Housing and Credit Markets: Bubbles and Crashes

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Abstract

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Prompted by the recent US experience, in this chapter, we study the interaction between cycles in credit markets and cycles in housing markets. There is a large growing literature exploring two different approaches: on the one hand, a boom-bust in house prices can generate a boom-bust in credit market and, on the other hand, a boom-bust in credit markets can generate a boom-bust in house prices. We start by presenting a stark mechanical model to formalize the interaction between housing prices and credit markets and explore these two channels in a mechanical way. Next, we present two simple models that highlight the two approaches. First, we propose a catastrophe model, where an increase in credit availability can generate first a boom and then a bust in mortgage markets because of multiple equilibria due to adverse selection: as lending expands, the composition of borrowers worsens and at some point this can generate a crash in credit market. Second, we propose a sentiment model, where house prices increase above fundamentals because investors buy assets under the irrational belief that there is always going to be an ever more foolish buyer, willing to buy at a higher price. In the course of the chapter, we relate our simple models to the large existing literature on these topics. At the end, we also point to some empirical papers that propose related facts.

**Keywords:** housing prices, credit markets, cycles, leverage, adverse selection, bubbles, sentiments

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1 Introduction

In the recent years, the United States has experienced, at the same time, a boom-bust episode in house price and a boom-bust episode in credit markets, as reflected in figures 1 and 2.

Figure 1: The S&P/Case-Shiller Home Price Indices

The purpose of this chapter is to explore the connection between financial markets and the housing market and its effects on the macroeconomic activity. There is a large and growing literature that separately explores credit cycles and house price bubbles and busts. In this chapter we will try to connect these two streams of literature and understand the potential feedbacks between the two.

In particular, we will explore two different broad approaches to think about this connection:
1. the house price boom-bust generates the credit boom-bust;

2. the credit boom-bust generates the house price boom-bust.

Moreover, we embrace the view that, in both cases, these connected boom-bust episodes generate a boom-bust episode in aggregate activity, which, in turn, can feedback and amplify the boom and bust in the financial and housing markets. Given that the relationship between the house price boom-bust episode and the credit boom-bust episode can be itself quite rich, in this chapter we will mostly focus on that, and less on the connection with the real economy.

We start the chapter by discussing a simple mechanical baseline model in section 2 which is meant to describe the interaction between the credit cycle and house prices, highlighted above. On purpose, it makes a number of
stark assumptions to avoid several thorny issues that arise in a fully specified equilibrium model. In particular, we take as given the dynamics of both leverage and house prices. We then perform two types of exercises: first, we keep house prices constant and study the response to a boom and bust in leverage; and second, we keep the leverage constant and study the response to a boom and bust in house prices. In the rest of the chapter, we try to go deeper in understanding where these dynamics are coming from and connect these two types of exercises to the existing literature.

We next explore the idea that the boom and bust in the credit market was the fundamental shock that spilled over the housing market and the real economy. We first discuss several papers related to this idea. In section 3, we discuss a large class of papers that explore in particular the role of leverage in the households’ sector. We focus on papers that study the effects of credit constraints on house prices and, more generally, on the real economy.

In this spirit, in section 4, we completely abstract from the housing price dynamics and focus on the boom and crash in the credit market. In particular, we propose a stylized static model of households who borrow to become homeowners and intermediaries who lack information about the quality of the borrowers. The main idea is that a simple increase in the credit availability in the economy, what we interpret as “saving glut”, can endogenously generate first a boom and then a crash in lending activity because of multiple equilibria due to adverse selection issues. The basic idea that a saving glut can generate an endogenous credit cycle because of multiplicity of equilibria is inspired by Boissay, Collard, and Smets (2016), although the mechanism is quite different. In our model, the increase in credit availability first increases the lending activity and hence increases the “sub-prime market”. However, as the quality of the pool of borrowers decreases, good borrowers may decide to pay a cost to separate themselves and get better credit terms. This, in turns, may make the sub-prime market to collapse.
Next, we move to the idea that the boom and crash in housing prices is the key element of the interaction between housing and credit markets. In particular, if house prices are expected to rise, banks are more willing to lend, although this means to lend to worse creditors, e.g. sub-prime borrowers. Moreover, speculating households are more willing to buy as house prices appreciate. While appealing, formalizing this intuition tends to run into the “conundrum of the single equilibrium”: if prices are expected to be high tomorrow, then demand for credit and thus housing demand should be high today, and that should drive up prices today, making it less likely that prices will increase. Or, put differently, as prices rise, they eventually must get to a near-maximum at some date: call it “today”. At that point, prices are expected to decline in the future. But if so, banks and speculating households are less likely to buy today: but then, the price should not be high today. The issue is that in a rational expectations equilibrium there should be no “fool” willing to buy at the highest price, when prices can only go down from there.

In section 5 we discuss two strands of literature that focus on two types of bubble models. In subsection 5.1 we refer to a class of bubble models, where the interest rate demanded on assets is below or at most equal to the growth rate of the economy. This can give rise to rational bubbles and stochastically bursting bubble in, essentially, dynamically inefficient (or borderline efficient) overlapping generations models, as in Carvalho, Martin, and Ventura (2012) or Martin and Ventura (2012). These authors employ various versions of OLG models, in which, ideally, resources should be funneled from inefficient investors or savers to efficient investors or entrepreneurs. They assume that there is a lending friction: entrepreneurs cannot promise repayment. They can only issue securities, where the buyer hopes that someone else buys them: call them “bubble”, “cash” or “worthless pieces of paper”. Equilibria then exist, where newborn entrepreneurs create “bubble” paper. The existing
bubble paper in the hands of old agents and bubble paper created by newborn entrepreneurs get sold to savers. Savers find investing in these bubbles more attractive than investing in their own inefficient technology. This technology needs to be inefficient enough so that its return is on average below the growth rate of the economy, creating the dynamic inefficiency for bubbles to arise. In that case, the “fundamental” value of any asset paying even a tiny amount per period is actually infinite. Or, put differently, the last fool to buy the bubble at the highest price is happy to do so, since the value of the bubble next period will not have gone down too much and since that fool is desperate to save.

In subsection 5.2, we discuss a second class of bubble models, where the interest rate demanded on assets is above the growth rate of the economy. Here, an aggregate bubble eventually must stop growing, being bounded by the resources in the hands of “newborn” agents purchasing these assets. Rationality considerations typically rule out such bubbles, see the “conundrum” above. We therefore investigate models with irrational optimism and changing sentiments. A benchmark example in the literature, exploiting changing sentiments, is the “disease” bubble model of Burnside, Eichenbaum, and Rebelo (2013). There is some intrinsically worthless bubble component, which could be part of the price of a house. An initially pessimistic population may gradually become infected to be “optimistic” and believe that the bubble component actually has some intrinsic value: once, everyone is optimistic (forever, let’s say), there is some constant price that everyone is willing to pay. However, “truth” may be revealed with some probability every period and reveals that the bubble component is worthless indeed. Then, during the pessimistically dominated population epoch, prices rise during the non-revelation phase, since the rise in prices compensates the pessimistic investors for the risk of ending up with a worthless bubble piece, in case the truth gets revealed. The price will rise until the marginal investor is optimistic: at that point,
the maximum price may be reached.

In the spirit of this stream of sentiment literature, in section 6, we propose a simple model where prices are above fundamentals because investors buy assets under the irrational belief that there is always going to be an ever more foolish buyer, willing to buy for a higher price.

Finally, in section 7, we juxtapose our findings to some lessons we have drawn from the existing literature regarding the empirical evidence.

We wish this chapter will trigger further research and thinking on this important connection. As shall become clear, the issues are far from resolved.

2 A stark model

In the financial crisis of 2008, the following interplay might be at work, amplifying any initial shock: 1) as house prices fell, banks became more reluctant to lend to new home buyers, and 2) as banks became more reluctant to lend to new home buyers, demand for houses and thus prices for houses fell as a consequence. In particular, banks became more reluctant to lend because the drop in house prices negatively impacted their balance sheets, hence generating a more general credit crunch, depressing real activity.

In this section, we introduce a very simple mechanical model, featuring some of that interplay, but without describing the deep reasons for some key elements. The model features a potential mismatch between the long-term assets in the form of a pool of mortgages and the short-term assets in the form of saver deposits. It allows us to study the evolution of bank balance sheets during a house price boom. The model is useful for providing some key insights regarding price crashes and leverage crashes, and their impact on the financial system. In particular, we will use it to conduct numerical experiments, illustrating the two channels emphasized in the introduction:

1. House prices are constant and there is a boom and bust in the leverage
2. The leverage ratio is constant, and there is a boom and bust in house prices.

Furthermore, the model and its analysis sets the stage for discussing the related literature and for the latter sections of the chapter.

Time is discrete and infinite, $t = \ldots, -1, 0, 1, \ldots$. There is a continuum of households, who are the borrowers. There is a competitive sector of bankers, each operating a bank. There is a group of savers who exogenously supply deposits to the banks, and a government which assumes the role of a special saver. Finally, there is a numeraire consumption good and a housing good (or simply houses).

Each period, a fraction $\lambda$ of the households exit (or “die”), and they are replaced with a fraction $\lambda$ of newborn households. Each period all alive households earn some exogenously fixed income $y$ and consume a non-negative amount of goods, while newborn households earn some initial income $\tilde{y}$ and buy a house. We allow $\tilde{y}$ to differ from $y$, reflecting a potential period of saving-up before purchasing the first home. Just before a household exits, it sells its house. Houses are in fixed supply and are identical to each other.

We assume that a household born at time $s$ is willing to buy a house for any price $p_s \leq \bar{p}_s$, where the process for $\bar{p}_s \geq 0$ is exogenously given. When $\tilde{y}$ is not large enough, households have to borrow in order to make that purchase. Restrictions to borrowing may then imply that the newly born households have less than $\bar{p}_s$ resources at hand. We assume that the sellers get to extract all the rents, i.e., we assume that the newly born households pay the lesser between the resources available and $\bar{p}_s$. Thus, only borrowing restrictions may force the market price $p_s$ below $\bar{p}_s$.

To buy their house, households borrow from a banking sector. Consider

\footnote{In principle, one could introduce preferences giving rise to this behavior.}
a household born at date $s$ who buys the house at the prevailing market price $p_s$. We assume that the following mortgage contract is the only type of contract offered by banks and available to households. In the initial period, households have to make a downpayment of $\theta < \bar{y}$ and borrow the remainder $l_s = \max\{p_s - \theta, 0\}$. We shall focus on parameter specifications such that in equilibrium $p_s \geq \theta$ for all $s$. The contract requires that households repay the principal $l_s$ when they exit and sell their house. Failing that, they pay all resources available to them in that exiting period. In all other periods, including the period of purchase, households pay a flow interest $r$ per unit of principal borrowed. We treat $\theta$ and $r$ as parameters of the model. We assume that $r > 0$, while we do not necessarily restrict $\theta$ to be positive, allowing for a cash out at the time of purchase of a house when $\theta < 0$.

We will focus the analysis on equilibria with $l_s = p_s - \theta \geq 0$, where equality is the autarkic case when households do not borrow from banks. Hence, the consumption of a household born at time $s$ in her first period of life is equal to $c_{s;1} = \bar{y} - \theta - r(p_s - \theta)$, where the first index of $c_{s;1}$ refers to the date of consumption and the second index refers to the year of birth. As we do not allow for negative consumption, that is, $c_{s;1} \geq 0$, prices are bounded above by

$$p_s \leq p^{\text{max}} = \frac{\bar{y} - (1 - r)\theta}{r}. \quad (1)$$

In any subsequent period, the household will learn if she exits at the end of that period. The non-exiting households then consumes $c_{t;1} = y - r(p_s - \theta) \geq 0$, imposing another constraint on house prices:

$$p_s \leq \frac{y}{r} + \theta. \quad (2)$$

We will concentrate on parameter specifications, where (2) is tighter than (1), that is, we assume that $\bar{y} - y > \theta$. If the household exits at time $t$, she sells her house at current market price $p_t$. If $p_t + y \geq (1 + r)(p_s - \theta)$,
she can repay the interest and the principal, and before exiting can consume
c_{ts}^f = p_t + y - (1 + r)(p_s - \theta), where \( f \) is meant to indicate her “final period”.
If \( p_t + y < (1 + r)(p_s - \theta) \), the household defaults, consumes zero, and the
bank receives \( p_t + y \) in total, which one can split into \( r(p_s - \theta) \) as the interest
portion and \( p_t + y - r(p_s - \theta) \) as the partial repayment of principal. One
can then calculate the fraction \( \phi_{t,s} \) of principal repaid by households born at
date \( s \) and exiting at date \( t \) by solving for \( \phi_{t,s} \) the following equation:

\[ \phi_{t,s}(p_s - \theta) = \min\{p_s - \theta, p_t + y - r(p_s - \theta)\}. \tag{3} \]

The default rate is then \( 1 - \phi_{t,s} \).

We assume that banks discount future periods at the same rate \( r \) that
they charge as interest payments on the mortgages: this is the easiest case to
analyze. Consider a scenario in which households never default. Then, the
date-\( t \) value \( v_{t,s} \) of a contract signed at date \( s \) is independent of \( t \), \( v_{t,s} \equiv v_s \)
and satisfies the recursion

\[ v_s = \frac{1}{1 + r} \left( r(p_s - \theta) + (1 - \lambda)v_s + \lambda(p_s - \theta) \right), \]

which gives

\[ v_s = p_s - \theta. \tag{4} \]

Banks only invest in mortgages. We assume that banks allow newly born
households to borrow as much as they wish to borrow, provided banks have
the resources to let them do that.

On the liability side, we assume that banks have deposits \( d_t \) by a group
of savers as well as a deposit or loan \( L_t \) by the government. The bank
pays some rate \( r_D \) per unit of deposit by savers. On the government loans,
the bank pays an interest \( r_L \), which is treated as an exogenous parameter.
Additionally, banks are required to repay an exogenously given fraction \( \mu \) of
the principal.

To close the model, we need to specify the evolution of \(d_t\) and \(L_t\). We choose an exogenous process for the banks’ leverage ratio and set \(d_t\) to match such a process, given the endogenous value of the banks’ assets. This is meant to be a simple stand-in for the view that banks finance projects by maximizing the amount of outside financing, subject to constraints on their leverage from regulatory restrictions or repayment concerns by depositors.

To calculate the value of a bank’s assets, we need to take a stand on how the bank or, implicitly, some (unmodeled) regulator values the portfolio of its mortgages. We shall assume \(v_s\) to be the book value of a mortgage issued in period \(s \leq t\) and which has not been repaid, even if the expected value or market value of this mortgage has been declining, due to house price decline and default considerations. Let \(a_t\) be the sum of all end-of-period book values of remaining mortgages, that is,

\[
a_t = \sum_{j=0}^{\infty} \lambda (1 - \lambda)^j (p_{t-j} - \theta) \\
= \lambda (p_t - \theta) + (1 - \lambda) a_{t-1},
\]

(5)
given that only young households, that is, a fraction \(\lambda\) of the population, purchase a home in each period and given equation (4). For example, if prices are constant forever, \(p_t \equiv p^*\), then

\[
a_t \equiv p^* - \theta.
\]

(6)

Consider the balance sheet at the end of the period. We assume that the liabilities are recorded at their face value. The differences between assets and liabilities is the net worth \(n_t\) of the bank. The (book-value) net worth \(n_t\) of
the bank then results from the balance sheet equation

\[ a_t = d_t + L_t + n_t \]  

(7)

We define the capital requirement or net worth requirement \( \kappa_t \) per

\[ \kappa_t a_t = n_t + L_t, \]  

(8)

or

\[ (1 - \kappa_t) a_t = d_t, \]  

(9)

effectively treating the government loan \( L_t \) as a perfect substitute for net worth. We choose an exogenous stochastic process for \( \kappa_t \in [0, 1] \) and assume that \( d_t \) is set so as to satisfy equation (9). Note that \( 1/\kappa_t \) is the book-value leverage ratio on \( n_t + L_t \).

For the evolution of \( L_t \), we consider two alternative versions of the model. The central issue is how to treat a shortfall of funds, should it occur. For simplicity, we seek specifications of the model that avoid potential defaults on depositors, although it would be interesting to explore an extension of the model with default. In the baseline version of the model, we assume that bankers themselves inject any needed funds and hence we assume \( L_t \equiv 0 \). In the alternative version of the model, we shut down the channel of the injection of bank equity, and instead assume that the government provides loans, if necessary, to avoid a default on depositors and to avoid a shortfall of regulatory capital. For both versions, we need to calculate the evolution of the balance sheet.

