On the Stewardship and Valuation Implications of Accrual Accounting Systems

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Abstract

In this paper we explore both the stewardship implications of accrual accounting systems with reversals over multiple time periods. We show that either aggressive or conservative accounting strategies are preferred by shareholders both from a stewardship and a valuation perspective. In the process, we also show that the optimal contract is robust with respect to the accounting strategy choice - it is always the same. Further, this choice can be delegated to the agent here who be persuaded to do what is best for the shareholders. Finally, that standard empirical constructs designed to measure value relevance generally respond negatively to the kind of strategies that actually make accounting earnings more correlated with real economic earnings.

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1 Introduction

We aim to provide a theoretical evaluation of the recent push by accounting (in particular US) standard setters to abandon a stewardship focus and fully concentrate on developing a reporting regime that provides shareholders with the best primary source of value relevant financial information. Consequently, to move away from backward-looking historical cost accounting towards presumably less biased estimated fair values. To that end we explore the optimal properties of an accounting system with reversing accruals in a multi-period model where the financial statements are a primary source of information about the reporting entity. Our key results are as follows. There is no conflict between the stewardship and the valuation functions of accruals nor are fair value estimates superior. Indeed, whether one takes a "valuation" or a "stewardship" perspective, either aggressive or conservative accrual strategies are actually (equally) preferred by shareholders.

No general consensus exists as to what exactly accrual accounting does or is intended to do, which accounting choices are conservative or aggressive, who the main users of financial reports are, or even what users actually use such reports for. The following stylized facts seem somewhat well accepted, however. First, accrual accounting involves procedures to assign cost and benefits to periods in such a way that individual periods' results reflect their overall (perhaps relative) economic contribution. Second, a significant user group of financial statements is existing as well as potential shareholders, and that among their primary uses are valuation of the firm and evaluation of managerial performance. Third, providing information useful for evaluation performance is in conflict with providing value relevant information; for valuation, "timely" forward looking financial statements are preferred while for evaluation of performance, the past matters more.

Traditional accrual accounting methods and procedures may not be designed to produce unbiased measures of the “true” economic value created in any given period. Still, we suggest that if accrual accounting under current standards is practiced in what is considered a “neutral” way, periods that contribute the same in real economic term, they should (at least in expectation) also yield the same amount of accounting. Aggressive or conservative reporting in one period, in contrast, involves the expectation of some reversal in the future. Accordingly, in the special case where all

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1 Indeed most would probably argue that traditional accrual accounting is inherently conservative as reflected for example in that book-to-market typically is expected to be less than "one."

2 In other words, while traditional accounting constructs in all periods (except the last perhaps) may generally be biased downwards, (overly) conservative accounting here is taken to mean that earlier periods a biased more down
periods over the life-span of a firm are expected to contribute equally, neutral reporting implies that accounting income also is expected to be the same each period. Likewise, (overly) conservative (aggressive) accounting implies earlier periods are expected to appear worse (better) than later.

Generally, it seems, accounting choices that are intended not to produce periodic earnings that fairly track the underlying economics but rather to distort performance through time either aggressively or conservatively (as per the discussion here) are generally viewed as “bad” and damaging to the reporting quality. This actually seems to be the case whether one prefers a stewardship or a valuation perspective. Perhaps (overly) conservative choices are viewed less unfavorably than aggressive ones, but nonetheless - such deliberate misrepresentations do not appear to be viewed as beneficial to outside shareholders at least. Yet at the same time it appears that managers continue to be left with all the tools to engage in such timing games and, significantly, seem to put them to good use.

If one accept these characterizations as reasonable, or even plausible, it seems pretty obvious why manages would engage in such behavior when allowed. The interesting question then of course remains why would managers be allowed to do it? Why wouldn’t shareholders or regulators concerned about the well-being of the investing public eliminate at least some of the more obvious options for seeming misrepresentations? Of course one can always point to the need for flexibility. However, flexibility presumably can only be a good thing if it is (mainly) used for the benefit of the users. If instead it is used regularly for timing games, be they conservative or aggressive, it seems harder to justify their continued availability.

As we demonstrate here, however, the flexibility actually is the most beneficial when used either to produce either an aggressive or (overly) conservative time profile into the expected earnings stream - not to produce unbiased fair estimates. Our findings therefore suggest that managers that pursue seemingly volatility-inducing accounting activities, such as big bath-type write-offs to concentrate expenses in the current period at the benefit of future periods earnings or aggressive accrual strategies aimed at booking sales in the current period at the expense of future periods sales, act in the best interest of shareholders. The implication of such behavior is not only a decrease in the agency cost when earnings is used for managerial performance evaluation, but also an improvement in valuation properties of financial statements due to reduced the volatility of the
aggregate earnings distribution.

