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"Preference Heterogeneity and Optimal Capital Income Taxation"

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Location: HC 3B

For any other information regarding the Applied Economics Workshop, please contact Tamara Lingo (AEW Administrator) at 773-702-2474, tammy.lingo@ChicagoBooth.edu, or stop by HC448.
Preference Heterogeneity and Optimal Capital Income Taxation

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Abstract

We analytically and quantitatively examine a prominent justification for capital income taxation: goods preferred by those with high ability ought to be taxed. We study an environment where commodity taxes are allowed to be nonlinear functions of income and consumption and show that a relationship between ability and preferences over goods provides a novel reason for sophisticated distortionary commodity taxation. For a two-type economy, we show that optimal relative marginal commodity taxes on a good preferred by high ability individuals are regressive: i.e., declining with income. For an economy with a continuum of types, we derive and explain an analytical expression for optimal commodity taxation, showing how preference variation allows for better screening of ability types. We then calibrate the model to evidence on the relationship between skills and preferences and examine the quantitative case for taxes on future consumption (saving). The relationship between skill and time preference delivers quantitatively small, generally regressive capital income taxes and would justify only a fraction of the prevailing level of capital income taxation.

Introduction

One justification for positive capital income taxation is that the goods preferred by high-ability individuals ought to be taxed because the consumption of these goods provides a signal of individuals’ otherwise unobservable ability.1,2 If individuals’ abilities are positively related to preferences for saving, this argument implies that capital income should be taxed. Two prominent expositions of this justification are Saez (2002) and Banks and Diamond (2009). Saez shows that a small linear tax on a commodity preferred by individuals with higher ability generates a smaller efficiency loss than does an increase in the optimal nonlinear income tax rate.

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1This result originated in Mirrlees (1976) and also appears in Mirrlees (1986). Nearly all comprehensive treatments of modern tax policy contain a section on this result as a deviation from the standard Atkinson-Stiglitz (1976) recommendation of uniform commodity taxation. Tuomala (1991) writes "...the marginal tax rates on commodities that the more able people tend to prefer should be greater." Salanie (2003) warns "If there is a positive correlation between the taste for fine wines and productivity, then fine wines should be taxed relatively heavily (God Forbid)!" while Kaplow (2008) argues "it tends to be optimal to impose a heavier burden on commodities preferred by the more able and a lighter burden on those preferred by the less able." Enthusiasm for this result may be because, as Mirrlees put it "This prescription is most agreeable to common sense."

2A different justification for the positive capital wedge is the New Dynamic Public Finance literature (see e.g., Golosov, Kocherlakota, and Tsyvinski 2003; Golosov, Tsyvinski and Werning 2006; Kocherlakota 2009).
tax that raises the same revenue from each individual. He applies this logic to capital income taxation and concludes "...the discount rate \( \delta \) is probably negatively correlated with skills. This suggests that interest income ought to be taxed even in the presence of a non-linear optimal earnings tax." Banks and Diamond (2009) is the chapter on direct taxation in the Mirrlees Review. Commissioned by the Institute for Fiscal Studies, the Mirrlees Review is the successor to the influential Meade Report of 1978 and is the authoritative summary of the current state of tax theory as it relates to policy. Their chapter concludes:

"With the plausible assumption that those with higher earnings abilities discount the future less (and thus save more out of any given income), then taxation of saving helps with the equity-efficiency tradeoff by being a source of indirect evidence about who has higher earnings abilities and thus contributes to more efficient redistributive taxation."

In this paper, we analytically and quantitatively study this justification for taxing goods preferred by those with high ability, in particular future consumption (i.e., saving), when commodity taxes are allowed to be nonlinear functions of both income and consumption.³

We first derive analytical expressions that indicate the shape of optimal commodity taxation. We start in a two-type, two-commodity economy and demonstrate that the high ability type faces no distortion to its chosen commodity basket while the low type faces a distortion away from consumption of the good preferred by the high type. In other words, relative marginal taxes are regressive on the good preferred by those with high ability.⁴ We then derive the condition describing optimal commodity taxes in an economy with two goods and a continuum of types where the relative preference for one good rises with ability. The marginal commodity tax on the good preferred by the able is again equal to zero for the highest type, and it is positive for lower-ability types. As is common in Mirrleesian models (e.g., Saez 2001) we then analytically study the forces driving the optimal distortions to commodity choices.

The intuition for why optimal nonlinear commodity taxation would more strongly discourage low earners from consuming a good preferred by high earners starts with the realization that the goal of optimal tax policy (in the Mirrleesian framework) is to redistribute from high-ability workers without discouraging their work effort. With this as the goal, the optimal use of commodity taxation is to increase the attractiveness of earning a high income. Marginal commodity tax rates that decline with income on those goods most valued by high-ability individuals will encourage them to earn more, allowing the tax authority to levy higher income taxes on them and redistribute more resources to those with lower ability.⁵

We also examine the quantitative case for capital income taxation due to the relationship between ability and preferences. We use data from the National Longitudinal Survey of Youth (NLSY) to show a positive correlation between ability and relative preference for future consumption. Our results are consistent with those of Benjamin, Brown, and Shapiro (2006), who find a positive relationship between ability and the

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³Though most research on this issue has focused on the linear tax problem, Mirrlees (1976, 1986) is clear that his results apply to nonlinear marginal commodity tax rates. A few later authors also noted the potential for optimal nonlinear rates: e.g., Kaplow (2008). Banks and Diamond (2009) look for but find no work on the nonlinear problem. They write: "In the context of this issue, how large the tax on capital income should be and how the marginal capital income tax rates should vary with earnings levels has not been explored in the literature that has been examined." ⁴The flip side of this regressive tax on the good preferred by the high ability is a progressive subsidy on the good not preferred by the high ability. Throughout the paper, we describe the optimal policy in terms of its effects on the goods preferred by the high ability, in keeping with the existing literature’s emphasis on capital taxation. ⁵The standard argument against nonlinear commodity taxation is arbitrage or retrading (see Hammond 1987, Golosov and Tsyvinski 2006). That may be an appropriate restriction for many goods, but important categories of personal expenditure can feasibly be taxed nonlinearly or as a function of income. ⁶We measure ability by the survey respondent’s score on the cognitive ability portion of the Armed Forces Qualification Test (AFQT). While it is impossible to measure ability perfectly, the AFQT score is commonly used, such as in the study of the returns to education.
holding of positive net assets. Using these data to estimate the relationship between time preference and wages, we find that optimal capital income tax rates are quantitatively small and generally flat or declining with income. For the baseline example, the maximal capital income tax rate in the nonlinear case is everywhere less than 6%, and the constrained-optimal linear capital income tax rate is 3.1%. Moreover, welfare gains from these optimal capital income taxes are negligible. These results suggest that the empirical relationship between ability and time preference justifies only a small fraction of the prevailing level of capital income taxation in developed economies.

This paper also studies the importance of preference normalization in our optimal taxation model. We normalize preferences over commodities in two ways. These normalizations are similar to two assumptions made by Saez (2002) in his analysis of optimal commodity taxes with preference heterogeneity. First, we normalize preferences to eliminate any incentive for the planner to redistribute across agents based simply on their preferences over goods. Specifically, the marginal social value to a Utilitarian planner of allocating resources to an undistorted individual is independent of that individual’s preferences over consumption goods. We also normalize preferences in a second way. We model preferences over commodities, including future and current consumption, as having no direct effect on the labor supply decisions of individuals. This normalization contrasts with the approach in recent work by Diamond and Spinnewijn (2009), who model preferences such that more patient individuals are more willing to work. Because the challenge of optimal tax policy is to encourage the high-ability to work despite redistributive taxation, our normalization increases the role for capital taxation as part of the optimal policy.

Finally, we extend our analysis of optimal capital taxation to a stochastic setting in which there is a relationship between ex post ability and preferences over goods consumed within a period. We show that this relationship does not affect the optimal intertemporal distortion: i.e., the inverse Euler equation of Golosov, Kocherlakota, and Tsyvinski (2003) continues to hold. Optimal distortions within the second period are similar to the results from the static model.