Consider the beginning of a new period, after exiting households have sold their houses to newly born households. Let us trace out the impact of each transaction on the residual net worth. The bank receives interest payments \( ra_{t-1} \) on all outstanding mortgages, increasing net worth by that
A fraction \( \lambda \) of outstanding mortgages exits. Let us define \( \phi_t \) the fraction of principal exiting mortgages that is repaid to the bank, so that the bank receives \( \phi_t \lambda a_{t-1} \) in total. Using equation (3), we obtain

\[
\phi_t a_{t-1} = \sum_{j=0}^{\infty} \lambda (1 - \lambda)^j \min\{p_{t-1-j} - \theta, p_t + y - r(p_{t-1-j} - \theta)\}
\]  

(10)

The resulting net worth loss is \((1 - \phi_t) a_{t-1}\), as the book value \( a_{t-1} \) of the exiting mortgages is replaced by their payoff \( \phi_t a_{t-1} \). In particular, if the current market price is at least as high as all past market prices, then \( \phi_t = 1 \) and there is no change in net worth.

The bank also receives an inflow of new deposits \( d_t - d_{t-1} \), new government loans \( L_t \), and makes new mortgage investments \( \lambda (p_t - \theta) \) which do not change net worth, but just lengthen the balance sheet.

On the liability side, banks pay the market interest rate \( r_D \) per unit of deposit, so that net worth decreases by the total payments \( r_D d_{t-1} \). Furthermore, the bank pays \((r_L + \mu)L_{t-1}\), the interest and a fraction \( \mu \) of the principal on the beginning-of-period government loans. After all these transactions, but excluding the new government loan position \( L_t \), the bank has a residual cash position \( m_t \) on the asset side, expressed in units of the consumption good. This position may be negative and can be expressed as follows:

\[
m_t = (r + \phi_t \lambda)a_{t-1} + d_t - (1 + r_D)d_{t-1} - (\mu + r_L)L_{t-1} - \lambda(p_t - \theta).
\]

Finally, we assume that the banker consumes some amount \( c_{b,t} \), reducing the net worth of its bank by that amount. It may be useful to think of this consumption as a payment to bank shareholders. In the baseline version of the model, we assume that \( L_t = 0 \) and that \( c_{b,t} = m_t \), that is it exactly equals the cash position, so that the post-banker-consumption cash position is equal to zero. Since that cash position can be negative, we must allow \( c_{b,t} \)
to be negative as well. One might wish to think of this as an injection of equity by the existing bank owners.

The equilibrium of the baseline model with the assumption that $L_t = 0$ can be characterized by the following equations:

\begin{align*}
    a_t &= \lambda(p_t - \theta) + (1 - \lambda)a_{t-1}, \\
    d_t &= (1 - \kappa_t)a_t, \\
    m_t &= (r + \phi_t \lambda)a_{t-1} + d_t - (1 + r_D)d_{t-1} - \lambda(p_t - \theta), \\
    c_{b,t} &= m_t, \\
    n_t &= a_t - d_t,
\end{align*}

where $\phi_t$ is given by equation (10). Note that substituting for $a_t$ and $d_t$ using equations (11) and (12) in (13) we obtain

\begin{align*}
    m_t &= [r - r_D + r_D \kappa_{t-1} + \lambda \kappa_t + \kappa_{t-1} - \kappa_t - (1 - \phi_t) \lambda] a_{t-1} \\
          &\quad - \kappa_t \lambda(p_t - \theta) \\
\end{align*}

This equation has an intuitive appeal. Consider the bracket, multiplying $a_{t-1}$. The first term, $r - r_D$ is the interest arbitrage collected. The second term $r_D \kappa_{t-1}$ is the interest earned on the net worth portion of $a_{t-1}$. The third term, $\lambda \kappa_t$ concerns the repayment of principal. The difference $\kappa_{t-1} - \kappa_t$ means that cash is freed up, if the capital requirement $\kappa_t$ decreases. The final term $(1 - \phi_t) \lambda$ reduces cash flow only if there are defaults, $\phi_t < 1$.

Moreover, substituting for $a_t$ using equation (11) into (15), and using equation (15) one period backward, after some manipulation we obtain

\begin{align*}
    n_t &= n_{t-1} - \lambda a_{t-1} + \lambda(p_t - \theta) - d_t + d_{t-1}.
\end{align*}

One can use this equation to examine the evolution of net worth. As one
special case, suppose that the evolution for the exogenous process $\kappa_t$ implies that deposits are constant, $d_{t-1} = d_t = d$. Then,

$$n_t = n_{t-1} - \lambda a_{t-1} + \lambda (p_t - \theta),$$  \hspace{1cm} (18)

i.e., the change in net worth is given by the book value difference between newly created and exiting mortgages. At a superficial look, it would appear that net worth is “magically” created by higher prices and that default on exiting mortgages does not matter. However, it needs to be recognized that these movements find their counterpart in the banker’s consumption $c_{b,t}$, see (14): to keep $d$ unchanged, higher prices for new houses as well as larger defaults on old mortgages reduce these shareholder pay-outs or even require equity injection.

Equation (18) also reveals that net worth stays constant, if deposits are constant and prices are constant, as, according to equation (6), constant prices imply $a_{t-1} = p^* - \theta$. In this case, equation (14) implies that bankers’ consumption is equal to

$$c^*_b = r (p^* - \theta) - r_D d^*. \hspace{1cm} (19)$$

This simply says that the interest payments on the assets, reduced by the interest payments on the liabilities, are the flow profits in this steady state situation.

Finally, the house price is easy to characterize in this baseline version of the model:

**Proposition 1.** Assume that $\bar{p}_t \leq p_{\text{max}}$, defined in equation (1). In the baseline version of the model, the house price is then always equal to the exogenous process, that is, $p_t = \bar{p}_t$.

**Proof.** This follows from the assumption that sellers extract all the rent from
buying households, i.e., newly born households are willing to borrow up to \( \bar{p}_t - \theta \), and the assumption that banks let them do so, potentially financing the needed resources with negative banker consumption.

For the alternative version of the model, we impose the restriction that \( c_{b,t} \geq 0 \), that is, the banks cannot raise equity from their owners. We assume that the government provides loans \( L_t \), making up for any potential shortfall. The equations characterizing the equilibrium are now:

\[
\begin{align*}
    a_t &= \lambda(p_t - \theta) + (1 - \lambda)a_{t-1}, \\
    d_t &= (1 - \kappa_t)a_t, \\
    m_t &= (r + \phi_t \lambda)a_{t-1} + d_t - (1 + r_D)d_{t-1}, \\
    c_{b,t} &= \max\{0, m_t\}, \\
    L_t &= (1 - \mu)L_{t-1} - \min\{0, m_t\}, \\
    n_t &= a_t - d_t - L_t,
\end{align*}
\]

where again \( \phi_t \) is given by equation (10). Equation (22), compared to (13), includes the payment of the interest and of a portion \( \mu \) of the principal on the outstanding government loans. Equation (23) encodes the non-negativity of \( c_{b,t} \), compared to (14). With that, one needs to add equation (24) for the evolution of the government loans, which are reduced by the repayment of the principal portion, but are increased by any need of repayment for a shortfall of funds \( m_t < 0 \).

The alternative model is not yet complete, however. Note that larger \( p_t \) in (22) can now be compensated for by correspondingly larger loans \( L_t \) by the government. Indeed, there is potentially an interesting range of policies to consider. At the one and most generous extreme, the government may provide sufficiently large loans so as to re-establish the maximal price \( p_t = \bar{p}_t \).
which households are willing to pay. At the other and most stingy extreme, the government may only provide loans to assure non-negative banker consumption, with house prices reduced all the way to \( p_t = \theta \) and thus not requiring bank loans for purchases (assuming \( \theta \geq 0 \)). In the numerical exercise for the exogenous price crash below, we shall investigate the implications of the latter extreme. Put differently, we pick the highest price \( p_t \) with \( \theta < p_t \leq \bar{p}_t \), subject to the restriction that the resulting \( m_t \) in (22) is non-negative, provided such a price exists. Note that this price will either equal \( \bar{p}_t \) or result in \( m_t = 0 \). If no such price exists, then \( p_t = \theta, m_t < 0 \) and the newly issued loan will equal \(-\min\{0, m_t\}\), as stated in equation (24).

With that, the house price becomes endogenous, and proposition 1 ceases to hold. In the case of a bust in the leverage ratio, we assume that the government provides a loan to the banks to make up the missing equity. A better interpretation is to view this as a partial stake in the banking system, at a required rate of return for the government. This stake is then reduced over time at the assumed required rate of the loan repayment.

### 2.1 Numerical experiments: overview

We now conduct two sets of numerical experiments to highlight the two approaches we discussed in the introduction:

1. We assume that house prices are constant and assume a boom and bust in the leverage ratio \( \kappa \);

2. We assume that the leverage ratio is constant, and assume a boom and bust in house prices dynamics.

For both exercises, we consider the implications both for the baseline specification, when bankers can inject fresh equity, and the alternative specification, when they cannot and when, potentially, government loans are required to cover shortfalls of resources.
Table 1: Parameter values for the numerical experiments

| $y$ | 1  |
| $p^*$ | 5 $y$ |
| $\kappa$ | 0.05 (pre-crash, experiment 1) |
| | 0.2 (post-crash, experiment 1) |
| | 0.1 (always, experiment 2) |
| $\theta$ | 2 $y$ |
| $r$ | 0.04 |
| $r_D$ | 0.03 |
| $r_L$ | 0.03 |
| $\mu$ | 0.05 |
| $\lambda$ | 0.1 |
| $\gamma$ | 1.13 (experiment 2) |
| $\alpha$ | 19 $y$ (experiment 2) |

The numerical exercises are meant to be illustrative, and are not intended as careful calibrations. The parameters are picked to be broadly reasonable, but the results are quite sensitive to their choices. We shall think of a period as one year. An overview of the parameters is in table 1.

Everything scales with income $y$, so we arbitrarily set income $y = 1$. Hence, one can read all quantities such as banker consumption, government loans or assets, as multiples of annual GDP. We assume a downpayment (relative to income) of $\theta = 2$, which should be assumed to be “saved up” from prior income before agents are born and enter the housing market. That is, we assume that $\tilde{y}$ is high enough, so that (2) is tighter than (1). Since $\tilde{y}$ does not play a role otherwise, we have not listed an explicit value in table 1. The exit probability $\lambda$ has been set equal to 0.1, implying a turnover of a house on average every 10 years. We assume that banks earn 4% on their assets and pay 3% on their liabilities, be they depositors or government loans. We assume that government loans have a maturity of 20 years, i.e.,
that the fraction $\mu = 0.05$ of the outstanding bonds need to be repaid each period.

For the first set of numerical experiments, we set the maximal willingness to pay constant at $\bar{p}_t \equiv p^*$, where $p^* = 5y$, and thus as five times (annual) income. To model the boom and subsequent bust in leverage, we assume that the required capital ratio is initially at $\kappa = 0.05$ until some date $t = -1$, implying a leverage ratio of 20, and then unexpectedly rises to $\kappa = 0.2$ at date $t = 0$, implying a leverage ratio of 5.

For the second set of numerical experiments, we keep the required capital ratio constant at $\kappa = 0.1$. To capture an initial run-up of house prices and subsequent crash, we assume that the maximal house prices $\bar{p}_t$ increase exponentially until $t = -1$ and then drop to some constant level $\bar{p}_t \equiv p^* \geq \theta$, where $p^*/y = 5$, which is also comparable to the distant past. That is, we assume that

$$\bar{p}_t = p^* + \alpha \gamma^t$$

for $t < 0$, and $\bar{p}_t = p^*$ for $t \geq 0$, where $\gamma \geq 1$.

### 2.2 An exogenous crash in leverage.

Let us examine the first set of numerical experiments, with a constant maximal price $\bar{p}_t = p^*$ and an exogenous crash in leverage. “Case A” is the benchmark version of the model, where we assume that bankers supply fresh equity, if needed. This is shown in figure 3. There is a single period, when the leverage ratio suddenly changes, necessitating an infusion of extra cash, modelled as negative banker consumption. Once the fresh equity is injected, everything continues as before, except that banker consumption is now higher, given the new and lower leverage. The price for houses remains at $p_t = \bar{p}$.

Matters are more dramatic for “case B”, the alternative specification of the model, where there is no fresh infusion of bank equity. The results are
shown in figure 4. Prices crash endogenously, as can be seen in the top left panel. There is a fairly brief period of default, as shown in the top right panel. The government makes up the missing equity by, essentially, obtaining a partial stake in the banking system, at a required rate of return for the government. This stake is then reduced over time at the assumed required rate of the loan repayment. If the payments for interest and repayments are less than the revenue of the banking system, the consumption of the bankers are positive, as can be seen here. Finally, banks gradually rebuild their net worth to the required new ratio, as indicated by the red-dashed line in the left panel of the third row.
Figure 4: An exogenous crash in leverage without infusion of bank equity. Prices crash endogenously, as can be seen in the top left panel. There is a fairly brief period of default, as shown in the top right panel. For the parameterization here, the banks gradually rebuild their net worth to the required new ratio, as indicated by the red-dashed line in the left panel of the third row.
2.3 An exogenous crash in house prices

For the second set of numerical experiments, we seek to investigate an exogenous crash in house prices, following a phase of increasing house prices, given by (26), while keeping leverage $\kappa$ constant.

Consider first the run-up phase for house prices, $t < 0$. In the benchmark specification of the model, houses are always sold at the maximum price that newborn home buyers are willing to spend, i.e. $p_t = \bar{p}_t$. This will also be true in the alternative specification of the model, provided that banks are able to build up net worth fast enough to finance the new loans, without necessitating the injection of further equity. This imposes some constraints on the parameters, which we shall illuminate.

Using (26) and $p_t = \bar{p}_t$ for all $t < 0$, assets $a_t$ per equation (11) can be rewritten as

$$a_t = p^* - \theta + \frac{\lambda \gamma}{\gamma + \lambda - 1} (p_t - p^*)$$

for $t < 0$.

As a useful benchmark, assume $p^* = 0$ and $\theta = 0$. The asset-to-price ratio then is

$$\frac{a_t}{p_t} = \frac{\lambda \gamma}{\gamma + \lambda - 1}.$$

This relationship between current price and the stock of outstanding assets for $t < 0$ is plotted in figure 5. As one can see, higher house price growth makes assets look small compared to current house prices. One may interpret this as a reason, why financial institutions are less concerned about default risks during price booms. With (27) and $a_t = \gamma a_{t-1}$ for $t < 0$, equation (13) implies

$$\frac{m_t}{a_t} = \frac{1}{\gamma} (r - r_D (1 - \kappa) - \kappa (\gamma - 1))$$

The balance sheets are growing for $t < 0$. If the banks finance this growth
exactly out of earnings, so that \( c_{b,t} = m_t = 0 \), one obtains

\[
\gamma = 1 + \frac{1}{\kappa} (r - r_D (1 - \kappa))
\]  

(28)

which is intuitive. In particular, if \( r = r_D \), then

\[
\gamma = 1 + r
\]  

(29)

so that the interest paid on net worth must exactly finance its growth. Put differently, the values calculated for \( \gamma \) in (28) or (29) are the upper bounds for the price growth rates \( \gamma \) to avoid that banker consumption falls into negative territory during the price growth phase, when holding leverage constant, and when assuming that \( p^* = 0 \) and \( \theta = 0 \).

With the other parameters as listed in table 1, equation (28) implies that \( \gamma = 1.13 \) or a 13 percent appreciation of maximal house prices, during the run-up phase. While the numerical experiments are intended as illustrations only, this number strikes us as perhaps a high, but not entirely unreasonable value during a house boom phase. Indeed, during the pre-2008 years, house prices grew even faster, towards the end, according to the Case-Shiller index. One may also wish to read this as a reasonable upper bound of long-time house price growth, when banks are constrained from raising new equity for financing new mortgages. Consider then the implications for banker consumption in figures 6 and 7. For this parameter choice, they are almost flat in the pre-crash phase, since the rise in higher interest payments on old mortgages is now nearly offset by the rise in resources needed for paying for new mortgages. For other choices for \( \gamma \), one should not expect nearly flat banker consumption during this run-up phase.

Per equation (2), we must be careful in letting prices grow too large. Examining the restriction at the last pre-crash price \( \bar{p}_{-1} \), this equation implies
that

\[ \alpha \leq \frac{1}{\gamma} \left( \frac{y}{r} + \theta - p^* \right) \approx 19.5 \ y \]  

We set \( \alpha = 19 \ y \), so that prices crash in the last possible period. These values for \( \gamma \) and \( \alpha \) are listed in table 1. Arguably, these are pretty much at the extreme end, and chosen to provide the most dramatic numerical experiment.

Consider now the post-crash phase, \( t \geq 0 \). Here, numerical calculations are required. The results are in figures 6 and 7. Figure 6 shows what happens in the benchmark specification “case A” of the model, when bank equity can be injected, i.e., when banker consumption can become negative. House prices trade at the exogenously given levels \( \bar{p}_t = p^* \). There is a temporary
dip in repayments, but they recover gradually, as the top right panel shows. No government loans are necessary or provided in this case.

