Micro Data for Macro Models

Topic 2: Capital Investment and Adjustment Costs

Thomas Winberry

January 14th, 2019
Plan for this Topic

1. An unfair summary of the empirical investment literature

2. Accounting for micro-level investment behavior with nonconvex adjustment costs

3. Macro implications of nonconvex adjustment costs
Plan for this Topic

1. An unfair summary of the empirical investment literature

2. Accounting for micro-level investment behavior with nonconvex adjustment costs

3. Macro implications of nonconvex adjustment costs
The empirical investment literature is full of disappointments. From time to time waves of new ideas challenge the aggregate investment equation, but these challenges are rarely successful, and progress is, at best, slow. There are serious theoretical obstacles, stemming mostly from the richness of the cross-sectional and time-series scenarios faced by actual investors, from the complexity of the investment technologies available to them, and from the myriad incentive problems that these agents face. There are at least as complex, and perhaps insurmountable, data problems. Both right- and left-hand side variables are seldom measured properly.

Caballero, Engel, and Haltiwanger, “Plant-Level Adjustment and Aggregate Investment Dynamics”
Empirical Investment Literature

• Many early papers focus on neoclassical model
  • “User cost” and “q theory” formulations
  • Finds model does not fit the data well at micro or macro level

• Two main responses:
  1. Real adjustment frictions with nonconvexities
  2. Financial frictions to acquiring investment funds are important
The Neoclassical Model

Consider individual firm investment problem:

- Firm $i$ with production function
  
  $$y_{it} = k_{it}^\alpha, \alpha \leq 1$$

- Invest to accumulate capital
  
  $$k_{it+1} = (1 - \delta)k_{it} + i_{it}$$

- Quadratic adjustment costs
  
  $$-\frac{\phi}{2} \left(\frac{i_{it}}{k_{it}}\right)^2 k_{it}$$

- Discount future at constant rate $r$
The Neoclassical Model

Consider individual firm investment problem:

- **Firm** \( i \) with production function
  \[ y_{it} = k_{it}^\alpha, \alpha \leq 1 \]

- **Invest** to accumulate capital
  \[ k_{it+1} = (1 - \delta)k_{it} + i_{it} \]

- **Quadratic adjustment costs**
  \[ -\frac{\phi}{2} \left( \frac{i_{it}}{k_{it}} \right)^2 k_{it} \]

- **Discount future** at constant rate \( r \)

\[
v(k_{it}) = \max_{i_{it},k_{it+1}} k_{it}^\alpha - i_{it} - \frac{\phi}{2} \left( \frac{i_{it}}{k_{it}} \right)^2 k_{it} + \frac{1}{1+r} v(k_{it+1})
\]

such that \( k_{it+1} = (1 - \delta)k_{it} + i_{it} \)
The Neoclassical Model

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such that \( k_{it+1} = (1 - \delta)k_{it} + i_{it} \quad (\times q_{it}) \)
The Neoclassical Model

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such that \( k_{it+1} = (1 - \delta) k_{it} + i_{it} \quad (\times q_{it}) \)

Take **first order conditions**:

\[ 1 + \phi \left( \frac{i_{it}}{k_{it}} \right) = q_{it} \]

\[ q_{it} = v'(k_{it+1}) \]

\[ q_{it} = \frac{1}{1 + r} \sum_{s=0}^{\infty} \left( \frac{1 - \delta}{1 + r} \right)^s \left( \alpha k_{it+s+1}^{\alpha-1} + \phi_{it+s+1} \right) \]
The User Cost Model: $\phi = 0$

With $\phi = 0$, first order conditions simplify to

$$q_{it} = 1$$

$$\alpha k_{it+1}^{\alpha-1} = r + \delta$$

- $r$ = user cost
The User Cost Model: $\phi = 0$

With $\phi = 0$, first order conditions simplify to

$$\begin{align*}
q_{it} &= 1 \\
\alpha k_{it+1}^{\alpha-1} &= r + \delta \\
\text{MPK}_{it} &= \text{user cost}
\end{align*}$$

- The **user cost of capital** is the implicit rental rate on capital.
- Typically extended to incorporate other empirically relevant features:

$$u_{st} = p_t \times \frac{1 - m_{st} - z_{st}}{1 - \tau_t} \times (r_t + \delta_s)$$

relative price of capital
taxes
Empirical Performance of the User Cost Model

• Typical regression takes the form

\[
\frac{i_{it}}{k_{it}} = \alpha_i + \beta u_{it} + \Gamma \text{other variables}_{it} + \epsilon_{it}
\]

• Two main failures of user cost model:

1. Estimated user cost elasticity \( \beta \) small \((\approx 0 \text{ to } -0.5)\)
2. Coefficients on other variables, especially cash flow, large and significant

• Hall and Jorgensen (1967); Cummins, Hassett, and Hubbard (1994); Chirinko, Fazarri, and Meyer (1999)
The Q-Theory Model: \( \phi \geq 0 \)

\[
q_{it} = 1 + \phi \left( \frac{i_{it}}{k_{it}} \right)
\]

\[
q_{it} = v'(k_{it+1}) = \frac{1}{1 + r} \sum_{s=0}^{\infty} \left( \frac{1 - \delta}{1 + r} \right)^s \left( \alpha k_{it+s+1}^{\alpha - 1} + \Phi_{it+s+1} \right)
\]
The Q-Theory Model: $\phi \geq 0$

\[ q_{it} = 1 + \phi \left( \frac{i_{it}}{k_{it}} \right) \]

\[ q_{it} = v'(k_{it+1}) = \frac{1}{1 + r} \sum_{s=0}^{\infty} \left( \frac{1 - \delta}{1 + r} \right)^s \left( \alpha k_{it+s+1}^{\alpha - 1} + \Phi_{it+s+1} \right) \]

• Two key implications of the model:
  1. $q_{it}$ is the **marginal value of capital** to the firm
  2. Investment positively related to $q_{it}$: $\frac{i_{it}}{k_{it}} = \frac{1}{\phi} (q_{it} - 1)$

Hayashi (Ѱ8Ѱ): under constant returns, $v'(k_{it}) = v(k_{it})$  
Marginal $q$ = average $q$ (sometimes called Tobin’s $q$)  
Extend to include relative price, taxes, etc.
The Q-Theory Model: $\phi \geq 0$

\[ q_{it} = 1 + \phi \left( \frac{i_{it}}{k_{it}} \right) \]

\[ q_{it} = v'(k_{it+1}) = \frac{1}{1 + r} \sum_{s=0}^{\infty} \left( \frac{1 - \delta}{1 + r} \right)^s \left( \alpha k_{it+s+1}^{\alpha-1} + \Phi_{it+s+1} \right) \]

- Two key implications of the model:
  1. $q_{it}$ is the marginal value of capital to the firm
  2. Investment positively related to $q_{it}$: $\frac{i_{it}}{k_{it}} = \frac{1}{\phi} (q_{it} - 1)$

- Hayashi (1982): under constant returns, $v'(k_{it}) = \frac{v(k_{it})}{k_{it}}$
  - Marginal q = average q (sometimes called Tobin’s q)
  - Extend to include relative price, taxes, etc.
Empirical Performance of the Q Model

• Typical regression takes the form

\[
\frac{i_{it}}{k_{it}} = \alpha_i + \beta q_{it} + \Gamma \text{other variables}_{it} + \varepsilon_{it}
\]

• Two main failures of the Q model:

  1. Estimated coefficient $\beta$ small and unstable
  2. Coefficients on other variables, especially cash flow, large and significant

• Summers (1981); Cummins, Hassett, and Hubbard (1994); Erickson and Whited (2000)
Doms and Dunne (1998)

Two responses to failure of neoclassical model:

1. Nonconvex adjustment costs are important
2. Financial frictions to acquiring investment funds are important
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1. Nonconvex adjustment costs are important
2. Financial frictions to acquiring investment funds are important

Doms and Dunne (1998):

- Landmark descriptive study of investment in LRD
- Shows micro-level investment is lumpy, i.e., occurs mainly along extensive margin
  - Fluctuations in total investment mainly due to extensive margin
- Suggests important role for fixed adjustment costs
Measurement

- Use **Census data** from LRD, 1972 - 1988
  - After 1988, stopped collecting book value of capital

- Construct capital stock using perpetual inventory method
  - Focus on balanced panel

- Analyze the **growth rate of capital** for plant $i$ at time $t$

  $GK_{it} = \frac{i_{it} - \delta k_{it-1}}{0.5 \times (k_{it-1} + k_{it})}$
Plant-Level Investment is Lumpy Across Plants

FIG. 1.  Capital growth rate ($GK$) distributions: Unweighted and weighted by investment.
Plant-Level Investment is Lumpy Across Plants

- 51.9% of plants increase capital ≤ 2.5%
- 11% of plants increase capital ≥ 20%

**FIG. 1.** Capital growth rate ($GK$) distributions: Unweighted and weighted by investment.
Plant-Level Investment is Lumpy Within Plants

- Capital growth in largest investment episode nearly 3/4%
- In median investment episode approximately 3/4%
Plant-Level Investment is Lumpy Within Plants

- Capital growth in largest investment episode nearly 50%
- In median investment episode approximately 0%
Plant-Level Investment is Lumpy Within Plants
Plant-Level Investment Lumpier than Firm-Level
Frequency of Spikes Correlated with Aggregate Investment
Zwick and Mahon (2016)

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Zwick and Mahon (2016)

Two responses to failure of neoclassical model:

1. Nonconvex adjustment costs are important
2. Financial frictions to acquiring investment funds are important

Zwick and Mahon (2016):

- Clean study exploiting policy-induced variation in cost of capital
- Shows investment very responsive to cost, especially for small/non-dividend paying firms
- Suggests important role for financial frictions (and potentially fixed costs)
### Bonus Depreciation Allowance

**Table 1: Regular and Bonus Depreciation Schedules for Five Year Items**

<table>
<thead>
<tr>
<th>Normal Depreciation</th>
<th>Year</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Deductions (000s)</td>
<td></td>
<td>200</td>
<td>320</td>
<td>192</td>
<td>115</td>
<td>115</td>
<td>58</td>
<td>1000</td>
</tr>
<tr>
<td>Tax Benefit ($\tau = 35%$)</td>
<td>70</td>
<td>112</td>
<td>67.2</td>
<td>40.3</td>
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<td>20.2</td>
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<td></td>
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<table>
<thead>
<tr>
<th>Bonus Depreciation (50%)</th>
<th>Year</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Deductions (000s)</td>
<td></td>
<td>600</td>
<td>160</td>
<td>96</td>
<td>57.5</td>
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</tr>
<tr>
<td>Tax Benefit ($\tau = 35%$)</td>
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<td>56</td>
<td>33.6</td>
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<td>10</td>
<td>350</td>
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Bonus Depreciation Allowance

• Bonus shifts depreciation allowances from future to present
• With discounting, lowers the total cost of investment
  ➞ Bonus more valuable for longer-lived investment

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$$Z_{s0} = \sum_{t=0}^{T_s} \frac{1}{(1+r)^t} D_t$$
Bonus Depreciation Allowance

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\[
Z_{s0} = \sum_{t=0}^{T_s} \frac{1}{(1+r)^t} D_t
\]

\[
Z_{st} = \theta \times 1 + (1 - \theta) \times Z_{s0}
\]
The most complete dataset yet applied to study business investment incentives.
Data

The most complete dataset yet applied to study business investment incentives.