The paper proceeds as follows. Section 1 provides an illustrative example of our theoretical results in an economy with two ability types and heterogeneity in preferences over two goods. Section 2 derives conditions on the optimal policy in a general model of optimal taxation with a continuum of ability types and heterogeneity in preferences. In Section 3, we calibrate the model to data from the NLSY on heterogeneous time preferences and calculate optimal distortions. We also test the robustness of our results to variation in individual risk aversion and labor supply elasticity. In Section 4, we compare these results to prevailing capital income tax rates in developed economies and characterize the relationship between ability and time preference that would be required for prevailing rates to be optimal. Section 5 discusses the importance of preference normalization in these models. Section 6 considers the dynamic, stochastic model. An Appendix contains technical details referred to in the text.

1 A simple example

In this section we introduce a simple example that captures the main intuition behind the more general model. We show that, in this setting, the optimal relative commodity tax is regressive for the good preferred

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7If we took into account variation around mean preference values within ability levels, the optimal taxes and welfare gains are likely to be even smaller.

8Similar examples are found in Diamond (2007) and Diamond and Spinnewijn (2008). However, as discussed in Section 6, we normalize preferences in important ways that these other examples do not. This normalization has direct effects on the optimal policies we derive.
by those with high ability. In particular, the relative marginal tax (wedge) is positive on this good for the low-ability individual, while the high-ability individual faces no distortion.

There is a continuum of measure one of two types of individuals indexed by \( i = \{ l, h \} \). The size of each group is equal to \( 1/2 \). These individuals differ in wage (skill) \( w^i \), where \( w^h > w^l > 0 \). The wage is private information to the agent. There are two commodities. The consumption of each commodity by an agent of type \( i \) is denoted by \( c_1 \) and \( c_2 \). The utility function for an individual \( i = \{ l, h \} \) is given by:

\[
 u \left( c_1^i, c_2^i, \frac{y}{w^i} \right),
\]

where \( y_i \) denotes the amount of output (income) produced by the agent. That is, the agent provides the amount of labor \( l^i \geq 0 \) to produce output \( y^i = w^i l^i \geq 0 \). The planner observes output \( y^i \) but not the wage \( w^i \) or effort \( l^i \). Agents’ consumption in each period \( t \) is observable. Let \( u_n \) be the partial derivative of \( u(c_1^i, c_2^i, l) \) with respect to the \( n \)th argument. Note that these partial derivatives may depend on the wage rate.\(^9\) We assume that \( u_n > 0 \), for \( n = \{1, 2\} \) and \( u_3 < 0 \).

The planner’s problem is a mechanism design problem in which the mechanism assigns consumption and income allocations to each wage type reported by agents. The planner designs the mechanism to maximize a Utilitarian social welfare function.

**Problem 1** Planner’s problem in two-type example

\[
\max_{\{c_1^i, c_2^i, y^i\}_{i=h}} \sum_i u \left( c_1^i, c_2^i, \frac{y^i}{w^i} \right)
\]

subject to the incentive compatibility constraint

\[
u \left( c_1^h, c_2^h, \frac{y^h}{w^h} \right) \geq u \left( c_1^l, c_2^l, \frac{y^l}{w^l} \right), \quad (2)
\]

and the feasibility constraint

\[
\sum_i y^i - c_1^i - c_2^i \leq 0. \quad (3)
\]

Constraint (2) is an incentive compatibility constraint stating that an individual of type \( i = 2 \) prefers the consumption and income bundle intended for it by the planner, \( \{c_1^h, c_2^h, y^h\} \), to a bundle \( \{c_1^l, c_2^l, y^l\} \) allocated to an individual of type \( i = 1 \).\(^9\) Constraint (3) is feasibility, where we assume that the marginal rate of transformation of consumption commodities is equal to one.

Consider first a benchmark environment in which the wage \( w^i \) is observable to the planner. Then the constrained efficient problem does not have the incentive compatibility constraint (2). The solution is then undistorted for both ability types – the marginal rates of substitution across commodities are equal to the marginal rate of transformation, which is equal to one:\(^11\)

\[
\frac{u_1 \left( c_1^i, c_2^i, \frac{y^i}{w^i} \right)}{u_2 \left( c_1^i, c_2^i, \frac{y^i}{w^i} \right)} = 1. \quad (4)
\]

\(^9\)For example, using the utility function from the general model stated later, (21), \( u_1 \left( c_1^i, c_2^i, l^i \right) = \frac{\alpha(w^i)}{1 + \alpha(w^i)} \frac{1}{c_1^i} \).

\(^10\)Writing this constraint we assumed that only an individual of type \( i = h \) can misrepresent his type. This is easy to ensure if the ratio \( w^h/w^l \) is high enough.

\(^11\)The consumption-labor margin is also undistorted.
Now, consider a program with unobservable wages. Let $\mu \geq 0$ be the multiplier on constraint (2). From the first order conditions for consumption, we obtain the following expressions for the marginal rate of substitution between consumption commodities for the high-wage individual, type $i = h$:

$$\frac{u_1(c^h_1, c^h_2, \frac{y^h}{w^h})}{u_2(c^h_1, c^h_2, \frac{y^h}{w^h})} = 1,$$

and for the low-wage individual, type $i = l$:

$$\frac{u_1(c^l_1, c^l_2, \frac{y^l}{w^l})}{u_2(c^l_1, c^l_2, \frac{y^l}{w^l})} = \frac{1 - \frac{u_2(c^l_1, c^l_2, \frac{y^l}{w^l})}{u_2(c^l_1, c^l_2, \frac{y^h}{w^h})} \mu}{1 - \frac{u_1(c^l_1, c^l_2, \frac{y^l}{w^l})}{u_1(c^l_1, c^l_2, \frac{y^h}{w^h})} \mu}.$$

Equation (5) shows that the consumption choices of the high-ability individual are undistorted. The marginal rate of substitution $\frac{u_1}{u_2}$ is equal to the marginal rate of transformation, which is equal to one. Equation (6) shows that if the multiplier $\mu$ on the incentive compatibility constraint is not equal to zero, then the consumption choices of the low-ability individual are distorted.

Now, suppose we impose a condition requiring that if all individuals are given the same consumption and income allocation, $(c_1, c_2, y)$, the marginal utility of good 2 relative to good 1 is higher for the high-ability individual (type $i = h$) than for the low-ability individual (type $i = l$).

**Assumption 1** The utility function $u$ satisfies:

$$\frac{u_2(c_1, c_2, \frac{y}{w})}{u_1(c_1, c_2, \frac{y}{w})} > \frac{u_2(c^l_1, c^l_2, \frac{y^l}{w^l})}{u_1(c^l_1, c^l_2, \frac{y^l}{w^l})}$$

for any $(c_1, c_2, y) \geq 0$.

The first order conditions (5) and (6) together with Assumption 1 then imply a proposition characterizing the distortions in the optimal allocation.

**Proposition 2** Suppose that $\{c^l_1, c^l_2, y^l\}_{i=h}$ is an optimal allocation solving (1) through (3). Then the optimal choice of consumption for the high-ability individual ($i = h$) is not distorted. Suppose that Assumption 1 holds. Then the optimal choice of consumption for the low-ability agent ($i = l$) is distorted away from good 2 in favor of good 1:

$$\frac{u_1(c_1, c_2, \frac{y}{w})}{u_2(c_1, c_2, \frac{y}{w})} < 1.$$

This Proposition states that if good 2 is particularly enjoyed by high-ability workers, the planner should impose a distortion (i.e., a positive relative tax)$^{12}$ on the consumption of good 2 by the low-ability workers (but not on consumption of that good by high-ability workers).$^{13}$ The intuition for this result is as follows. The planner wants to discourage a high-ability individual from deviating and claiming that he is a low type. A high-ability agent will find deviating less attractive if doing so causes him to face a positive relative tax

$^{12}$In this model, the implementation of the optimal allocation with taxes is easy as the taxes correspond to wedges. We interchangeably use terms wedges and taxes.

$^{13}$Mirrlees (1986) shows a similar relationship between the marginal rates of substitution by ability level and optimal distortions. He does not characterize the optimal pattern of these distortions, however, our central goal in this paper.
on the good that he values highly. The cost to the planner of such a positive relative tax is a distortion in the consumption choices by the low-ability agent. Assumption 1 ensures that the costs of such distortion are smaller than the gain from relaxing the incentive compatibility constraint.

It is important to be clear that this result depends on preferences varying by ability level, not income. In particular, it does not apply to goods with an income elasticity of demand greater than 1 but for which preferences are unrelated to ability. For those goods, the inequality in (7) would be an equality because each type would have the same ratio of marginal utilities given the same consumption and income bundle. Instead, the case for regressive taxes requires the high-ability to prefer good 2 even when at the same income level as the low-ability.