“Case B” is the alternative specification of the model, when no fresh injection of bank equity is available. The results are now more dramatic, and shown in figure 7. Now, when prices crash, they crash to the down-payment level $\theta$ and stay there, for the chosen parameter configuration. Banker consumption never recovers. There is continued default on banker assets. These are the legacy assets of pre-crash assets, which gradually disappear over time: households with pre-crash loans continue to have difficulties repaying these loans. Eventually, the government holds a bond position offset by a negative amount of net worth of bankers, without any corresponding assets.

2.4 Remarks

The previous numerical experiments are useful to highlight how a boom and bust in housing prices and financial markets can be qualitatively driven either by anything that affects directly house prices or by anything that affects directly banks’ leverage. One important ingredient of the model that generates an interesting interaction between house prices and banks’ leverage is the presence of long-term loans.

In the experiments above, a permanent decrease in leverage has more modest effects overall than an exogenous crash in house prices. However, as we repeatedly mentioned, these are only illustrative example, and the size of the exogenous crash that we imposed on leverage in the first exercise is difficult to compare to the size of the price crash that we imposed in the second exercise. It would be interesting in future work to calibrate a more realistic version of the model and try to do a horse-race between the two types of shocks.

It is also interesting to highlight that in both types of numerical experi-
Figure 6: House price boom and crash: implications in the benchmark specification, when bankers inject fresh equity to cover shortfalls of funds.
Figure 7: House price boom and crash: implications in the alternative specification, when bankers do not inject equity and the government provides the minimal loan to keep banks from defaulting.
ments, events unfold always more dramatically in “case B”, without the fresh injection of bank equity, than in “case A”. This indicates that a quick recapitalization of the banking system may be important in getting things back on track and avoiding long and persistent slumps. There is a self-feeding crisis here: without such an equity infusion, banks cannot fund new mortgages, house prices may remain low, leading to further defaults and leading to further impairments on bank balance sheets. A generous government loan program (not shown here), which supports house prices at the maximum willingness that households are willing to pay, will likewise insulate the housing markets from the drop in bank equity, but may result in keeping the government involved in the banking sector for a long time to come. Clearly, we do not model the costs of a possible government intervention, so conclusive policy recommendations are beyond the scope of this section. This is another interesting avenue for future work.

3 Related Literature: Households’ Leverage

The simple model we introduced in the previous section highlights the interplay between credit cycles and boom-bust cycles in housing prices. One key ingredient in that interplay is that households borrow to become homeowners and are subject to financial constraints. Indeed one defining feature of the recent US experience is a dramatic increase in the households’ gross debt to GDP ratio, which reached roughly 128% by 2008, and then sharply dropped. This drew a lot of attention on the effects of households’ leveraging and deleveraging not only on housing markets, but, more generally, on aggregate activity.
3.1 Financial frictions in macro models

There is a large growing literature that embeds financial frictions in macro model. Brunnermeier, Eisenbach, and Sannikov (2011) is a comprehensive survey on this matter. We will just refer here to the seminal papers in this literature and focus next on the specific link between credit markets and house prices. One of the first papers that started a literature of macroeconomics models with financial frictions is Bernanke and Gertler (1989). They focus on long-lasting effects of temporary shocks through the feedback effect of a tightening of the financial frictions. In this model, as well as in Carlstrom and Fuerst (1997) and Bernanke, Gertler, and Gilchrist (1999), the key friction is the assumption of costly verification of the entrepreneur’s type. Another seminal paper that had a huge impact on the macroeconomic literature is Kiyotaki and Moore (1997) who model financial frictions with a collateral constraint on borrowing rather than with a costly state verification framework. They propose a dynamic economy where durable assets play the dual role of factor of production and collateral for producers’ loans. In their model, credit limits are endogenously determined and the interaction between them and asset prices generates a powerful transmission mechanism that allow temporary shocks to technology and income distribution to have large and persistent effects on asset prices and output. The mechanism is the following: after a temporary shock to productivity that reduces firms’ net worth, constrained firms have to cut back their investment, hence reducing land value, and this hurts their future borrowing capacity, and reduces investment further down. This mechanism has been largely incorporated in macro models to study the real effects of financial shocks and the amplification of other types of shocks. Another influential paper on financial frictions and macro is Geanakoplos (2009) who focus on the role of leverage in boom and bust episodes. The key idea is that some investors are more optimistic than others and in good times they will lever up and drive asset prices up.
However, if bad states realized, they may lose their wealth and the assets may shift in more pessimistic hands, and leverage and prices go down. This is what a leverage cycle is. Another related paper is Myerson (2012) who propose a model of credit cycles generated by moral hazard in financial intermediation.

There is a large recent literature that build on these models to think about the role of firms’ balance sheets in the macroeconomy. See for example Lorenzoni (2008), Mendoza and Quadrini (2010), Geanakoplos (2011), Brunnermeier and Sannikov He and Krishnamurthy (2013), Bocola (2014), Gilchrist and Zakrajšek (2012) construct a new credit spread index and show that indeed a reduction in credit availability can have adverse macroeconomic consequences. However, in this chapter we focus more on the households’ side and hence we tilt also the discussion of the literature in this direction.

3.2 The effect of credit constraints on house prices

There is a large strand of literature building models of the housing market where households’ credit constraints play a crucial role in affecting house prices. Davis and VanNieuwerburgh (2015) also offers a nice overview of part of this literature. To the best of our knowledge, Stein (1995) is the first paper to explore the effects of down-payment requirements on house price volatility, as well as on the correlation between prices and trading volume. In particular, the paper highlights the self-reinforcing effect that runs from house prices to down payments and housing demand, back to house prices: if house prices decline, the value of households’ collateral declines, depressing housing demand and hence pushing house prices further down. This multiplier effect can generate multiple equilibria and account for house price boom-bust episodes. This self-reinforcing effect is in the same spirit of the transmission mechanism in the seminal paper of Kiyotaki and Moore (1997).
In a related paper, Ortalo-Magne and Rady (2006) also explore the key role of down-payment requirements to explain house price volatility, although they focus on a different mechanism. They propose a life-cycle model of the housing market with credit constraints where there are two types of homes, “starter homes” and “trade-up homes.” This allows them to focus on the key role of first-time buyers and show that income volatility of young households or relaxation of their credit constraints can explain excess volatility of house prices. Their model also delivers positive correlation between house prices and transaction volume.

More recently, Kiyotaki, Michaelides, and Nikolov (2011) develop a quantitative general equilibrium life-cycle model where land is a limited factor of production and is used as collateral for firms’ loans. They show that, the more important is land relative to capital in the production of tangible assets, the more housing prices are sensitive to fundamental shocks as productivity growth rate or the world interest rate. Moreover, these type of shocks affect wealth and welfare of different households differently, typically making net house buyers the winners and net house sellers the losers during a housing boom. In contrast, financial innovation that relaxes collateral constraints turn out to have small effects on house prices. Similarly, Sommer, Sullivan, and Verbrugge (2013) develop a quantitative general equilibrium model with housing and financial constraints and argue that a relaxation of financial constraints has only small effects on house prices, while movements in interest rates have large effects.

In related work, Favilukis, Ludvigson, and Nieuwerburgh (2016) also develop a quantitative general equilibrium model with housing and collateral constraints to explore what drives fluctuations in house prices to rent ratio, but draw very different conclusions. Relative to previous quantitative papers, this model has two new features: aggregate business cycle risk and bequest heterogeneity to generate a realistic wealth distribution. In contrast to the
previous literature, the authors find that a relaxation of collateral requirements can generate a large housing boom, while lower interest rates, due to an inflow of foreign capital in the domestic bond market, cannot. In particular, they show that the mechanism through which financial liberalization can generate a house price boom is by reducing the housing risk premium. In a similar spirit, Kermani (2012) propose a model to emphasize the importance of financial liberalization and its reversal to explain the housing boom and bust. Chao He and Zhu (mining) also propose a model where housing collateralizes loans and house price boom and bust can be generated by financial innovation because the liquidity premium on housing is non-monotone in the loan-to-equity ratio. In their paper, even without a change in fundamentals, house prices can be cyclical because of self-fulfilling beliefs. In a related paper, Huo and Ríos-Rull (2014) propose a model with heterogenous households, housing and credit constraints, and also show that financial shocks can generate large drops in housing prices.

In a more recent paper, Justiniano, Primiceri, and Tambalotti (2014a) ask what is the best way of formalizing the “credit easing” shock behind the recent US housing boom. Their objective is to model the shock in a way to be able to match a number of stylized facts about the housing and mortgage markets: not only the rise in house prices and households’ debt, but also the fairly stable loan-to-value ratio and the decline in mortgage rates. In particular, they distinguish between a loosening of “lending constraints,” i.e. an increase in the availability of funds to be borrowed for the purpose of home mortgages, and a loosening of “borrowing constraints,” i.e. the lessening of collateral requirements. They argue that a loosening of the collateral requirements alone cannot explain the recent housing boom in the US, but there must have been an expansion in the credit supply.
3.3 The effect of credit constraints and housing prices on macro

The impact of changes in credit conditions in the housing market on the overall economy and on economic policy is obviously an important question and the focus on a significant portion of the literature. Iacoviello (2005) has become a work horse in this literature. The paper embeds nominal households’ debt and collateral constraints tied to real estate values, as in Kiyotaki and Moore (1997), into a new-keynesian model. The paper shows that demand shocks move housing and consumer prices in the same direction and hence are amplified. When demand rises due to some exogenous shock, consumer and asset prices increase. The rise in asset prices increases the borrowing capacity of the debtors, allowing them to spend and invest more. The rise in consumer prices reduces the real value of their outstanding debt obligations, positively affecting their net worth. Given that borrowers have a higher propensity to spend than lenders, the net effect on demand is positive. Thus the demand shock is amplified. Guerrieri and Iacoviello (2014) emphasize that collateral constraints drive an asymmetry in the relationship between house prices and economic activity. Michal Brzoza-Brzezina (2014) examine a DSGE model with housing and financial intermediaries. They evaluate the impact of having multi-period vs. one-period contracts on monetary and macroprudential policy, and the role of fixed-rate vs variable-rate mortgages. Garriga, Kydland, and Sustek (2016) also explore the interaction among long-term mortgages, nominal contracts and monetary policy in a similar general equilibrium model. Benes, Kumhof, and Laxton (2014a) offers a richer structure yet for studying the interplay between the housing market and economic performance and its implications for macroprudential policies. Applications and extensions are in Benes, Kumhof, and Laxton (2014b) and Clancy and Merola (2015).
Corbae and Quintin (2014) are interested in assessing the role of high-leverage mortgages to explain the foreclosure crisis. They propose a model with heterogenous agents who can choose between a mortgage contract with a 20% down-payment and one with no downpayment and can choose to default. The model show that the increase in number of high-leverage loans can explain more than 60% of the increase in foreclosure rates.

There is another strand of literature that focus on macroeconomic models with a housing sector and collateral constraints, but take house prices as given. Among them, Campbell and Hercowitz (2006) explores the macroeconomic consequences of the relaxation of households’ collateral constraints that followed the US financial reforms in the early ’80s. They propose a general equilibrium model with heterogenous households who have access to loan contracts that require a down payment and rapid amortization. House prices are taken as given. Reducing the down-payment rate or extending the term of the loans reduces macroeconomic volatility. In particular, they show that the reforms of the early ’80s can explain a large fraction of the volatility decline in hours worked, output, households debt and durables’ consumption. In a similar spirit, Iacoviello and Pavan (2013) embed housing in a life-cycle general equilibrium business cycle model where households face collateral constraints. They show that higher income risk and lower downpayments can explain the reduced volatility of housing investment, the procyclicality of debt and part of the reduced output volatility during the Great Moderation. They also show that looser credit conditions can make housing and debt more stable in response to small shocks but more fragile in

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2In their work, house prices are constant, as in the early literature that included housing in one-sector real business cycle models in the form of capital used for home production, following the seminal papers by Benhabib, Rogerson, and Wright (1991) and Greenwood and Hercowitz (1991). More recently, Fisher (2007) extends these models by making household capital complimentary to business capital and labor in market production to reconcile the fact that household investment leads nonresidential capital over the business cycle.
response to large negative shock, as it happened in the Great Recession.

Since the recent boom and bust in housing prices and subsequent long recession, there has been a new wave of macro models that take households’ leveraging and deleveraging as the fundamental shock affecting economic activity, even without explicitly modeling the housing market. In his 2011 Presidential Address, Hall (2011) emphasized that the “long slump” that recently hit the US was driven by a severe decline in aggregate demand, which he attribute to the large deleveraging wave that on the onset of the 2007-08 financial crisis followed a large buildup of consumer debt at the beginning of 2000. On the empirical side, Mian and Sufi (2014) use US zip code data to argue that demand shocks were the main source of the employment decline in the recent recession. In the same spirit, there has been a growing body of work that consider a credit crunch as the fundamental shock of the economy and explores how the subsequent deleveraging affects the overall economy, and the housing market in particular. Together with Hall (2011), the first papers that develop macro models where the fundamental shock is a credit crunch type of shock (instead for example of a productivity shock) are Eggertsson and Krugman (2012) and Guerrieri and Lorenzoni (2011). Both papers propose an incomplete market model with households facing a borrowing constraint and represent a credit crunch as an unexpected tightening in the borrowing limit. In order to focus on households’ gross debt positions, both papers need to introduce some form of households’ heterogeneity into the model: Eggertsson and Krugman (2012) use a keynesian model with two types of agents, borrowers and lenders, while Guerrieri and Lorenzoni (2011) use a Bewley type of model with uninsurable idiosyncratic income risk, so that households delever not only when they hit the borrowing limit, but also for precautionary reason when they are close enough to it. Both paper show that a credit crunch type of shock can have large (also persistent in Guerrieri and Lorenzoni (2011)) effects on the real economy, especially in
the presence of sizeable nominal rigidities.

There has been a growing group of papers working on related incomplete market models with heterogeneous households and focusing on a similar “credit tightening” shock. On a more quantitative side, Justiniano, Primiceri, and Tambalotti (2014b) and Del Negro, Eggertsson, Ferrero, and Kiyotaki (2011) quantify the real effects of this type of shock, using different general equilibrium models and reaching different conclusions. On the one hand, Justiniano, Primiceri, and Tambalotti (2014b) builds on Iacoviello (2005) and Campbell and Hercowitz (2006) and propose a model with two types of households who can borrow using their house as collateral. They show that the leveraging and deleveraging cycle recently experienced by the US did not have significant real effects. On the other hand, Del Negro, Eggertsson, Ferrero, and Kiyotaki (2011) introduce liquidity frictions in an otherwise standard DSGE model and show that the effects of a liquidity shock can be large.

There is a number of papers exploring the aggregate effects of a similar shock, focusing on transmission mechanisms that do not rely on nominal rigidities. Huo and Ríos-Rull (2014) study an incomplete market economy where heterogeneous households face a borrowing constraint and the fundamental shock is a tightening in the borrowing limit. The new ingredient in the model that makes the financial shock having real effects is the introduction of search frictions in some consumption markets. That is, households need to engage in costly search to purchase some type of goods and hence, when the borrowing constraint tightens and households want to save more, they will also search less intensively. This will reduce demand and hence generate a recession. Moreover, there is an amplifying effect coming from the fact that consumption tilts more towards the wealthier households who are farther away from the constraint and who are the ones who exert less search effort.

3The introduction of search frictions in consumption markets builds on Bai et al. Similar frictions are key in the transmission of financial shocks in Huo and Ríos-Rull (2013) who focus on a small open economy.
Another related paper is Kehoe, Midrigan, and Pastorino (2014) who propose a search and matching model a la Diamond-Mortensen-Pissarides with upward-sloping wage profiles and risk-averse consumers who face borrowing constraints. In their model, a tightening in the borrowing limit raise workers’ and firm’ discount rates, hence reducing vacancy creation and employment, with a similar mechanism as in Hall (2014). This effect is amplified by the presence of on-the-job human capital accumulation and workers’ debt constraints. Macera (2015) studies a model with both heterogenous households and heterogenous producers and explores the aggregate effects of a tightening in the borrowing capacity of both types of agents.

Another important related paper is Midrigan and Philippon (2011) who study a cash-in-advance economy with housing, where transactions can be conducted not only with money but also with home equity borrowing. In their economy, there is a continuum of islands that are subject to different collateral constraints. The authors parameterize the model to match the empirical evidence from Mian and Sufi (2011) at the MSA level. When house prices decline in one island, the cash-in-advance constraint tightens reducing aggregate demand in that island. This leads to a recession, thanks to nominal wage rigidities and frictions for the reallocation of labor from different sectors, which prevent households to work harder or to move to tradable sectors. The authors also consider an extension of the model with two types of households, patient and impatient, so that patient households lend to impatient households who can use housing as collateral. The authors distinguish between “liquidity shocks,” i.e. a tightening in the cash-in-advance constraint which affect all households, and “credit shocks,” i.e. a tightening the borrowing constraint which affect only impatient households, and show that liquidity shocks are very powerful. The distinction between the two types of constraints is useful to capture the empirical evidence in Johnson, Parker, and Souleles (2006), Parker, Souleles, Johnson, and McClelland (2013),
Kaplan and Violante (2014) showing that there is a large fraction of wealthy households who are liquidity constrained. In many macro models, as the ones I described above, there is only one collateral constraint that typically captures both types of shocks.