- Representative panel drawn from universe of corporate firms in US
  - Released by Statistics of Income division of IRS
  - Available to researchers through proposal application system
- Also used by BEA to finalize national income statistics
Data

The most complete dataset yet applied to study business investment incentives.

- Representative panel drawn from universe of corporate firms in US
  - Released by Statistics of Income division of IRS
  - Available to researchers through proposal application system
  - Also used by BEA to finalize national income statistics
- Investment $i_{it}$ measured as expenditures on equipment eligible for Bonus
- PV of depreciation allowances $z_{st}$ constructed at four digit level using $r = 7\%$
Identification Strategy

- Identify effect of policy using \textit{difference-in-differences} design
  - Treatment group: firms in long-lived industries
  - Control group: firms in short-lived industries

- Regression specification

\[
f(i_{it}, k_{it}) = \alpha_i + \delta_t + \beta g(z_{st}) + \gamma X_{it} + \varepsilon_{it}
\]

- \( f(i_{it}, k_{it}) \): \( \log i_{it}, \log \frac{p_{st}}{1-p_{st}}, \) or \( \frac{i_{it}}{k_{it}} \)
- \( g(z_{st}) \): \( z_{st} \) or \( \frac{1-\tau z_{st}}{1-\tau} \)

- Key assumption for difference-in-differences: parallel trends holds
Graphical Evidence

**Intensive Margin: Bonus I**

- Log(Investment)
- Year: 1996 to 2004
- Treatment Group (Long Duration Industries)
- Control Group (Short Duration Industries)

**Intensive Margin: Bonus II**

- Log(Investment)
- Year: 2005 to 2010
- Treatment Group (Long Duration Industries)
- Control Group (Short Duration Industries)

**Extensive Margin: Bonus I**

- Log(Outs-Rate)
- Year: 1996 to 2004
- Treatment Group (Long Duration Industries)
- Control Group (Short Duration Industries)

**Extensive Margin: Bonus II**

- Log(Outs-Rate)
- Year: 2005 to 2010
- Treatment Group (Long Duration Industries)
- Control Group (Short Duration Industries)
### Overall Effect of Bonus on Investment

The equation for the overall effect of bonus on investment is:

\[ f(i_{it}, k_{it}) = \alpha_i + \delta_t + \beta g(z_{st}) + \gamma X_{it} + \varepsilon_{it} \]

#### Intensive Margin: LHS Variable is log(Investment)

<table>
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<tr>
<th></th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
<th>(5)</th>
<th>(6)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\pi_{N,t}$</td>
<td>3.69**</td>
<td>3.78**</td>
<td>3.07**</td>
<td>3.02**</td>
<td>3.73***</td>
<td>4.69***</td>
</tr>
<tr>
<td></td>
<td>(0.53)</td>
<td>(0.57)</td>
<td>(0.69)</td>
<td>(0.81)</td>
<td>(0.70)</td>
<td>(0.62)</td>
</tr>
<tr>
<td>$C_{F_{it}}/K_{it-1}$</td>
<td>0.44***</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.016)</td>
<td></td>
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</table>

**Observations:** 735341, 580422, 514035, 221306, 585914, 722262

**Clusters (Firms):** 128001, 100883, 109678, 63699, 107985, 124962

**$R^2$:** 0.71, 0.74, 0.73, 0.80, 0.72, 0.71

#### Extensive Margin: LHS Variable is log(P(Investment > 0))

<table>
<thead>
<tr>
<th></th>
<th>(1)</th>
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<th>(4)</th>
<th>(5)</th>
<th>(6)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\pi_{N,t}$</td>
<td>3.79**</td>
<td>3.87**</td>
<td>3.12**</td>
<td>3.59**</td>
<td>3.99*</td>
<td>4.00***</td>
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<tr>
<td></td>
<td>(1.24)</td>
<td>(1.21)</td>
<td>(2.00)</td>
<td>(1.14)</td>
<td>(1.69)</td>
<td>(1.13)</td>
</tr>
<tr>
<td>$C_{F_{it}}/K_{it-1}$</td>
<td>0.029**</td>
<td></td>
<td></td>
<td></td>
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</tr>
<tr>
<td></td>
<td>(0.0100)</td>
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</tbody>
</table>

**Observations:** 803659, 641173, 556011, 247648, 643913, 803659

**Clusters (Industries):** 314, 314, 314, 274, 277, 314

**$R^2$:** 0.87, 0.88, 0.88, 0.93, 0.90, 0.90

#### Tax Term: LHS Variable is Investment/Lagged Capital

<table>
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<tr>
<th></th>
<th>(1)</th>
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<th>(3)</th>
<th>(4)</th>
<th>(5)</th>
<th>(6)</th>
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<tr>
<td>$1 - k_{it}$</td>
<td>-1.60***</td>
<td>-1.53***</td>
<td>-2.00***</td>
<td>-1.42***</td>
<td>-2.27***</td>
<td>-1.50***</td>
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<tr>
<td></td>
<td>(0.096)</td>
<td>(0.095)</td>
<td>(0.16)</td>
<td>(0.13)</td>
<td>(0.14)</td>
<td>(0.10)</td>
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<tr>
<td>$C_{F_{it}}/K_{it-1}$</td>
<td>0.043***</td>
<td></td>
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<tr>
<td></td>
<td>(0.0023)</td>
<td></td>
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</table>

**Observations:** 637243, 633598, 426214, 211029, 510653, 631295

**Clusters (Firms):** 103890, 103220, 87939, 57343, 90145, 103565

**$R^2$:** 0.43, 0.43, 0.48, 0.54, 0.45, 0.44

**Controls:** No, No, No, No, Yes, No

**Industry Trends:** No, No, No, No, Yes, Yes
Larger Effect Than Existing Literature
### Heterogeneity Suggestive of Financial Frictions

<table>
<thead>
<tr>
<th></th>
<th>Sales</th>
<th>Div Payer?</th>
<th>Lagged Cash</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Small</td>
<td>Big</td>
<td>No</td>
</tr>
<tr>
<td>$z_{N,t}$</td>
<td>6.29***</td>
<td>3.22***</td>
<td>5.98***</td>
</tr>
<tr>
<td></td>
<td>(1.21)</td>
<td>(0.76)</td>
<td>(0.88)</td>
</tr>
<tr>
<td>Equality Test</td>
<td>$p = 0.030$</td>
<td>$p = 0.079$</td>
<td>$p = 0.000$</td>
</tr>
<tr>
<td>Observations</td>
<td>177620</td>
<td>255266</td>
<td>274809</td>
</tr>
<tr>
<td>Clusters (Firms)</td>
<td>29618</td>
<td>29637</td>
<td>39195</td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.44</td>
<td>0.76</td>
<td>0.69</td>
</tr>
</tbody>
</table>
Heterogeneity Explains Larger Estimate than Literature
Unfair Review of Empirical Investment Lit

- Neoclassical model predicts investment very responsive to cost
  - **User cost** formulation: capital stock responds to implied rental rate
  - **Q theory** formulation: investment responds to marginal value of capital
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  - Capital/investment unresponsive to cost
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  - Other variables (cash flow) significant

- Two responses to failure of neoclassical model
  1. Adjustment costs feature nonconvexities
  2. Financial frictions influence investment behavior
The Rest of This Topic

Focus on role of nonconvex adjustment costs in explaining micro and macro investment dynamics
The Rest of This Topic

Focus on role of nonconvex adjustment costs in explaining micro and macro investment dynamics

1. **Models of micro-level investment behavior**

2. **Aggregate implications of these models**
   - Aggregation
   - General equilibrium
Plan for this Topic

1. An unfair summary of the empirical investment literature

2. Accounting for micro-level investment behavior with nonconvex adjustment costs

3. Macro implications of nonconvex adjustment costs
Cooper and Haltiwanger (2006)

- What types of \textit{adjustment costs} do we need to match micro-level investment behavior?
  - Pays special attention to lumpy nature of investment
Cooper and Haltiwanger (2006)

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- Answer using an **estimated structural model**
  - Simulated method of moments
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- A note on terminology in this literature:
  - **Partial equilibrium** = analyzing decision rules with fixed prices
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LRD Data

Sample

- Establishment-level observations
- **Balanced panel**: model abstracts from entry and exit
- 1972 - 1988: want to use data on expenditures and retirements
LRD Data

Sample

- Establishment-level observations
- **Balanced panel**: model abstracts from entry and exit
- 1972 - 1988: want to use data on expenditures and retirements

Measurement

- **Investment** $i_{it}$: expenditure$_{it} -$ retirments$_{it}$
- **Capital** $k_{it}$: $k_{it+1} = (1 - \delta_{it})k_{it} + i_{it}$
- **Depreciation** $\delta_{it}$: constructed to reflect in-use depreciation
Cross-Sectional Distribution of Investment Rates

- Large mass of observations near zero
- Highly skewed and fat right tails
Cross-Sectional Distribution of Investment Rates

- Large mass of observations near zero
- Highly skewed and fat right tails
## Cross-Sectional Distribution of Investment Rates

<table>
<thead>
<tr>
<th>Variable</th>
<th>LRD</th>
</tr>
</thead>
<tbody>
<tr>
<td>Average Investment Rate</td>
<td>12.2% (0.10)</td>
</tr>
<tr>
<td>Inaction Rate: Investment</td>
<td>8.1% (0.08)</td>
</tr>
<tr>
<td>Fraction of Observations with Negative Investment</td>
<td>10.4% (0.09)</td>
</tr>
<tr>
<td>Spike Rate: Positive Investment</td>
<td>18.6% (0.12)</td>
</tr>
<tr>
<td>Spike Rate: Negative Investment</td>
<td>1.8% (0.04)</td>
</tr>
<tr>
<td>Serial correlation of Investment Rates</td>
<td>0.058 (0.003)</td>
</tr>
<tr>
<td>Correlation of Profit Shocks and Investment</td>
<td>0.143 (0.003)</td>
</tr>
</tbody>
</table>
Bellman equation

\[ v(z_{it}, k_{it}) = \max_{i_{it}} e^{z_{it}k_{it}} - p(i_{it})i_{it} - c(i_{it}, k_{it}, z_{it}) \]

\[ + \frac{1}{1 + r} \mathbb{E}_t[V(z_{it+1}, (1 - \delta)k_{it} + i_{it})] \]
General Investment Model