We draw our specific conclusions from the study of a reasonably simple and straightforward $n$-period principal-agent model. We construct the basic production setting such that in equilibrium, the distributions of unobservable (stochastic) economic income for the $n$ periods are always identical (and statistically independent). We couple that with an imperfect financial reporting system that produces (noisy and potentially biased) publicly disclosed information about economic income in each of the periods. The baseline system is designed here to be neutral in the sense discussed earlier: because the economic income distributions are (in equilibrium) the same in each the periods, so are the distributions of accounting earnings as they are linked to the underlying economics of the firm through the same stochastic structures. Accordingly, if the financial reporting system is left neutral, expected accounting income is the same in each period with the same volatility and information content.

The flexibility that allows for timing games we introduce here by allowing for the earnings distribution to be deliberately altered in any given period but the last. However, in keeping with the spirit of accrual accounting (the way we understand it), any deliberate earnings alteration made in one period will be reversed in subsequent. The specific way we structure the reversal here is that it mechanically takes place in the period immediately following the one where the deliberate alteration was undertaken.\(^3\) That we choose to work with alterations of the bias in the distribution of expected earnings over time reflect our view that in any given period the outcomes of all accounting decisions are uncertain themselves. Auditing is an obvious reason, but random events in general make it implausible that period-by-period implications can be predicted with certainty. The advantage of our model setup is that it admits only the flexibility to move expected accounting income between periods relative to the neutral case but offers no guarantee that it will work as intended.

Our analysis of our model proceeds as follows. We first derive the optimal performance evaluation system for the manager. Significantly, we demonstrate that while the agent here can be evaluated on each period’s realized accounting earnings,\(^4\) the compensation contract he faces is completely immune to how the available flexibility is employed (if at all) and can actually be ex-

\(^3\)Because of the flexibility to alter we allow for in our model, this simply simplifies the analysis somewhat but is with no loss of generality.

\(^4\)Being the only observable performance measures.
pressed as an increasing convex function of aggregate earning for the contracting horizon.\(^5\) How much he is going to be paid for each *given* earnings realization doesn’t depend at all on how the earnings distribution is altered relative to the neutral accrual accounting case; nor does it depend on when a particular earnings realization took place.

This key independence-result is important in its own right for several reasons. First, it makes the contracting problem simple in that it can simply be solved based on the properties of a neutral accounting treatment without having to consider whether neutrality actually prevails in equilibrium. Second, agency theory is often criticized for producing predictions that lack robustness even to miniscule variations in the information environment. In our straightforward setting, however, the opposite actually turns out to be the case. Despite the fact that different accounting treatments alter both the expectation and the distribution of accounting earnings in each period and may (actually most likely will) even do so asymmetrically, the optimal contract here is fully robust; it always remains the same.

For our purposes this independence is particularly significant as it ties the stewardship value of the financial accounting system directly to the observed in-equilibrium earnings distribution as opposed to some hypothetical out-of-equilibrium one! Indeed, while the optimal contract doesn’t change as a result of accounting choices, the volatility of aggregate earnings does and thus the uncertainty surrounding the manager’s compensation. The expected cost of administering the contract thus does as well. Accordingly, in our setting the stewardship properties of accounting earnings depend exclusively on the in-equilibrium distribution of earnings.

Given this key result, our next steps towards our overall conclusion are then relatively straightforward. First we show that the introducing negative auto-correlation in the (noisy) earnings stream “tightens” the distribution of aggregate earnings. Moreover, that this tightening makes aggregate earnings a better construct to rely on for linking managers’ pay to performance. Significantly, we establish that even though the manager is evaluated and paid based on earnings, the task of choosing the time-allocation of earnings can be left with him at no cost. This follows directly from our main independence result too as while from a stewardship perspective the shareholders prefer either aggressive or conservative accounting, the manager is indifferent given the optimal contract. Presumably, then, he will always choose to implement the accrual strategy shareholders’ wish.