Incomplete markets model have also been used to emphasize the effect of house prices on consumption, that is sizable according to Mian, Rao, and Sufi (2013). There is a large empirical literature that has tried to estimate the effect of house price changes on consumption, using different data samples and different identification strategy, such as Campbell and Cocco (2007), Attanasio, Blow, Hamilton, and Leicester, Carroll, Otsuka, and Slacalek (2011), Case, Quigley, Shiller, et al. (2013), Ströbel and Vavra (2015) (see Iacoviello (2012) for a more comprehensive survey on this topic). A standard permanent income hypothesis model typically delivers small consumption responses to house prices, as house prices affect households’ wealth but also households’ implicit rental rates. Berger, Guerrieri, Lorenzoni, and Vavra (2015) show that a simple incomplete market model with heterogeneous agents, housing and collateral constraints, can deliver sizable consumption elasticity to house prices consistent with the empirical evidence. They show that the size of such an elasticity is determined by the correlation of marginal propensity to consume out of temporary income shocks and housing values, by deriving a simple sufficient-statistic formula for the individual elasticity. It follows that more levered economy are typically more responsive. They also analyze a boom-bust episode in the house prices similar to the one recently experience by the US and show that a shock to expected house price appreciation can generate a large boom and bust in consumption and in residential investment at the same time. Kaplan, Mitman, and Violante (2015) use a general equilibrium incomplete markets model with heterogeneous agents also to look at the recent boom and bust in house prices and consumption. They allow for different types of shocks: productivity shocks, taste shocks, shocks to the credit markets, and shocks to beliefs about future price appreciation. They
show that this last type of shock is the most important to explain the movements in house prices, while shocks to credit conditions are important to explain homeownership, leverage and foreclosure.

Finally, there is another strand of literature that is more interested in understanding fluctuations in residential investment. Most of this literature takes house prices as exogenous. One of the seminal papers in this area is [Davis and Heathcote (2005)](https://www.jstor.org/stable/3620488) who actually feature both endogenous housing investment and endogenous house prices. They build a neoclassical multi-sector stochastic growth model where one sector produces residential structures that, together with land, are used to produce houses. The model does not feature credit constraints, but already capture many facts about dynamics of residential investment. [Iacoviello and Neri (2010)](https://www.jstor.org/stable/41108696) extend the multi-sector structure of [Davis and Heathcote (2005)](https://www.jstor.org/stable/3620488) by adding, in particular, nominal rigidities and borrowing constraints. They show that demand shocks, such as housing preference shocks, are important in accounting for fluctuations in house prices. In a more recent paper, [Rognlie, Shleifer, and Simsek (2015)](https://www.jstor.org/stable/43783341) propose a model where a house price boom generates overbuilding of residential capital that would require a reallocation of resources among sectors. The authors use this model to think about the Great Recession and argue that, in the presence of a liquidity trap, this “investment hangover” can generate a recession. They show that their model is consistent with an asymmetric recovery where the residential sector has been left behind. In a related paper, [Boldrin, Christiano, and Fisher (2001)](https://www.jstor.org/stable/3620488) use input-output tables to recover the linkages between the construction sector and the other sectors of the economy and evaluating the contribution of the construction sector to the Great Recession. This review has not included the large literature, examining the housing market in the absence of financial frictions. For example, [Magnus (2011)](https://www.jstor.org/stable/41108696) has argued that search frictions may well be key to understanding many of the housing market phenomena such as liquidity, prices
and vacancies.

4 A Simple Model of Catastrophes

In this section, we focus on the boom and bust in the credit cycle, abstracting from the dynamics of house prices. The main idea is that, if credit markets are affected by private information about the quality of the borrowers, a credit cycle can arise endogenously simply because of an increase in credit availability, which can be interpreted as a “saving glut.” The idea is that when banks have easier access to credit, for example because the interest rate they face is lower, at first they will offer cheaper loans and increase their lending. However, due to the presence of adverse selection, when borrowing is cheaper worse borrowers will take loans and this can endogenously generate a crash of the credit market.

This idea is inspired by Boissay, Collard, and Smets (2016), but the model that we present here is quite different. The main mechanism in our model is based on adverse selection in the mortgage market, while Boissay, Collard, and Smets (2016) relies on a model of the interbank market affected by moral hazard. In their paper, banks are heterogeneous for their intermediation efficiency and their quality is private information. At the same time, borrowing banks can divert some of the funds to low return assets that cannot be recovered by the lending banks. This mechanism also generates endogenous credit cycles as a result of an increase in credit availability: as interest rates go down, the more efficient banks increase their activity, generating a boom of the banking sector, but as interest rate keep decreasing, worse banks may have an higher incentive to divert their funds, increasing counterparty risk and possibly generating an interbank market freeze. Moreover, Boissay, Collard, and Smets (2016) embed their basic interbank market model in a standard DSGE model. Instead, we reduce the dynamics to 2 periods only and leave richer dynamic settings.
4.1 Model

There are two periods $t = 1, 2$. The economy is populated by a continuum of three types of agents: households, lenders, and banks. Lenders and banks are homogenous, while households are heterogeneous.

Households enjoy utility $u(c, h)$ in period 2, where $c$ is consumption of a non-durable good and $h$ is housing consumption. For simplicity, let us assume that utility is linear, that is,

$$u(c, h) = c + \gamma h.$$  

Houses come in fixed size equal to $\bar{h}$, so that $h \in \{0, \bar{h}\}$, and their price is fixed to 1. Households have no endowment in period 1 but receive an income draw $y$ in period 2. They have to decide whether to buy a house or not in period 1, so if they decide to buy a house they have to borrow the full amount.

Households are heterogenous with respect to their income process. Let $\nu \in [0, 1]$ be the household’s type. Assume that $\nu$ is distributed according to some distribution function $G(\nu)$ and affects the distribution of the household’s income $F_\nu(y)$. Throughout, we shall assume

**Assumption A. 1.** $F_{\nu_B}(y)$ first order stochastic dominates $F_{\nu_A}(y)$ whenever $\nu_B > \nu_A$.

Thus, higher household types have a “better” income distribution.

To buy a house in period 1, a household has to borrow 1 unit of funds from the banks at some mortgage “price” $p$ and promise to repay $1/p$ in period 2. Let us note that the label “price” (and notation $p$) might be a bit confusing. It does not refer to the price of the house (which remains fixed at
1), but is period-1 price for one unit of period-2 resources. Alternatively, one may wish to think of \( p \) as the “payment” the household receives in period \( t = 1 \) for each unit of promised repayment at date \( t = 2 \). We shall continue to refer to it as the mortgage price.

At the beginning of period 2, the household’s income \( y \) is realized and then the household decides whether to repay its debt or not. If it does default on its debt it does not pay anything back to the lender, but it suffers a penalty \( \delta > 0 \). This implies that households with higher \( \nu \) are better potential borrowers, in the sense that they have a lower probability of bad income realizations. Let us denote by \( \chi \in \{0, 1\} \) the repayment decision.

The household’s type \( \nu \) is private information of the household. However, at the beginning of period 1, households have the option to verify their type at a utility cost \( \kappa > 0 \). Let \( v(\nu) \in \{0, 1\} \) be the decision of verifying their type (\( v = 1 \)) or not (\( v = 0 \)). If a household verifies its type, banks can make the lending terms type-contingent, so that the mortgage price \( p \) is going to be equal to \( \tilde{p}(\nu) \). If instead a household does not verify its type, banks do not know the type of the borrower and will offer a pooling mortgage price \( p^P \). Note that we assume that banks are restricted to offer only one mortgage price to all that have not verified. It would be interesting to extend the model allowing banks to offer more general contracts.

To sum up, households have three options: 1) do not verify their type and borrow accepting a pooling contract, that is, \( h(\nu) = 1 \) and \( v(\nu) = 0 \) and borrow at the pooling mortgage price \( p = p^P \); 2) verify their type and

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\[4\] The assumption that a defaulting household does not pay anything back to the lender and only suffers a penalty is stark, but simplifies the analysis. The idea is that the household could run away, at a cost which is summarized by \( \delta \). One could relax that and assume that the lender can seize only part of the income of the borrower, as it is reasonable to assume that part of it must be lost in legal fees.

\[5\] The household decides whether to verify its type or not before banks make their offers. Also, we assume that the verification costs is in terms of utility, because for simplicity we assume that the households start period 1 with no endowment. However, it would be easy to extend the model to make it a monetary cost.
access type-contingent contracts, that is, \( h(\nu) = v(\nu) = 1 \) and borrow at the type-contingent mortgage price \( p = \tilde{p}(\nu) \); 3) do not borrow at all, that is, \( h(\nu) = v(\nu) = 0 \).

Let us proceed backward and consider the repayment decision, conditional on borrowing in the first period, that is, on \( h(\nu) = 1 \). Recall that the household suffers a penalty \( \delta \), if it defaults, and that the lender does not get anything back. Then, a household with realized income \( y \) who borrows at the mortgage price \( p \) would like to repay if

\[
y - 1/p + \gamma \bar{h} \geq y - \delta.
\]

We assume throughout that \( \delta \) is large enough so that in equilibrium any household would like to pay back its debt if it can. However, it may not be able to repay because its realized income is not high enough, so that \( \chi(y, p) = 1 \) iff \( y \geq 1/p \). Let \( \pi(\nu, p) \) be the ex-ante repayment probability of a household of type \( \nu \) who borrows at the mortgage price \( p \), that is,

\[
\pi(\nu, p) = E[\chi(y, p) = 1|\nu, p] = 1 - F_{\nu}\left(\frac{1}{p}\right)
\]

Then we can show the following proposition.

**Proposition 2.** The repayment probability \( \pi(\nu, p) \) is increasing in \( \nu \) and increasing in \( p \).

**Proof.** First, \( \pi(\nu, p) \) is increasing in \( \nu \), since \( F_{\nu} \) are ordered by first-order stochastic dominance. Second, \( \pi(\nu, p) \) is increasing in \( p \), since \( 1 - F_{\nu}(y) \) is decreasing in \( y \) for any \( \nu \).

This means that a household with higher type has a higher repayment probability, for any given mortgage price \( p \).

\footnote{We will show below that this is a necessary condition to have a non-empty set of borrowers at the pooling mortgage price.}

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Let us now focus on the lending market. Let us assume that the banks can borrow from the lenders at some rate $R$, which is exogenously given.\footnote{The interest rate $R$ can be interpreted as the rate at which lenders can borrow in the international market.} Also, they trade the loans, which we will refer to as assets from now on, on the secondary market.\footnote{One can potentially generalize the model to create MBS that pool different loans and a similar mechanism would go through as long as there is a constraint on the measure of types that can be pooled together.} Each asset is characterized by the type of the associated borrower $\nu$ and has a different repayment probability $\pi(\nu, p) \in [0, 1]$. This implies that the pooling mortgage price is determined by the no-arbitrage condition

$$p^p = \frac{E \left[ 1 - F_\nu \left( \frac{1}{p^p} \right) | \nu \in S \right]}{R},$$

(31)

where $S \equiv \{ \nu | \nu(\nu) = 0 \}$ is the set of households who decide not to verify their type. Likewise, the type-contingent mortgage price is determined by the no-arbitrage equation

$$\bar{p}(\nu) = \frac{1 - F_\nu \left( \frac{1}{\bar{p}(\nu)} \right)}{R}.$$  

(32)

In principle, there may be none, one or several solutions to this equation. Borrowing the logic in Mankiw (1986), we assume that the highest of these prevails in equilibrium: at a lower mortgage price and thus higher promised return to all other banks, a bank could profitably deviate by offering a higher mortgage price and a better deal to the household, under mild conditions. Define $\nu_L$ as the lowest type, beyond which a type-contingent mortgage price exists for some types,

$$\nu_L = \inf \{ \nu | \text{there is a solution to equation (32)} \}$$

We can then show that a type-contingent mortgage price exists for all types
better than $\nu_L$.

**Proposition 3.** There exist a type-contingent mortgage price $\tilde{p}(\nu)$ for any $\nu > \nu_L$.

*Proof.* Consider some $\nu$. Let $\nu' \in [\nu_L, \nu]$ be such that there is a solution $\tilde{p}(\nu')$ to equation (32). Define

$$\mathcal{P}(\nu) = \{p \geq \tilde{p}(\nu') \mid p \leq \frac{1 - F_\nu \left( \frac{1}{\tilde{p}} \right)}{R} \}$$

Since $F_\nu$ second-order stochastically dominates $F_{\nu'}$, $\tilde{p}(\nu') \in \mathcal{P}(\nu)$ and $\mathcal{P}(\nu)$ is therefore non-empty. Let $\bar{p} = \sup \mathcal{P}(\nu)$, the supremum of $\mathcal{P}(\nu)$. Note that $\bar{p} \leq 1/R < \infty$. Consider an increasing sequence $p_j \to \bar{p}$ with $p_j \in \mathcal{P}(\nu)$. Calculate that

$$\bar{p} = \lim_{j \to \infty} p_j \leq \lim_{j \to \infty} \frac{1 - F_\nu \left( \frac{1}{p_j} \right)}{R} \leq \frac{1 - F_\nu \left( \frac{1}{\bar{p}} \right)}{R}$$

and thus $\bar{p} \in \mathcal{P}$. This shows that $\bar{p}$ is a maximum and that there is therefore at least one solution to equation (32).

Obviously, households who verify their type will be able to borrow at terms that are more favourable the better their type is. That is, we can prove the following proposition.

**Proposition 4.** For $\nu > \nu_L$, the type contingent mortgage price $\tilde{p}(\nu)$ is increasing in $\nu$ and decreasing in $R$.

*Proof.* First, let us make a change of variable and define $\tilde{R}(\nu) \equiv 1/\tilde{p}(\nu)$. Rewrite equation 32 as

$$1 - \frac{R}{x} = F_\nu(x).$$ (33)

which we seek to solve for $\tilde{R}(\nu) = x$, for a given $\nu$. Assume then that there is at least one solution for (32), i.e. that the curves defined by the left-hand
side and right-hand side of that equation cross at least once. As assumed above per the logic in Mankiw (1986), pick the lowest solution to (33) or, equivalently, the highest of the solutions for (32), if there are several. This pins down a unique $\tilde{R}(\nu)$ and unique $\tilde{p}(\nu)$ for each $\nu$. Fix some $\nu$ and its solution $\tilde{R}(\nu) = x$. Note that the left-hand side of (33) diverges to $-\infty$, as $x \to 0$, while the right-hand side converges to a nonnegative number. Thus, at the lowest solution and as a function of $x$, the right-hand side of (33) approaches and then either crosses or touches the left-hand side from above, as $x$ approaches the solution from below. Per the definition of second-order stochastic dominance, the right-hand side shifts to the right, as $\nu$ is increased. Therefore a solution continues to exist for higher $\nu$ and they are to the left of the solution fixed at the beginning of this argument. As $\nu$ is decreased, the right-hand side function shifts to the left. By the similar logic, the solution either moves to the right, when the intersection between the two sides moves locally, or will jump to a solution at a higher value, if the current intersection disappears or a solution will cease to exist altogether. In sum, if a solution $\tilde{R}(\nu)$ exists, it is decreasing in $\nu$. Equivalently, if a solution $p(\nu)$ exists, it is increasing in $\nu$. Likewise, consider now an decrease in $R$. This shifts the left-hand side of (33) upward, a solution will continue to exist and will be lower than the previously fixed solution. If $R$ increases, the current intersection may move locally or disappear: in either case, if a solution continues to exist, it will be higher than the previously fixed solution. This shows that $\tilde{R}(\nu)$ is increasing and $p(\nu)$ therefore decreasing, as a function of $R$.

Consider now the household’s problem. Define

$$U^B(\nu, p) = \int_{\frac{1}{p}}^{\infty} (y - \frac{1}{p} + \gamma \bar{h})dF_\nu(y) + \int_{0}^{\frac{1}{p}} (y - \delta)dF_\nu(y),$$

(34)

which is the expected utility of a household of type $\nu$ who decides to buy
a house at the mortgage price $p$ and does not verify its type ($h(\nu) = 1$ and $v(\nu) = 0$). For $\nu > \nu_L$, define

$$U^V(\nu) = U^B(\nu, \tilde{p}(\nu)) - \kappa,$$

which is the expected utility of a household who decides to borrow at mortgage price $p = \tilde{p}(\nu)$ and verify its type ($h(\nu) = v(\nu) = 1$). For $\nu < \nu_L$, no such type-contingent mortgage price exists: thus define

$$U^V(\nu) = -\infty$$

in that case. For $\nu = \nu_L$, use (35), if there is a solution to (32), and (36), if not. Finally, define

$$U^N(\nu) = \int ydF_\nu(y) = E[y \mid \nu],$$

which is the expected utility of a household of type $\nu$ who decides not to buy a house ($h(\nu) = v(\nu) = 0$), and equal to the expected income, given our assumption of linear utility. For a given pooling mortgage price $p^P$, the utility of the household of type $\nu$ and its maximization problem is now

$$\bar{U}(\nu, p^P) = \max \left\{ U^B(\nu, p^P), U^V(\nu), U^N(\nu) \right\},$$

To make more progress, we need an assumption regarding the income uncertainty as expressed by $F_\nu$.