Bellman equation

\[
\nu(z_{it}, k_{it}) = \max_{i_{it}} e^{z_{it}} k_{it}^\alpha - p(i_{it})i_{it} - c(i_{it}, k_{it}, z_{it}) \\
+ \frac{1}{1 + r} \mathbb{E}_t [\nu(z_{it+1}, (1 - \delta)k_{it} + i_{it})]
\]

Adjustment costs

\[
c(i_{it}, k_{it}, z_{it}) = \frac{\gamma}{2} \left( \frac{i_{it}}{k_{it}} \right)^2 k_{it} + \mathbb{1} (i_{it} \neq 0) (Fk_{it} + \lambda e^{z_{it}} k_{it}^\alpha)
\]

[convex]

[nonconvex]
General Investment Model

Bellman equation

\[ v(z_{it}, k_{it}) = \max_{i_{it}} e^{z_{it}} k_{it}^\alpha - p(i_{it}) i_{it} - c(i_{it}, k_{it}, z_{it}) \]

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Irreversibilities

\[ p(i_{it}) = \mathbb{1} (i_{it} \geq 0) + p_s \times \mathbb{1} (i_{it} < 0) \]
No Adjustment Costs

\[ v(z_{it}, k_{it}) = \max_{i_{it}} e^{z_{it}} k_{it}^\alpha - i_{it} + \frac{1}{1 + r} \mathbb{E}_t[v(z_{it+1}, (1 - \delta)k_{it} + i_{it})] \]
No Adjustment Costs

\[ v(z_{it}, k_{it}) = \max_{i_{it}} e^{z_{it}} k_{it}^\alpha - i_{it} + \frac{1}{1 + r} \mathbb{E}_t[v(z_{it+1}, (1 - \delta)k_{it} + i_{it})] \]

Optimal Behavior

\[ 1 = \frac{1}{1 + r} \mathbb{E}_t[v_2(z_{it+1}, k_{it+1})] \]

\[ \rightarrow \text{user cost model: } r + \delta = \mathbb{E}_t[\alpha k_{it+1}^{\alpha - 1}] \]
Convex Costs Only

\[ v(z_{it}, k_{it}) = \max_{i_{it}} e^{z_{it}k_{it}^\alpha} - i_{it} - c(i_{it}, k_{it}, z_{it}) + \frac{1}{1 + r} \mathbb{E}_t[v(z_{it+1}, k_{it+1})] \]

\[ c(i_{it}, k_{it}, z_{it}) = \frac{\gamma}{2} \left( \frac{i_{it}}{k_{it}} \right)^2 k_{it} \]
Convex Costs Only

\[ v(z_{it}, k_{it}) = \max_{i_{it}} e^{z_{it}} k_{it}^\alpha - i_{it} \]  
\[ - c(i_{it}, k_{it}, z_{it}) + \frac{1}{1 + r} \mathbb{E}_t[v(Z_{it+1}, k_{it+1})] \]

\[ c(i_{it}, k_{it}, z_{it}) = \frac{\gamma}{2} \left( \frac{i_{it}}{k_{it}} \right)^2 k_{it} \]

Optimal Behavior

\[ 1 + \gamma \left( \frac{i_{it}}{k_{it}} \right) = \frac{1}{1 + r} \mathbb{E}_t[v_2(Z_{it+1}, k_{it+1})] \]

\[ \rightarrow \text{Q-theory model:} \quad \frac{i_{it}}{k_{it}} = \frac{1}{\gamma} \left( \mathbb{E}_t[v_2(Z_{it+1}, k_{it+1})] - 1 \right) \]
Nonconvex Costs

\[ v(z_{it}, k_{it}) = \max_{i_{it}} e^{z_{it}} k_{it}^{\alpha} - i_{it} - c(i_{it}, k_{it}, z_{it}) + \frac{1}{1 + r} \mathbb{E}_t[v(z_{it+1}, k_{it+1})] \]

\[ c(i_{it}, k_{it}, z_{it}) = \frac{\gamma}{2} \left( \frac{i_{it}}{k_{it}} \right)^2 k_{it} + 1 (i_{it} \neq 0) (Fk_{it} + \lambda e^{z_{it}} k_{it}^{\alpha}) \]
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Optimal Behavior

\[ v^a(z_{it}, k_{it}) = \max_{i_{it}} \ e^{z_{it}} k_{it}^{\alpha} - i_{it} - c(i_{it}, k_{it}, z_{it}) + \frac{1}{1 + r} \mathbb{E}_t[v(z_{it+1}, k_{it+1})] \]

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→ Adjust iff \( v^a(z_{it}, k_{it}) > v^n(z_{it}, k_{it}) \)

- Depreciation
- Productivity shock
Irreversibility

\[ v(z_{it}, k_{it}) = \max_{i_{it}} e^{z_{it} k_{it}^{\alpha}} - i_{it} + \frac{1}{1 + r} \mathbb{E}_t[v(z_{it+1}, k_{it+1})] \]

\[ p(i_{it}) = \mathbb{1} (i_{it} \geq 0) + \rho_s \times \mathbb{1} (i_{it} < 0) \]
Irreversibility

\[ v(\mathbf{z}_{it}, k_{it}) = \max_{i_{it}} e^{\mathbf{z}_{it}k_{it}^\alpha} - i_{it} + \frac{1}{1 + r} \mathbb{E}_t[v(\mathbf{z}_{it+1}, k_{it+1})] \]

\[ p(i_{it}) = 1 \times 1 (i_{it} \geq 0) + p_s \times 1 (i_{it} < 0) \]

Optimal Behavior

\[ v^b(\mathbf{z}_{it}, k_{it}) = \max_{i_{it} > 0} e^{\mathbf{z}_{it}k_{it}^\alpha} - i_{it} + \frac{1}{1 + r} \mathbb{E}_t[v(\mathbf{z}_{it+1}, k_{it+1})] \]

\[ v^s(\mathbf{z}_{it}, k_{it}) = \max_{i_{it} < 0} e^{\mathbf{z}_{it}k_{it}^\alpha} - p_s i_{it} + \frac{1}{1 + r} \mathbb{E}_t[v(\mathbf{z}_{it+1}, k_{it+1})] \]

\[ v^n(\mathbf{z}_{it}, k_{it}) = e^{\mathbf{z}_{it}k_{it}^\alpha} + \frac{1}{1 + r} \mathbb{E}_t[v(\mathbf{z}_{it+1}, (1 - \delta)k_{it})] \]
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→ Also generates inaction
Illustration of Various Frictions

<table>
<thead>
<tr>
<th>Moment</th>
<th>LRD</th>
<th>No AC</th>
<th>CON</th>
<th>NC-F</th>
<th>NC-(\lambda)</th>
<th>TRAN</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fraction of inaction</td>
<td>0.081</td>
<td>0.0</td>
<td>0.038</td>
<td>0.616</td>
<td>0.588</td>
<td>0.69</td>
</tr>
<tr>
<td>Fraction with positive investment bursts</td>
<td>0.18</td>
<td>0.298</td>
<td>0.075</td>
<td>0.212</td>
<td>0.213</td>
<td>0.120</td>
</tr>
<tr>
<td>Fraction with negative investment bursts</td>
<td>0.018</td>
<td>0.203</td>
<td>0.0</td>
<td>0.172</td>
<td>0.198</td>
<td>0.024</td>
</tr>
<tr>
<td>(\text{corr}(i_{it},i_{it-1}))</td>
<td>0.058</td>
<td>-0.053</td>
<td>0.732</td>
<td>-0.057</td>
<td>-0.06</td>
<td>0.110</td>
</tr>
<tr>
<td>(\text{corr}(i_{it},a_{it}))</td>
<td>0.143</td>
<td>0.202</td>
<td>0.692</td>
<td>0.184</td>
<td>0.196</td>
<td>0.346</td>
</tr>
</tbody>
</table>
Model Quantification

**Overall strategy**

1. Fix a subset of parameters
2. Estimate shock process using measured TFP-type approach
3. Estimate adjustment costs to match moments
Model Quantification

Overall strategy

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Fixed parameters

- Depreciation rate $\delta = 6.9\%$
- Discount rate $r = 5.25\%$
Model Quantification

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2. Estimate shock process using measured TFP-type approach
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**Fixed parameters**
- Depreciation rate $\delta = 6.9\%$
- Discount rate $r = 5.25\%$

**Estimate idiosyncratic shocks**
- Assume $z_{it} = \varepsilon_{it} + b_t$
- Assume AR(1) and use GMM on
  \[
  \log(\pi_{it}) = \rho \varepsilon \log(\pi_{it-1}) + \theta k_{it} - \rho \varepsilon \theta k_{it-1} + b_t - \rho \varepsilon b_{t-1} + \eta_{it}
  \]
- See paper for details
Estimating Adjustment Cost Parameters

• Estimate parameters for two separate cases:
  1. **Fixed cost case**: estimate $\Theta = (\gamma, F, p_s)$, set $\lambda = 1$
  2. **Opportunity cost case**: estimate $\Theta = (\gamma, \lambda, p_s)$, set $F = 0$
Estimating Adjustment Cost Parameters

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  2. **Opportunity cost case**: estimate $\Theta = (\gamma, \lambda, p_s)$, set $F = 0$

- **Simulated Method of Moments (SMM)**

  $$\hat{\Theta} = \arg \min_{\Theta} [\psi_d - \psi_s(\Theta)]^T W [\psi_d - \psi_s(\Theta)]$$

  - Data moments $\psi_d$: drawn from data
  - Model moments $\psi_s(\Theta)$: simulated panel of firms from model
  - Weighting matrix $W$: efficient matrix from GMM
  - Standard errors: GMM formulas plus factor for Monte Carlo error
## Estimation Results: Fixed Cost Case

<table>
<thead>
<tr>
<th>Spec.</th>
<th>Structural Parm.</th>
<th>Est. (s.e.)</th>
<th>( \gamma )</th>
<th>( F )</th>
<th>( p_s )</th>
<th>( corr(i, i_{-1}) )</th>
<th>( corr(i, a) )</th>
<th>( spike^+ )</th>
<th>( spike^- )</th>
<th>( L(\hat{\Theta}) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>LRD</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.058</td>
<td>0.143</td>
<td>0.186</td>
<td>0.018</td>
<td></td>
</tr>
<tr>
<td>all</td>
<td>0.049 (0.002)</td>
<td>0.039 (0.001)</td>
<td>0.975 (0.004)</td>
<td>0.086</td>
<td>0.31</td>
<td>0.127</td>
<td>0.030</td>
<td>6399.9</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \gamma ) only</td>
<td>0.455 (0.002)</td>
<td>0</td>
<td>1</td>
<td>0.605</td>
<td>0.540</td>
<td>0.23</td>
<td>0.028</td>
<td>53182.6</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( p_s ) only</td>
<td>0</td>
<td>0</td>
<td>0.795 (0.002)</td>
<td>0.113</td>
<td>0.338</td>
<td>0.132</td>
<td>0.033</td>
<td>7673.68</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( F ) only</td>
<td>0</td>
<td>0.0695 (0.00046)</td>
<td>1</td>
<td>-0.004</td>
<td>0.213</td>
<td>0.105</td>
<td>0.0325</td>
<td>7390.84</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Estimation Results: Fixed Cost Case

