Standard free-entry condition for vacancies

- $k = \text{Vacancy posting cost per period}$

- $\beta = \text{Discount factor (risk-neutral agents)}$
Asset Value Conditions

(2) \[ U_s = \lambda + \beta \sum_{s'} \pi_{s,s'} \left[ \phi(x_s)(w_{s'} + V_{s'}) + (1 - \phi(x_s))U_{s'} \right]. \]

Value of Unemployment

(3) \[ V_s = \beta \sum_{s'} \pi_{s,s'} \left[ (1 - \delta)(w_{s'} + V_{s'}) + \delta U_{s'} \right]. \]

Value of Employment, Excluding Current Wage

(4) \[ J_s = z_s + \beta(1 - \delta) \sum_{s'} \pi_{s,s'} (J_{s'} - w_{s'}). \]

Value of Filled Job After Current Wage Payment

(5) \[ 0 = -k + \beta \rho(x_s) \sum_{s'} \pi_{s,s'} (J_{s'} - w_{s'}). \]

Value of Unfilled Job (Vacancy)
Wages and the Job Creation Incentive

• From (5): As firms create more vacancies, tightness ratio rises and fill-rate $\rho(x_s)$ falls until expected payoff equals recruiting cost. This pins down the state-contingent tightness ratio, $x_s$.

• Conditional on the state-contingent wage, (4) determines value to employer of a filled job.

• Equation (5) then determines recruiting effort (vacancies) and tightness ratio for each state.

• (2) and (3) do not directly affect the job creation incentive, but they are needed to verify that the wage lies in the bargaining set.
The Bargaining Set

- Assumption: new and continuing matches involve the same wage-bargaining problem.
- The worker’s reservation wage equates asset value of employment and unemployment:
  \[ w_s = U_s - V_s \]  
- Employer’s reservation wage is value of filled job:
  \[ \bar{w}_s = J_s \]  
- Taken together, these reservation values determine the bargaining set:
  \[ B_s = [w_s, \bar{w}_s] \]
The Bargaining Set, Continued

• Any wage in the bargaining set yields privately efficient match formation and retention.

• That is, for any wage in the bargaining set, both worker and employer receive a match value at least as large as the non-match values represented by the reservation wages.

• The frictions in the search and matching process are what expand the bargaining set from a point to an interval.
**Equilibrium Wages, 1**

- **Assumption**: Wages are determined according to a demand game or auction as follows:
  - Worker & employer know the reservation wages
  - Worker & employer simultaneously propose wages $w_L$ and $w_H$, respectively, without knowing the other's proposal.
  - If $w_l \leq w_H$, the match forms or continues, and the wage is agreed to be $w = \kappa w_L + (1 - \kappa)w_H$, for $0 < \kappa < 1$.

- For this auction, any wage in the bargaining set is a Nash equilibrium
In this context, the generalized Nash bargain is one equilibrium selection mechanism.

There are others. Hall considers selection rules that assign wages in the bargaining set as a function of the state of the economy, $s$.

To be an equilibrium, such a wage rule must satisfy (2)-(5) with $w_s \in B_s$ for all $s$. 

Equilibrium Wages, 2
Equilibrium Wages, 3

- Consider the class of constant wages that satisfy these requirements. Define $\tilde{J}_s$ as the solution to the linear system,

$$\tilde{J}_s = z_s + \beta(1 - \delta) \sum_{s'} \pi_{s,s'} \tilde{J}_s,$$

which is the value an employer would attach to a new worker who never receives any wage. That is, it is the expected present value of revenue generated by a worker hired when the economy is in state $s$. 

PROPOSITION: A constant wage \( w \) is an equilibrium of the model if

\[
\lambda \leq w \leq \min_s \left[ 1 - \beta(1 - \delta) \right] \tilde{J}_s
\]

that is, the wage lies between the flow value of being unemployed, \( \lambda \), and the annuity value, \( [1 - \beta(1 - \delta)]\tilde{J}_s \), of the lowest-expected-profit state.
Two Remarks on the Proposition:

• See Lorenzoni’s note on Hall’s website for a correction to Hall’s proof of the proposition.

• The proposition does not preclude an equilibrium with no employment (asymptotically).
  
  – To see this point, consider the case with constant $z$.
  
  – Solve (10) for $\tilde{J}_s$ and plug into (11) to deduce that the upper value in the constant wage set is $w=z$.
  
  – But, if $w=z$, an employer can never recover the upfront cost of posting a vacancy and filling a job. So no job creation occurs.
Interpreting a Constant-Wage Rule

• A constant-wage rule can be interpreted as a focal point or wage norm.