**Assumption A. 2.** There is some $x^* \in \mathbb{R}$ so that $F_\nu(x)$ has nondecreasing slopes above $x^*$: for all $x_1$ and $x_2$ with $x^* \leq x_1 \leq x_2$ and all $\nu_A$ and $\nu_B$ with $\nu_L \leq \nu_A \leq \nu_B$, we have

$$F_{\nu_A}(x_2) - F_{\nu_A}(x_1) \leq F_{\nu_B}(x_2) - F_{\nu_B}(x_1).$$

(38)
The assumption is trivially satisfied at some \( x^\ast \), where \( F_1(x^\ast) = 1 \), if such a value \( x^\ast \) exists, i.e., if the income distribution is bounded. Obviously then, this assumption is only useful, if \( x^\ast \) is fairly small and smaller than some upper bound on income. Indeed, it will be particularly convenient to assume that \( x^\ast = R \), the safe return.

The following lemma is a bit technical, and useful for an intermediate step in the proof of the next proposition.

**Lemma 1.**

1. Define
   \[
   Z(\nu, p) \equiv \left( 1 - F_\nu \left( \frac{1}{p} \right) \right) \left( \gamma \bar{h} - \frac{1}{p} + \delta \right). \tag{39}
   \]
   and suppose that \( \gamma \bar{h} - (1/p) + \delta > 0 \). Then \( Z(\nu, p) \) is increasing in both \( \nu \) and \( p \).

2. For \( \nu > \nu_L \), define
   \[
   g(\nu, p) = Z(\nu, \tilde{p}(\nu)) - Z(\nu, p) \tag{40}
   \]
   For \( \nu < \nu_L \), define \( g(\nu, p) = -\infty \). For \( \nu = \nu_L \), define \( g(\nu_L, p) \) as in equation \( 40 \), if there is a solution to equation \( 32 \) and \( g(\nu_L, p) = -\infty \) otherwise. Suppose that \( p \leq \tilde{p}(\nu) \leq 1/x^\ast \) for all \( \nu > \nu_L \) and that \( \gamma \bar{h} - (1/p) + \delta > 0 \). Impose the assumption \( A.2 \) of nondecreasing slopes. Then \( g \) is increasing in \( \nu \) and decreasing in \( p \).

**Proof.** It is easy to see that \( Z(\nu, p) \) is increasing in both \( \nu \) and \( p \). It follows that \( g(\nu, p) \) is decreasing in \( p \). It remains to show that \( g \) is increasing in \( \nu \) for \( \nu \geq \nu_L \). Suppose that \( \nu_A \leq \nu_B \). We need to show that

\[
g(\nu_A, p) \leq g(\nu_B, p) \tag{41}
\]

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Excluding the trivial case of \( \nu_B = \nu_L \) with \( g(\nu_L, p) = -\infty \), calculate

\[
g(\nu_B, p) - g(\nu_A, p) = Z(\nu_B, \bar{p}(\nu_B)) - Z(\nu_A, \bar{p}(\nu_A)) - (Z(\nu_B, p) - Z(\nu_A, p))
\]

where we have exploited that \( \bar{p}(\nu_A) \leq \bar{p}(\nu_B) \) per proposition [4] and that \( Z(\nu, p) \) is increasing in \( p \). Define \( x_1 = 1/\bar{p}(\nu_A) \) and \( x_2 = 1/p \) and note that \( x^* \leq x_1 \leq x_2 \). With that, rewrite the right-hand side of the last equation as

\[
g(\nu_B, p) - g(\nu_A, p) \\
\geq (F_{\nu_A}(x_1) - F_{\nu_B}(x_1)) \left( \gamma \bar{h} - x_1 + \delta \right) - (F_{\nu_A}(x_2) - F_{\nu_B}(x_2)) \left( \gamma \bar{h} - x_2 + \delta \right)
\]

with assumption A[2].

The following proposition shows that the households’ optimal behavior can be characterized by two cutoffs values, such that, households with low types do not buy a house, households with high types buy a house and may verify their type, and households in the middle range, buy a house but choose to borrow at the pooling mortgage price. The logic is illustrated in figure 8.

**Proposition 5.** Impose the assumption A[2] of nondecreasing slopes. Assume that \( x^* = R \), the safe return. Then, there exists a value \( p \) so that no household buys a house at a mortgage price \( p < p \), whereas for all \( p^P > p \) there are two cutoffs \( \nu(p^P) \leq \bar{\nu}(p^P) \) such that

1. \( h(\nu) = 0 \), i.e. no house purchase, if \( \nu < \nu(p^P) \).
2. \( h(\nu) = 1, \nu(\nu) = 0 \), i.e. house purchase without verification at the pooling mortgage price \( p^P \), if \( \nu(p^P) < \nu < \bar{\nu}(p^P) \).
Figure 8: Households’ problem (curves are linear just for illustration).
3. \( h(\nu) = 1, v(\nu) = 1 \), i.e. house purchase with verification at the type-contingent mortgage price \( \bar{p}(\nu) \), if \( \nu > \bar{\nu}(p^P) \).

4. \( S = [\nu, \bar{\nu}] \) or \( S = (\nu, \bar{\nu}] \) or \( S = [\nu, \bar{\nu}) \) or \( S = (\nu, \bar{\nu}) \), for \( \nu = \nu(p^P) \) and \( \bar{\nu} = \bar{\nu}(p^P) \).

Moreover, \( \nu(p^P) \) and \( \bar{\nu}(p^P) \) are respectively decreasing and increasing in \( p^P \).

**Proof.** Let us rewrite

\[
U^B(\nu, p) = E[y \mid \nu] + Z(\nu, p) - \delta,
\]

\[
U^V(\nu) = E[y \mid \nu] + Z(\nu, \bar{p}(\nu)) - \delta - \kappa,
\]

and

\[
U^N(\nu) = E[y \mid \nu],
\]

where \( Z(\nu, p) \) is defined in equation (39).

Note that a type \( \nu \)-household will choose to buy a house at mortgage price \( p \) without verification, iff \( U^B(\nu, p) \geq U^N(\nu) \), that is, iff

\[
Z(\nu, p) \geq \delta. \tag{42}
\]

Let \( \underline{p} \) be the infimum of all \( p \), so that there exists a \( \nu \), satisfying equation (42). Therefore, no household of any type will purchase a house at a mortgage price \( p < \underline{p} \) without verification and certainly not either, when paying the verification cost.

Consider now any \( p > \underline{p} \). Since \( Z(\nu, p) > 0 \) for some \( \nu \), it follows that \( \gamma \bar{h} - (1/p) - \delta > 0 \). Since \( Z(\nu, p) \) is increasing in \( \nu \) per Lemma 1, there is a unique cut-off \( \nu(p) \) such that such that \( U^B(\nu, p) \geq U^N(\nu) \) if \( \nu > \nu(p) \). If \( Z(1, p) > \delta \) and \( Z(0, p) < \delta \), the cutoff \( \nu \) is implicitly defined by the infimum of all \( \nu \in [0, 1] \) such that

\[
Z(\nu, p) \geq \delta. \tag{43}
\]
If $Z(1, p) < \delta$, then $\nu(p) = 1$ and if $Z(0, p) > \delta$, then $\nu(p) = 0$. Since $Z(\nu, p)$ is increasing in $p$, it follows that $\nu(p)$ is decreasing (more precisely: non-increasing) in $p$.

Note that a type $\nu$-household will choose to verify and buy a house at the type-contingent mortgage price $\tilde{p}(\nu)$ rather than a mortgage pooling price $p$, iff $U^V(\nu) \geq U^B(\nu, p)$, that is, iff

$$Z(\nu, \tilde{p}(\nu)) - \kappa \geq Z(\nu, p)$$  \hspace{1cm} (44)

provided a type-contingent mortgage price $\tilde{p}(\nu)$ exists, or, equivalently, iff

$$g(\nu, p) \geq \kappa$$  \hspace{1cm} (45)

where $g$ is defined in equation (40). Let $\tilde{\nu}(p)$ be the infimum over all $\nu \in [0, 1]$, for which (45) holds, with the convention that $\tilde{\nu}(p) = 1$, if no such $\nu$ exists. Consider some $\nu_A > \tilde{\nu}(p)$ such that (45) holds. Since $Z(\nu, p)$ is increasing in $p$, it follows that $p \leq \tilde{p}(\nu)$. Equation (32) implies that $\tilde{p}(\nu) \leq 1/x^*$ for all $\nu$. Let $\nu_B > \nu_A$. Lemma 1 now implies that $g(\nu_B, p) \geq g(\nu_A, p) \geq \kappa$, i.e., equation (45) also holds at $\nu_B$. This proves that equation (45) holds for all $\nu > \tilde{\nu}(p)$.

Finally, recall that $g(\nu, p)$ is decreasing in $p$, per Lemma 1. Therefore, if (45) holds at some $\nu$ and $p$, it continues to hold at some $p' < p$. It follows that $\tilde{\nu}(p) \geq \tilde{\nu}(p')$, i.e. that $\tilde{\nu}(p)$ is increasing in $p$. \hfill \Box

We have been careful to allow for discontinuities in all equations, and expressing solutions as infima or suprema for variables appearing in inequalities. In practice, it may be simpler to proceed with enough continuity and to assume that these equations hold with equality at the limiting points. Furthermore, it may be best to impose that $G(\nu)$ has no mass points. With that and in sum, an equilibrium can be represented by a separating mort-
gage price schedule $\bar{p}(\nu)$ solving equation (32) for $\nu \geq \nu_L$, a pooling mortgage price $p^P$, which satisfies

$$p^P = \int_\nu^{\bar{\nu}} \frac{1 - F_\nu \left( \frac{1}{p^P} \right) G(d\nu)}{R},$$

(46)

and two cutoffs $\nu$ and $\bar{\nu}$ satisfying the two conditions

$$\left[ 1 - F_\nu \left( \frac{1}{p^P} \right) \right] \left( \gamma \bar{h} - \frac{1}{p^P} + \delta \right) = \delta,$$

(47)

and

$$\left[ 1 - F_{\bar{\nu}} \left( \frac{1}{p^P} \right) \right] \left( \gamma \bar{h} - \frac{1}{p^P} + \delta \right) = R \bar{p}(\bar{\nu}) \left( \gamma \bar{h} + \delta \right) - R - \kappa.$$ (48)

We next want to show that multiple equilibria can arise in our model performing some simple numerical exercises.

### 4.2 Multiple equilibria

In our model, good households may decide to costly verify their type to signal that they are good and so not to be pooled with bad households. This feature of the model is key to generate multiple equilibria. For some parameters, we can have two equilibria: a good equilibrium where good households do not verify their type, the mortgage price is high and hence it is indeed optimal not to suffer the verification cost; and a bad equilibrium, where good households do verify their type, hence lowering the pooling mortgage price and making it indeed optimal to costly verify their type.

The possibility of multiple equilibria can generate an endogenous credit cycle, driven by a simple increase in credit availability, that is, a decrease in $R$. As we highlighted earlier, this can be thought as an episode of “saving glut”, using Ben Bernanke language: “I will argue that over the past decade a combination of diverse forces has created a significant increase in the global
supply of saving—a global saving glut—which helps to explain both the increase in the U.S. current account deficit and the relatively low level of long-term real interest rates in the world today. The prospect of dramatic increases in the ratio of retirees to workers in a number of major industrial economies is one important reason for the high level of global saving. However, as I will discuss, a particularly interesting aspect of the global saving glut has been a remarkable reversal in the flows of credit to developing and emerging-market economies, a shift that has transformed those economies from borrowers on international capital markets to large net lenders.”

In the next subsection we will show a numerical example where this is the case. Let us first describe the mechanics behind such an endogenous cycle.

1. Let us imagine that we start in an equilibrium where \( R \) is relatively high so that the pool of borrowers is relatively good, that is, \( \nu \) is large, and all borrowers are pooled together, that is, \( \bar{\nu} = 1 \).

2. Then, assume that \( R \) declines, pushing both \( p \) and \( \tilde{p}(\nu) \) up and hence increasing both \( U^B \) and \( U^V \). This implies that \( \nu \) decreases and more bad households become borrowers. However, let us assume that it is still the case that \( \bar{\nu} = 1 \). The change in the composition of the loans tends to depress mortgage prices. However, mortgage prices have to go up on net, so the interest rate effect has to dominate.

3. If \( R \) decreases further, at some point \( \bar{\nu} \) will become smaller than 1 and some borrowers will decide to verify that they are good types, hence worsening the pool of households who borrow at the pooling mortgage price, \( p^P \). There are two possibilities:

   (a) \( p^P \) increases, then both \( U^B \) and \( U^V \) shift further up and both \( \nu \) and \( \bar{\nu} \) decline, dampening the increase in \( p^P \).

   Imagine, by contradiction that \( p \) declines, then \( \nu \) has to increase, but then \( p \) has to increase, generating a contradiction.

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(b) $p^P$ declines, then $U^B$ has to shift down and $\nu$ increases. In this case, it must be that the decline in $\bar{\nu}$ is strong enough to more than compensate the pressure upward on $p^P$ played by the increase in $R$ and in $\nu$.

Let us imagine that the second case arises.

4. If $R$ decreases even further, the economy is now stuck in a bad equilibrium with some separation.

The shift from a good equilibrium to a bad equilibrium can be interpreted as a market crash, as mortgage prices suddenly drop or, equivalently, required interest payments on mortgages suddenly increase.

4.3 Some numerical examples

In this subsection, we show some simple numerical examples to illustrate how our model can generate an endogenous credit cycle.

For simplicity, let us assume that the income process follows a binary distribution with $y \in \{0, \bar{y}\}$, where $\bar{y} > R$ is sufficiently high that repayment is guaranteed. Let $\nu$ be the probability for the high outcome, that is, $\nu = Pr(y = \bar{y})$. The income distribution $F_\nu$ is then given by

$$F_\nu(x) = \begin{cases} 
0, & \text{if } x < 0 \\
1 - \nu, & \text{if } 0 \leq x < \bar{y} \\
1, & \text{if } x \geq \bar{y}
\end{cases}$$

Let $x^*$ be some small, but positive real number, $0 < x^* < R$. Let $\nu_A < \nu_B$. For all $x_1$ and $x_2$ with $x^* \leq x_1 \leq x_2 < \bar{y}$ or with $\bar{y} \leq x_1 \leq x_2$, we have

$$F_{\nu_A}(x_2) - F_{\nu_A}(x_1) = 0 = F_{\nu_B}(x_2) - F_{\nu_B}(x_1) \quad (49)$$

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Suppose then that \( x^* \leq x_1 < \bar{y} \leq x_2 \). Now,

\[
F_{\nu A}(x_2) - F_{\nu A}(x_1) = \nu_A \leq \nu_B = F_{\nu B}(x_2) - F_{\nu B}(x_1)
\]

(50)

Thus, assumption \(2\) is satisfied and proposition \(3\) applies.

Equation (31) now reduces to

\[
\tilde{p}(\nu) = \frac{\nu}{R}.
\]

The two cut-off \( \nu \) and \( \bar{\nu} \) are given by

\[
\nu = \frac{\delta}{\gamma h + \delta - p^P - 1}.
\]

(51)

and

\[
\bar{\nu} = (R - \kappa)p^P,
\]

(52)

where the pooling mortgage price \( p^P \) is given by condition (46), which can be rewritten as

\[
p^P = E[\nu|\nu \in [\nu, \bar{\nu}]].
\]

For all the numerical examples, we set \( \gamma \bar{h} = 2 \) and \( \delta = 0.1 \). We then experiment with different distributions \( H \) for \( \nu \).

