Micro Data for Macro Models

Topic 4: Firm Lifecycle

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Stylized Facts About Firm Dynamics

1. New entrants smaller than the average firm

2. Young firms more likely to exit than average firm

3. Conditional on survival:
   • Young firms grow faster than average
   • Small firms grow faster than average, conditional on age

4. Distribution of firm size has fat tail
Benchmark Model of Firm (Non) Dynamics

- Consider firms $i$ with production function
  $$y_{it} = e^{z_{it} k_{it}^{\theta} n_{it}^{\nu}}, \text{ where } \theta + \nu < 1$$

- Productivity shocks feature mean reversion
  $$z_{it+1} = \rho z_{it} + \varepsilon_{it+1}$$

- Suppose firms enter with average productivity $z_{i0}$ and low capital $k_{i0} < \mathbb{E} [k_{it}]$

- Investment satisfies
  $$1 = \mathbb{E} [MPK_{it+1} + (1 - \delta)]$$

$\implies$ All growth occurs in first period
Mechanisms Generating Firm Dynamics

1. **Selection** upon entry and exit (Hopenhayn 1992)
   - Surviving firms have higher productivity than entrants
   - Mean reversion + selection $\rightarrow$ growth patterns

2. **Capital adjustment costs** (Clementi and Palazzo 2015)
   - Keeps investment from all happening in first period

3. **Financial frictions** (Ottonello and Winberry 2017)
   - Costly to finance investment

4. **Demand accumulation** (Foster, Haltiwanger, and Syverson 2016)
   - Another form of investment
Clementi and Palazzo (2015)

- Adds *capital adjustment costs* to Hopenhayn (1992) model
- Model matches *key stylized facts* about firm dynamics
  1. New entrants smaller than average
  2. Young firms more likely to exit
  3. Conditional on survival, young/small firms grow faster
- Generates *persistence* in response to aggregate shocks
Model Overview

**Incumbent Firms**
- Idiosyncratic + aggregate productivity shocks
- Fixed operating cost + exit
- Fixed + quadratic adjustment costs

**Potential Entrants**
- Fixed mass
- Signal of productivity if enter
- Fixed entry cost

**Partial equilibrium**
- Fixed discount factor $1/R$
- Labor supply function $L(w) = w^\gamma$
Incumbent Firms

- **Production function** \( y_{it} = e^{Z_t} e^{z_{it}} k_{it}^\theta n_{it}^{\nu} \)
  - Aggregate shock \( Z_{t+1} = \rho_Z Z_t + \varepsilon_{t+1}^Z \)
  - Idiosyncratic shock \( z_{it+1} = \rho_z Z_{it} + \varepsilon_{it+1}^Z \)

- **Capital accumulation** follows \( k_{it+1} = (1 - \delta) k_{it} + i_{it} \)
  - Fixed cost \( c_0 k_{it} \) if \( i_{it} \neq 0 \)
  - Quadratic adjustment cost \(-\frac{c_1}{2} \left( \frac{i_{it}}{k_{it}} \right)^2 k_{it} \)

- To continue into next period, must pay fixed operating cost \( c_f \sim \log N(\mu_{c_f}, \sigma_{c_f}) \)
Timing and Decision Problem

\[
v^1(z, k; s) = \max_n e^Z e^Z k^\theta n^\nu - w(s)n + E_c f \left[ \max \{ v^0(k), v^2(z, k; s) - c_f \} \right]
\]

\[
v^2(z, k; s) = \max_{k'} - (k' - (1 - \delta)k) - AC(k', k) + \frac{1}{R} E_{z', z'|z, s} \left[ v^1(z', k'; s') \right]
\]
Potential Entrants

• **Fixed mass** $M$ of potential entrants
• **Draw signal** of future productivity $q \sim \text{Pareto}(q)$
  - $z' = \rho_s q + \eta', \eta' \sim \mathcal{N}(0, \sigma_s)$
• To become an incumbent, pay fixed entry cost $c_e$
Potential Entrants

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$$ v(q; s) = -k' + \frac{1}{R} \mathbb{E}_{z', z'|z, q} \left[ v^1(z', k'; s') \right] $$
Calibration