2 Model

In this section, we set up a model with a continuum of ability types, as in the classic Mirrlees (1971) framework. Agents are heterogeneous in their preferences. We derive a formula for optimal relative commodity taxes that are allowed to be nonlinear in consumption and to depend on income. Our results formally resemble those of Diamond (1998) and Saez (2001) on optimal income taxes.

There is a continuum of measure one of individual agents. Agents are indexed by \( i \in [0, 1] \). Individuals differ in their abilities, which we measure with their wages, denoted by \( w^i \) and distributed according to the density function \( f(w) \) over the interval \([w_{\text{min}}, w_{\text{max}}]\). Ability is private information to the agent. The utility function of an individual depends on \( w^i \), so that the preference parameter for an individual depends directly on his or her wage.

Each individual has the continuous and differentiable utility function:

\[
U(w^i) = u(c_1^i, c_2^i, l^i, \alpha(w^i)).
\] (8)

Utility is a function of the consumption of good 1, \( c_1 \geq 0 \), and the consumption of good 2, \( c_2 \geq 0 \), as well as of labor effort \( l \geq 0 \), and the preference parameter \( \alpha(w^i) \). We assume \( \alpha(w^i) > 0 \) for all \( w^i \) and that the partial derivatives of utility take the following signs: \( u_{c_1}(\cdot) > 0, u_{c_2}(\cdot) > 0, u_l(\cdot) < 0 \). Superscripts \( i \) on consumption and labor denote the values of these variables for the individual of wage \( w^i \). To capture preference heterogeneity, we assume that preferences across consumption goods are a function of ability. This simplifies the planner’s problem by retaining a single dimension of heterogeneity. Two or more dimensions introduce a multiple screening problem for which a tractable analytical approach at this level of generality has not been developed.\(^\text{14}\) The output \( y^i = w^i l^i \geq 0 \).

A social planner maximizes a utilitarian social welfare function. The planner’s problem is given as follows.

Problem 3

\[
\max \left\{ c_1^i, c_2^i, y^i \right\}_{i \in [0, 1]} \int_{w_{\text{min}}}^{w_{\text{max}}} u(c_1^i, c_2^i, y^i, \alpha(w^i)) f(w^i) \, dw^i \tag{9}
\]

subject to the feasibility constraint

\[
\int_{w_{\text{min}}}^{w_{\text{max}}} (y^i - c_1^i - c_2^i) f(w^i) \, dw^i \geq 0, \tag{10}
\]

\(^\text{14}\) See Kleven, Kreiner, and Saez (2009), Tarkiainen and Tuomala (2007), and Judd and Su (2008) for discussions of the approach to optimal taxation with multi-dimensional heterogeneity.
and the incentive compatibility constraint

$$u \left( c_1^i, c_2^i, y_i^i, w_i^i, \alpha (w^i) \right) \geq u \left( c_1^j, c_2^j, y_i^j, w_i^j, \alpha (w^i) \right),$$  \hspace{1cm} (11)$$

for all \( i, j \in [0, 1]. \)

Constraint (11) is the incentive compatibility constraint ensuring that an individual of type \( i \) prefers the consumption and income allocation intended for it by the planner to the allocations intended for any other individual of type \( j \).

Solving the planner’s problem in equations (9) through (11) can yield insights into the wedges that optimal policy drives into private optimization.

It is standard to rewrite the planner’s problem with explicit tax functions. To characterize the form of these optimal tax functions, we follow the formal techniques of the Mirrleesian literature. We start with the statement of the problem solved by each individual, who takes the tax functions as given.

**Problem 4 Individual’s Problem, \( i \in [0, 1]: \)**

$$\max_{\{c_1^i, c_2^i, l^i\}} U \left( c_1^i \right)$$  \hspace{1cm} (12)

subject to the individual’s after-tax budget constraint,

$$l^i w^i - T \left( w^i l^i \right) - (c_1^i + t^1 \left( w^i l^i, c_1^i \right)) - (c_2^i + t^2 \left( w^i l^i, c_2^i \right)) \geq 0.$$  \hspace{1cm} (13)

The budget constraint requires careful examination. The nonlinear income tax \( T \left( w^i l^i \right): R_+ \rightarrow R \) is a continuous, differentiable function of income \( y^i = w^i l^i \). The two other tax functions, \( t^1 \left( w^i l^i, c_1^i \right), t^2 \left( w^i l^i, c_2^i \right) : R_+ \times R_+ \rightarrow R \) are commodity tax functions that we also assume to be continuous and differentiable. Importantly, note that we explicitly allow for the taxation of each commodity to be nonlinear in consumption of that good and to depend on income.\(^{15}\) The budget constraint (13) has the multiplier \( \mu (i) \geq 0. \)

In this formal approach, the social planner’s problem is as follows:

**Problem 5 Planner’s Problem**

$$\max_{\{T(\cdot), t^1(\cdot), t^2(\cdot)\}_{i \in [0, 1]}} \int_{w_{\min}}^{w_{\max}} U \left( w^i \right) f \left( w^i \right) dw^i$$  \hspace{1cm} (14)

subject to the feasibility constraint

$$\int_{w_{\min}}^{w_{\max}} \left( T \left( w^i l^i \right) + t^1 \left( w^i l^i, c_1^i \right) + t^2 \left( w^i l^i, c_2^i \right) \right) f \left( w^i \right) dw^i \geq 0,$$  \hspace{1cm} (15)

and incentive compatibility, which is that each individual \( i \in [0, 1] \) solves the optimization problem in (12), given tax policies \( T \left( w^i l^i \right), t^1 \left( w^i l^i, c_1^i \right), \) and \( t^2 \left( w^i l^i, c_2^i \right). \)

In words, the social planner chooses a tax system to maximize Utilitarian social welfare subject to two constraints. First, the budget constraint requires that total tax revenue be non-negative (we assume no

\(^{15}\)These tax instruments are notationally redundant, in that a single tax function of the consumption of one good and income would be sufficient to characterize the full policy. Separating taxes into these functions aids interpretation and has no effect on the analytical or quantitative results of the paper.
government spending for simplicity). Second, each individual will respond to the tax system by choosing labor supply and a consumption bundle that maximize his or her utility.

2.1 The optimal commodity choice wedge

We now derive a formula for the optimal commodity wedge, i.e., the wedge distorting commodity choices. We formulate the Hamiltonian from the planner’s problem above, using the envelope condition and the first order condition with respect to labor \( l^i \) from the individual’s utility maximization problem:

\[
H \left( w^i \right) = (U \left( w^i \right) + \lambda \left( w^i l^i - c_1^i - c_2^i \right)) f \left( w^i \right) + \phi \left( u_{w^i} \left( \cdot \right) - \frac{l^i u_{w^i} \left( \cdot \right)}{w^i} \right),
\]

where subscripts denote partial derivatives and \( \cdot \) denotes the set of arguments of the utility function, \((c_1^i, c_2^i, l^i, \alpha \left( w^i \right))\). The first term of the Hamiltonian is the utility of the individual with wage \( w^i \). The second is government’s budget constraint multiplied by its multiplier \( \lambda \). The third term is the evolution of the state variable \( U \left( w^i \right) \) with respect to \( w^i \), as derived above, and is multiplied by the costate variable \( \phi \).

To solve for the optimal policy, choose \( l \) and \( c_1^i \) as the control variables, with \( c_2^i \) an implicit function defined by the budget constraint. The first order condition with respect to \( c_1^i \) combined with the condition that individuals will set the ratio of marginal utilities from the consumption goods equal to the marginal tax ratio, yields the following expression for the distortion to individual \( i \)'s consumption basket:

\[
\frac{1 + t_1^i \left( w^i l^i, c_1^i \right)}{1 + t_2^i \left( w^i l^i, c_2^i \right)} = \frac{\lambda f \left( w^i \right) - \phi \left( w^i \right) \left( u_{w^i c_1^i} \left( \cdot \right) - \frac{l^i u_{w^i c_1^i} \left( \cdot \right)}{w^i} \right)}{\lambda f \left( w^i \right) - \phi \left( w^i \right) \left( u_{w^i c_2^i} \left( \cdot \right) - \frac{l^i u_{w^i c_2^i} \left( \cdot \right)}{w^i} \right)}.
\]

(16)

To fully characterize the optimal distortion to commodity purchases given by (16), we solve for the multipliers \( \lambda \) and \( \phi \left( w^i \right) \) under the following assumption:

**Assumption 2** Utility function \( u \) in (8) is separable in consumption and labor, that is it satisfies:

\[
u_{w^i c_1^i} \left( c_1^i, c_2^i, l^i, \alpha \left( w^i \right) \right) = u_{w^i c_2^i} \left( c_1^i, c_2^i, l^i, \alpha \left( w^i \right) \right) = 0
\]

(17)

The following proposition derives an expression for optimal commodity taxes.