<table>
<thead>
<tr>
<th>Spec.</th>
<th>Structural Parm.</th>
<th>Est. (s.e.)</th>
<th>moments</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>$corr(i,i_{-1})$</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
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</tr>
<tr>
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<td>0</td>
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</tr>
</tbody>
</table>

Estimated fixed cost $F \approx 4\%$ of capital stock
### Estimation Results: Disruption Cost Case

<table>
<thead>
<tr>
<th>Spec.</th>
<th>Structural Parm.</th>
<th>Est. (s.e.)</th>
<th>moments</th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\gamma$</td>
<td>$\lambda$</td>
<td>$p_s$</td>
<td>$corr(i, i-1)$</td>
<td>$corr(i, a)$</td>
<td>$spike^+$</td>
<td>$spike^-$</td>
</tr>
<tr>
<td>LRD</td>
<td></td>
<td></td>
<td></td>
<td>0.058</td>
<td>0.143</td>
<td>0.186</td>
<td>0.018</td>
</tr>
<tr>
<td>$\lambda$ only</td>
<td>0</td>
<td>0.796 (0.0040)</td>
<td>1.0</td>
<td>-0.009</td>
<td>0.06</td>
<td>0.107</td>
<td>0.042</td>
</tr>
<tr>
<td>all</td>
<td>0.153 (0.0056)</td>
<td>0.796 (0.0090)</td>
<td>0.981 (0.0090)</td>
<td>0.148</td>
<td>0.156</td>
<td>0.132</td>
<td>0.023</td>
</tr>
</tbody>
</table>
## Estimation Results: Disruption Cost Case

<table>
<thead>
<tr>
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<th>moments</th>
</tr>
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<tbody>
<tr>
<td></td>
<td>$\gamma$</td>
<td>$\lambda$</td>
<td>$p_s$</td>
</tr>
<tr>
<td>LRD</td>
<td>0</td>
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<td>0.153 (0.0056)</td>
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</tr>
</tbody>
</table>

Estimated disruption cost $1 - \lambda \approx 20\%$ of profits

On average, pay 3.1% of profits in AC when adjust
Cooper and Haltiwanger (2006): Wrapping Up

- What types of adjustment costs do we need to match micro data?
Cooper and Haltiwanger (2006): Wrapping Up

- What types of adjustment costs do we need to match micro data? **Non-convexities:**
  - Fixed costs
  - Disruption costs
  - Irreversibilities
Cooper and Haltiwanger (2006): Wrapping Up

- What types of adjustment costs do we need to match micro data? Non-convexities:
  - Fixed costs
  - Disruption costs
  - Irreversibilities

- Nice illustration of Simulated Method of Moments (SMM) methodology
  - Specify moments of the data you think are important
  - Select parameters which are well-identified by those moments
  - Choose parameters to get model as close as possible to data
Asker, Collard-Wexler, and De Loecker (2014)

Asker, Collard-Wexler, and De Loecker (2014)

Shows Cooper-Haltiwanger (2006) model also explains much of MRPK$_{it}$ dispersion documented by Hsieh and Klenow (2009)

**Data:** LRD, 1972 - 1997
- Also use cross-country data for analysis in paper

**Model:** Cooper-Haltiwanger (2006) opportunity cost model

\[
v(z_{it}, k_{it}) = \max_{i_{it}} e^{z_{it} k_{it}^\alpha} - i_{it} - c(i_{it}, k_{it}, z_{it}) + \frac{1}{1 + r} \mathbb{E}_t [v(z_{i_{t+1}}, k_{i_{t+1}})]
\]

\[
c(i_{it}, k_{it}, z_{it}) = \frac{\gamma}{2} \left( \frac{i_{it}}{k_{it}} \right)^2 k_{it} + \mathbb{1} (i_{it} \neq 0) \lambda e^{z_{it} k_{it}^\alpha}
\]
Estimation

**Estimate** $\Theta = (\gamma, \lambda)$ using SMM

$$\hat{\Theta} = \arg\min_{\Theta} [\psi_d - \psi_s(\Theta)]^T W [\psi_d - \psi_s(\Theta)]$$
Estimation

**Estimate** $\Theta = (\gamma, \lambda)$ using SMM

$$\hat{\Theta} = \arg \min_\Theta [\psi_d - \psi_s(\Theta)]^T W [\psi_d - \psi_s(\Theta)]$$

### Adjustment Cost Estimates and Moments by Country

<table>
<thead>
<tr>
<th>Country</th>
<th>Convex</th>
<th>Fixed</th>
<th>Less than 5%</th>
<th>More than 20%</th>
<th>Standard Deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>United States</td>
<td>8.80</td>
<td>.09</td>
<td>.39</td>
<td>.09</td>
<td>.21</td>
</tr>
<tr>
<td>Chile</td>
<td>4.10</td>
<td>.07</td>
<td>.19</td>
<td>.11</td>
<td>.28</td>
</tr>
<tr>
<td>India</td>
<td>3.46</td>
<td>.12</td>
<td>.29</td>
<td>.19</td>
<td>.30</td>
</tr>
<tr>
<td>France</td>
<td>.21</td>
<td>.00</td>
<td>.13</td>
<td>.57</td>
<td>.57</td>
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<tr>
<td>Spain</td>
<td>.74</td>
<td>.00</td>
<td>.20</td>
<td>.41</td>
<td>.59</td>
</tr>
<tr>
<td>Mexico</td>
<td>1.15</td>
<td>.22</td>
<td>.08</td>
<td>.73</td>
<td>.66</td>
</tr>
<tr>
<td>Romania</td>
<td>.66</td>
<td>.03</td>
<td>.08</td>
<td>.61</td>
<td>.72</td>
</tr>
<tr>
<td>Slovenia</td>
<td>.35</td>
<td>.00</td>
<td>.15</td>
<td>.52</td>
<td>.76</td>
</tr>
</tbody>
</table>

**Note.**—Standard errors were computed using the usual formula for minimum-distance estimators. However, because of the large size of the data sets we employ, the standard errors are of the order of $1 \times 10^{-3}$ or smaller, and so we do not report them. Adjustment costs for Slovenia are based on a model with production function coefficients set to the mean US coefficients (see the discussion in Sec. V.B).
Higher Idiosyncratic Volatility $\rightarrow$ Higher MRPK Dispersion
Higher Idiosyncratic Volatility $\rightarrow$ Higher MRPK Dispersion

- $\text{MRPK}_{it} = \alpha \frac{y_{it}}{k_{it}}$
- Time to build $\rightarrow$ ex-post dispersion
- Adjustment costs $\rightarrow$ ex-ante dispersion
Idiosyncratic Volatility and MRPK Dispersion in Data

Fig. 2.—Volatility and the dispersion in MRPK: US plant data, 1972–97. The unit of observation is the industry. The line is generated by an OLS regression on 188 industries, in which the estimated slope is 0.73 (0.08) and the constant is 0.57 (0.03), and the $R^2 = .3$, where the standard errors are in parentheses.
## Idiosyncratic Volatility and MRPK Dispersion in Data

<table>
<thead>
<tr>
<th>Country</th>
<th>Coefficient</th>
<th>$R^2$</th>
<th>Industry-Year Observations</th>
</tr>
</thead>
<tbody>
<tr>
<td>United States: Plants</td>
<td>.76*** (.04)</td>
<td>.47</td>
<td>4,037</td>
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<tr>
<td>United States: Firms</td>
<td>.68*** (.07)</td>
<td>.44</td>
<td>4,037</td>
</tr>
<tr>
<td>Chile</td>
<td>.54* (.29)</td>
<td>.13</td>
<td>55</td>
</tr>
<tr>
<td>France</td>
<td>1.03*** (.33)</td>
<td>.28</td>
<td>167</td>
</tr>
<tr>
<td>India</td>
<td>.61** (.17)</td>
<td>.28</td>
<td>279</td>
</tr>
<tr>
<td>Mexico</td>
<td>.19** (.07)</td>
<td>.07</td>
<td>296</td>
</tr>
<tr>
<td>Romania</td>
<td>.44*** (.13)</td>
<td>.21</td>
<td>126</td>
</tr>
<tr>
<td>Slovenia</td>
<td>.53** (.21)</td>
<td>.09</td>
<td>108</td>
</tr>
<tr>
<td>Spain</td>
<td>.56* (.33)</td>
<td>.35</td>
<td>181</td>
</tr>
<tr>
<td>All: Unweighted</td>
<td>.55*** (.15)</td>
<td>.67</td>
<td>5,326</td>
</tr>
<tr>
<td>All: Weighted</td>
<td>.74*** (.03)</td>
<td>.50</td>
<td>5,326</td>
</tr>
</tbody>
</table>
# Quantitative Amount of Dispersion Explained

<table>
<thead>
<tr>
<th>Country</th>
<th>Specification</th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
<th>(5)</th>
</tr>
</thead>
<tbody>
<tr>
<td>United States</td>
<td></td>
<td>.223</td>
<td>.806</td>
<td>.806</td>
<td>.643</td>
<td>.820</td>
</tr>
<tr>
<td>France</td>
<td></td>
<td>.892</td>
<td>.702</td>
<td>.899</td>
<td>.944</td>
<td>.651</td>
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<tr>
<td>Chile</td>
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<td>.994</td>
<td>.983</td>
<td>.987</td>
<td>.963</td>
<td>.785</td>
</tr>
<tr>
<td>India</td>
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<td>.984</td>
<td>.941</td>
<td>.964</td>
<td>.976</td>
<td>.596</td>
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<tr>
<td>Mexico</td>
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<td>.879</td>
<td>.813</td>
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<td>.908</td>
<td>.634</td>
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<tr>
<td>Romania</td>
<td></td>
<td>.983</td>
<td>.923</td>
<td>.817</td>
<td>.702</td>
<td>.846</td>
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<tr>
<td>Slovenia</td>
<td></td>
<td>.967</td>
<td>.774</td>
<td>.967</td>
<td>.984</td>
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<tr>
<td>Spain</td>
<td></td>
<td>.718</td>
<td>.627</td>
<td>.600</td>
<td>.530</td>
<td>.495</td>
</tr>
<tr>
<td>All (excluding United States)</td>
<td></td>
<td>.879</td>
<td>.777</td>
<td>.820</td>
<td>.800</td>
<td>.640</td>
</tr>
<tr>
<td>All</td>
<td></td>
<td>.674</td>
<td>.786</td>
<td>.816</td>
<td>.748</td>
<td>.696</td>
</tr>
</tbody>
</table>