We start by a baseline example where we assume that \( \nu \) is uniformly distributed on \([0, 1]\). In this case, the pooling mortgage price can be solved for in closed form and is equal to

\[
p^P = \left( \frac{\delta}{R - \kappa} + 1 \right) \frac{1}{\gamma h + \delta}.
\]

This implies that in this simple benchmark the pooling mortgage price is monotonically decreasing in \( R \), and hence a reduction in \( R \) is always going to increase \( p^P \) and decrease the two cutoffs \( \nu \) and \( \bar{\nu} \), so there is no possibility
of multiple equilibria. Figure 9 shows the results for this numerical case. The top panel on the left shows the equilibrium manifold for mortgage prices as a function of the exogenous interest rate $R$ and clearly shows that in this case multiple equilibria never arise. The top panel on the right just shows the distribution for $\nu$. The two panels in the middle illustrate that there is a unique equilibrium for any level of $R$, showing in particular the case of $R = 1.1$, $R = 1.4$, and $R = 1.7$. Finally the bottom panel on the left shows the two cutoffs, $\underline{\nu}$, in red, and $\bar{\nu}$, in blue, as a function of the pooling mortgage price for a specific $R$. Finally, the bottom right panel shows the volume of loans offered in equilibrium, again for $R = 1.1$, $R = 1.4$, and $R = 1.7$, and shows that, as expected, it is increasing both in the pooling mortgage price and in $R$.

We then explore the following two alternative distributions:

1. a mixture of two exponential densities,

   $$h(\nu) = \omega \frac{\lambda_1 e^{-\lambda_1 \nu}}{1 - e^{-\lambda_1}} + (1 - \omega) \frac{\lambda_2 e^{-\lambda_2 \nu}}{1 - e^{-\lambda_2}};$$

2. a mixture of an exponential density and a truncated normal (where, in terms of the parameterization, we have not normalized the latter to integrate to unity, just the density $h(\nu)$ as a whole),

   $$h(\nu) \propto \omega \frac{\lambda e^{-\lambda \nu}}{1 - e^{-\lambda}} + (1 - \omega) \frac{e^{-(\nu - \nu^e)^2/(2\sigma^2)}}{\sqrt{2\pi}\sigma}.$$ 

The first example we consider assumes that $H$ is a mixture of an exponential density and a truncated normal, where $\kappa = 0.25$, $\lambda = -20$, $\nu^e = 0.1$, $\sigma = 0.2$, and $\omega = 0.6$. Figure 10 shows the results for this case. The top panel on the left shows again the equilibrium manifold for mortgage prices as a function of the exogenous interest rate $R$ and shows that for middle-range levels of $R$, multiple equilibria can arise. The top panel on the right
just shows again the distribution for $\nu$. The two panels in the middle show that the number of equilibrium pooling mortgage prices depends on the level of $R$. For example, we obtain a unique pooling equilibrium when $R = 1.5$, multiple equilibria when $R = 1.4$, and a unique separating equilibrium when $R = 1.3^{10}$. In the case of multiple pooling mortgage prices, we have two stable ones and one unstable in the middle. Finally the bottom panel on the left shows again the two cutoffs, $\underline{\nu}$ in red, and $\bar{\nu}$, in blue, as a function of the pooling mortgage price, for the case $R = 1.4$. The bottom right panel shows the volume of loans offered in equilibrium as a function of the pooling mortgage price, for the three different levels of $R$ considered above. This illustrates that if the economy starts at $R = 1.5$ and then $R$ declines, the pooling mortgage price and the loan volume can first increase and then drop as a result of a shift from a good to a bad equilibrium.

For the second case, a mixture of two exponentials, we set the verification cost $\kappa = 0.15$, and the parameters of the $H$ distribution to $\lambda_1 = -20$, $\lambda_2 = 5$, and $\omega = 0.8$. Figure 11 shows the results for this numerical case. The plots are analogous to the one described above. In this case, we show that a unique pooling equilibrium arises when $R = 1.65$, multiple equilibria arise when $R = 1.58$ and a unique separating equilibrium arises when $R = 1.4$. This implies that, also in this case, a decline in $R$ can generate an endogenous boom and bust in the credit markets, represented by an initial increase and a following decline in the pooling mortgage price and in the loan volume. Again, the bust is originated by a shift from a good to a bad equilibrium.

---

$^{10}$This is clearly a numerical example, not a calibration. In any case, high interest rates might be justified by the fact that mortgages are long-period contracts.
Figure 9: Uniform distribution for $\nu$ on $[0, 1]$. No multiplicity
Figure 10: For the mixture of an exponential density and a truncated normal, where $\gamma h = 2, \delta = 0.1, \kappa = 0.25, \lambda = -20, \nu^e = 0.1, \sigma = 0.2, \omega = 0.6$, we obtained a unique separating equilibrium for $R = 1.3$, multiple equilibria for $R = 1.4$ and a unique pooling equilibrium for $R = 1.5$. 
Figure 11: For the mixture of two exponentials, where $\gamma = 2, \delta = 0.1, \kappa = 0.15, \lambda_1 = -20, \lambda_2 = 5, \omega = 0.8$ (with $\kappa$ the cost for verification), we obtained a unique separating equilibrium for $R = 1.4$, multiple equilibria for $R = 1.58$ and a unique pooling equilibrium for $R = 1.65$. 

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5 Related Literature: Sentiments and Bubbles

One story about run-ups in house prices and subsequent crashes, which appears to be popular in journalistic descriptions of financial crises, runs as follows: as prices rise, speculators are drawn to the market, hoping to sell at a higher price tomorrow. Eventually, this comes to an end, the speculators withdraw from the market and prices come crashing down. While appealing, formalizing this intuition tends to run into the obstacle that if prices are expected to be high tomorrow, then demand for credit and thus houses should be high today, and that should drive up prices today, making it less likely that prices will increase. Or, put differently, as prices rise, they eventually must get to a near-maximum at some date (unless for some reason agents can buy on credit against the future resale): call that date “today”. At that point, prices are expected to decline in the future. But if so, banks and speculating households are less likely to buy today and prices should not be high today to start with. These types of bubbles are typically ruled out by thinking about a “last fool” who is willing to buy at the highest price, when prices can only go down from there: such fools should not exist in rational expectations equilibria. Agents should realize that prices cannot outgrow the economy forever: using backward induction, the bubble then gets stopped dead in its tracks before it can get going at all. That is why formalizing this popular story indeed presents a challenge.

Two strands of the literature have evolved to address this challenge. One strand of the literature, that we refer to as “bubbles”, keeps the expected growth rate of bubbles bounded by the growth rate of the economy, thus circumventing the backward induction logic. These models may additionally invoke irrational beliefs or differences in sentiments, but many do not. Another strand of literature, that we define “sentiments”, allows agents to
believe that bubbles will grow faster than the economy, but then also needs to invoke irrational beliefs for at least some portion of the agents in order to disable the backward induction logic described above.


5.1 Bubbles

Perhaps the most prominent and earliest example of a model with bubbles is the celebrated overlapping generations model of money by Samuelson (1958). If other means of savings do not produce a rate of return higher than the growth rate of the economy, then an intrinsically worthless asset (“fiat money”) can have a nonzero price in terms of goods, as it gets sold by the currently old agents to the next generation of currently young agents. Such economies need to be dynamically inefficient, satisfying the Cass criterion or Balasko-Shell criterion, see Cass (1972) and Balasko and Shell (1980). In such economies, the first welfare theorem might not hold, competitive equilibria might not be Pareto optimal. One may achieve a Pareto-improvement by giving resources to the current old from the current young, who in turn receive resources, when they are old from the next young generation, ad infinitum. There are various ways to implement or interpret such a transfer scheme. Samuelson interpreted the scheme as fiat money, issued perhaps by the initially old generation. Others have interpreted it as government debt, to be rolled over forever, or as an unfunded pension system.

Another stream of literature that features rational bubbles is the search
literatures with fiat money. The seminal paper in this literature is Kiyotaki and Wright (1989) that proposes a model of decentralized trade where agents meet randomly and fiat money can arise as general medium of exchange.

The literature of greatest interest to us here has interpreted this transfer scheme as a bubble, and has discussed how such bubbles might get introduced by various generations. In some of these papers, the transfer scheme is stochastic, and may end in any given period with some probability, generating a crash. For example Carvalho, Martin, and Ventura (2012), Martin and Ventura (2010), Martin and Ventura (2012), Martin and Ventura (2014), Martin and Ventura (2010) employ various versions of OLG models, in which, ideally, resources should be funneled from inefficient investors or savers to efficient investors or entrepreneurs. In particular, Martin and Ventura (2010) have used this framework to think about the financial crisis of 2008. There may, for example, be some lending friction, where entrepreneurs cannot promise repayment. They may be limited in how much paper they can issue against future cash flow from the project, or perhaps they need more financing than can be achieved by issuing such paper. They can additionally issue intrinsically worthless “bubble” securities, valued only because the buyer hopes that someone else buys them in the future. The issuance of such bubble paper starts another sequence of the intergenerational transfer scheme described above. The existing bubble paper in the hands of old agents as well as those created by newborn entrepreneurs get sold to savers. Savers find investing in these bubbles more attractive than investing in their own, inefficient technologies. This technology needs to be inefficient enough so that its return is on average below the growth rate of the economy, creating the dynamic inefficiency for bubbles to arise.

Chao He and Zhu (ming) likewise focus on houses to facilitate intertemporal transactions when credit markets are imperfect, and the resulting liquidity premium for house prices. They obtain deterministic cyclic and chaotic
dynamics as self-fulfilling prophecies, though their equilibria do not display an extended price run-up followed by collapse. Differences in beliefs are at the heart of trading in Scheinkman and Xiong (2003): the belief differences create a bubbly, but (on average) non-growing component of asset prices.

Another paper that generate bubbles with fully rational agents and perfect foresight is Wright and Wong (2014). Here bubbles arise in a model of bilateral exchange that involve chains of intermediaries in markets with search frictions and bargaining problems.

5.2 Sentiments

The other strand of literature we are going to focus in this section, is what we called “sentiment literature”. In Section 6 we are going to propose a simple model that captures the “sentiment” idea that we mentioned in the introduction: asset prices may be above fundamental value because agents “irrationally” believe that there is always a “greater fool” who is going to be willing to buy at an even higher price. and discuss the related literature there. Here, we summarize a number of variants of this story formalized in different ways in the literature, where assets are trading above fundamental values due to diverse beliefs between optimists and pessimists, creating an “add-on” above the fundamental value, possibly requiring short-sale constraints on the pessimists. Static versions can be found in Geanakoplos (2002) or as in Simsek (2013). Dynamic versions are in Harrison-Kreps (1978) or in Scheinkman and Xiong (2003): in the latter paper, agents do understand, however, that the bubble will not grow faster than the economy, on average. Edward L. Glaeser (2008) study bubbles and their role for the housing market. They claim that rational bubbles can obtain, if there is no new construction. They do not provide a full general equilibrium formulation or description of the underlying credit market for this claim: one interpreta-
tion may be that agents can buy on credit against the future resale or have unlimited “deep pockets” to buy at any price. They rule out bubbles with elastic housing supply, and then proceed to use a model of irrational, exuberant buyers to study housing bubbles, relating the length and frequency of bubbles and their welfare consequences to the elasticity of housing supply.

Should asset price movements be taken into account in the central-bank interest rate setting? And if so, how? To answer this question, Adam and Woodford (2012) study the optimal monetary policy in a New Keynesian model with a housing sector, using near-rational equilibria, as developed in Woodford (2010) and allowing for a set of possible and internally coherent probability beliefs, that are not too different from the benchmark.

The papers incorporating diverse beliefs probably come closest to our model in section 6. However, these models typically focus on the case where agents are either pessimistic or optimistic: by construction then, the optimists must be the “greatest fools”. We instead wish to incorporate the idea that the more optimistic buyers are typically not yet the greatest fools themselves, but simply betting on even greater fools out there. At the extreme end, the “greatest fool” must be someone willing to pay for something that is intrinsically worthless (to all others), without ever being able to sell to someone at an even higher price, and there may be quite foolish people out there with overly strong optimism of being able to sell to such “greatest fools”. So, at that end, the model may require substantial irrationality. The key here is, however, how this suspected strong irrationality at the upper end of the potential price distribution trickles down to the price and sales dynamic among the less foolish or even rational part of the population.

The model shares many elements with Golosov, Lorenzoni, and Tsyvinski (2014). There, assets are traded in a sequence of bilateral meetings between agents having different information regarding the fundamental value of the asset. By contrast here, everyone understands the asset to be intrinsically valueless:
the differences arise in beliefs regarding the optimism of others. As such, our chapter is more closely related to Abreu and Brunnermeier (2003). A benchmark example for a dynamic model exploiting heterogeneous beliefs and changing sentiments, is the “disease” bubble model of Burnside, Eichenbaum, and Rebelo (2013). There is some intrinsically worthless bubble component, which could be part of the price of a house. An initially pessimistic population may gradually become infected to be “optimistic” and believe the bubble component actually has some intrinsic value: once, everyone is optimistic (forever, let’s say), there is some constant price that everyone is willing to pay. However, “truth” may be revealed at some probability every period, and clarify that the bubble component is worthless indeed. Then, during the pessimistically dominated population epoch, prices rise during the non-revelation phase, since the rise in prices there compensates the pessimistic investor for the risk of ending up with a worthless bubble piece, in case the truth gets revealed. The price will rise until the marginal investor is optimistic: at that point, the maximum price may be reached.

Another related recent paper is Bordalo, Gennaioli, and Shleifer (2016), where credit cycles arise from “diagnostic expectations,” that is, from the assumption that when they form expectations agents overweight future outcomes that seems more likely in light of the recent data. This can generate excess volatility, over-reaction to news and predictable reversals.

6 A Simple Model of Sentiments

In this section, we are going to propose a simple model to formalize and examine the following and often-told story about buyers and sellers in asset markets. We wish to use it in particular for thinking about the housing market, but it may apply more generally to the stock market or any other market in which assets get re-traded.
The story we have in mind is as follows. Prices for assets sometimes bubble above their fundamental value and then come crashing down. They do so due to buyers betting on greater fools. More precisely, when a buyer buys an asset, she may realize that the price is above its fundamental value, but is betting on being able to sell the asset at a future date at an even higher price to a greater fool. What matters to the buyer is not, how foolish it is to keep the asset itself, but how foolish other participants are.

There are variants of this story formalized in various way in the literature, as we discussed in the literature review in subsection 5.2. For our showcase model below and in contrast to the models of rational bubbles discussed in the literature review in subsection 5.1, we do not assume that the economy is, effectively, dynamically inefficient. That is, we do not wish to assume that bubbles can be traded forever, because the return to be earned on these assets, as perceived by the agents, does not exceed the growth rate of the economy. It may be important, however, to examine models in which the required rate of return is higher than the growth rate. It is then clear from the start that the price eventually must hit a ceiling: say, when the value of the asset exceeds all resources in the hands of the buyers. Usual backward induction arguments then rule out such bubbles in the first place, see Tirole (1985). The purpose of this section is to tweak the rationality argument per introducing a mythical “greatest fool”, thwarting that backward induction. This “greatest fool” can alternatively be interpreted as a rational “collector”, who just happens to value an asset at high price, while nobody else does. We will consider environments where this person is a myth indeed. Agents falsely believe, however, that this mythical collector is out there. Some particularly optimistic believers will buy the assets and hold it, in the hope of ultimately selling to the collector, but more importantly, some traders will buy the assets in the hope of selling to an agent who has an even more optimistic beliefs about the existence of a collector. This is what we
mean by a sentiment-driven bubble. Note that it does not actually matter whether such collector agents are present: all that matters is the beliefs by the various agents in the presence of such agents. We allow for the belief in such collectors to suddenly disappear: if that happens, the price crashes.

It should be clear that one can construct higher-layer type theories too. For a second-layer theory, all agents may agree that there are no collectors. However, they may all believe that a certain fraction of “first-layer” agents out there does believe such collectors to be there, and the more optimistic agents may believe that fraction to be higher. Agents in such an economy will then not per se wait to sell to a collector (they know they cannot), but wait to sell to an agent who believes such collectors to be present. Furthermore, the agents that are less optimistic regarding the existence of such believers will sell to agents who are more optimistic regarding such first-layer believers. Once again, a bubble can arise, this time even if actually neither the collector nor first-layer believers are present in the economy. A third-layer theory would be about agents differing in their beliefs of meeting agents who believe that they can meet an original believer, etc. We feel that it would be fascinating to explore the ramifications and variations of the simple model below a lot further than we do. It is just meant as an inspiration and starting point.