**Proposition 6** Given Assumption 2 on the individual utility function, the solution to the Planner’s Problem (14) satisfies:

\[
\frac{1 + t_1^i \left( w^i l^i, c_1^i \right)}{1 + t_2^i \left( w^i l^i, c_2^i \right)} = A_1 \left( w^i \right) B \left( w^i \right) + C \left( w^i \right)
\]

where

\[
A_1 \left( w^i \right) = w^i u_{w^i c_1^i}, \quad A_2 \left( w^i \right) = w^i u_{w^i c_2^i}
\]

\[
B \left( w^i \right) = \left( \frac{1}{1 - F \left( w^i \right)} \int_{w^i = w^i_{\min}}^{w^i = w^i_{\max}} \frac{1}{u_{c_2^i}^i} f \left( w^j \right) dw^j - \frac{1}{1 - F \left( w^i_{\min} \right)} \int_{w^j = w^i_{\min}}^{w^j = w^i_{\max}} \frac{1}{u_{c_2^i}^i} f \left( w^j \right) dw^j \right)
\]

(18)

\[16\] The above procedure uses the so-called first order approach, where the first-order conditions of the individual’s problem are assumed to be sufficient, not just necessary, conditions for a maximum. In the Appendix, we show that our assumptions on the utility function of individuals allows us to reduce the second-order condition’s requirements for a global maximum to two inequalities: \( y' \left( w^i \right) \geq 0 \) and \( c_2^i \left( w^i \right) \geq 0 \). Both of these hold in all numerical simulations we perform in Section 3, and they can be checked to ensure optimality in any simulation.
\[ C(w^i) = \frac{w^i f(w^i)}{1 - F(w^i)} \]

**Proof.** In the Appendix, we derive the following expressions for \( \lambda \) and \( \phi(w^i) \):

\[ \lambda = \frac{1}{\int_{w^j = w_{\min}}^{w^j = w_{\min}} \frac{1}{u_{c_2}} f(w^j) \, dw^j} \]

\[ \phi(w^i) = (1 - F(w^i)) \left( 1 - \frac{\int_{w^j = w_{\min}}^{w^j = w_{\max}} \frac{1}{u_{c_2}} f(w^j) \, dw^j}{\int_{w^j = w_{\min}}^{w^j = w_{\max}} \frac{1}{u_{c_2}} f(w^j) \, dw^j} \right). \]

Using these results in expression (16), we obtain (18). \( \blacksquare \)

As with the conditions for optimal marginal income tax rates from, e.g., Diamond (1998), Saez (2001), and Golosov, Tsyvinski, and Troshkin (2009), expression (18) is not a fully closed-form solution as it depends on optimal utility and consumption levels. Instead, it is a combination of the first order conditions of the optimal problem allowing us to examine the forces affecting optimal taxes.

Importantly, an examination of the terms in (18) gives details about the determinants of the optimal distortions. We identify three important forces at play. Two are familiar from previous results in Mirrleesian optimal taxation, for instance from the formulas for the income tax in Diamond (1998) and Saez (2001). However, they have no impact in our model without the existence of an additional, novel, force.

The novel force affecting distortions in result (18) is the disparity between \( A_1(w^i) \) and \( A_2(w^i) \), which are the derivatives of the marginal utility of consumption of goods 1 and 2 with respect to the wage, multiplied by the wage \( w^i \). This disparity determines whether policy discourages consumption of good 1 or good 2.

If \( A_1(w^i) \) and \( A_2(w^i) \) are equal (for instance, if they are both zero as in most optimal tax models), there is no distortion to consumption choices in the optimal policy. If, instead, higher-ability workers relatively prefer good 2, then \( u_{w^i c_2^1} < 0 \) while \( u_{w^i c_2^2} > 0 \), so \( A_1(w^i) < 0 \) and \( A_2(w^i) > 0 \). In that case, because both \( B(w^i) \) and \( C(w^i) \) are non-negative, the ratio on the right-hand side of (18) is less than one, and the optimal distortion discourages marginal consumption of good 2. That is, the good preferred by the more able workers ought to be marginally taxed.

Whether the distortion to consumption choices increases or decreases with wages depends on the behavior of \( A_1(w^i) \) and \( A_2(w^i) \) as the wage level increases. Though a full characterization depends on the specific form of the utility function, both higher \( w^i \) and the rising consumption levels that come with higher \( w^i \) will push \( A_1(w^i) \) and \( A_2(w^i) \) toward zero as the wage level increases, reducing the size of this distortion at higher wages.

The two forces familiar from previous models are as follows.

First, \( B(w^i) \) measures the redistributive benefit of a distortion at wage \( w^i \). That distortion allows the planner to shift income from those with wages above \( w^i \) to the population as a whole. Formally, due to this distortion the planner can extract an average revenue per high-skilled individual of:

\[ \frac{1}{1 - F(w^i)} \int_{w^j = w_{\min}}^{w^j = w_{\max}} \frac{1}{u_{c_2}} f(w^j) \, dw^j \]

while retaining incentive compatibility. The planner can then distribute this income across the population in an incentive compatible way at an average cost per individual of:

\[ \frac{1}{1 - F(w_{\min})} \int_{w^j = w_{\min}}^{w^j = w_{\max}} \frac{1}{u_{c_2}} f(w^j) \, dw^j. \]

The more that this average revenue exceeds this average cost (i.e., the more that \( B(w^i) \geq 0 \)), the more valuable is this distortion to the planner. Intuitively, higher-ability workers have lower marginal utilities of consumption, and the more concave is utility in good 2 above wage \( w^i \),
the more valuable is the redistribution made possible (i.e., incentive compatible) by the commodity choice distortion at \( w^i \).

Second, \( C (w^i) \) depends on the ratio of the share of the population distorted by a commodity tax at \( w^i \) to the share of individuals with higher wages who are encouraged to exert more effort due to the distortion. When this ratio is low, the optimal consumption distortion (if non-zero) is larger, as the planner wants to concentrate distortions on small sub-populations and use distortions that will encourage large populations to work harder (i.e., when \( f (w^i) \) and \( F (w^i) \) are small). The term \( C (w^i) \) also contains the wage \( w^i \), which allows us to interpret \( C (w^i) \) as an inverse hazard ratio. Following Saez (2001), a Pareto distribution of high abilities would imply a constant value for \( C (w^i) \) above a certain level of \( w^i \).

We can derive several specific results that characterize the optimum and aid intuition. First, on the top type, \((1 - F (w_{\text{max}}))\) is zero, and the result (18) reduces to

\[
\frac{1 + t_{c_1}^1 (w_{\text{max}})}{1 + t_{c_2}^2 (w_{\text{max}})} = 1.
\]

so the commodity distortion is zero on the highest ability worker. Second, the distortion is also zero on the lowest ability worker, as \( B (w_{\text{min}}) = 0 \). Third, if we restrict attention to commodity taxes that are a linear function of the consumption of the good, a modification of result (18) confirms the results of the previous literature (e.g., Saez 2002, Salanie 2003) that goods preferred by the highly able ought to be taxed.