Specification details:
- All US adjusted costs: X
- Own-country adjusted costs: X
- All 2 × US adjusted costs: X
- 1-period time to build only: X
- US average β’s: X
- Industry-country β’s: X

**Note.**—The unit of observation is the country-industry. Specifications are as follows: (1) All countries have the United States’ estimated adjustment costs and production coefficients equal to the US averages across industries; (2) industry-country-specific production coefficients (except for Slovenia; see Sec. III.B), country specific adjustment costs, and industry-country-specific AR(1); (3) same as for 2, but with the United States’ estimated adjustment costs for all countries; (4) same as for 3, but with twice the United States’ estimated adjustment costs for all countries; and (5) same as for 3, but with zero adjustment costs (other than the one-period time to build) for all countries. In all specifications, the
Plan for this Topic

1. An unfair summary of the empirical investment literature

2. Accounting for micro-level investment behavior with nonconvex adjustment costs

3. Macro implications of nonconvex adjustment costs
Aggregate Implications of Micro Investment Models

1. Aggregation of micro-level models holding prices fixed (partial equilibrium)

   • Response of aggregate investment to shocks depends on number of firms who adjust
   • Aggregate investment features time-varying elasticity w.r.t. shocks
   • Representative firm instead predicts constant elasticity
1. Aggregation of micro-level models holding prices fixed (partial equilibrium)
   - Response of aggregate investment to shocks depends on number of firms who adjust
   - Aggregate investment features time-varying elasticity w.r.t. shocks
   - Representative firm instead predicts constant elasticity

2. Endogenize prices in general equilibrium
   - In benchmark RBC framework, procyclical real interest rate eliminates time-varying elasticity
   - Modifications to benchmark model can break this irrelevance result
General Lessons

1. Anytime you go from micro to macro, need to think about
   - Aggregation
   - General equilibrium

2. Macro models with micro heterogeneity are hard
   - Entire cross-sectional distribution of agents part of state vector
   - Difficult to numerically compute and estimate
General Lessons

1. Anytime you go from micro to macro, need to think about
   - Aggregation
   - General equilibrium

2. Macro models with micro heterogeneity are hard
   - Entire cross-sectional distribution of agents part of state vector
   - Difficult to numerically compute and estimate

   • Aggregate implications of lumpy investment models good illustration of these more general issues
     • Each of these steps has been extensively studied
Outline of Next Steps

1. Benchmark general equilibrium model with lumpy investment: Khan and Thomas (2008)
   • Aside: how to numerically compute heterogeneous agent models

2. Model generates time-varying elasticity in partial equilibrium

3. Model generates constant elasticity in general equilibrium

4. Two broad responses to irrelevance result in literature
   • Specification of micro-level adjustment costs
   • Specification of general equilibrium
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Model Overview

Heterogeneous Firms

• Fixed mass
• Idiosyncratic + aggregate productivity shocks
• Fixed capital adjustment costs

Representative Household

• Owns firms
• Supplies labor
• Complete markets
Heterogeneous Firms

Production technology \( y_{jt} = e^{z_t} e^{\varepsilon_{jt}} k_j^\theta n_j^\nu, \theta + \nu < 1 \)

- Idiosyncratic productivity shock \( \varepsilon_{jt+1} = \rho_\varepsilon \varepsilon_{jt} + \omega^\varepsilon_{jt+1} \) where \( \omega^\varepsilon_{jt+1} \sim N(0, \sigma_\varepsilon^2) \)
- Aggregate productivity shock \( z_{t+1} = \rho_z z_t + \omega^z_{t+1} \) where \( \omega^z_{t+1} \sim N(0, \sigma_z^2) \)
Heterogeneous Firms

**Production technology** \( y_{jt} = e^{z_t} e^{\epsilon_{jt}} k_{jt}^{\theta} n_{jt}^{\nu}, \theta + \nu < 1 \)

- Idiosyncratic productivity shock \( \epsilon_{jt+1} = \rho_{\epsilon} \epsilon_{jt} + \omega_{jt+1}^{\epsilon} \) where \( \omega_{jt+1}^{\epsilon} \sim N(0, \sigma_{\epsilon}^2) \)
- Aggregate productivity shock \( z_{t+1} = \rho_z z_t + \omega_{t+1}^{z} \) where \( \omega_{t+1}^{z} \sim N(0, \sigma_{z}^2) \)

**Firms accumulate capital** according to \( k_{jt+1} = (1 - \delta) k_{jt} + i_{jt} \)

- If \( \frac{i_{jt}}{k_{jt}} \notin [-a, a] \), pay fixed cost \( \xi_{jt} \) in units of labor
- Fixed cost \( \xi_{jt} \sim U[0, \bar{\xi}] \)
Firm Optimization Problem: Recursive Formulation

\[
v(\varepsilon, k, \xi; \mathbf{s}) = \max_n e^{z_n} e^{\varepsilon k \theta_n} n^\nu - w(\mathbf{s}) n
\]

\[
+ \max \left\{ v^A(\varepsilon, k; \mathbf{s}) - w(\mathbf{s}) \xi, v^N(\varepsilon, k; \mathbf{s}) \right\}
\]
Firm Optimization Problem: Recursive Formulation

\( v(\varepsilon, k, \xi; s) = \max_n e^n \varepsilon k^n - w(s) \ n \)

\[ + \max \left\{ v^A(\varepsilon, k; s) - w(s) \xi, v^N(\varepsilon, k; s) \right\} \]

\( v^A(\varepsilon, k; s) = \max_{i \in \mathbb{R}} -i + \mathbb{E} \left[ \Lambda(s') v(\varepsilon', k', \xi'; s') \right] | \varepsilon, k; s \]

\( v^N(\varepsilon, k; s) = \max_{i \in [-ak, ak]} -i + \mathbb{E} \left[ \Lambda(s') v(\varepsilon', k', \xi'; s') \right] | \varepsilon, k; s \)
Firm Optimization Problem: Recursive Formulation

\[ v(\varepsilon, k, \xi; s) = \max_n e^z e^\xi n^\nu - w(s) n \]

\[ + \max \left\{ v^A(\varepsilon, k; s) - w(s) \xi, v^N(\varepsilon, k; s) \right\} \]

\[ \hat{v}(\varepsilon, k; s) = \max_n e^z e^\xi n^\nu - w(s) n \]

\[ + \frac{\hat{\xi}(\varepsilon, k; s)}{\xi} \left( v^A(\varepsilon, k; s) - w(s) \frac{\hat{\xi}(\varepsilon, k; s)}{2} \right) \]

\[ + \left( 1 - \frac{\hat{\xi}(\varepsilon, k; s)}{\xi} \right) v^N(\varepsilon, k; s) \]
Household

**Representative household** who owns all firms in the economy

$$\max_{C_t, \Pi_t} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t (\log C_t - aN_t) \text{ such that }$$

$$C_t = w_t N_t + \Pi_t$$
**Representative household** who owns all firms in the economy

\[
\max_{C_t, N_t} E_0 \sum_{t=0}^{\infty} \beta^t (\log C_t - aN_t) \quad \text{such that}
\]

\[
C_t = w_t N_t + \Pi_t
\]

**Complete markets** implies that \( \Lambda_{t,t+1} = \beta \left( \frac{C_{t+1}}{C_t} \right)^{-1} \)

- Firms maximize their market value
- Market value given by expected present value of dividends using stochastic discount factor
- With complete markets, SDF is household’s *intertemporal marginal rate of substitution*
Defining Recursive Competitive Equilibrium

What is the aggregate state $s$?
Defining Recursive Competitive Equilibrium

What is the aggregate state $s$?

- Aggregate shock $z$
Defining Recursive Competitive Equilibrium

What is the aggregate state $s$?

- Aggregate shock $z$
- Firm’s individual states: productivity $\varepsilon$ and capital $k$
Defining Recursive Competitive Equilibrium

What is the aggregate state $s$?

- Aggregate shock $z$
- Firm’s individual states: productivity $\varepsilon$ and capital $k$
  $\rightarrow$ need distribution of firms $g(\varepsilon, k)$
Defining Recursive Competitive Equilibrium

What is the aggregate state $s$?

- Aggregate shock $z$
- Firm’s individual states: productivity $\varepsilon$ and capital $k$
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What is the law of motion for the $s$?
Defining Recursive Competitive Equilibrium

What is the aggregate state $s$?

- Aggregate shock $z$
- Firm’s individual states: productivity $\varepsilon$ and capital $k$
  $\rightarrow$ need distribution of firms $g(\varepsilon, k)$

What is the law of motion for the $s$?

$$g_{t+1}(\varepsilon', k') = \int \left[ \times \int 1\{\rho_\varepsilon \varepsilon + \sigma_\varepsilon \omega' = \varepsilon' \} \int 1 \{k'_t(\varepsilon, k, \xi) = k' \} dG(\xi) \right] \times p(\omega'_\varepsilon) g_t(\varepsilon, k) d\omega'_\varepsilon d\varepsilon dk$$
Recursive Competitive Equilibrium

A set of $v(\varepsilon, k; z, g)$, $C(z, g)$, $w(z, g)$, $\Lambda(z'; z, g)$, and $g'(z, g)$ such that
Recursive Competitive Equilibrium

A set of $v(\varepsilon, k; z, g)$, $C(z, g)$, $w(z, g)$, $\Lambda(z'; z, g)$, and $g'(z, g)$ such that

1. **Firm optimization**: Taking $\Lambda(z'; z, g)$ and $w(z, g)$ as given, $v(\varepsilon, k; z, g)$ solves Bellman equation
Recursive Competitive Equilibrium

A set of $v(\varepsilon, k; z, g)$, $C(z, g)$, $w(z, g)$, $\Lambda(z'; z, g)$, and $g'(z, g)$ such that

1. **Firm optimization**: Taking $\Lambda(z'; z, g)$ and $w(z, g)$ as given, $v(\varepsilon, k; z, g)$ solves Bellman equation

2. **Household optimization**: $w(z, g)C(z, g)^{-1} = a$
Recursive Competitive Equilibrium

A set of $v(\epsilon, k; z, g)$, $C(z, g)$, $w(z, g)$, $\Lambda(z'; z, g)$, and $g'(z, g)$ such that

1. **Firm optimization**: Taking $\Lambda(z'; z, g)$ and $w(z, g)$ as given, $v(\epsilon, k; z, g)$ solves Bellman equation

2. **Household optimization**: $w(z, g)C(z, g)^{-1} = a$

3. **Market clearing + consistency**:

$$\Lambda(z'; z, g) = \beta \left( \frac{C(z', g'(z, g))}{C(z, g)} \right)^{-1}$$

$$C(z, g) = \int (y(\epsilon, k, \xi; z, g) - i(\epsilon, k, \xi; z, g))dG(\xi)g(\epsilon, k)d\epsilon dk$$