<table>
<thead>
<tr>
<th>Description</th>
<th>Symbol</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Capital share</td>
<td>(\alpha)</td>
<td>0.3</td>
</tr>
<tr>
<td>Span of control</td>
<td>(\theta)</td>
<td>0.8</td>
</tr>
<tr>
<td>Depreciation rate</td>
<td>(\delta)</td>
<td>0.1</td>
</tr>
<tr>
<td>Interest rate</td>
<td>(R)</td>
<td>1.04</td>
</tr>
<tr>
<td>Labor supply elasticity</td>
<td>(\gamma)</td>
<td>2.0</td>
</tr>
<tr>
<td>Mass of potential entrants</td>
<td>(M)</td>
<td>1,766.29</td>
</tr>
<tr>
<td>Persistence idiosync. shock</td>
<td>(\rho_s)</td>
<td>0.55</td>
</tr>
<tr>
<td>Variance idiosync. shock</td>
<td>(\sigma_s)</td>
<td>0.22</td>
</tr>
<tr>
<td>Operating cost – mean parameter</td>
<td>(\mu_{c_f})</td>
<td>-5.63872</td>
</tr>
<tr>
<td>Operating cost – var parameter</td>
<td>(\sigma_{c_f})</td>
<td>0.90277</td>
</tr>
<tr>
<td>Fixed cost of investment</td>
<td>(c_0)</td>
<td>0.00011</td>
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<tr>
<td>Variable cost of investment</td>
<td>(c_1)</td>
<td>0.03141</td>
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<tr>
<td>Pareto exponent</td>
<td>(\xi)</td>
<td>2.69</td>
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<tr>
<td>Entry cost</td>
<td>(c_e)</td>
<td>0.005347</td>
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<tr>
<th>Statistic</th>
<th>Model</th>
<th>Data</th>
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<tr>
<td>Mean investment rate</td>
<td>0.153</td>
<td>0.122</td>
</tr>
<tr>
<td>Std. Dev. investment rate</td>
<td>0.325</td>
<td>0.337</td>
</tr>
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<td>Investment autocorrelation</td>
<td>0.059</td>
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<td>Inaction rate</td>
<td>0.067</td>
<td>0.081</td>
</tr>
<tr>
<td>Entry rate</td>
<td>0.062</td>
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</tr>
<tr>
<td>Entrants’ relative size</td>
<td>0.58</td>
<td>0.60</td>
</tr>
<tr>
<td>Exiters’ relative size</td>
<td>0.47</td>
<td>0.49</td>
</tr>
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Survival Probability by Age

- Survival probability decreases with age
  - Productivity increases with age
Growth by Age and Size

• Growth rate **decreasing in size**
  • Mean reversion in productivity

• Growth rate **decreasing in age**
  • Young firms have lower capital
Calibrating Aggregate TFP Shocks

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<td>Autocorrelation output growth</td>
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<td>Std. dev. employment growth (rel. to output growth)</td>
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Entry and Exit Over the Cycle

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<th>Entrants’ Size</th>
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<tr>
<td>0.402</td>
<td>-0.779</td>
<td>-0.725</td>
<td>-0.892</td>
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Aggregate Impulse Responses

- **Aggregate Productivity**
- **Wage**
- **Entry Rate**
- **Output**
- **Employment**
- **Exit Rate**
Aggregate Impulse Responses

Number of Firms

Entrants’ Avg Idiosync Prod

Exiters’ Avg Idiosync Prod

Avg. Idiosyncratic Productivity

Entrants’ Average Size Relative to Incumbents

Exiters’ Average Size Relative to Non-Exiters
Propagation

Output – Impulse Response

Percentage Deviation

Time

With entry & exit
Without
Three New Propagation Mechanisms in This Model

\[ Y_t = A_t K_t^{1-\alpha} L_t^\alpha, \text{ where} \]

\[ A_t = Z_t \]
Three New Propagation Mechanisms in This Model

\[ Y_t = A_t K_t^{1-\alpha} L_t^\alpha N_t^{1-\alpha}, \text{ where} \]

\[ A_t = Z_t \left( \left( \mathbb{E}_t \left[ Z_{jt}^{1/(1-\alpha)} \right] \mathbb{E}_t \left[ k_{jt}^{\theta/(1-\alpha)} \right] \right) + \text{Cov}_t(Z_{jt}^{1/(1-\alpha)}, k_{jt}^{\theta/(1-\alpha)}) / K_t \right)^{1-\alpha} \]
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1. **External propagation**: exogenous component of TFP
   \[ Z_t = \rho Z_{t-1} + \varepsilon_t \]
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   - Firm accumulation
   - Selection
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