Though these interpretations aid in understanding result (18), we may want to reformulate that result in terms of observable quantities in the spirit of Saez (2001). The Appendix derives the following version of result (18):

\[
\frac{1 + t_{c_1}^1}{1 + t_{c_2}^2} = \frac{C (w^i)}{(\varepsilon_{c_1} w^i - \varepsilon_{c_1} w^i) D (w^i) + C (w^i)}
\]

(19)

where

\[
D (w^i) = \left( \frac{1}{1 - F (w^i)} \int_{w^i = w_{\text{max}}}^{w^i = w_{\text{max}}} \frac{\tilde{y}_j}{y_i} f (w^j) dw^j - \frac{1}{1 - F (w_{\text{min}})} \int_{w^j = w_{\text{min}}}^{w^j = w_{\text{max}}} \frac{\tilde{y}_j}{y_i} f (w^j) dw^j \right),
\]

and where \( \varepsilon_{c_1} w^i \) is the (marginal-utility constant) elasticity of the consumption of good \( m \) with respect to the wage, \( \tilde{y}^i \) is the disposable income individual \( i \) would choose to earn in an economy with income taxes only (i.e., before the introduction of optimal commodity taxes, the planner can observe the distribution of \( \tilde{y}^i \)). This alternative representation of the main result on optimal commodity taxes can be more readily applied with observable data.

One may also be interested in the pattern of marginal tax rates on income that are the focus of the conventional Mirrleesian optimal tax literature. In this paper’s multiple-commodity setting, the marginal tax rate on income must be calculated relative to the distortion to consumption of one of the commodities. In the Appendix, we derive an expression analogous to (18) to describe these relative marginal income taxes. In addition, the numerical results of the next section can be used to calculate the optimal marginal income taxes (relative to commodity consumption) implied by the data.

The results of Sections 1 and 2 show the forces affecting the optimal commodity taxation of goods when preferences over goods vary with ability. The expression (18) makes it clear that the shape of optimal commodity taxes depends on many details of the economy. In the next two sections, we turn to a quantitative study of optimal commodity taxation when the commodities in the utility function are current and future
consumption (savings).

3 Optimal capital income taxes

We begin our quantitative analysis of optimal capital income taxation by discussing the existing literature on the relationship between time preferences and income. That relationship, though interesting on its own, is distinct from the relationship that matters for this paper: that between time preferences and ability. Given the scarcity of research on this topic, we generate new estimates on time preferences by ability level (and thus wages) using data from the National Longitudinal Survey of Youth. We then simulate the optimal capital income taxes justified by these estimates.

Throughout this section, we specify a utility function for numerical analysis. For consistency with Proposition 1 from the previous section, we consider a utility function that is separable in consumption and labor. In addition, we assume that utility from consumption is constant relative risk aversion (CRRA) and the disutility from labor effort is iso-elastic:

$$U = \frac{1}{\phi^i} \left( \left( \frac{\alpha (w^i)}{1 + \alpha (w^i)} \right)^{\gamma} \left( \frac{c^i_1}{1 - \gamma} - 1 \right) + \left( \frac{1}{1 + \alpha (w^i)} \right)^{\gamma} \left( \frac{c^i_2}{1 - \gamma} - 1 \right) - \frac{1}{\sigma} (l^i)^{\sigma} \right)$$  \hspace{1cm} (20)

As a baseline case, we assume $\gamma = 2$ and $\sigma = 3$. Note that if $\gamma = 1$, this utility function simplifies to

$$U = \frac{1}{\phi^i} \left( \left( \frac{\alpha (w^i)}{1 + \alpha (w^i)} \right) \ln c^i_1 + \left( \frac{1}{1 + \alpha (w^i)} \right) \ln c^i_2 - \frac{1}{\sigma} (l^i)^{\sigma} \right)$$  \hspace{1cm} (21)

It is important to note that these utility functions normalize preferences over consumption goods in the two ways mentioned in the Introduction. The first normalization ensures that the planner does not favor individuals based on their preferences over consumption goods. The second normalization separates heterogeneity in commodity preferences from the consumption-leisure choice of individuals. Derivations of the normalizations, including an expression for $\phi^i$ in (20), are found in the Appendix. We discuss the importance of and logic behind these normalizations in Section 5.

3.1 Evidence on ability and time preference

In this section, we calibrate the model of optimal commodity taxation from Section 2. In particular, we estimate the relationship between time discounting and cognitive ability using panel data from the National Longitudinal Survey of Youth (NLSY).

A sizeable literature exists on measuring and explaining differences in saving behavior across income groups. In general, this research has acknowledged the possible role of heterogeneous time preferences but has found little evidence that they are important in practice. Hubbard, Skinner, and Zeldes (1995) build a lifecycle model with individual uncertainty, precautionary saving, means-tested social insurance, and homogeneous time preferences. Using data from the Panel Study of Income Dynamics (PSID), they argue that their model can explain the wide variation in saving rates across education (as a proxy for income) levels without heterogeneity in time preferences. Samwick (1998) uses Survey of Consumer Finances data on wealth and income to estimate a lifecycle model and notes that variations in wealth profiles not explained by his model may be due to variation in time preferences, though he does not have direct evidence for them. Dynan, Skinner, and Zeldes (2004) find a *strong, positive relationship between saving rates and lifetime
income," using data from the PSID, but they argue that preference differences cannot explain their findings (at least, without a strong bequest motive). Lawrance (1991) calculates annual time preference rates using data on food consumption and finds that implied discount factors rise with income, but Dynan (1993) shows that Lawrance’s results are sensitive to the inclusion of controls.

While this literature casts doubt on the potential for heterogeneous time preferences to justify substantial capital taxation, little if any research exists on whether saving preferences are related to innate ability, the relationship of interest for our analysis. With the goal of calibrating our model, we need direct evidence on that relationship. Using data on income and net worth from the National Longitudinal Survey of Youth (NLSY) and a standard model of an individual’s intertemporal utility maximization problem, we compute a discount factor for each individual in the sample. Next, we regress these discount factors on the log of ability and other personal characteristics observed by the NLSY, where we measure ability with individuals’ scores on a widely-used aptitude test. The coefficient on ability in this regression allows us to predict, holding fixed other personal characteristics, a discount factor for each level of ability. Using NLSY data on wages by ability level, we are then able to estimate a functional relationship between discount factors and wages, the key inputs to our policy simulations. To summarize, we estimate an elasticity of the annual discount factor \( \beta \) to the wage \( w \) of 0.0037. For example, an increase in the wage from $20 to $24 per hour corresponds to an increase in the discount factor from 0.9603 to 0.9610, a 0.07 percent increase.

The NLSY consists of a nationally representative sample of individuals born between 1957 and 1964, first interviewed in 1979, and interviewed annually or biannually since. The NLSY contains data on individuals’ net worth and income over time, allowing us to roughly estimate saving rates as described below.

The key advantage of the NLSY for our purposes is that it includes a direct measure of ability. This allows us to relate a measure of ability, not income, to time preferences. In 1980, the NLSY administered the Armed Forces Qualification Test (AFQT) to 94 percent of its participants. This test measured individuals’ aptitudes in a wide range of areas, including some mechanical skills relevant to military service.

We use an aggregation of scores in some of the areas covered by the AFQT as the indicator of ability. This aggregation, the AFQT89, is calculated by the Center for Human Resource Research at Ohio State University, as follows:

Creation of this revised percentile score, called AFQT89, involves (1) computing a verbal composite score by summing word knowledge and paragraph comprehension raw scores; (2) converting subtest raw scores for verbal, math knowledge, and arithmetic reasoning; (3) multiplying the verbal standard score by two; (4) summing the standard scores for verbal, math knowledge, and arithmetic reasoning; and (5) converting the summed standard score to a percentile.

Our measure of preferences is based on the discount factor \( \delta \) implied by using NLSY data on individuals’ income paths and net worth in a simple model of individual optimization described in the Appendix. Intuitively, the higher is an individual’s final net worth relative to his or her cumulative value of income, the greater the estimated \( \delta \). To give a sense for the data, in Table 1 we show the mean and standard deviations of \( \delta \) by AFQT quintile. Table 1 also shows the implied values of \( \alpha (w^k) \), the parameter of interest from

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17 The AFQT most likely measures some combination of innate ability and accumulated achievement. To the extent that more innately patient individuals invest more in human capital and thereby have higher AFQT scores because of achievement, not ability, our analysis will be biased toward finding a stronger relationship between ability and time preferences than that which truly holds.

18 We lack data on individuals’ expected future income flows from sources such as Social Security. To the extent that these flows are greater relative to past income for low earners, we are underestimating the true \( \delta \) for low earners and thereby overestimating the strength of the relationship between ability and savings preferences. Similarly, we are not taking into account the existing tax system when estimating \( \delta \).
the model of Section 2, and \( \beta (w^i) \), the standard annual discount factor. The variation in \( \delta \) within AFQT quintiles is large relative to the variation across wage levels. These results have their limitations for use in calibrating our model. The data are likely to be very noisy, and our inference of \( \delta \) is based on a simplified model. Moreover, simple AFQT quintile means of \( \delta \) are likely to be misleading, as they fail to control for variables correlated with both ability and saving behavior.