$g'(\epsilon, k)$ satisfies law of motion for distribution
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   - Aside: how to numerically compute heterogeneous agent models

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4. Two broad responses to irrelevance result in literature
   - Specification of micro-level adjustment costs
   - Specification of general equilibrium
Computing Equilibrium

• Key challenge: aggregate state $g$ is infinite-dimensional
Computing Equilibrium

• Key challenge: aggregate state $g$ is infinite-dimensional

• Two steps:
  1. Compute **steady state without aggregate shocks** → distribution constant at $g^*$
  2. Compute **full model with aggregate shocks** → distribution varies over time
Computing Equilibrium

• Key challenge: aggregate state $g$ is infinite-dimensional

• Two steps:
  1. Compute steady state without aggregate shocks → distribution constant at $g^*$
  2. Compute full model with aggregate shocks → distribution varies over time

• Today will give you an overview to help you read papers
  • My HW2: solve steady state
  • Aggregate dynamics: Khan and Thomas (2008); Winberry (2016); Terry (2016)
Steady State Recursive Competitive Equilibrium

A set of $v^*(\epsilon, k)$, $C^*$, $w^*$, and $g^*(\epsilon, k)$ such that
Steady State Recursive Competitive Equilibrium

A set of $v^*(\varepsilon, k), C^*, w^*$, and $g^*(\varepsilon, k)$ such that

1. **Firm optimization**: Taking $w^*$ as given: $v^*(\varepsilon, k)$ solves Bellman equation
Steady State Recursive Competitive Equilibrium

A set of $v^*(\epsilon, k)$, $C^*$, $w^*$, and $g^*(\epsilon, k)$ such that

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Steady State Recursive Competitive Equilibrium

A set of $v^*(\varepsilon, k), C^*, w^*$, and $g^*(\varepsilon, k)$ such that

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2. **Household optimization**: Taking $w^*$ as given: $w^*(C^*)^{-1} = a$

3. **Markets clearing + consistency**:

$$C^* = \int (y(\varepsilon, k, \xi) - i(\varepsilon, k, \xi))dG(\xi)g^*(\varepsilon, k)d\varepsilon dk$$

$g^*(\varepsilon, k)$ satisfies law of motion for distribution given $g^*$
Hopenhayn-Rogerson (1993) Algorithm

Start with guess of $w^*$
Hopenhayn-Rogerson (1993) Algorithm

Start with guess of $w^*$

- Solve firm optimization problem $\rightarrow v^*(\varepsilon, k)$
Hopenhayn-Rogerson (1993) Algorithm

Start with guess of $w^*$

- Solve firm optimization problem $\rightarrow v^*(\epsilon, k)$
- Compute stationary distribution $g^*(\epsilon, k)$
- Compute implied aggregate consumption $C^*$
Hopenhayn-Rogerson (1993) Algorithm

Start with guess of $w^*$

- Solve firm optimization problem $\rightarrow v^*(\varepsilon, k)$
- Compute stationary distribution $g^*(\varepsilon, k)$
- Compute implied aggregate consumption $C^*$
- Check household optimization $w^*(C^*)^{-1} = a$
Hopenhayn-Rogerson (1993) Algorithm

**Start with guess of** $w^*$

- Solve **firm optimization** problem $\rightarrow v^*(\varepsilon, k)$
- Compute **stationary distribution** $g^*(\varepsilon, k)$
- Compute implied aggregate consumption $C^*$
- Check **household optimization** $w^*(C^*)^{-1} = a$

**Update guess of** $w^*$
Steady State Outcomes

Distribution in model with no idiosyncratic productivity shocks
Investment decision characterized by adjustment hazard
Full Model with Aggregate Shocks

- Outside of steady state, three key challenges

  1. Distribution $g$ varies over time! How to approximate distribution?
  2. Law of motion for $g$ is complicated! How to approximate law of motion?
  3. Prices are functions of distribution! How to approximate these functions?

- Will briefly describe two approaches to dealing with these challenges

  2. Winberry (2006): approximate distribution with flexible parametric family

- If curious: continuous time makes this easier (Ahn, Kaplan, Moll, Winberry, and Wolf, 2007)
Full Model with Aggregate Shocks

- Outside of steady state, three key challenges
  1. Distribution $g$ varies over time → how to approximate distribution?
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Full Model with Aggregate Shocks

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Krusell and Smith (1998)

- Approximate distribution with moments, e.g., \( g(\varepsilon, k) \approx \bar{K} \)
Krusell and Smith (1998)

- Approximate distribution with moments, e.g., \( g(\varepsilon, k) \approx \bar{K} \)
  - Law of motion: \( \log \bar{K}' = \alpha_0 + \alpha_1 z + \alpha_2 \log \bar{K} \)
Krusell and Smith (1998)

- Approximate distribution with moments, e.g., \( g(\varepsilon, k) \approx \bar{K} \)
- Law of motion: \( \log K' = \alpha_0 + \alpha_1 z + \alpha_2 \log K \)
- Pricing functions: \( \log C = \gamma_0 + \gamma_1 z + \gamma_2 \log K \)
Krusell and Smith (1998)

- Approximate distribution with moments, e.g., \( g(\varepsilon, k) \approx \bar{K} \)
  - Law of motion: \( \log \bar{K}' = \alpha_0 + \alpha_1 z + \alpha_2 \log \bar{K} \)
  - Pricing functions: \( \log C = \gamma_0 + \gamma_1 z + \gamma_2 \log \bar{K} \)

- **Given guess \( \alpha \) and \( \gamma \)**
  - Compute **individual decisions** \( v(\varepsilon, k; z, \bar{K}) \)
  - **Simulate** decision rules \( \rightarrow \{ \bar{K}_t, C_t, z_t \} \)

- **Update \( \alpha \) and \( \gamma \) using OLS**
Krusell and Smith (1998)

- Approximate distribution with moments, e.g., \( g(\varepsilon, k) \approx \bar{K} \)
  - Law of motion: \( \log \bar{K}' = \alpha_0 + \alpha_1 z + \alpha_2 \log \bar{K} \)
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- **Given guess** \( \alpha \) **and** \( \gamma \)
  - Compute **individual decisions** \( v(\varepsilon, k; z, \bar{K}) \)
  - Simulate decision rules \( \rightarrow \{\bar{K}_t, C_t, z_t\} \)

- **Update** \( \alpha \) **and** \( \gamma \) **using OLS**

- \( R^2 \) on regressions typical **accuracy measure**
  - Only \( \bar{K} \) matters \( \rightarrow \) distribution not important ("**approximate aggregation**")
  - Problems with this measure: Den Haan (2010)
Approximate distribution with \textbf{parametric family}:

\[
g(\epsilon, k) \approx g_0 \exp\{g_1^1 (\epsilon - m_1^1) + g_2^2 (k - m_2^2) + \sum_{i=2}^{n_g} \sum_{j=0}^{i} g_i^j \left[ (\epsilon - m_1^1)^{i-j} (k - m_2^2)^j - m_i^j \right] \}
\]

\[\rightarrow\text{Aggregate state approximated by } (z, g(\epsilon, k)) \approx (z, \mathbf{m})\]
Winberry (2018)

- Approximate distribution with parametric family:

\[
g(\epsilon, k) \approx g_0 \exp\{g_1^1 (\epsilon - m_1^1) + g_1^2 (k - m_1^2) + \sum_{i=2}^{n_g} \sum_{j=0}^{i} g_i^j \left[ (\epsilon - m_1^1)^{i-j} (k - m_1^2)^j - m_i^j \right] \}\]

→ Aggregate state approximated by \((z, g(\epsilon, k)) \approx (z, m)\)

- Compute law of motion + prices directly by integration
Winberry (2018)

• Approximate distribution with parametric family:

\[ g(\epsilon, k) \approx g_0 \exp\left\{ g_1^{1} (\epsilon - m_1^{1}) + g_2^{2} (k - m_2^{2}) + \sum_{i=2}^{n_g} \sum_{j=0}^{i} g_i^{j} \left[ (\epsilon - m_1^{1})^{i-j} (k - m_2^{2})^{j} - m_i^{j} \right] \right\} \]

→ Aggregate state approximated by \( (z, g(\epsilon, k)) \approx (z, m) \)

• Compute law of motion + prices directly by integration

• Compute aggregate dynamics using perturbation methods
  • Solve for steady state in Matlab
  • Solve for aggregate dynamics using Dynare
Winberry (2018)

Productivity

\[ n_g = 1 \]
\[ n_g = 2 \]
\[ n_g = 6 \]
Exact

Capital

• Run time \( \Psi_v - \Theta_v \) seconds for accurate approximation
• Fast enough for likelihood-based estimation
• Codes at my website
Winberry (2018)

- Run time $\approx$ 20 - 40 seconds for accurate approximation
- Fast enough for likelihood-based estimation
- Codes at my website
Outline of Next Steps

1. Benchmark general equilibrium model with lumpy investment: Khan and Thomas (2008)
   • Aside: how to numerically compute heterogeneous agent models

2. Model generates time-varying elasticity in partial equilibrium

3. Model generates constant elasticity in general equilibrium

4. Two broad responses to irrelevance result in literature
   • Specification of micro-level adjustment costs
   • Specification of general equilibrium
## Khan and Thomas (2008) Calibration

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Households</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\beta$</td>
<td>Discount factor</td>
<td>.961</td>
</tr>
<tr>
<td>$\psi$</td>
<td>Labor disutility</td>
<td>$N^* = \frac{1}{3}$</td>
</tr>
<tr>
<td><strong>Firms</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\nu$</td>
<td>Labor share</td>
<td>.64</td>
</tr>
<tr>
<td>$\theta$</td>
<td>Capital share</td>
<td>.256</td>
</tr>
<tr>
<td>$\delta$</td>
<td>Capital depreciation</td>
<td>.085</td>
</tr>
<tr>
<td>$\xi$</td>
<td>Fixed cost</td>
<td>.0083</td>
</tr>
<tr>
<td>$a$</td>
<td>No fixed cost region</td>
<td>.011</td>
</tr>
<tr>
<td>$\rho_\epsilon$</td>
<td>Idiosyncratic TFP AR(1)</td>
<td>.859</td>
</tr>
<tr>
<td>$\sigma_\epsilon$</td>
<td>Idiosyncratic TFP AR(1)</td>
<td>.022</td>
</tr>
<tr>
<td><strong>Aggregate shock</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\rho_z$</td>
<td>Aggregate TFP AR(1)</td>
<td>.859</td>
</tr>
<tr>
<td>$\sigma_z$</td>
<td>Aggregate TFP AR(1)</td>
<td>.014</td>
</tr>
</tbody>
</table>
Complicated Impulse Responses

Response of aggregate investment to shock depends on interaction of initial distribution and adjustment hazards.
Implication: Sign Dependence