• Akerlof et al. (1996), Bewley (1999) and others provide evidence that social forces constrain wage-setting behavior. Many researchers stress resistance to wage cuts, but Hall points out that similar ideas point to the absence of immediate upward wage adjustments as well.

• The new feature of Hall’s analysis is that wage norms are constrained to lie in the bargaining set
Constant-Wage Rule, 2

• Hall, page 56: “The wage norm I consider interferes neither with the formation of efficient matches once the parties are in touch with one another nor with the preservation of jobs with positive surplus. Inefficient separations cannot occur.”

• But there’s no useful allocative role for wages on the separation margin in this model, or on any margin other than the job creation/vacancy posting margin. So there’s little scope for a sticky wage to produce privately inefficient outcomes on other margins. See Hall (2009) for more on this issue.
The key feature of Hall’s analysis is the effect of wage stickiness on pre-match recruiting efforts.

Because the variations in unemployment and vacancies respond to expectations formed when jobs/vacancies are created, the essential stickiness in the model is in those expectations.

If only post-employment wages were sticky, and wages paid in the first period of employment fluctuated to offset anticipated later wages, the model would deliver much smaller fluctuations in labor-market conditions.
Wage Rules in a Non-Stationary Environment

• In a non-stationary environment with productivity growth over time, it becomes important to extend the idea of a wage norm.

• For example, suppose productivity evolves as

\[
(19) \quad z_t = z_t^P z_{s,t}^M.
\]

The first variable on the right is a slow-moving trend known to the public; the second is a mean-reverting process that evolves similarly to the single component considered above.
Wage Rules in a Non-Stationary Environment, 2

• The analog to the constant-wage rule above is:

\[(20) \quad w_t = wz_t^P.\]

• See Hall for a straightforward reformulation of the previous analysis to this case.

• However, some potential new issues arise:
  
  – Agents may differ in their views about the trend
  – The trend may involve nominal elements, partly reflecting drift in price level.
  
  – Wage determination adapts to persistent inflation and productivity growth, as in Friedman (1968) and Phelps (1967).
Wage Rules in a Non-Stationary Environment, 3

• Hall: “The huge literature on wage determination in the Phillips-curve and related frameworks has distinguished backward-looking or adaptive behavior from forward-looking behavior. My approach sidesteps this issue by associating the component of wages that represents shifts of the Phillips curve with the trend variable and the component that represents movements along the Phillips curve with the random variable.”
Productivity takes on five distinct values uniformly spaced in the \([1 - \gamma, 1 + \gamma]\).

The productivity transition matrix is

\[
\begin{align*}
\pi_{12} &= \pi_{45} = \pi_{21} = \pi_{54} = 2(1 - \theta), \\
\pi_{23} &= \pi_{34} = \pi_{32} = \pi_{43} = 3(1 - \theta), \\
0 & \text{ for other non-diagonal elements} \\
1 & \text{ sum of non-diagonals for diagonals}
\end{align*}
\]

The serial correlation of \(z\) is given by \(\theta\).

Calibrate to monthly data.
### Table 2—Calculations from JOLTS Data

<table>
<thead>
<tr>
<th></th>
<th>December 2000</th>
<th>December 2002</th>
</tr>
</thead>
<tbody>
<tr>
<td>New hires</td>
<td>4.070 million</td>
<td>3.187 million</td>
</tr>
<tr>
<td>Unemployed</td>
<td>5.264 million</td>
<td>8.209 million</td>
</tr>
<tr>
<td>Vacancies</td>
<td>4.036 million</td>
<td>2.558 million</td>
</tr>
<tr>
<td>Job-finding rate, $\phi$</td>
<td>0.773 per month</td>
<td>0.388 per month</td>
</tr>
<tr>
<td>Job-filling rate, $\rho$</td>
<td>1.008 per month</td>
<td>1.246 per month</td>
</tr>
<tr>
<td>Unemployment rate, $u$</td>
<td>3.6 percent</td>
<td>5.7 percent</td>
</tr>
<tr>
<td>Vacancy rate, $v$</td>
<td>2.8 percent</td>
<td>1.8 percent</td>
</tr>
<tr>
<td>$x$</td>
<td>0.767 vacancies per</td>
<td>0.312 vacancies per</td>
</tr>
<tr>
<td></td>
<td>unemployed worker</td>
<td>unemployed worker</td>
</tr>
<tr>
<td>$\alpha$, elasticity of job finding with respect to $x$</td>
<td>0.765</td>
<td></td>
</tr>
<tr>
<td>$\omega$, efficiency of matching</td>
<td>0.947</td>
<td></td>
</tr>
<tr>
<td>Parameter</td>
<td>Interpretation</td>
<td>Value</td>
</tr>
<tr>
<td>-----------</td>
<td>--------------------------------------------------------------------------------</td>
<td>---------</td>
</tr>
<tr>
<td>$\delta$</td>
<td>Separation rate</td>
<td>0.034</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>Flow value while searching (leisure or unemployment compensation)</td>
<td>0.4</td>
</tr>
<tr>
<td>$k$</td>
<td>Flow cost of a vacancy</td>
<td>0.986</td>
</tr>
<tr>
<td>$\beta$</td>
<td>Discount factor</td>
<td>0.995</td>
</tr>
<tr>
<td>$\theta$</td>
<td>Serial correlation of mean-reverting component of productivity</td>
<td>0.9899</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>Dispersion parameter for mean-reverting component of productivity</td>
<td>0.00565</td>
</tr>
</tbody>
</table>
## Table 4—Values of Endogenous Variables in the Median State