\(^5\) Retained earnings at the end of the second period plus aggregate dividends paid out over the two periods.
As we then proceed to show the accrual strategy that produces the highest stewardship value of earnings actually also improves the quality of earnings from a valuation perspective in terms of how correlated aggregate earnings are with the underlying economic value created over time. Much of our attention here, however, is directed at evaluating how well standard empirical measures of earnings quality (as assessed by the price-earnings relation) capture this. The answer turns out to be: at best - not at all. Faring best is the so-called "Earnings Response Coefficient" that turns out to be completely immune to the introduction of the (fundamentally beneficial) accrual strategy. Measuring "earnings quality" by the $R^2$ from a price-earnings regression turns out to provide exactly the wrong answer - the $R^2$ declines when earnings quality enhancing accruals are introduced. The same problem attaches to the coefficient of correlation. Thus, in addition to providing theoretical insights into the role of accruals in improving earnings quality, our study provides direct insight into the need for caution in interpreting standard empirical constructs intended to be related to actual earnings quality.

The remainder of this paper is organized as follows. In section 2 we introduce our basic model structure. Section 3 contains an analysis of the optimal contract and its relation to the accounting choice made. In section 4 we identify the (Pareto) optimal accounting choice(s) from a stewardship perspective and establish that its implementation can be delegated to the manager at no cost. Section 5 is dedicated to the valuation implications of the optimal accrual strategy as well as the implications for standard measures of earnings quality. Section 6 offers a brief conclusion.

## 2 Basic Model Structure

For convenience we start the construction of our $n$-period model with the relatively simple case of $n = 2$. Consider thus a setting in which an ongoing firm (the principal) hires a manager (the agent) to supply unobservable productive effort in each of two time periods ($t = 1, 2$). The principal is risk-neutral and the manager is risk-averse with additively separable utility in payoff and personal cost of effort:

$$U^A(I(\cdot), e) = H(I(\cdot)) - V(e),$$

where $I(\cdot)$ is the manager’s compensation, $V(e)$ is the manager’s personal cost of productive effort $e$, $H'(\cdot) > 0$, $H''(\cdot) < 0$ and $V'(\cdot) > 0$. 

6
The manager delivers productive effort $e_t$ at the beginning of each period $t$. In the interest of simplicity, effort is binary: $e_t \in \{e^h, e^l\}$, where $e^h > e^l$, and without any loss of generality we assume that only high productive effort is costly for the manager, $V(e^h_t) \equiv V > V(e^l_t) = 0.$ The manager’s productive effort results in firm output, $x_t$, at the end of each period $t$. Firm output is also binary: $x_t \in \{x^H, x^L\}$, where $x^H > x^L$. The manager’s effort increases the likelihood that high firm output realizes and we assume that the principal wants to induce high productive effort in both periods. The relationship between the manager’s effort and firm output is captured by

$$1 > pr(x^H_t | e^h_t) > pr(x^H_t | e^l_t) > 0,$$

for $t = 1, 2$. To economize on notation and for convenience we assume symmetry over time:

$$pr(x^H_t | e^h_t) \equiv p^h \text{ and } pr(x^H_t | e^l_t) \equiv p^l \text{ for } t = 1, 2.$$

Firm output is unobservable during the contracting horizon of the manager. For contracting purposes, the manager’s reward is based on noisy performance report about the firm’s actual output at the end of each period. We use $y_t$ to represent this performance report and assume that $y_t \in \{y^L, y^H\}$, where $y^H > y^L$. The relationship between the performance report and firm output is captured by $pr(y_t | x_t)$, where

$$pr(y^H_t | x^H_t) = \lambda^H \text{ and } pr(y^L_t | x^L_t) = \lambda^L.$$

Intuitively, $x$ represents the firm’s true economic income and $y$ represents accounting earnings.

The $pr(y|x)$ therefore determines how accurately the firm’s earnings represent the firm’s underlying economic value. In this sense, $\lambda$s determine the reporting system’s bias. Nevertheless, given the assumed symmetry over the two time periods, the expected accounting earnings at the end of each period should be the same, if the expected economic income is also the same. We term such a system as "neutral."

**Definition 1** A neutral accounting system produces the same expected accounting income in periods that are expected to contribute the same in real economic income.

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6 We label choice variables with lower case superscripts and the realization of random variables with upper case superscript.
The key feature of the model is that reporting choices affect how performance is measured in multiple periods. That is, reporting choices made by the principal or the manager in a particular period have an effect on how performance is measured in future periods. Most importantly, the impact of any reporting choices is temporal and is expected to reverse over time. For example, making reporting choices that result in accelerating revenue recognition in one period also means that there is less revenue to be recognized in future periods. If an income increasing accrual is recorded in a period, an income decreasing accrual must be recorded in a future period.