Aggregate investment more responsive to positive than negative shocks

Note true in frictionless model
Implication: State Dependence

From Bachmann, Caballero, and Engel (2013)

\[
\frac{l_t}{K_t} = \sum_{j=1}^{p} \phi_j \frac{l_{t-j}}{K_{t-j}} + \sigma_t e_t
\]

\[
\sigma_t = \alpha_1 + \eta_1 \frac{1}{p} \sum_{j=1}^{p} \frac{l_{t-j}}{K_{t-j}}
\]
Aggregate Nonlinearities

- Both of these are examples of **nonlinear aggregate dynamics**
  - Linear model has constant loading on aggregate shock
Aggregate Nonlinearities

- Both of these are examples of **nonlinear aggregate dynamics**
  - Linear model has constant loading on aggregate shock
- Some evidence in aggregate data
  - Sign and state dependence → distribution of $\frac{I_t}{K_t}$ positively skewed
  - State dependence → dynamics of $\frac{I_t}{K_t}$ feature **conditional heteroskedasticity**
Aggregate Nonlinearities

• Both of these are examples of nonlinear aggregate dynamics
  • Linear model has constant loading on aggregate shock

• Some evidence in aggregate data
  • Sign and state dependence → distribution of $\frac{I_t}{K_t}$ positively skewed
  • State dependence → dynamics of $\frac{I_t}{K_t}$ feature conditional heteroskedasticity

• My view: time series evidence is suggestive at best
  • Predictions are about extreme states, which are rare
  • But that is exactly when we care about these predictions!
    → rely on cross-sectional data + carefully specified general equilibrium model
Outline of Next Steps

1. Benchmark general equilibrium model with lumpy investment: Khan and Thomas (2008)
   • Aside: how to numerically compute heterogeneous agent models

2. Model generates time-varying elasticity in partial equilibrium

3. **Model generates constant elasticity in general equilibrium**

4. Two broad responses to irrelevance result in literature
   • Specification of micro-level adjustment costs
   • Specification of general equilibrium

5. If time, discuss policy implications
Distribution of Aggregate $\frac{I_t}{K_t}$ in Partial Equilibrium
Distribution of Aggregate $\frac{I_t}{K_t}$ in General Equilibrium
Distribution of Aggregate $\frac{I_t}{K_t}$ in General Equilibrium

**TABLE III**

**ROLE OF NONCONVEXITIES IN AGGREGATE INVESTMENT RATE DYNAMICS**

<table>
<thead>
<tr>
<th></th>
<th>Persistence</th>
<th>Standard Deviation</th>
<th>Skewness</th>
<th>Excess Kurtosis</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Postwar U.S. data</strong>a</td>
<td>0.695</td>
<td>0.008</td>
<td>0.008</td>
<td>−0.715</td>
</tr>
<tr>
<td><strong>A. Partial equilibrium models</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>PE frictionless</td>
<td>−0.069</td>
<td>0.128</td>
<td>0.358</td>
<td>0.140</td>
</tr>
<tr>
<td>PE lumpy investment</td>
<td>0.210</td>
<td>0.085</td>
<td>1.121</td>
<td>2.313</td>
</tr>
<tr>
<td><strong>B. General equilibrium models</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>GE frictionless</td>
<td>0.659</td>
<td>0.010</td>
<td>0.048</td>
<td>0.048</td>
</tr>
<tr>
<td>GE lumpy investment</td>
<td>0.662</td>
<td>0.010</td>
<td>0.067</td>
<td>−0.074</td>
</tr>
</tbody>
</table>

*a* Data are annual private investment-to-capital ratio, 1954–2005, computed using Bureau of Economic Analysis tables.
## Business Cycles Nearly Identical to Representative Firm

### Table IV

<table>
<thead>
<tr>
<th></th>
<th>Output</th>
<th>TFP&lt;sup&gt;a&lt;/sup&gt;</th>
<th>Hours</th>
<th>Consump.</th>
<th>Invest.</th>
<th>Capital</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>A. Standard deviations relative to output&lt;sup&gt;b&lt;/sup&gt;</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>GE frictionless</td>
<td>(2.277)</td>
<td>0.602</td>
<td>0.645</td>
<td>0.429</td>
<td>3.562</td>
<td>0.494</td>
</tr>
<tr>
<td>GE lumpy</td>
<td>(2.264)</td>
<td>0.605</td>
<td>0.639</td>
<td>0.433</td>
<td>3.539</td>
<td>0.492</td>
</tr>
<tr>
<td><strong>B. Contemporaneous correlations with output</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>GE frictionless</td>
<td>1.000</td>
<td>0.955</td>
<td>0.895</td>
<td>0.976</td>
<td>0.034</td>
<td></td>
</tr>
<tr>
<td>GE lumpy</td>
<td>1.000</td>
<td>0.956</td>
<td>0.900</td>
<td>0.976</td>
<td>0.034</td>
<td></td>
</tr>
</tbody>
</table>

<sup>a</sup> Total factor productivity.

<sup>b</sup> The logarithm of each series is Hodrick–Prescott-filtered using a weight of 100. The output column of panel A reports percent standard deviations of output in parentheses.
Why Do the Nonlinearities Disappear?

General equilibrium price movements

- Time-varying elasticity comes from large movements in adjustment hazard
- Procyclical real interest rate and wage restrain those movements

\[ 1 + r_t = \frac{1}{\mathbb{E}[\Lambda_{t,t+1}]} \]
Why Do the Nonlinearities Disappear?

**General equilibrium price movements**

- Time-varying elasticity comes from large movements in adjustment hazard
- Pro-cyclical real interest rate and wage restrain those movements

\[ 1 + r_t = \frac{1}{\mathbb{E}[\Lambda_{t,t+1}]} \]

**Specification of adjustment costs**

- Calibrated adjustment costs small
Outline of Next Steps

1. Benchmark general equilibrium model with lumpy investment: Khan and Thomas (2008)
   - Aside: how to numerically compute heterogeneous agent models

2. Model generates time-varying elasticity in partial equilibrium

3. Model generates constant elasticity in general equilibrium

4. Two broad responses to irrelevance result in literature
   - Specification of micro-level adjustment costs
   - Specification of general equilibrium
Outline of Next Steps

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   • Specification of general equilibrium
Bachmann, Caballero, and Engel (2013)

- Argue Khan and Thomas’ calibration of adjustment costs responsible for irrelevance result
- Calibrate larger adjustment costs and recover aggregate nonlinearities
Bachmann, Caballero, and Engel (2013)

- Argue Khan and Thomas’ calibration of adjustment costs responsible for irrelevance result
- Calibrate larger adjustment costs and recover aggregate nonlinearities
- Argument based on decomposition between AC smoothing and PR smoothing
  - Frictionless partial equilibrium model excessively volatile
  - **AC smoothing**: dampening due to adjustment costs
  - **PR smoothing**: dampening due to price movements
- Measure AC smoothing in data and target in calibration → higher adjustment costs
Model

**Production technology** \( y_{jt} = e^{z_t} e^{\epsilon_{st}} e^{\epsilon_{jt}} k_{jt}^\theta n_{jt}^\nu, \theta + \nu < 1 \)

- Idiosyncratic productivity shock \( \epsilon_{jt+1} = \rho_\epsilon \epsilon_{jt} + \omega_{jt+1}^{\epsilon} \) where \( \omega_{jt+1}^{\epsilon} \sim N(0, \sigma_\epsilon^2) \)
- Aggregate productivity shock \( z_{t+1} = \rho_z z_t + \omega_{t+1}^{z} \) where \( \omega_{t+1}^{z} \sim N(0, \sigma_z^2) \)
- Sectoral productivity shock \( \epsilon_{st+1} = \rho_\epsilon \epsilon_{st} + \omega_{st+1}^{\epsilon} \) where \( \omega_{st+1}^{\epsilon} \sim N(0, \sigma_\epsilon^{s}) \)
Model

**Production technology** \( y_{jt} = e^{zt} e^{\varepsilon_{st+1}} e^{\varepsilon_{jt}} k_{jt} n_{jt}^{\theta}, \theta + \nu < 1 \)

- Idiosyncratic productivity shock \( \varepsilon_{jt+1} = \rho_{\varepsilon} \varepsilon_{jt} + \omega_{jt+1}^{\varepsilon} \) where \( \omega_{jt+1}^{\varepsilon} \sim N(0, \sigma_{\varepsilon}^2) \)
- Aggregate productivity shock \( z_{t+1} = \rho_z z_t + \omega_{t+1}^{z} \) where \( \omega_{t+1}^{z} \sim N(0, \sigma_z^2) \)
- Sectoral productivity shock \( \varepsilon_{st+1} = \rho_{\varepsilon} \varepsilon_{st} + \omega_{st+1}^{\varepsilon} \) where \( \omega_{st+1}^{\varepsilon} \sim N(0, \sigma_{\varepsilon s}^2) \)

Firms **accumulate capital** according to \( k_{jt+1} = (1 - \delta) k_{jt} + i_{jt} \)

- If don’t pay fixed cost, must undertake maintenance investment \( \chi \times \delta k_{jt} \)
- Otherwise, pay fixed cost \( \xi_{jt} \) in units of labor
- Fixed cost \( \xi_{jt} \sim U[0, \bar{\xi}] \)
Calibration

Set most parameters \textit{exogenously}

Choose $\sigma_z$, $\bar{\xi}$, and $\chi$ to match degree of \textit{AC-smoothing}

- Identify AC-smoothing using \textit{volatility of sectoral investment rates}
  - Aggregated enough to capture interaction of distribution and hazards
  - Small enough to not generate price response
Calibration

Set most parameters \textit{exogenously}

Choose $\sigma_z$, $\bar{\xi}$, and $\chi$ to match degree of AC-smoothing

- Identify AC-smoothing using \textit{volatility of sectoral investment rates}
  - Aggregated enough to capture interaction of distribution and hazards
  - Small enough to not generate price response

- Targets:
  1. Volatility of aggregate investment rate
  2. Average volatility of sectoral investment rates
  3. Amount of conditional heteroskedasticity
AC vs. PR Smoothing Decomposition

\[ UB = \log \left( \frac{\sigma(\text{none})}{\sigma(\text{AC})} \right) / \log \left( \frac{\sigma(\text{none})}{\sigma(\text{both})} \right) \]

\[ LB = 1 - \log \left( \frac{\sigma(\text{none})}{\sigma(\text{PR})} \right) / \log \left( \frac{\sigma(\text{none})}{\sigma(\text{both})} \right) \]
Calibrated Adjustment Costs