6.1 The model

Time is continuous, \( t \geq 0 \). There is initially a continuum of agents of total mass one. There is distribution of agent types \( \theta \in [0, 1] \) in the economy, characterized by the distribution function \( H(\theta) \). We assume that \( H \) has a density \( h(\theta) \). We call agents of type \( \theta = 1 \) “collectors”. We shall assume that \( H(\theta_{\text{max}}) = 1 \) for some \( \theta_{\text{max}} < 1 \), and thus, the distribution \( H \) assigns no weight to collector types.
Agents differ in their beliefs about the distribution of beliefs in the population, with $\theta$ parameterizing that belief. Specifically, we shall assume that, initially, an agent of type $\theta$ believes that other agents’ type $x$ is drawn from

$$H_\theta(x) = (1 - \theta)H(x) + \theta 1_{x=1}. \tag{53}$$

In other words, agent $\theta$ uses a weighted average between the true distribution and a point mass at the collector type. Most of the analysis below carries over to a more general formulation: we leave these extensions to future research. Agents are aware of their differing beliefs, but they individually nonetheless insist on the beliefs they hold. We assume that an aggregate revelation event may arrive at the arrival rate $\alpha$ (or instantaneous probability $\alpha dt$), at which point all agents suddenly understand that there are no collector types and their beliefs switch to the true distribution $H$. One might wish to assume that agents are unaware that this revelation event could occur (“MIT shock”), but it turns out that the mathematics is not much different if they do: so we shall assume that. In the latter case, the better interpretation is that agents believe that, with some probability $\alpha dt$, the distribution of population types changes from $H_\theta$ to $H$, interpreting this as a taste and belief shift for other agents.

There is a single and indivisible asset (“coconut”), initially in the hand of an agent of type $\theta = 0$. There are random pairwise meetings between agents: due to our assumption of a single asset available for trading, it suffices to describe the meetings between the agent that currently has the asset and some other agent. If the agent currently holding the asset is of type $\theta$ and if the revelation event has not yet happened, then she will believe that she meets an agent drawn from the distribution $H_\theta$ at rate $\lambda$. The asset-holding agent (who we shall call the “seller”) posts a take-it-or-leave-it price $q_\theta$ (the posted contract can be generalized, and we leave this to future research). The
other agent (who we shall call the “buyer”) decides to accept or reject the trade. If the trade is rejected, the seller keeps the asset and keeps on waiting for the next pairwise meeting. If the trade takes place, the buyer produces $q_\theta$ units of a consumption good or “cash”, at instantaneous disutility $q_\theta$, which the seller consumes, experiencing instantaneous utility $q_\theta$. The future is discounted at rate $\rho$. The buyer receives the asset and then in turn waits for the next pairwise meeting. If the buyer turns out to be a collector, he will be willing to buy the asset at any price at or below some exogenously fixed value $v(1)$. The asset may provide some intrinsic value to the collector or the collector may simply be the “last fool”, failing to understand that he can sell the asset at an even higher value in the future. In any case, the asset offers no intrinsic benefit to any agent who is not a collector. In other words, we assume that non-collector agents have preferences given by

$$U = E\left[ \int_0^\infty e^{-\rho t} c_t dt \right]$$

where we allow $c_t$ to be negative, and where $c_t$ is the consumption flow resulting from these trades. We assume that the discount rate is strictly positive, $\rho > 0$.

A few brief remarks may be in order. We have not used time subscripts for $q_\theta$, though there may be equilibria, in which these prices do depend on time. Here, we shall concentrate on time-invariant solutions, for simplicity. More importantly, it may seem odd to consider only a single asset, given that we have a whole continuum of agents at our disposal. This assumption considerably simplifies the analysis, though, as it allows us not to distinguish between meetings, where the potential buyer already owns an asset or not. Furthermore, over time, agents would need to keep track of the distribution of asset-owning types. It is plausible that these distributions shift to the right over time, i.e., that it is the higher types holding assets, as time pro-
gresses. In the decision problem to be analyzed, selling agents would then need to forecast these evolutions, creating potentially intricate interactions and complications that go beyond the scope of this chapter. These would be good topics to pursue in future research.

Note that, in essence, the bubbly economies described in section ?? can be understood as featuring $\rho \leq 0$ with $\alpha = 0$ (and finite lives), so that agents are willing to agree to a trade, in which they give up more today than they receive later, or at least do not insist on getting more later on. Here, we rule out this channel. Note also that the search-theory models of money like Kiyotaki and Wright (1993) and their successors assume that the total sum of consumption is larger than zero, i.e., that the seller benefits more from the sale than the buyer is hurt. If trades can only take place, using the intrinsically worthless asset, the asset helps in achieving a better outcome than autarky. Related modelling devices are used in Harrison-Kreps (1978) or Scheinkman and Xiong (2003). Here, by contrast, we shut down any benefits from the trade per se.

6.2 Analysis

We formulate the strategies of buyers as threshold strategies. A buyer of type $\theta$ picks some value $v_\theta$, and purchases the asset, if the take-it-or-leave-it price is at or below that value, provided the revelation event has not yet taken place. For the collector type, $v_1 > 0$ is a parameter. A seller of type $\theta$ picks a take-it-or-leave-it price $q_\theta$ before the revelation event. After the revelation event has taken place, the asset is valued at zero by all and traded at zero price. A Nash equilibrium is then given by two functions $(v_\theta, q_\theta)_{\theta \in [0,1]}$, so that the strategies of agent $\theta$ maximize the utility function (54), given the strategies of all other agents. We shall additionally impose that $v_\theta$ is measurable. A seller can only hope to sell the asset in the future
before the revelation event takes place. Put differently, we can assume that the seller discounts the future at rate $\alpha + \rho$, and that any before-revelation value of the asset to the seller in the future is discounted at that rate too. We could introduce a new symbol for $\alpha + \rho$. In slight abuse of notation (or appealing to the “MIT shock logic”), we shall continue to use $\rho$ for that discount rate.

Consider now the before-revelation phase. We seek to characterize the Nash equilibrium or Nash equilibria. It is straightforward to see that a buyer of type $\theta$ will choose to buy at any price not bigger than $v_\theta$, where $v_\theta$ is his continuation value of holding the asset. Consider then a seller of type $\theta$, contemplating a sale price $0 \leq q \leq v(1)$ (obviously, it does not make sense to post a price above $v(1)$ or below zero). He assumes that his buyer’s type $x$ is drawn from the distribution $H_\theta$, and that buyers follow their equilibrium strategy $v_x$ and buy only if $q \leq v_x$. Hence, conditional on meeting a buyer, a seller of type $\theta$ who posts price $q$ expects to sell with probability

$$\phi_\theta(q) = (1 - \theta) \int 1_{v_x \geq q} h(x) dx + \theta$$

(55)

**Proposition 6.** The probability of a sale $\phi_\theta(q)$ is decreasing in $q$ and increasing in $\theta$.

**Proof.** The proof is immediate, once one rewrite equation (55) as

$$\phi_\theta(q) = (1 - \theta)\phi_0(q) + \theta.$$ 

If the trade takes place, the seller receives $q$. Trading possibilities arrive at rate $\lambda$, so in a time interval $dt$, the sale takes place with probability $\lambda \phi_\theta(q)$. Otherwise, the seller will remain owner of the asset at time $t+dt$, still valuing the asset at $V_\theta(q)$ then (provided the aggregate revelation event has not taken
place: remember, that we implicitly took care of that via our discount factor $\rho$). Therefore, the continuation value of a seller of type $\theta$ who chooses a sale strategy $q$, $V_\theta(q)$, is equal to

$$V_\theta(q) = \lambda \phi_\theta(q) q \, dt + (1 - \lambda \phi_\theta(q)) \, dt (1 - \rho \, dt) V_\theta(q), \quad (56)$$

or, canceling higher order terms,

$$V_\theta(q) = \frac{q}{\lambda \phi_\theta(q)} + 1. \quad (57)$$

The optimal selling strategy $q = q_\theta$ is the one maximizing $V_\theta(q)$, that is,

$$q_\theta \in \arg\max V_\theta(q) \quad (58)$$

delivering $v_\theta = V_\theta(q_\theta)$.

It is easy to construct two bounds for the optimal continuation value. On the one hand, consider the suboptimal strategy that agents will only attempt to sell to the collector, per posting the price $q = 1$. This strategy would give

$$v_\theta = \frac{v_1}{\lambda \phi_\theta} + 1. \quad (59)$$

Clearly the optimal value function cannot be lower than $v$, as in equilibrium there would be more trade for speculative reasons. On the other hand, consider the widely optimistic assumption, that any potential buyer is willing to purchase the asset at $q = v_1$. The value function would then be given by

$$\bar{v}_\theta = \frac{v_1}{\lambda} + 1 \quad (60)$$

where the omission of $\theta$ is the difference to (60). It is straightforward to show that the equilibrium value function in between these two bounds.
Proposition 7. Suppose that the function \( v : x \mapsto v_x \) used for calculating \( \phi_\theta(q) \) in equation (55) is measurable and satisfies \( v \leq v_x \leq \bar{v} \). Then \( V_\theta \) has a maximum.

Proof. Note that \( V_\theta(q) \) is bounded by \( \bar{v}_\theta \). Let \( q^{(j)}, j = 1, 2, \cdot \cdot \) be a sequence, so that \( V_\theta(q^{(j)}) \) is increasing, converging against \( \sup V_\theta(q) \). Since \( q^{(j)} \in [0, v(1)] \), we can find a convergent subsequence, which we can furthermore assume to be monotone. Wlog, assume that \( q^{(j)} \to q^* \) for some \( q^* \) and is monotonically increasing or decreasing. If the sequence \( q^{(j)} \) is monotonically increasing,

\[
\bigcap_j \{ x \mid v_x \geq q^{(j)} \} = \{ x \mid v_x \geq q^* \}
\]

Therefore \( \phi_\theta(q^*) = \lim_{j \to \infty} \phi_\theta(q^{(j)}) \) and hence

\[
\frac{q^*}{\lambda_{\phi_\theta(q^*)} + 1} = \lim_{j \to \infty} \frac{q^{(j)}}{\lambda_{\phi_\theta(q^{(j)})} + 1}
\]

If the sequence \( q^{(j)} \) are monotonously decreasing, then

\[
\bigcup_j \{ x \mid v_x \geq q^{(j)} \} \subseteq \{ x \mid v_x \geq q^* \}
\]

and therefore \( \phi_\theta(q^*) \geq \lim_{j \to \infty} \phi_\theta(q^{(j)}) \). Hence

\[
\frac{q^*}{\lambda_{\phi_\theta(q^*)} + 1} \geq \lim_{j \to \infty} \frac{q^{(j)}}{\lambda_{\phi_\theta(q^{(j)})} + 1}
\]

Here, though, “>” is ruled out, since the right-hand side is the supremum of \( V_\theta(q) \). We can conclude that \( q^* \) maximizes \( V_\theta(q) \).

The axiom of choice now implies that \( q_\theta \) is well defined.

Proposition 8. The value \( v_\theta \) of any Nash equilibrium is increasing in \( \theta \).
Proof. Let $\tilde{\theta} > \theta$. Note that $V_{\tilde{\theta}}(q) \geq V_{\theta}(q)$ for all $q$, since $\phi_{\tilde{\theta}}(q) \geq \phi_{\theta}(q)$. Since this is true in particular at $q = q_{\tilde{\theta}}$, the claim now follows. \hfill \square

Now we can define the set of potential value functions

$$\mathcal{V} = \{v : [0, 1] \to \mathbb{R} \mid v \text{ is increasing and } \underline{v} \leq v \leq \bar{v}\}.$$ 

Given that increasing functions are measurable, we can consider the mapping $T : \mathcal{V} \to \mathcal{V}$, defined by the following steps:

1. map $v \in \mathcal{V}$, into a function $\phi_{\theta}(q)$, using equation (55);
2. map $\phi$ into a function $V_{\theta}(q)$ using equation (56);
3. map $V_{\theta}(q)$ into the function $v_{\theta}$ that maximizes $V_{\theta}(q)$ (this maximum exists thanks to proposition 7).

**Proposition 9.** The mapping $T : \mathcal{V} \to \mathcal{V}$ is monotone and has a fixed point in $\mathcal{V}$. Therefore, a Nash equilibrium with $v \in \mathcal{V}$ exists.

Proof. For monotonicity, check that each step of the mapping is monotone. I.e., if $\tilde{v} \geq v$, then $\tilde{\phi} \geq \phi$ for the first step, and so forth, where the inequalities are understood to hold pointwise for all arguments. Note that $\mathcal{V}$ is a complete lattice, with the usual order structure. Tarski’s fixed point theorem now delivers the result that the set of fixed points of $T$ forms a non-empty complete sublattice of $\mathcal{V}$. \hfill \square

Next proposition shows that the equilibrium exhibits a threshold property.

**Proposition 10.** For each sale price $q$, there is a threshold buyer type $x(q)$ such that all buyers of type $x \geq x(q)$ will buy the asset and all buyers of type $x < x(q)$ will not, i.e.

$$x \geq x(q) \iff v_x \geq q.$$ (61)
The function \( \underline{x}(q) \) is increasing in \( q \). Furthermore, for \( q \leq v_1 \),
\[
\phi_\theta(q) = (1 - \theta)(1 - H(\underline{x}(q))) + \theta.
\] (62)

Proof. The proof follows immediately from proposition \([\blacksquare]\) \( \square \).

To obtain a bit more analytic insight, consider a price \( q \), where \( \underline{x}(q) \) is differentiable.

**Proposition 11.** Suppose \( \underline{x}(q) \) is differentiable at \( q = q_\theta \). Then, the optimal \( q_\theta \) satisfies the first-order condition
\[
0 = 1 + \frac{\lambda}{\rho} \phi_\theta(q) - \eta_\theta(q)
\] (63)
where \( \eta_\theta(q) \) is the elasticity of the sale probability,
\[
\eta_\theta(q) = -\frac{\phi_\theta'(q)q}{\phi_\theta(q)} = \frac{h(\underline{x}(q))\underline{x}(q)'q}{1 - \theta - H(\underline{x}(q))}
\] (64)

Proof. Differentiate \( V_\theta(q) \) with respect to \( q \), and note that \( V_\theta'(q) = 0 \) at \( q = q_\theta \).

One can rewrite the sales probability elasticity a bit further. Let
\[
\psi_\theta(x) = (1 - \theta)(1 - H(x)) + \theta
\]
be the probability of meeting a buyer of type \( x \) or better (including the collector), from the perspective of a type-\( \theta \) seller. Define its elasticity
\[
\eta_{\theta,\psi}(x) = \frac{\psi_\theta'(x)x}{\psi_\theta(x)} = -\frac{h(x)x}{1 - \theta - H(x)}
\]
Define the elasticity of the threshold buyer type,

\[ \eta_x(q) = \frac{x'(q)}{x(q)} \]

Then

\[ \eta_\theta(q) = \eta_{\theta,\psi}(x(q))\eta_x(q). \]

This is the usual chain rule for elasticities, of course, applied to \( \phi_\theta(q) = \psi_\theta(x(q)) \).

The results above suggest a strategy for characterizing an equilibrium. Suppose, one has some conjectured threshold buyer type function \( x(q) \), which is increasing and differentiable in \( q \). With that, solve (63) for the optimal strategy \( q_\theta \) and thereby for the value \( v_\theta = V_\theta(q_\theta) \). With the value, calculate the resulting buyer threshold type function

\[ x^*(x) = \arg\min_x v_x \geq q \]

If \( x^*(q) = x(q) \), one has obtained an equilibrium.

It may be possible to obtain analytical examples, for smart choices for \( H \), exploiting this strategy. We leave this to future research to pursue. Here instead, we shall provide a numerical example.

### 6.3 Numerical example

Rather than employing the first-order conditions above, we compute equilibria, using a rather brute-force grid-maximization algorithm. We create a suitable grid in \( q \) and \( \theta \). We start the iteration at the lower bound \( v^{(0)} = x \), defined over a grid in \( \theta \). We iterate on the mapping \( T : V \to V \) described above. Specifically for step \( j \), calculate \( \phi^{(j)}_\theta(q) \) on the \( q \)-grid, using \( x^{(j-1)} \) on the right hand side of equation (57). Now, calculate \( V^{(j)}_\theta(q) \) per (57) for all grid values \( \theta \) and \( q \). For each grid value \( \theta \), find \( v^{(j)}_\theta \) as the maximum of
$V^{(j)}_\theta(q)$ over the grid values $q$. For each grid value $q$, find the smallest $x$, so that $v^{(j)}_x \geq q$, exploiting (61). This is the new $x^{(j)}_x(q)$ and the next iteration step can commence. Iterate sufficiently often to obtain a reasonable degree of accuracy with the last solution.

As parameters, we chose $\lambda = 1$, $\rho = 0.1$ and let $H$ be a uniform distribution on $[0, 0.25]$. The “collector price” was normalized at $v_1 = 1$. We used an evenly spaced grid of 500001 points for $q$ and 1001 points for $\theta \in [0, 1]$ or 251 points in the relevant range $[0, 0.25]$. As a starting point, we set $v^{(0)}_x = v$, as defined in (60). For each grid value $q$, we then find the smallest $x = x^{(0)}_x(q)$, so that $v^{(0)}_x \geq q$, exploiting (61).