Table 2 shows the results of a regression of \( \delta \) on ability as well as other observable characteristics. In particular, we control for the cumulative value of income over the individual’s working life, age, age squared, and gender. Formally, we estimate:

\[
\ln \delta = \beta_1 \text{age} + \beta_2 \text{age}^2 + \beta_3 \text{gender} + \beta_4 \ln \text{(income)} + \beta_5 \ln \text{(AFQT)}
\]

where the calculation of "income" is described in the Appendix. This regression yields a highly significant estimate for \( \beta_5 \) of 0.026 (standard error of 0.004). In words, this coefficient implies an elasticity of 0.026 for the discount factor \( \delta \) with respect to ability as measured by the AFQT. For example, if ability increases by 10 percentile points from 50 to 60 (a twenty percent increase), the discount factor \( \delta \) would increase from 0.394 to 0.396 (i.e., by approximately 0.52 percent). These findings are consistent with the findings of the literature cited above that relates saving to income and with Benjamin, Brown, and Shapiro (2006), who find a "strong, statistically significant, and positive relationship between AFQT score and the propensity to have positive net assets" in the NLSY. Those authors, using a different measure of time preference, report "an additional 10 percentile points of AFQT is associated with an increase of about 1.5 percentage points in the propensity to have positive net assets."

The estimate of \( \beta_5 \) allows us to derive a value of \( \delta \) for each ability level holding fixed an individual’s age, gender, and cumulative income. In particular, we use

\[
\delta = 0.356 \text{(AFQT)}^{0.026},
\]

where the constant 0.356 is pinned down by matching the value of \( \delta \) for the middle AFQT quintile from Table 1 (0.394) with the mean AFQT score in that quintile (49.32). Expression (22) allows us to calculate, from the average AFQT score by quintile, a "regression-based \( \delta \)" for each quintile that can be compared to the simple means in Table 1. The results are shown in Table 3, along with the implied values of \( \alpha (w^i) \) and \( \beta (w^i) \).

The final step is to relate these discount factors to wages, as wage rates are the measure of ability in the model from Section 2 that we will use to simulate optimal policy. The NLSY provides data on individuals' reported wages, and we report the average of these wages by AFQT quintile in Table 3. Assuming the same measurement error likely affects both our estimates of ability and discounting, though bias would be introduced only by error in the former. While AFQT is an imperfect measure of ability, its retest reliability is very high. Moreover, if AFQT mismeasures ability, it is unclear whether that biases our results down or up. It may be that AFQT measures those parts of ability that are particularly highly correlated with preferences (i.e., ability to delay gratification, cognitive alacrity), and a more accurate measure of ability would show less relationship with preferences.

20We compute wages from the total wage and salary income divided by the total hours worked in 1992, as reported in 1993. We calculate mean wages by AFQT quintile limiting the sample to workers who reported more than 1,000 hours worked. Using
functional form as in expression (22), the values of $\alpha \left( w^i \right)$ and $w^i$ in Table 3 imply the following relationship between discounting and wages:

$$\alpha \left( w^i \right) = 1.0529 \left( w^i \right)^{-0.0037}. \tag{23}$$

or, as $\beta \left( w^i \right) = \frac{1}{\alpha \left( w^i \right)}$,

$$\beta \left( w^i \right) = 0.9498 \left( w^i \right)^{0.0037}. \tag{24}$$

Expression (23) allows us to derive $\alpha \left( w^i \right)$ and $\beta \left( w^i \right)$ for a wide range of wages. Figure 1 shows the results for $\beta \left( w^i \right)$ as expressed in (24).

### 3.2 Optimal capital income taxes

To simulate optimal capital income taxes using the estimated form for $\alpha \left( w^i \right)$ in expression (23), we specify a wage ($w^i$) distribution, calculate the implied values for $\alpha \left( w^i \right)$, and numerically simulate the planner’s problem in (9). We also simulate an augmented planner’s problem that limits the planner to a constant rate of capital income taxation to compare our results to Saez (2002). Recall that the assumed individual utility function is shown in (20).

We use a wage distribution from $4 to $100 with 25 equally-spaced discrete values. Based on Saez (2001), we assume that the distribution of the population across these wages is lognormal up to $62.50 and Pareto with a parameter value of two for higher wages. We calibrate the lognormal distribution with the 2007 wage distribution for full-time workers in the United States as reported in the Current Population Survey.

To measure the intertemporal wedge we use the expression:

$$\tau = 1 - \frac{\frac{u_{c1}}{w_{c1}} - 1}{r} \tag{25}$$

where $r$ is the annual rate of return to savings. The variable $\tau$ measures the relative distortion toward good 1 and away from good 2 at a given income level. Under the capital income tax interpretation, $\tau$ is the implicit tax on the interest income earned on good 2, i.e., capital. If this expression is positive, the tax policy is discouraging future consumption relative to current consumption. More informally, it is taxing the return to saving, so we will refer to it as the implied capital income tax.

Figure 2 shows optimal nonlinear capital income tax rates in the baseline case ($\gamma = 2$ and $\sigma = 3$). Optimal capital income tax rates are less than 6% everywhere, flat over much of the distribution, and declining at higher incomes. The average capital income tax rate is 2.8%, while the constrained-optimal linear capital income tax rate is 3.1% (not shown). The welfare gains from optimal capital income taxation given the calibrated $\alpha \left( w^i \right)$ are negligible.

The pattern of optimal tax rates shown in Figure 2 is robust to alternative values of the parameters of the utility function and form for the wage distribution. For a range of values for $\gamma$ and $\sigma$, as well for a specification of the wage distribution as lognormal, optimal rates are low and generally flat until high wage levels, above which they decline. The only noteable exceptions to these patterns are that optimal rates are higher when preferences exhibit greater risk aversion ($\gamma$) or elasticity of labor supply ($\frac{1}{\sigma - 1}$). Even in these all workers does not change the pattern, but all wage levels rise because some workers with low reported hours have high imputed hourly wages.

24In the simulations, we assume that $1 + r = 1 - \ln \int \beta \left( w^i \right) f \left( w^i \right)$, where we calculate the average instantaneous discount rate, $\rho = -\ln \int \beta \left( w^i \right) f \left( w^i \right)$, and set $r = \rho$. The implicit tax $\tau$ is on net capital income, i.e., the implicit after-tax return to saving is $(1 + r (1 - \tau))$.
cases, however, justified rates are small relative to prevailing rates (as will be discussed in detail below). Specifically, when $\gamma = 4$ (rather than the baseline value of 2), the average of the optimal nonlinear capital income taxes is 6.9%, which is also the best linear capital tax rate. When $\sigma = 1.5$ (rather than the baseline value of 3), the average of the optimal nonlinear capital income taxes is also 6.9%, while the best linear capital tax rate is 6.0%. Welfare gains from optimal capital taxation are negligible in all cases (less than 0.0001 percent of total output).

One variation to the model that has the potential to generate large optimal capital income taxes is to make the social welfare function more concave. In the extreme case of a Rawlsian social welfare function, for example, the optimal capital income tax rate averages 37.1% across the population, and the constrained-optimal linear capital income tax rate is 32.9%.

In sum, these results suggest that the relationship between time preferences and ability generates only modest rates of optimal capital income taxation. Moreover, these optimal rates are flat for much of the ability distribution and fall at higher wages.

4 Comparing optimal to existing capital income taxes

In this section, we compare the empirical relationship between time preferences and ability to that which would be required to justify all or a portion of prevailing levels of capital income taxes in developed economies, using our baseline model specification.

One natural measure of the taxation of capital income is the level of statutory tax rates on various forms of capital income. The Organization for Economic Cooperation and Development (OECD 2008) reports tax rates on corporate profits and capital income earned by individuals. The average combined corporate and personal statutory rate on distributed corporate profits reported by the OECD was 42.4 percent in 2007, down from 50 percent in 2000.

An alternative measure is the "tax ratio" of capital income tax revenue to total capital income. This measure has weaknesses—for example, it is backward-looking—but it has the virtue of implicitly controlling for the complicated exemptions, definitional variations, and tax incentives that cause the economic extent of capital taxation to differ from that implied by statutory rates. Carey and Rabesona (2004) calculate the tax ratio for capital income across sixteen OECD countries in 2000 to be 46.3.