<table>
<thead>
<tr>
<th>Variable</th>
<th>Interpretation</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$U$</td>
<td>Value while searching</td>
<td>229.34</td>
</tr>
<tr>
<td>$V$</td>
<td>Value while working</td>
<td>229.28</td>
</tr>
<tr>
<td>$J$</td>
<td>Value of worker to the firm</td>
<td>1.8698</td>
</tr>
<tr>
<td>$w$</td>
<td>Wage</td>
<td>0.96572</td>
</tr>
</tbody>
</table>
Although unemployment is a state variable in the model, the job-finding rate is very high. Thus, the model’s unemployment rate departs only slightly from the SS approximation we have used before:

\[(23) \quad u^*_s = \frac{\delta}{\delta + \phi(x_s)}\]

The figure on the next page shows some properties of the model using the approximation.
Figure 2. Job Finding, Vacancy, and Unemployment Rates, Fixed Wage
\( \bar{w} \) and \( \underline{w} \) are functions of endogenous asset values -- see (6)-(8). They vary with contemporaneous productivity.

- \( \bar{w} \): Upper end of bargaining set
- \( \underline{w} \): Lower end of bargaining set
- Leisure Value + Unemp. Benefits
- Annuity value of state with lowest expected revenues from (11)

**Figure 3. Wage Elements**
Quantitative Exercises, 3

- In the fixed-wage model, when productivity rises from state 3 to state 4, \( J \) rises by 21 units per unit rise in \( z \). This large increase in the value of a filled job induces a large increase in recruiting activity (job creation/vacancy posting).

- By way of contrast, in the version of the model with a symmetric Nash wage bargain, \( J \) rises by only 1.4 units per unit rise in \( z \). See next slide.

- Thus, relative to Nash bargaining, the fixed wage amplifies the impact on the value of a filled job by a factor of \( (21/1.4)=15 \).
Figure 4. Job Finding, Vacancy, and Unemployment Rates, Nash-Bargain Wage
• The sensitivity of job creation/recruiting effort to productivity depends on the distribution of rents.
  – For example, if employers get (almost) all the rents, then they have large incentives to recruit workers at all times. The elasticity of the value of filled jobs to productivity is unity.

• The sensitivity also depends on the gap between unemployment income and market wages. See Hagedorn and Manovskii (2008).

• The model has weak internal propagation mechanisms. We’ve made this point before, but see Hall’s Section IV.B
Many other wage-setting rules are consistent with equilibrium in this setup. Consider the following hybrid wage rule:

\[(24) \quad w^P_s = \alpha w + (1 - \alpha)w^N_s\]

where \(w^N_s\) is the state-contingent Nash wage bargain, and \(\alpha\) is a parameter between 0 and 1 that governs the volatility of wages. Closing the model with (24) yields equilibrium behavior that is in between that of the fixed-wage and Nash-bargain versions of the model.
As another example, consider

\[
(25) \quad w_t^A = (1 - \alpha)w_{t-1}^A + \alpha w_s^N
\]

In this adaptive wage-setting formulation, the wage becomes an additional state variable with potentially important implications for dynamics. The wage must be constrained to lie within the bargaining set.

See Thomas and Worrall (1988) and the w.p. version of Hall (2005) for more on this case.
III. The Limited Influence of Unemployment on the Wage Bargain

Robert E. Hall and Paul R. Milgrom

American Economic Review,
September 2008
A. Main Point of the Paper

An Amplification Result

Relative to the Nash Bargain, the alternating-offer bargaining solution of Binmore, Rubinstein and Wolinksy (BRW, 1986) dampens feedback from current unemployment to the current wage in an MP-type model.