The results are in figures 12 and 13. As one can see, agents with low $\theta$ pursue a strategy of seeking to sell to higher-$\theta$ agents, but beyond (approximately) $\theta = 0.1$, agents now only wait for the collector to make the sale. This can also be seen from the probability of sales. The black-dashed horizontal line shows the price chosen by the $\theta = 0$ types. This indicates that this market proceeds in two stages only, starting from the asset initially in the hands of a $\theta = 0$ agent (or an agent with a low $\theta$). That agent will charge a price $q_\theta$ such that a sale only takes place, when meeting an agent with a fairly high $\tilde{\theta}$, who in turn hopes to sell to the collector, at $q_{\tilde{\theta}} = v_1 = 1$. It would be interesting to find examples, in which there are several stages of sale and re-sale to ever-more optimistic agents: we leave this to future research on this topic.

Consider now averaging across many simulations or individual markets, where the asset is initially held by the least optimistic agent $\theta = 0$. In principle, one can obtain the average price as well as the average hazard rate of a sale by simulation, using the results calculated thus far. Due to the two-stage structure of the sales process, it is easier to proceed analytically instead (and these arguments can be generalized to a multi-stage structure as well). If the asset is still in the hands of the initial $\theta = 0$ agent, it will
be sold at the hazard rate $\xi = \phi_0(q_0)$, where we introduced the new symbol $\xi$ for this hazard rate, to save on subsequent notation. Once the asset is sold, it will be posted at price $q_\theta = 1$ and not trade again, since there is no collector in the market. The unconditional date-$t$ probability $\pi_t$, that the asset remains in the hands of initial $\theta = 0$ agent, solves the linear differential equation $\dot{\pi}_t = -\xi \pi_t$, with the solution given by $\pi_t = \exp(-t\xi)$. The average price is given by

$$E[q_t] = v_1 - (v_1 - q_0(q_0)) \exp(-t\xi)$$

The unconditional or average hazard rate of a sale occurring is $\xi \pi_t$. Figure 13 shows the resulting average price path and average sales hazard rate $E[\phi_\theta(q_\theta)]$. As one can see, the average posted price rises, while the average sales probability falls over time. The price path is conditional on the revelation event not occurring. Once the revelation event happens, the price crashes to zero. This captures the original story, with which this section got started.

The logic of the calculation just presented can also be used to calculate the results in 12 directly. Calculate first $v_\theta^{(0)} = v_\theta$ for all $\theta$ per equation (60). Invert that function and use the distribution function $H$ to calculate $\phi_\theta(q)$. With that, calculate $V_\theta(q)$ and find its maximum, for each $q$ and the resulting $v_\theta^{(1)}$. The last step appears tedious, but may be solvable in closed form. This is the final result, in the situation that there are at most two stages of selling (first, sell to a more optimistic agent, second, attempt to sell to the collector), as here. One has to verify that indeed there are no further stages. Put differently, one now has to check that less optimistic agents would not now want to “change their mind” and sell to even more optimistic agents, who now value the assets higher, due to their reselling to optimistic agents. We leave the details of the calculations and the verification condition
to the interested reader.

7 Evidence

What caused the subprime crisis and, by extension, the financial crisis of 2008? What moved first, what moved later? What was cause and what was effect? The chapter has focused on two possible stories. One possibility is that house prices fell first for exogenous reasons, impairing bank balance sheets and leading to a financial collapse. Another possibility is that the banking system collapsed, leading to a reduction in mortgage lending and a fall in house prices. Perhaps, the fall in house prices triggered greater reluctance by banks to issue subprime loans, or perhaps and conversely, mortgage lending and, in particular, subprime borrowing, was reduced first, triggering a fall in house prices. Perhaps, subprime lending was reduced in the wake of higher delinquency rates on subprime mortgages or perhaps subprime lending was reduced, and the subsequent fall in house prices triggered delinquencies. Perhaps delinquencies rose because the pool of borrowers worsened or perhaps short-term interest rates rose, leading to higher rates for ARM mortgages and thereby higher delinquency rates. There are various ways of thinking through the interactions and tell the story. And how much did the interplay and feedback loop enhance the original shock?

Considerable research has been undertaken to seek to sort out these channels empirically: much more work still awaits to be done. We shall not attempt to give a full-fledged overview of the existing and large literature. We instead select some figures and facts from parts of the literature, and give them a somewhat impressionistic interpretation. Clearly, this is no substitute for careful empirical research on these data, but it may provide a good guide to questions and to developments to look at in the raw data. Most of the facts concern the United States. This generates a frontier of research interest.
Figure 12: Results from a numerical example. As parameters, we chose $\lambda = 1$, $\rho = 0.1$ and $H$ to be a uniform distribution on $[0, 0.25]$. The “collector price” was normalized at $v_1 = 1$. In the top left panel, we compare the optimal value function to the value function $v$ obtained per only selling to the collector. As one can see, agents with low $\theta$ pursue a strategy of seeking to sell to higher-$\theta$ agents, but beyond $\theta = 0.1$, agents now only wait for the collector to make the sale. This can also be seen from the probability of sales. The black-dashed horizontal line shows the price chosen by the $\theta = 0$ types. The top-right panel is essentially the top-left panel, flipped at the 45 degree line.
Figure 13: Averaging over many posted price paths. The left panel shows the average price and how it is increasing over time. The right panel shows the hazard rate of a transaction, and how it is decreasing over time.
and common ground for researchers to discuss, but it may miss important relationships and facts, compared to employing a world-wide perspective. We return to the latter towards the very end.

Figure 14: The S&P/Case-Shiller Home Price Indices

Figures [14] and [15] show the S&P/Case-Shiller Home Price Indices. There is a run-up in house prices up to somewhere in the middle of 2006. According to the 20-city index in [15], the peak is in July 2006. From there, prices started to drop, falling by 6.5% in October 2007, a relatively small drop. However, by October 2008, the date of the Lehman Brothers crisis and, in essence, the date of the financial collapse, house prices were at 25% below their July-2006 peak level, having fallen rather quickly and continuously from October 2007. House prices fell a bit more subsequently, reaching their bottom in April 2009, having fallen 32.6% from the original peak. From the sequence of these events, it appears plausible that house prices fell first, and the financial
system collapsed subsequently.

However, the share of mortgages in the form of subprime fell quite substantially much earlier, as figure 16 reveals. Again, the peak subprime lending share of all mortgage originations occurred in 2006, at 23.5%, with a dramatic fall to 9.2% in 2007 and a near-zero in 2008. The peak occurred roughly at the same time as the peak of the S&P/Case-Shiller index in 14 and 15 and one might even wish to argue that the hump near the peak looks rather similar here. The decline in the peak subprime lending share from 2006 to 2007 was very sharp.

From this comparison, it appears plausible that subprime lending rose and fell together with house prices. If anything, subprime lending collapsed and decreased sharply, before house prices did. Thus, it may have been the reduction in subprime lending, causing the fall in house prices rather than
vice versa.

One might wish to blame the decline in subprime lending on delinquency rates. Here, figure [17] is revealing. First, it shows that delinquency rates on fixed rate mortgages, be they prime or subprime, did not noticeably increase in 2006 and 2007: if anything, subprime fixed rate delinquency rates were near their all-time low of 2005.

The story is different for adjustable rate mortgages or ARMs. Here, rates did go up somewhat in 2006 and then somewhat more from their all-time low in 2004, but even there, the level in 2007 is rather comparable to the levels before 2002, both for prime adjustable rates as well as subprime adjustable rates, as figure [17] shows.

These movements are important for interpreting the course of events leading up to the crisis, but they are fairly small, compared to the subsequent
development of delinquency rates shown in figure 18. Delinquencies later rose to unprecedented levels (at least for this time interval), peaking at rates somewhat above 40% for subprime adjustable rate mortgages around the end of 2009. In particular, delinquencies on subprime mortgages and prime adjustable rate mortgages had risen already considerably until October 2008, the date of the financial collapse. Overall, though, it does not seem plausible to argue that subprime lending was reduced in 2007, because delinquency rates had increased already at that point.

If anything, perhaps the delinquency rates in 2007 and the overall movement of delinquency rates on adjustable rate mortages up to 2007 are linked to short-term interest rates. These rates are shown in figure 19. The Federal Reserve Bank increased the Federal Funds Rate in a sequence of small steps, starting at 1 percent in June 2004 to 5.25 in July 2006, and then lev-
eling off, before dramatically reversing course at the end of 2007. It is fairly plausible that the rise in short-term market interest rates from mid-2004 to mid-2006 resulted in the rise of delinquencies on adjustable rate mortgages from mid-2004 to mid-2006, seen in the previous figures.

Justiniano, Primiceri, and Tambalotti (fort) have argued that house prices rose from 2000 to 2007 without an expansion of leverage, i.e. at rather constant rates of mortgages to real estate, see the bottom-right panel of figure 20. The subsequent fall in house prices then went along with an increase in leverage and not necessarily a reduction in the volume of outstanding mortgages, see the top left panel.

These authors point to the fact that, first, there was an increase in available funds without an increase in leverage, leading to more mortgages at stable interest rates and stable leverage ratios, leading to a run-up in house
prices. For 2007 and beyond, they argue that the collateralizability of houses relative to available funds increased (or that available funds for lending decreased), leading to a rise in mortgage rates and a collapse in house prices. There certainly are some interesting comovements in 20 that deserve explanation, though not all readers may buy into the hypothesis that the collapse in house prices was caused by their relatively better collateralizability.

Most notably, Jorda, Schularick and Taylor and their co-authors have investigated the interplay between credit booms, house price booms and economic performance in a series of papers, constructing and providing new data sets along the way. Schularick and Taylor (2012) provide “a new long-run historical dataset for 14 developed countries over almost 140 years” and show, how credit growth is a powerful predictor of financial crises. Jorda, Schularick, and Taylor (2013) subsequently argue that financial crisis
recession are costlier than typical recession. These data sets are updated and extended in Jorda, Schularick, and Taylor (2016) for 17 advanced economies from 1870, covering disaggregated bank credit to the domestic nonfinancial private sector, with special attention to mortgage lending. They claim that "mortgage lending booms were only loosely associated with financial crisis before WWII, but ... [have] become a more important predictor of impeding financial fragility” subsequently. Knoll, Schularick, and Steger (2014) construct a house price index for 14 advanced economies from 1870 to 2012, assembling a variety of data sources. They argue that real house prices have largely followed a “hockey stick” pattern: fairly constant for a long time initially, followed a pronounced appreciation towards the end of the sample. They furthermore say that most of the price increase can be attributed to the increase in the price of land. Knoll (2016) subsequently argues that the rise in
house prices coincides with a rise in the price-rent ratio. Combining data from these papers for 14 advanced economies, Jorda, Schularick, and Taylor (2015) claim that the 20th century has been an era of increasing “bets on the house.” They write that “mortgage credit has risen dramatically as a share of banks’ balance sheets from about one third at the beginning of the 20th century to about two thirds today.” Using IV regressions, they show that “mortgage booms and house price bubbles have been closely associated with a higher likelihood of a financial crisis.” Jorda, Schularick, and Taylor (2017) provide further insights into the nature of these interactions, extending their data sets once more. They point to the fact that the build-up of leverage leads to higher tail risk. In a related paper, Mondragon, Wieland, and Yang (2016), using a spatial IV-strategy, document empirical evidence that local credit supply shocks generate quantitatively significant boom-bust cycles in local house prices. In a similar spirit, Favara and Imbs (2015) show that an expansion in mortgage credit has significant effects on house prices, using the US branching deregulation between 1994 and 2005 as an instrument for credit. More recently, DiMaggio and Kermani (2014) show that a credit expansion can generate a boom and bust in house prices and real activity, using the change in national banks’ regulation in 2004 by banning the anti-predatory lending laws that a number of states adopted in 1999.

These series of papers and insights are completely in line with the fact that “a rise in household debt to GDP ratio predicts lower output growth”, as shown by Mian, Sufi, and Verner (2015).

We now show some figures taken from these papers to highlight some of these insights. Figure 21 from Jorda, Schularick, and Taylor (2017), show the “hockey stick” both for real house prices and mortgages.

The data can be sliced in other ways too, as figure 22 shows. That figure plots results both for the US alone, their “benchmark economy”, and for the sample of 17 countries investigated in Jorda, Schularick, and Taylor (2017).
House price growth and mortgage growth generally comove. In relation to real GDP, the real house price hockey stick, visible in figure 21, now becomes a downward trend, while the mortgage hockey stick becomes an upward trend. These figures raise intriguing, additional issues, concerning the attribution of changes in these series to their underlying causes.

Knoll, Schularick, and Steger (2014) use figure 23 and additional analysis to show that house prices have risen faster than income in recent decades, while they have fallen relative to income in the first half of the 20th century and especially in the interwar period. Knoll (2016) argues that the rise in house prices coincided with a rise in the house price to rent ratio, as shown in figure 24. The price-to-rent ratio is similar to the often used price-dividend ratio for stocks, which has been shown to be useful for predicting stock returns. It is plausible that a similar phenomenon is at work for the housing market, as Knoll (2016) investigates.

The data sets created by these authors will be useful for further empirical investigations of the issues at hand. Luigi Bocola, in his discussion of a first draft of this chapter, combined the quarterly house price dataset from 1975 for a number of advanced countries, described by Mack and Martínez-García (2011), with the data for the 19 crisis events for advanced economies after 1975, de-
Figure 22: Growth and Trends in Mortgages and House Prices. These figures have been provided by Helen Irvin and Oscar Jorda.

scribed in Schularick and Taylor (2012). He constructed figure 25 to shed light on the relationship between house price growth, credit growth and GDP performance. The figure compares all crisis events (blue line) to the 5 events with the highest house price drop (red line) in the group: Denmark-87, Spain-08, Uk-91, Norway-88, Swe-91. This figure indicates once more the co-movement of house prices and credit growth, but may suggest that the size of house price boom does not matter much for the average size of the subsequent recession.

Once more, we want to stress that this section is not meant to be a comprehensive review of the empirical literature on this topic. Our objective was just to report a subsample of facts and empirical papers that relate to the theoretical literature we have focused on in this chapter. These are all suggestive figures. However, the debate on whether house prices have been the main driving source of the credit cycle or financial conditions the main
Figure 23: House prices have risen faster than incomes in recent decades, while they have fallen relative to incomes in the first half of the 20th century and especially in the interwar period.

The driving force of house price cycle is still open and hopefully future research will shed more light on this topic.

8 Conclusions

The purpose of this chapter was to explore a key connection between boom-bust episodes in housing markets and boom-bust episodes in credit markets and to point to their effects on macroeconomic activity. To do so, we investigated several benchmark approaches and channels, and related them to the existing literature. It is already a challenge to understand the house price boom-bust together with the credit boom-bust, without analyzing the aggre-
Figure 24: The rise in house prices coincided with a rise in the house price to rent ratio.
Figure 25: Date “T” denotes the Schularick-Taylor crisis dates. The panels compare all crisis events (blue line) to the 5 events with the highest house price price drop (red line) in the group: Denmark-87, Spain-08, Uk-91, Norway-88, Swe-91. The lines are cross-country averages for the four variables, and normalized to equal unity at $T - 5$. 
gate activity repercussions. We therefore mostly focused on the interaction of the first two. In particular, there are two broad possible approaches to think about this interaction:

1. The house price boom-bust generates the credit boom-bust.

2. The credit boom-bust generates the house price boom-bust.

We started the chapter by proposing a stark mechanical model to think about this interaction. On purpose, it is designed to avoid several thorny issues that arise in a fully specified equilibrium model. Next, we explored these two main approaches in more detail.

First, we proposed a simple model of catastrophes, where we focused on the credit cycle. The idea is that an increase in credit availability can generate first a boom and then a bust in mortgage markets because of adverse selection issues. In particular, in a world where banks do not know the quality of their borrowers, that is, their expected default rate, and borrowers can either pool or pay a cost to verify their type, multiple equilibria can arise. If we start from an equilibrium with pooling, an increase in credit supply translates into a decrease in the quality of the pool of active borrowers (like “subprime borrowers”). This, in turns, can generate a switch to an equilibrium where good borrowers separate themselves and the pooling market crashes.

Second, we proposed a simple model of sentiments, where we focus on the housing price cycle. The main idea is that a house price bubble can arise when speculating households believe that there is always a “bigger fool” out there that is going to be willing to buy housing at a higher price. While appealing, formalizing this intuition tends to run into the “conundrum of the single equilibrium”: if prices are expected to be high tomorrow, then demand for credit and thus houses should be high today (see above), and that should drive up prices today, making it less likely that prices will increase. Or, put differently, as prices rise, they eventually must get to a near-maximum at
some date: call it “today”. At that price, prices are expected to decline in the future. But if so, banks and speculating households are less likely to buy today: but then, the price should not be high today. That is, in rational expectations models, there should not be anybody willing to buy at the highest price when prices can only go down from there. We break the curse of the conundrum by departing from the rational expectations framework and assuming that households always believe that with some positive probability there is a bigger fool, although he does not really exists.

In the course of the chapter, we related our simple models to the large literature on these topics. At the end, we also point to some empirical papers that propose facts related to these two theoretical approaches.

We wish this chapter is going to trigger further research and thinking on this important connection. As has become clear, the issues are far from resolved.

References