<table>
<thead>
<tr>
<th>Model</th>
<th>Adjustment costs/unit’s output (in percent)</th>
<th>Adjustment costs/unit’s wage bill (in percent)</th>
</tr>
</thead>
<tbody>
<tr>
<td>This paper ($\chi = 0$)</td>
<td>38.9</td>
<td>60.9</td>
</tr>
<tr>
<td>This paper ($\chi = 0.25$)</td>
<td>12.7</td>
<td>19.8</td>
</tr>
<tr>
<td>This paper ($\chi = 0.50$)</td>
<td>3.6</td>
<td>5.6</td>
</tr>
<tr>
<td>Caballero-Engel (1999)</td>
<td>16.5</td>
<td>—</td>
</tr>
<tr>
<td>Cooper-Haltiwanger (2006)</td>
<td>22.9</td>
<td>—</td>
</tr>
<tr>
<td>Bloom (2009)</td>
<td>35.4</td>
<td>—</td>
</tr>
<tr>
<td>Khan-Thomas (2008)</td>
<td>0.5</td>
<td>0.8</td>
</tr>
</tbody>
</table>

Notes: This table displays the average adjustment costs paid, conditional on adjustment, as a fraction of output (left column) and as a fraction of the wage bill (right column), for various models. Rows 4–6 are based on table IV in Bloom (2009). For Cooper and Haltiwanger (2006) and Bloom (2009) we report the sum of costs associated with two sources of lumpy adjustment: fixed adjustment costs and partial irreversibility. The remaining models only have fixed adjustment costs.
### Table 5—Heteroscedasticity Range

<table>
<thead>
<tr>
<th>Model</th>
<th>log ($\sigma_{95}/\sigma_{5}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Data</td>
<td>0.3021</td>
</tr>
<tr>
<td>This paper ($\chi = 0$)</td>
<td>0.1830</td>
</tr>
<tr>
<td>This paper ($\chi = 0.25$)</td>
<td>0.2173</td>
</tr>
<tr>
<td>This paper ($\chi = 0.50$)</td>
<td>0.2901</td>
</tr>
<tr>
<td>Quadratic adj. costs ($\chi = 0$)</td>
<td>0.0487</td>
</tr>
<tr>
<td>Quadratic adj. costs ($\chi = 0.25$)</td>
<td>0.0411</td>
</tr>
<tr>
<td>Quadratic adj. costs ($\chi = 0.50$)</td>
<td>0.0321</td>
</tr>
<tr>
<td>Frictionless</td>
<td>0.0539</td>
</tr>
<tr>
<td>Khan-Thomas (2008)</td>
<td>0.0468</td>
</tr>
</tbody>
</table>

Notes: This table displays heteroscedasticity range ($\log(\sigma_{95}/\sigma_{5})$) for the data (row 1) and various model specifications that vary in terms of the maintenance parameter $\chi$ and the adjustment technology for capital: fixed adjustment costs (rows 2–4), quadratic adjustment costs (rows 5–7), a frictionless model, and the Khan-Thomas (2008) model. The adjustment costs for the models in rows 2–7 have been calibrated to match aggregate and sectoral investment rate volatilities.
Aggregate Nonlinearities

**Time-series model**

**Lumpy model**

**FL-model**
Aggregate Nonlinearities

**Figure 3. Time Paths of the Responsiveness Index**
Aggregate Nonlinearities

Figure 7. Impulse Responses of the Aggregate Investment Rate in the 2000 Boom-Bust Cycle
Outline of Next Steps

1. Benchmark general equilibrium model with lumpy investment: Khan and Thomas (2008)
   • Aside: how to numerically compute heterogeneous agent models

2. Model generates time-varying elasticity in partial equilibrium

3. Model generates constant elasticity in general equilibrium

4. **Two broad responses to irrelevance result in literature**
   • Specification of micro-level adjustment costs
   • Specification of general equilibrium: Winberry (2018), Bachmann and Ma (2016), Cooper and Willis (2014)
Winberry (2018)

- Argues that procyclical interest rate in Khan and Thomas’ model inconsistent with data
  - Cooper and Willis (2014): feed in from data
  - Winberry (2018): general equilibrium model
- When consistent with data recover aggregate nonlinearities
Winberry (2018)

- Argues that procyclical interest rate in Khan and Thomas’ model inconsistent with data
  - Cooper and Willis (2014): feed in from data
  - Winberry (2018): general equilibrium model
- When consistent with data recover aggregate nonlinearities

<table>
<thead>
<tr>
<th></th>
<th>$\sigma (r_t)$</th>
<th>$\rho (r_t, y_{t-1})$</th>
<th>$\rho (r_t, y_t)$</th>
<th>$\rho (r_t, y_{t+1})$</th>
</tr>
</thead>
<tbody>
<tr>
<td><em>T-bill</em></td>
<td>2.18%</td>
<td>-0.08</td>
<td>-0.17</td>
<td>-0.251</td>
</tr>
<tr>
<td><em>AAA</em></td>
<td>2.34%</td>
<td>-0.29</td>
<td>-0.37</td>
<td>-0.40</td>
</tr>
<tr>
<td><em>BAA</em></td>
<td>2.43%</td>
<td>-0.32</td>
<td>-0.41</td>
<td>-0.45</td>
</tr>
<tr>
<td><em>Stock</em></td>
<td>24.7%</td>
<td>-0.24</td>
<td>-0.14</td>
<td>0.02</td>
</tr>
<tr>
<td><em>RBC</em></td>
<td>0.16%</td>
<td>0.61</td>
<td>0.97</td>
<td>0.74</td>
</tr>
</tbody>
</table>
Rolling Windows of $r_t$ Dynamics
IRF of $r_t$ to TFP Shock
Model

Firms as in Khan and Thomas except:

- Corporate tax code
- Temporary investment stimulus policy
- Quadratic adjustment costs
Firms as in Khan and Thomas except:

- Corporate tax code
- Temporary investment stimulus policy
- Quadratic adjustment costs

Household preferences feature habit formation:

\[
\max_{C_t, N_t} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \log \left( C_t - H_t - \chi \frac{N_t^{1+\eta}}{1 + \eta} \right) \\
S_t = \frac{C_t - H_t}{C_t} \quad \text{and} \quad \log S_t = (1 - \rho_S) \log \bar{S} + \rho_S \log S_{t-1} + \lambda \log \frac{C_t}{C_{t-1}}
\]
Set most parameters *exogenously*. 
Calibration

Set most parameters *exogeneously*

Choose parameters governing *micro heterogeneity* and *habit formation* to match *micro investment data* and *real interest rate dynamics*

- **Real interest rate dynamics** pin down capital supply and demand curves
  - Capital supply: households smoothing consumption → habit formation
  - Capital demand: firms demanding future capital → shocks and adjustment costs
- **Micro investment data** pins down shocks and adjustment costs
## Calibration

### Table 3
**Empirical Targets**

<table>
<thead>
<tr>
<th>Micro Investment Target</th>
<th>Data</th>
<th>Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>Average investment rate (%)</td>
<td>10.4%</td>
<td>10.7%</td>
</tr>
<tr>
<td>Standard deviation of investment rates</td>
<td>0.16</td>
<td>0.15</td>
</tr>
<tr>
<td>Spike rate (%)</td>
<td>14.4%</td>
<td>19.0%</td>
</tr>
<tr>
<td>Positive investment rates (%)</td>
<td>85.6%</td>
<td>81.0%</td>
</tr>
</tbody>
</table>

**Interest Rate Dynamics**

<table>
<thead>
<tr>
<th>Target</th>
<th>Data</th>
<th>Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cumulative impulse response</td>
<td>−0.49</td>
<td>−0.31</td>
</tr>
<tr>
<td>$\sigma(I_t)/\sigma(Y_t)$</td>
<td>2.87</td>
<td>2.88</td>
</tr>
</tbody>
</table>
Figure 3: Identification of Habit Formation and Adjustment Costs
Calibration

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\xi$</td>
<td>Upper bound on fixed costs</td>
<td>0.53</td>
</tr>
<tr>
<td>$\varphi$</td>
<td>Convex adjustment cost</td>
<td>2.34</td>
</tr>
<tr>
<td>$\rho_\varepsilon$</td>
<td>Idiosyncratic productivity AR(1) (fixed)</td>
<td>0.90</td>
</tr>
<tr>
<td>$\sigma_\varepsilon$</td>
<td>Idiosyncratic productivity AR(1)</td>
<td>0.056</td>
</tr>
</tbody>
</table>

**Habit Formation**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\lambda$</td>
<td>Sensitivity of habit w.r.t. consumption bundle</td>
<td>0.73</td>
</tr>
</tbody>
</table>
State Dependence of TFP Shocks

**Table 6**

<table>
<thead>
<tr>
<th></th>
<th>95-5 ratio</th>
<th>90-10 ratio</th>
<th>75-25 ratio</th>
<th>$\rho(RI_t, \log Y_t)$</th>
<th>$\rho(RI_t, \text{adj}_t)$</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Benchmark Calibration</strong> (PE interest elasticity $d \log I_t/dr_t = -7.55$)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Partial Equilibrium</td>
<td>64%</td>
<td>50%</td>
<td>25%</td>
<td>0.93</td>
<td>0.93</td>
</tr>
<tr>
<td>General Equilibrium</td>
<td>31%</td>
<td>23%</td>
<td>15%</td>
<td>0.99</td>
<td>0.78</td>
</tr>
<tr>
<td><strong>Khan and Thomas (2008) Calibration</strong> (PE interest elasticity $d \log I_t/dr_t = -1055.41$)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Partial Equilibrium</td>
<td>49%</td>
<td>38%</td>
<td>18%</td>
<td>0.92</td>
<td>0.94</td>
</tr>
<tr>
<td>General Equilibrium</td>
<td>7%</td>
<td>5%</td>
<td>3%</td>
<td>0.98</td>
<td>0.93</td>
</tr>
</tbody>
</table>

$Rl_t = 100 \times \log \left( \frac{l(Z_t + \sigma_Z, X_t, \mu_t) - l(Z_t, X_t, \mu_t)}{l(\sigma_Z, X^*, \mu^*) - l(0, X^*, \mu^*)} \right)$
### Table 8

**Responsiveness Index for Investment Stimulus Shock**

<table>
<thead>
<tr>
<th></th>
<th>95-5 ratio</th>
<th>90-10 ratio</th>
<th>75-25 ratio</th>
<th>$\rho(RI_t, \log Y_t)$</th>
<th>$\rho(RI_t, \text{adj}_t)$</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Impact</strong></td>
<td>22%</td>
<td>15%</td>
<td>6%</td>
<td>0.86</td>
<td>0.60</td>
</tr>
<tr>
<td><strong>Cumulative</strong></td>
<td>17%</td>
<td>11%</td>
<td>5%</td>
<td>0.78</td>
<td>0.66</td>
</tr>
</tbody>
</table>

price of investment = 1 – sub$_t$
Conclusion: Takeaways from Topic 2

1. **Investment is lumpy** in the microdata

2. Structural micro models provide evidence for **nonconvex adjustment costs**
   - SMM estimation

3. Calibrated macro models indicate possibly generates **time-varying aggregate elasticity**
   - Aggregation and general equilibrium both important
   - Solving models with distribution in state vector