As a result, closing the MP model with the BRW solution leads to greater sensitivity of employer surplus to productivity shocks, greater responsiveness of employer recruiting efforts, and bigger responses in vacancies, the job-finding rate and the unemployment rate.
B. Nash Bargaining in the MP Model

• Threat points for bargaining are the payoffs that result when the job-seeker returns to the market and the employer waits for another applicant.

→ Bargained wage = weighted mean of applicant’s productivity in job and value of unemployment. Recall equation (3) on slide 8.

The latter value, in turn, depends in large part on the wages offered in other jobs.
Nash Bargaining in the MP Model, 2

• If a unit productivity shock reduces every employer’s reservation wage by one unit, then both terms fall by almost equal amounts. If both changed by exactly one unit, then the employer’s recruiting effort would be unchanged and unemployment would not fluctuate.

• The actual equilibrium is similar to this approximate outcome and cannot explain realistic employment fluctuations with shocks of plausible size.
C. BRW Bargaining in the MP Model

• Hall and Milgrom close the MP model with the BRW bargaining solution. Now the threat points are to extend, rather than terminate, bargaining.

• Relative to Nash, BRW bargaining loosens the connection of wages to outside conditions.

• The bargain is controlled by the job’s productivity and the job-seeker’s patience as a bargainer relative to the employer.

• The job opportunity may disappear during bargaining so the outside option remains relevant, but its force is weaker than with Nash.
Weaker force of outside option $\Rightarrow$ equilibrium wages are less sensitive to productivity shocks.

In turn, less sensitivity of the wage to productivity shocks means more responsiveness of vacancies, job-finding and unemployment through the same mechanism as in Hall (2005).
BRW Bargaining in the MP Model, 3

• BRW distinguish between the *outside-option payoff* the parties get by quitting the negotiation to seek other opportunities and the *disagreement payoff* they get in the period before an agreement is reached.

• Unless the outside option is especially favorable, it is the disagreement payoff—not the outside option—that determines the bargaining outcome.
BRW Bargaining in the MP Model, 4

• In the BRW equilibrium, the parties do not actually spend any time bargaining.

• They think through the consequences of a sequence of offers and counteroffers and then move immediately to an agreement at the unique subgame perfect equilibrium of the bargaining game. No time and resources are lost in haggling over the wage.
D. Another Route to Highly Responsive Unemployment in MP Models

- MP models with Nash wage bargaining can deliver high shock sensitivity of vacancies, job-finding and unemployment, if LS is sufficiently elastic. In this context, high LS elasticity involves workers who are nearly indifferent between working and not.

- Hagedorn and Manovskii (2008) develop an MP-Nash model with high LS elasticity and argue that it resolves the unemployment volatility puzzle.

- Hall and Milgrom adopt the HagMan calibration as one of their models.
E. The Environment

• The same as Hall (2005), except for wage-setting behavior.
• Small changes in notation, introduced below.
• Hall-Milgrom (HM) consider three variants of the MP model with productivity shocks:
  – MP-Nash with standard calibration
  – MP-Nash with HagMan calibration
  – MP-BRW, which involves some new parameters
F. Asset Value Condition for Job Seekers

(1) \[ U_i = z + \frac{1}{1 + r} \sum \pi_{i,i'}[\phi(\theta_i)(W_{i'} + V_{i'}) + (1 - \phi(\theta_i))U_{i'}]. \]

Value of Unemployment, the outside option  \hspace{2cm} \hspace{2cm} PV of Wage Contract on a Job  \hspace{2cm} \hspace{2cm} Value of Additional Utility Flows Following End of a Job

(2) \[ V_i = \frac{1}{1 + r} \sum \pi_{i,i'}[sU_{i'} + (1 - s)V_{i'}]. \]
Standard free-entry condition + optimizing behavior by firms yields:

\[ P_i = p_i + \frac{1}{1 + r} \sum_{i'} \pi_{i,i'} (1 - s) P_{i'} . \]

PV of Worker’s Output Over Life of Match

\[ q(\theta_i)(P_i - W_i) = c. \]

Per-period Recruiting Cost, Payable at end of period
H. Closing the Model with Nash

\[ W_i = \beta P_i + (1 - \beta)(U_i - V_i). \]

Worker’s share of surplus in the generalized Nash bargain

N values for the productivity shock

The MP-Nash model has 5N endogenous variables:

- \( U_i \) = Worker's value of being unemployed
- \( V_i \) = Value after end of job
- \( \theta_i \) = Vacancy-Unemployment Ratio
- \( P_i \) = PV of productivity
- \( W_i \) = PV of Wages for \( i = 1, \ldots, N \)