Formally, the firm’s reporting policies are determined by parameter $\delta$ that affects the conditional probabilities of high and low accounting earnings as follows:

$$
pr (y_1^H | x_1^H, \delta) = \lambda^H + \delta,
$$

$$
pr (y_2^H | x_2^H, \delta) = \lambda^H - \delta,
$$

$$
pr (y_1^L | x_1^L, \delta) = \lambda^L - \delta,
$$

$$
pr (y_2^L | x_2^L, \delta) = \lambda^L + \delta,
$$

where $\delta \in [\underline{\delta}, \bar{\delta}]$, $\underline{\delta} < 0$ and $\bar{\delta} > 0$. In other words the choice of $\delta$ affects the expected accountings earnings in each period in exactly opposite directions but doesn’t change total expected earnings over the horizon we consider.

An important feature of the model is the stochastic nature of how reporting choices affect the manager’s performance evaluation. It should be noted that reporting choices do not guarantee that the manager’s evaluation will be "good" or "bad." Also, any reporting choices made during the first period will have an effect (stochastic) on the distribution of earnings in the second period, regardless of whether reporting choices actually affected earnings or not. Intuitively, recognizing an accrual during the first period affects the likelihood that the firm’s accounting performance in that period is considered “good”. However, it does not guarantee that the manager’s evaluation will be different than his or her evaluation had no accrual been recognized. Even if the manager’s evaluation is unaffected by the accrual in the first period, his or her evaluation in the second period may be affected by the accrual recognized in the first period.

Whereas the manager’s productive effort has a direct impact on the firm’s expected real output, the reporting choices have a "window dressing" effect that only affect the timing of accounting
earnings between two periods. To simplify the analysis, we also assume that \( \delta \) can only be set at the beginning of the first period prior to the realization of firm output. We therefore exclude cases where the manager uses reporting choices to continuously manipulate or "cover up" past performance.

Intuitively, a \( \delta > 0 \) means that the firm adopts an "aggressive" revenue recognition policy during the first period. Conversely, a \( \delta < 0 \) suggests a "conservative" revenue recognition policy. Importantly, taking action to make the firm’s reporting system more "aggressive" or "conservative" in the first period means that the system will report earnings more "conservatively" or "aggressively," respectively, the following period.

**Definition 2** An aggressive (conservative) accounting system produces higher (lower) expected accounting income, relative to a neutral system, in periods that are expected to contribute the same in real economic income.

To maintain consistent labeling of "good" and "bad" news, and avoid moving support cases we limit the range of possible values for \( \delta \):

\[
\delta \in \left[ \max \{ \lambda^L - 1, -\lambda^L, -\lambda^H, \lambda^H - 1 \}, \min \{ \lambda^L, 1 - \lambda^L, 1 - \lambda^H, \lambda^H \} \right].
\]

This ensures that the probabilities of a high or low report are always between zero and one, regardless of the reporting choices.

The sequence of events is as follows (Figure 1). At the beginning of the game, \( t = 0 \), the principal offers a compensation contract to the manager. If the manager agrees to participate in the game, reporting choices are made by the principal or the manager at \( t = 1 \). At \( t = 2 \), the manager supplies productive effort \( e_1 \). Next, at \( t = 3 \), the first period accounting earnings are observed. The manager then supplies productive effort \( e_2 \) for the second period at \( t = 4 \). Lastly, at \( t = 5 \), second period earnings are measured and payments are made as promised.
3 Optimal Contract Analysis

We start the analysis with the case where the reporting choices \( \delta \) are made by the principal. In this setting, the principal’s optimization program is as follows:

\[
\begin{align*}
\min_{\delta, I(y_1, y_2)} & \quad E \left[ I(y_1, y_2) \mid e^h_1, e^h_2, \delta \right] \\
\text{s.t.} & \quad E \left[ U^A \mid e^h_1, e^h_2, \delta \right] \geq \bar{U} \quad (IR) \\
& \quad E \left[ U^A \mid e^h_1, e^h_2, \delta \right] \geq E \left[ U^A \mid e^l_1, e^l_2, \delta \right] \quad (IC_1) \\
& \quad E \left[ U^A \mid e^h_1, e^h_2, \delta, y^H \right] \geq E \left[ U^A \mid e^h_1, e^l_2, \delta, y^H_1 \right] \quad (IC_2^H) \\
& \quad E \left[ U^A \mid e^h_1, e^h_2, \delta, y^L \right] \geq E \left[ U^A \mid e^l_1, e^l_2, \delta, y^L_1 \right] \quad (IC_2^L)
\end{align*}
\]

The principal’s objective is to minimize expected compensation costs while ensuring that the manager supplies high productive effort in both periods. The \( IR \) constraint ensures the manager receives at least his or her reservation utility and agrees to participate in the game. The \( IC_1 \) constraint ensures that the manager supplies high productive effort in the first period. The \( IC_2 \) constraints ensure that the manager supplies high productive effort in the second period, regardless of the realized performance at the end of the first period.