To find the $\alpha(w^i)$ functions that yield constrained-optimal linear intertemporal wedges corresponding to prevailing capital income tax rates, we continue to model (as in expression 23) the function $\alpha(w^i)$ as a two-parameter power function

$$\alpha(w^i) = \psi (w^i)^\varepsilon,$$

where $\psi$ and $\varepsilon$ are scalars. We fix $\alpha(w^i = $28$) = 1.0413$, the value implied by our analysis of the NLSY data, to ensure comparability of these preferences to our empirical estimates. Note that this value for $\alpha(w^i)$ corresponds to an annual discount factor $\beta(w^i)$ of 0.96, in line with standard estimates. Then, we use the wage ($w^i$) distribution and utility function (20) from Section 3 with $\gamma = 2$ and $\sigma = 3$, and we vary the values of $\psi$ and $\varepsilon$ in (26) while simulating the planner’s problem in (9), (10), and (11) augmented by a constraint that commodity distortions be linear.

The measures above suggest that tax rates on capital income in developed economies today are over 40 percent. Figure 3 plots the $\alpha(w^i)$ required for the best linear intertemporal wedge to imply capital income tax rates.
tax rates of 40%, 20%, and 10% as well as the values for $\alpha(w^i)$ from our analysis of the NLSY data. To aid intuition, Figure 4 plots the conventional annual discount factor $\beta(w^i)$ implied by these $\alpha(w^i)$. As these Figures make clear, the empirical relationship between time preferences and ability is far weaker than that which would justify the capital income tax rates prevailing in developing economies today. For example, to justify a 20% capital income tax rate, the discount rate would need to be nearly 80 percent higher for an individual at the twentieth percentile of the ability distribution than for an individual at the eightieth percentile. The NLSY data implies only a 10 percent gap between these two individuals.

5 Role of preference normalization

In this section, we explore the role of preference normalization in the study of optimal commodity taxation. In this paper, we normalize preferences in two ways: to neutralize the role of preferences over goods in how much the planner values individuals; and to neutralize the effect of preferences over goods on the labor supply choices of individuals.

First, we normalize so as to eliminate any incentive for the planner to redistribute across agents based simply on their preferences over goods. Compare the representation of preferences in expression (21) to the following representation of the same preferences

$$U = \ln c_1^i + \frac{1}{\alpha(w^i)} \ln c_2^i - \frac{1}{\sigma} (\ell^i)\sigma,$$

Unlike (21), (27) causes the planner to favor individuals with low $\alpha(w^i)$. Intuitively, a planner using (27) obtains a larger increase in social welfare from allocating a marginal unit of resources to the individual with lower $\alpha(w^i)$. Therefore, optimal tax policy will favor individuals with lower $\alpha(w^i)$. To see this another way, note that if we were to multiply the first two terms of expression (27) by $\alpha(w^i)$, the planner would then gain more social welfare from allocating resources to the high $\alpha(w^i)$ type even though we would be using an observationally equivalent representation of preferences.

Formally, this normalization is derived as follows. First, we compute the allocations of consumption and labor individuals would choose if solving the problem in (12) and (13) with all taxes set to zero. Next, starting at those allocations, we calculate the marginal social welfare (i.e., the increase in the value of the planner’s objective function (14)) if each individual received a unit of income as an endowment (this is equivalent to a marginal relaxation of the individual’s budget constraint). The normalization in (21) ensures that this marginal social welfare is identical across all individuals with the same wage $w^i$. For utility that is of the constant relative risk aversion form (rather than logarithmic) over consumption, as in expression (20), the Appendix shows the derivation of an analogous normalizing factor $\varphi^i$.

Second, we use a representation of preferences that implies no relationship between preferences across goods and the willingness to work. For example, suppose the right-hand side of (21) were not divided by $(1 + \alpha(w^i))$. Starting from a common level of $\ell^i$, individuals with higher $\alpha(w^i)$ would derive greater marginal utility from the consumption they could purchase with the income earned from an increase in hours worked.

To derive this normalization, we compute the allocations of consumption and labor individuals would choose if solving the problem in (12) and (13) with all taxes set to zero. Then, we normalize the utility function to ensures that the same $\ell^i$ is chosen by all individuals with the same wage $w^i$, regardless of their

$^{26}$That is, $\rho(w^i)$ where $\rho(w^i) = -\ln(\beta(w^i))$. 


preferences $\alpha(w)$, in that setting.

These normalizations are similar to two assumptions Saez (2002) states in his analysis of this topic. His Assumption 1 is that the planner’s marginal social welfare weights on individuals are independent of their tastes for goods, conditional on their incomes. Our first normalization pursues the same neutrality of marginal social welfare weights, though we use the laissez-faire allocations rather than the optimal allocations as the starting point for the normalization. This normalization captures the idea that the government does not want to redistribute resources across individuals simply because they will spend them on different consumption baskets. Our second normalization parallels Saez’s Assumption 2, which states that, conditional their income, individuals’ labor supply responses to tax changes are unaffected by their preferences. Though our normalization focuses on isolating labor supply from preferences rather than tax changes, the idea of the two approaches is similar. Intuitively, this normalization means that individuals choose how much to work without regard to how they plan to spend their disposable income. Saez notes that both of his Assumptions seem like reasonable ones in the context of capital income taxation. We take a similar perspective that our normalizations provide a neutral starting point for modeling preference heterogeneity and its effects on optimal commodity taxation.

In a valuable contemporaneous analysis of the impact of preference heterogeneity on optimal capital income taxation, Diamond and Spinnewijn (2009) study optimal income-dependent but linear capital income taxes. Their analysis focuses on variation in preferences conditional on ability rather than on a systematic relationship between the two. They use a representation of preferences similar to (27), which by itself causes the planner to want to redistribute to the more patient individuals at each ability level. Their social welfare function, however, puts a small enough weight on those with high ability to ensure redistribution away from them regardless of preferences. These welfare weights address the issues motivating our first normalization in an alternative way. Related to our second normalization, their preferred functional form, expression (27), means that more patient individuals have a greater willingness to work. That is, in a competitive equilibrium, of two individuals with the same wage the individual who puts more weight on future consumption will supply more labor effort. If we simulate the optimal policy model of Section 2 with the functional form in (27), and therefore without our second normalization, the average optimal capital tax rate falls from 3.1% to 1.6%. Intuitively, the relationship between patience and willingness to work reduces the distortionary impact of labor income taxes on the patient and, because patience and ability are positively related, high income earners. This makes the benefits of capital taxes smaller, so the optimal policy avoids them. Therefore, our second normalization causes us to estimate higher optimal capital taxes than if we apply the preferences used in Diamond and Spinnewijn (2009).

6 Optimal Capital Taxation when Stochastic Abilities are Related to Preferences

In this section, we extend our analysis of optimal capital taxation when preferences vary with ability to a stochastic setting in a simple environment. The environment below parallels the standard dynamic Mirrlees model based on Golosov, Kocherlakota, and Tsyvinski (2003). While different from the model analyzed in previous sections, the model here addresses an additional aspect of how optimal taxes ought to respond to a relationship between preferences and ability.

In period $t = 1$, agents have a common wage $w = 1$, consume a good $x$, and produce income $y$. In period $t = 2$, agents have wages $w_i$ that take one of two values ($i = \{l, h\}$ for low and high) with probability
\( \pi^i : \sum \pi^i = 1, \) consume two goods \( c_1 \) and \( c_2, \) and produce income \( y^i. \) Let \( w^h > w^l > 0. \) Wages are private information to the agent. Importantly, agents with the high second-period wage have a relative preference for \( c_2 \) over \( c_1 \) measured by \( \alpha^i(w^i) \) where \( \alpha'(w^i) < 0, \) just as in the previous sections.

An individual’s problem in this setting with no policy is, for \( i \in \{l, h\}: 
\[
\max_{x,y,\left\{c_1^i, c_2^i, y^i\right\}_{i=1,2}} \left[ u(x) - v(y) + \sum_{i=l,h} \pi^i \beta \left[ \frac{\alpha^i(w^i)}{1 + \alpha^i(w^i)} \ln c_1^i + \frac{1}{1 + \alpha^i(w^i)} \ln c_2^i - v \left( \frac{y^i}{w^i} \right) \right] \right]
\]
subject to the budget constraint
\[
(1 + r)(y - x) + (y^i - c_1^i - c_2^i) \geq 0.
\]

where \((1 + r)\) is the marginal rate of transformation of goods across periods.