The model also has 5N equations:

(1), (2), (3), (4) & (8)
I. The BRW Bargaining Game

- Bargaining takes place over time
- The parties alternate in making proposals
- After a proposer makes an offer, the responder has three options:
  - Accept the current proposal
  - Reject, and make a counter-proposal
  - Abandon the bargain, and take the outside option
- If either party abandons, employer gets 0 and worker gets $U$
If responder makes a counter-proposal, each party gets its disagreement payoff for the period and the bargaining game continues. Disagreement payoffs:

- Employer incurs a cost, $\gamma$
- The worker gets the flow benefit, $z$

When the joint payoff from matching, $V_i + P_i$, exceeds both $U_i$ and the capitalized flow, $(z - \gamma)(1 + r) / r$, the parties agree on a wage $W_i < P_i$. We assume that these conditions hold in what follows.
Assume, provisionally, that the subgame perfect equilibrium of the bargaining game is unique.

– Subgame perfection is a refinement concept in dynamic games. A strategy profile in the dynamic game is said to be subgame perfect if it is a Nash equilibrium in every subgame of the larger dynamic game.

– The main force of subgame perfection is to rule out threats that are not credible, e.g., an employer threatening to rescind a job offer on the next round if the job seeker does not accept a low-ball offer now.

– HM later prove this uniqueness condition.
The indifference condition for a worker, when contemplating an offer, $W_i$, from an employer is:

(10)  \[ W_i + V_i = \delta U_i + (1 - \delta) \left[ z + \frac{1}{1 + r} \sum_{i'} \pi_{i,i'}(W_{i'} + V_{i'}) \right]. \]

Probability that job offer disappears (exogenously) in a given sub-period

Anticipated counter offer if the worker rejects and job offer remains open.

The left side of (10) is what the worker gets if he accepts the current offer, $W_i$. The right side is what he gets if he rejects and continues bargaining.
The condition for an employer contemplating a counteroffer, $W'_i$, from the worker is:

\[(11) \quad P_i - W'_i = (1 - \delta) \left[ -\gamma + \frac{1}{1 + r} \sum_{i'} \pi_{i,i'}(P_{i'} - W_i) \right] \]

The wage offer proposed by the employer when contemplating the counteroffer, $W'_i$.

- The worker accepts the offer in equilibrium.
- **Assumption**: The employer proposes first.
- So $W_i$ is the equilibrium wage offer.
• Although the employer makes the first offer and the worker accepts, the wage is higher than if the employer could make a take-it-or-leave-it offer. The worker’s right to respond to a low offer with a counter offer—though not used in equilibrium—gives the worker part of the surplus.

• By assumption, the employer cannot commit to ignoring a counteroffer.
J. Closing the Model with BRW

• (10) and (11) play the same role in the MP-BRW model as (8) plays in the MP-Nash model.
• These two ways of closing the MP model differ in the threats that drive the wage bargain
  – For the worker, its $U$ with Nash and (10) with BRW
  – For the employer, its 0 with Nash and (11) with BRW
  – If $\delta = 1$, the threats are the same in the two bargaining models.
• The MP-BRW model has 6N equations in 6N unknowns.
Closing the Model with BRW, 2

• HM note that the solution for $W_i$ is “somewhat complicated” but very close to $(1/2)(W_i + W'_i)$

$$\begin{align*}
\frac{1}{2}(W + W') &= \frac{1}{2} \left\{ P + \left( I - \frac{1 - \delta}{1 + r \pi} \right)^{-1} \left[ \delta U + (1 - \delta)(z + \gamma) \right] - V \right\}
\end{align*}$$

where $\left( I - \frac{1 - \delta}{1 + r \pi} \right)^{-1}$ is the value of an income stream discounted at the rate $r$ and subject to declines at the rate $1-\delta$. $\pi$ is the transition matrix for the exogenous shocks.
Closing the Model with BRW, 3

• (12) gives the same role to productivity $P$ as in the Nash bargain (8)...

• “.. and to the worker’s subsequent career value $V$. ”

• Relative to (8), the role of the worker’s outside option, $U$, is scaled by $\delta$, the probability that the job offer disappears in a sub-period, because the outside option is relevant only when the worker is forced to return to search.

• The term $(z+\gamma)$ is not sensitive to current market conditions, unlike $U$. Smaller $\delta \rightarrow$ bigger role for this disagreement payoff, smaller role for current market conditions (the outside option)
K. Some Key Parameters

- Greater hiring costs, $c$, reduce unemployment sensitivity to productivity shocks.
- $z$, the flow value of nonmarket activity; see HM and Hagman for a discussion of evidence on this parameter and its implications.
- $\gamma$, the employer’s cost of delay
- $\delta$, the probability of a bargaining breakdown
- The elasticity of matching w.r.t. vacancies
- The separation rate, $s$
- The portion of unemployment fluctuations attributed to productivity shocks.
## APPENDIX: CALIBRATION

### TABLE 5—VALUES OF ENDOGENOUS VARIABLES FOR CALIBRATION

<table>
<thead>
<tr>
<th>Variable</th>
<th>Interpretation</th>
<th>Value</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>$u$</td>
<td>Unemployment rate, percent</td>
<td>5.5</td>
<td>Current Population Survey, long-run average</td>
</tr>
<tr>
<td>$\phi$</td>
<td>Job-finding rate, percent per day</td>
<td>2.4</td>
<td>JOLTS hiring flow rate divided by unemployment rate</td>
</tr>
<tr>
<td>$\theta$</td>
<td>Vacancy/unemployment ratio</td>
<td>0.5</td>
<td>Vacancy rate from JOLTS</td>
</tr>
<tr>
<td></td>
<td>Standard deviation of unemployment related to productivity fluctuations, percent</td>
<td>0.68</td>
<td>See text, Section IIG</td>
</tr>
<tr>
<td>Parameter</td>
<td>Interpretation</td>
<td>Model</td>
<td>Value</td>
</tr>
<tr>
<td>-----------</td>
<td>--------------------------------------------------------------------------------</td>
<td>-------</td>
<td>-----------</td>
</tr>
<tr>
<td>$s$</td>
<td>Exogenous separation rate</td>
<td>All</td>
<td>0.001</td>
</tr>
<tr>
<td></td>
<td>Elasticity of job-finding rate</td>
<td>All</td>
<td>0.5</td>
</tr>
<tr>
<td>$\phi_0$</td>
<td>Coefficient of matching efficiency</td>
<td>All</td>
<td>0.034</td>
</tr>
<tr>
<td>$r$</td>
<td>Daily interest rate</td>
<td>All</td>
<td>0.000192</td>
</tr>
<tr>
<td>$\beta$</td>
<td>Worker’s bargaining power</td>
<td>MP</td>
<td>0.5445</td>
</tr>
<tr>
<td>$\delta$</td>
<td>Daily probability that job opportunity disappears during bargaining</td>
<td>CB</td>
<td>0.0055</td>
</tr>
<tr>
<td>$c$</td>
<td>Cost of maintaining vacancy, in days of productivity per day</td>
<td>All</td>
<td>0.433</td>
</tr>
<tr>
<td>$b$</td>
<td>UI benefit replacement rate</td>
<td>MP and CB</td>
<td>0.25</td>
</tr>
<tr>
<td>$z$</td>
<td>Flow value of nonwork</td>
<td>MP and CB</td>
<td>0.71</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>Cost to employer of delaying bargaining by one day and formulating a counteroffer, in days of productivity</td>
<td>CB</td>
<td>0.932</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Calibrated Values: Flow Cost of Vacancies

We follow José I. Silva and Manuel Toledo (2007) in measuring the flow cost of maintaining a vacancy. They report that recruiting costs are 14 percent of quarterly pay per hire, or 9.1 days of pay per hire, based on data collected by PricewaterhouseCoopers. The days in our model are weekdays. As described below, we take the daily probability of filling a vacancy as 4.7 percent. The flow cost is the product,

\[ c = (4.7 \text{ percent}) \times (9.1 \text{ days of pay}) = 0.43 \text{ days of pay}. \]

Hagedorn and Manovskii (2008) use a much lower figure for noncapital hiring cost of 0.11, based on data for management time alone. On the other hand, they include a cost of idle capital of 0.47, so their value for \( c \) is 0.58, somewhat above ours, but not enough to alter any of our conclusions. We are skeptical of capital costs for recruiting, which rest on two hypotheses: first, that a vacancy involves a shortfall in employment and, second, that the shortfall causes the corresponding fraction of capital to be idle. An optimizing firm will generate a flow of hires so that the resulting level of employment is close to the optimum, not chronically below because of outflows, so vacancies should not be interpreted as shortfalls in employment. And even if there is a shortfall, the capital should be spread over the available workers as much as possible, rather than leaving it all idle.
Calibrated Values: Employer Cost of Delay

We choose $\gamma$ in the stationary version of the credible-bargaining model so that it generates the observed average level of unemployment, 5.5 percent. The resulting value of $\gamma$ is 0.23 days of worker productivity per day of delay. This value is about half of Hagedorn and Manovskii’s estimate of the cost of idle capital, which is more consistently used as cost of bargaining delay than as vacancy cost, for the following reason: in equilibrium no delay occurs. Hence, the delay is not one for which a rational manager would prepare. Unless investment is observed by the applicant and influences bargaining, capital will be installed at the moment a qualified applicant appears, not held off until bargaining is completed.