The individual allocations satisfy standard stochastic Euler equations:
\[
u' (x) = \beta (1 + r) \sum \pi^i \frac{\alpha^i (w^i)}{1 + \alpha^i (w^i)} u' (c_1^i) = \beta (1 + r) \sum \pi^i \frac{1}{1 + \alpha^i (w^i)} u' (c_2^i)
\]

The social planner’s problem is similar to the static problem in Section 2:

\[\text{Problem 7}\]
\[
\max_{x,y,\left\{c_1^i, c_2^i, y^i\right\}_{i=1,2}} \left[ u(x) - v(y) + \sum_{i=l,h} \pi^i \beta \left[ \frac{\alpha^i (w^i)}{1 + \alpha^i (w^i)} \ln c_1^i + \frac{1}{1 + \alpha^i (w^i)} \ln c_2^i - v \left( \frac{y^i}{w^i} \right) \right] \right]
\]
subject to the feasibility constraint
\[
(1 + r)(y - x) + \sum_{i=l,h} \pi^i (y^i - c_1^i - c_2^i) \geq 0,
\]
and incentive compatibility in period \( t = 2: \)
\[
\frac{\alpha^i(w^h)}{1 + \alpha^i(w^h)} \ln c_1^h + \frac{1}{1 + \alpha^i(w^h)} \ln c_2^h - v \left( \frac{y^h}{w^h} \right) \geq \frac{\alpha^i(w^h)}{1 + \alpha^i(w^h)} \ln c_1^l + \frac{1}{1 + \alpha^i(w^h)} \ln c_2^l - v \left( \frac{y^l}{w^h} \right),
\]
which says that the high-wage agents do not choose to mimic the low-wage agents in the second period.

Let \( \mu \) denote the multiplier on the incentive compatibility constraint (30).

The first-order conditions of the planner’s problem yield a condition describing optimal policy:
\[
\beta (1 + r) \frac{1}{w'(x)} = \sum_{i=l,h} \pi^i \left( \frac{1}{w'(c_1^i)} + \frac{1}{w'(c_2^i)} \right)
\]

Result (31) resembles the Inverse Euler Equation derived in Golosov, Kocherlakota, and Tsyvinski (2003) for a multiple-good economy, and it is the same condition that describes optimal policy when preferences do not

\[\text{An equivalent expression combining the second-period commodities could be used, instead.}\]
\[\text{As in the previous Sections, we assume that only the high ability agent can pretend to be the low ability agent and not vice versa.}\]
vary across types. Thus, the planner’s optimal policy toward saving is unaffected by the relationship between stochastic ability and the preference for goods when the planner is able to use nonlinear, income-dependent commodity taxes.

How does the optimal policy treat consumption choices in the second period? Define \( \theta^i \) as the distortion to second-period consumption choices for an individual of second-period type \( i \):

\[
\theta^i = \frac{\alpha(w^i)}{1+\alpha(w^i)} \frac{u'(c^i_1)}{u'(c^i_2)}.
\]

In the Appendix, we show that the optimal policy mimics that from our analysis of optimal capital taxation in previous sections. In particular, the high type faces no distortion to its commodity choices (\( \theta^h = 1 \)), while the low type is distorted away from the good that the high-type relatively prefers (\( \theta^l > 1 \)).

7 Conclusion

Among others, Mirrlees (1976) and Saez (2002) have argued that goods preferred by the high-ability ought to be taxed as part of an optimal tax policy that seeks to redistribute toward the (unobservably) low-ability. We analyze this argument by characterizing optimal nonlinear and income-dependent distortions to consumption choices across goods when preferences are related to ability. We derive a novel implication for these sophisticated relative marginal commodity taxes: they may be decreasing with income on those goods preferred by the highly able. This result has a straightforward intuition: high-ability individuals will be encouraged to exert effort if earning low incomes will subject them to higher marginal taxes on those goods they value highly.

Nonlinear, income-dependent commodity taxes may be applicable to many important categories of consumption such as education, health, or housing. Most prominently, however, the logic for taxing goods preferred by those with high ability has been used to argue for positive capital income taxation, for example by Banks and Diamond (2008). We examine data on preferences for current relative to future consumption and find that the relationship between ability and time discounting is unlikely to justify more than a small fraction of the substantial capital income taxation prevailing in developed economies today.

References


### Table 1. Summary of $\delta$ by AFQT quintile

<table>
<thead>
<tr>
<th>AFQT89 quintile</th>
<th>Bottom</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>Top</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean $\delta$</td>
<td>0.336</td>
<td>0.374</td>
<td>0.394</td>
<td>0.418</td>
<td>0.466</td>
</tr>
<tr>
<td>Std. Dev. of $\delta$</td>
<td>0.160</td>
<td>0.180</td>
<td>0.180</td>
<td>0.210</td>
<td>0.250</td>
</tr>
<tr>
<td>Implied $\alpha(w^i)$</td>
<td>1.0486</td>
<td>1.0437</td>
<td>1.0413</td>
<td>1.0387</td>
<td>1.0338</td>
</tr>
<tr>
<td>Implied $\beta(w^i)$</td>
<td>0.9536</td>
<td>0.9581</td>
<td>0.9603</td>
<td>0.9628</td>
<td>0.9673</td>
</tr>
<tr>
<td>Average $w^i$</td>
<td>12.27</td>
<td>16.21</td>
<td>19.20</td>
<td>21.62</td>
<td>25.73</td>
</tr>
</tbody>
</table>

### Table 2. Results of regression of log of discount factor, $\ln(\delta)$, on ability and controls

<table>
<thead>
<tr>
<th>Variable</th>
<th>Coefficient</th>
<th>Standard Error</th>
<th>T-statistic</th>
</tr>
</thead>
<tbody>
<tr>
<td>age</td>
<td>-2.62E-02</td>
<td>2.97E-02</td>
<td>-0.88</td>
</tr>
<tr>
<td>agesq</td>
<td>8.80E-04</td>
<td>8.36E-04</td>
<td>1.05</td>
</tr>
<tr>
<td>sex</td>
<td>1.16E-02</td>
<td>8.15E-03</td>
<td>1.42</td>
</tr>
<tr>
<td>$\ln(fvincome)$**</td>
<td>1.69E-01</td>
<td>7.61E-03</td>
<td>22.15</td>
</tr>
<tr>
<td>$\ln(afqt)$**</td>
<td>2.60E-02</td>
<td>4.46E-03</td>
<td>5.82</td>
</tr>
</tbody>
</table>

Note: ** indicates significance at the 1% level or lower

<table>
<thead>
<tr>
<th>Summary</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Observations</td>
<td>7,008</td>
</tr>
<tr>
<td>F-statistic</td>
<td>203.98</td>
</tr>
<tr>
<td>R-squared</td>
<td>0.127</td>
</tr>
</tbody>
</table>

### Table 3. Regression-based $\delta$ by AFQT quintile

<table>
<thead>
<tr>
<th>AFQT89 quintile</th>
<th>Bottom</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>Top</th>
</tr>
</thead>
<tbody>
<tr>
<td>Regression-based $\delta$</td>
<td>0.377</td>
<td>0.389</td>
<td>0.394</td>
<td>0.398</td>
<td>0.400</td>
</tr>
<tr>
<td>Implied $\alpha(w^i)$</td>
<td>1.0435</td>
<td>1.0418</td>
<td>1.0413</td>
<td>1.0410</td>
<td>1.0406</td>
</tr>
<tr>
<td>Implied $\beta(w^i)$</td>
<td>0.9583</td>
<td>0.9599</td>
<td>0.9603</td>
<td>0.9606</td>
<td>0.9610</td>
</tr>
<tr>
<td>Average $w^i$</td>
<td>12.27</td>
<td>16.21</td>
<td>19.20</td>
<td>21.62</td>
<td>25.73</td>
</tr>
</tbody>
</table>
Figure 1: Estimated $\beta(w_i)$

Figure 2: Optimal capital income distortion
Figure 3: Time preferences, $\alpha(w)$

Figure 4: Discount factors, $\beta(w)$

This point (w=28) held constant