We could, alternatively, interpret $\gamma$ as the cost that the employer incurs in formulating a counteroffer. The employer avoids this cost by accepting the worker’s offer. If the worker produces $20 per hour or $160 per day, then $\gamma = 0.23$ implies a cost of $37 to produce the counteroffer. Notice that the value of $\gamma$ only comes into play two steps off the equilibrium path. First, the worker makes a counteroffer to the employer’s starting offer, which never happens on the equilibrium path. Second, the firm counters the counter offer, which also never happens, even one step off the equilibrium path. Nonetheless, the value of $\gamma$ has an important role in determining the equilibrium bargain.
Calibrated Values: Probability that a Job Opportunity Disappears while Bargaining

• Higher values of $\delta$ lead to lower volatility of unemployment. “Our knowledge of $\delta$ is limited to the view that the probability cannot be smaller than the probability $s$ that a job itself loses its productivity. We believe that the situation during bargaining is more fragile.”

• HM set $\delta$ to match the observed volatility of unemployment $\Rightarrow \delta=0.0055$ per day, or four times the probability that a job will end.
# Stochastic Productivity Process

<table>
<thead>
<tr>
<th>Category</th>
<th>Productivity deviation (percent)</th>
<th>Daily transition probability to new category</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>(1)</td>
</tr>
<tr>
<td>1</td>
<td>−2.69</td>
<td>0.9944</td>
</tr>
<tr>
<td>2</td>
<td>−0.95</td>
<td>0.0057</td>
</tr>
<tr>
<td>3</td>
<td>0.07</td>
<td>0.0000</td>
</tr>
<tr>
<td>4</td>
<td>1.21</td>
<td>0.0000</td>
</tr>
<tr>
<td>5</td>
<td>2.43</td>
<td>0.0000</td>
</tr>
</tbody>
</table>
How Much Unemployment Volatility to Attribute to Productivity?

- S.D. of unemployment rate is 1.5 percentage points per quarter over the sample.
- S.E. of residuals in a regression of the unemployment rate on productivity growth rates is 1.34 pps per quarter. This S.E. is a measure of the part of unemployment fluctuations beyond the reach of a productivity explanation.
- $\sqrt{(1.5)^2 - (1.34)^2} \approx 0.68$ -- what HM take as the appropriate target for the unemployment volatility in a model driven entirely by productivity shocks.
## L. Comparing Outcomes Across Models

### Table 2—Comparison of Three Models

<table>
<thead>
<tr>
<th>Measure</th>
<th>Our estimate</th>
<th>Mortensen-Pissarides</th>
<th>Hagedorn-Manovskii</th>
<th>Credible bargaining</th>
</tr>
</thead>
<tbody>
<tr>
<td>Flow value of non-work, ( z )</td>
<td>0.71</td>
<td>Input</td>
<td>Output</td>
<td>Input</td>
</tr>
<tr>
<td>Worker’s share of surplus</td>
<td>0.54</td>
<td>Output</td>
<td>Output</td>
<td>Output</td>
</tr>
<tr>
<td>Productivity component of unemployment volatility, standard deviation in percentage points</td>
<td>0.68</td>
<td>Output</td>
<td>Input&lt;sup&gt;a&lt;/sup&gt;</td>
<td>Input</td>
</tr>
<tr>
<td>Labor supply elasticity</td>
<td>1.0</td>
<td>Input</td>
<td>Output</td>
<td>Input</td>
</tr>
</tbody>
</table>
Comparing Outcomes Across Models, 2

Table 3—Responses to Changes in Productivity

<table>
<thead>
<tr>
<th></th>
<th>Slope with respect to $P$</th>
<th>Elasticity Unemployment rate</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$U$</td>
<td>$V$</td>
</tr>
<tr>
<td>Nash bargaining, Mortensen-Pissarides calibration</td>
<td>1.14</td>
<td>0.30</td>
</tr>
<tr>
<td>Nash bargaining, Hagedorn-Manovskii calibration</td>
<td>0.87</td>
<td>0.23</td>
</tr>
<tr>
<td>Credible bargaining</td>
<td>1.20</td>
<td>0.32</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>Slope of $P$’s contribution to $W$ via $P$</th>
<th>Slope of $W$ with respect to $P$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$P$</td>
<td>$U$</td>
</tr>
<tr>
<td>Nash bargaining, Mortensen-Pissarides calibration</td>
<td>0.54</td>
<td>0.52</td>
</tr>
<tr>
<td>Nash bargaining, Hagedorn-Manovskii calibration</td>
<td>0.19</td>
<td>0.70</td>
</tr>
<tr>
<td>Credible bargaining</td>
<td>0.50</td>
<td>0.35</td>
</tr>
</tbody>
</table>

These derivatives are calculated by comparing values in state 2 to values in state 4.
1. Closing the basic MP model with BRW bargaining delivers sizable unemployment fluctuations from productivity shocks of plausible magnitude.

2. BRW bargaining generates greater responsiveness to shocks than Nash, because the BRW threat points are shaped by the “disagreement payoffs”.

3. The outside option remains relevant under BRW, because a job opportunity may disappear while bargaining. But the force of the outside option is weaker under BRW than under Nash bargaining.
4. BRW offers a richer description of the bargaining process than Nash, and it seems like a more apt model for many real-world bargaining situations.

5. However, the BRW bargaining model also introduces important new parameters:
   - Probability a job opportunity disappears while bargaining.
   - Disagreement payoff to the employer – the cost of delay or the cost of reformulating an offer in HM
   - These parameters are not easily calibrated, and evidence on their appropriate values is scarce
Other key parameters are shared by the MP-BRW and MP-Nash models:

- Flow value of nonwork, which strongly affects labor supply behavior in the MP model
- Flow cost of maintaining a vacancy
- Elasticity of the job-finding rate to market tightness
- These parameters are also not easy to calibrate
7. Setting aside issues of calibration, casual evidence suggests that the nature of bargaining between worker and employer varies widely by education, experience, industry and occupation.

– This evidence suggests that it would be useful to allow for cross-sectional differences in the bargaining game, or in parameter values for a given bargaining game.

– The analysis of Hall (2005) and Hall-Milgrom (2008) suggest that this type of heterogeneity would generate important cross-sectional differences in the quantity responses to common shocks.
8. The models considered in this lecture focus on incentives and outcomes on the job creation/vacancy posting margin, and the follow-on consequences for job-finding rates and unemployment.

- These models neglect the role of time variation in job destruction, layoffs and unemployment inflows that we considered in Lecture 3. See Davis (2006) and Elsby et al. (2009) for more on this point.

- Mortensen and Pissarides (1994) is the seminal analysis of an MP-type model with interesting dynamics on the job destruction margin.
Wage stickiness amplifies the aggregate output response to exogenous shocks in canonical economic models.

- We saw a powerful example of this claim in our discussion of Hall (2005).
- The amplification role of wage stickiness is not limited to search and matching environments. It is a feature of many environments. This is a key theme in Shimer’s paper, “Wage Rigidities, Reallocation Shocks, and Jobless Recoveries.”
However, as we have discussed, search and matching environments naturally provide some scope to introduce wage stickiness without violating individual rationality constraints.
Wage stickiness expands the scope for monetary policy to have real effects and to function as a useful tool for countercyclical stabilization policy.

• I will not develop this claim in the course. If you are interested, see Gali (2008, chapters 3 and 6) and Gali (2010).
Wage stickiness raises the government spending multiplier in standard economic models.

• See Woodford (2011)
Many workers suffer large and persistent earnings loss in the wake of job loss. (We will see some of the evidence in our discussion of Davis and von Wachter, 2011.)

Firms sometimes (often?) rely on layoffs rather than wage cuts to reduce costs, even when the opportunity cost of time for laid off workers appears to be far below their pre-layoff wages.
Why Study Wage Stickiness, 6

• Evidence of wage stickiness on the separation margin (e.g., Bewley, 1999) points to departures from bilateral efficiency.

• Can wage stickiness on separation margin help to explain large earnings losses of (some) job losers?

• Is wage stickiness on the separation margin an important amplification mechanism?

• What causes wage stickiness on the separation margin? Stickiness on this margin is harder to square with IR than stickiness on the hiring margin
Why Study Wage Stickiness, 7

• There is much anecdotal and systematic evidence that wages are slow to change in the wake of real and nominal disturbances.

• We will review some of the evidence supporting this claim, if time permits.

• Understanding the implications of this wage stickiness for aggregate fluctuations is an important item on the macro research agenda.

• Developing and testing theories of wage stickiness is another important research area.
References 2