We proceed by deriving the optimal compensation contract. The latter is expressed in utiles as it facilitates the analysis.
Lemma 1 The optimal compensation contract, expressed in utiles, is defined as follows:

\[ U_{HH} = \left( 2 - P_1^h - P_2^h \right) \Omega + \bar{U} + 2V, \]
\[ U_{HL} = \left( 1 - P_1^h - P_2^h \right) \Omega + \bar{U} + 2V, \]
\[ U_{LH} = \left( 1 - P_1^h - P_2^h \right) \Omega + \bar{U} + 2V, \]
\[ U_{LL} = \left( -P_1^h - P_2^h \right) \Omega + \bar{U} + 2V, \]

where \( \Omega = \frac{V}{(p^h - p^l)(\lambda^H + \lambda^L - 1)} \).

**Proof of Lemma 1.** Note that the best solution when \( \delta \) can be chosen freely by the principal binds both the IC\(_1\) and the IC\(_2\) constraints. Thus, from the IC\(_1\) constraint:

\[
E \left[ U^A | e^h_1, e^h_2, \delta \right] = E \left[ U^A | e_1^1, e_2^1, \delta \right] \implies \\
P_1^h \cdot E \left[ U \left( y^H, \cdot \right) \right] + \left( 1 - P_1^h \right) \cdot E \left[ U \left( y^L, \cdot \right) \right] - V = \\
P_1^l \cdot E \left[ U \left( y^H, \cdot \right) \right] + \left( 1 - P_1^l \right) \cdot E \left[ U \left( y^L, \cdot \right) \right] \implies \\
\left( P_1^h - P_1^l \right) \{ E \left[ U \left( y^H, \cdot \right) \right] - E \left[ U \left( y^L, \cdot \right) \right] \} = V \implies \\
\Omega_1 = \frac{V}{P_1^h - P_1^l} = \frac{V}{(p^h - p^l)(\lambda^H + \lambda^L - 1)},
\]

where \( \Omega_1 \equiv \left\{ E \left[ U \left( y^H, \cdot \right) \right] - E \left[ U \left( y^L, \cdot \right) \right] \right\} \) and \( P_1^j \) denotes the probability of \( y^H_j \) given productive effort \( e^j \). Similarly, from the IC\(_2\) constraints:

\[
P_2^h \cdot U \left( y^j, y^H \right) + \left( 1 - P_2^h \right) \cdot U \left( y^j, y^L \right) - V = \\
P_2^l \cdot U \left( y^j, y^H \right) + \left( 1 - P_2^l \right) \cdot U \left( y^j, y^L \right) \implies \\
\left( P_2^h - P_2^l \right) \left[ U \left( y^j, y^H \right) - U \left( y^j, y^L \right) \right] = V \implies \\
\Omega_2^j = \frac{V}{P_2^h - P_2^l} = \frac{V}{(p^h - p^l)(\lambda^H + \lambda^L - 1)},
\]

where \( \Omega_2^j \equiv \left[ U \left( y^j, y^H \right) - U \left( y^j, y^L \right) \right] \) for \( j \in \{ H, L \} \).

Note that \( \Omega_2^H \) and \( \Omega_2^L \) do not depend on the realization of accounting earnings at the end of the first period. Thus, \( \Omega_2^H = \Omega_2^L \), so we use \( \Omega \) to economize on notation. Also note that \( \Omega_1 = \Omega_2^j \); thus, \( \Omega_2^H = \Omega_2^L = \Omega_1 = \Omega \).
We also know that the principal will hold the agent to his or her reservation utility level:

\[ E \left[ U^A | e_1^h, e_2^h, \delta \right] = \bar{U}. \]

It can then be verified that the compensation contract that satisfies the above conditions is the following:

\[
\begin{align*}
U_{HH} &= \left( 2 - P_1^h - P_2^h \right) \Omega + \bar{U} + 2V, \\
U_{HL} &= \left( 1 - P_1^h - P_2^h \right) \Omega + \bar{U} + 2V, \\
U_{ LH} &= \left( 1 - P_1^h - P_2^h \right) \Omega + \bar{U} + 2V, \\
U_{LL} &= \left( -P_1^h - P_2^h \right) \Omega + \bar{U} + 2V,
\end{align*}
\]

where \( U_{jk} \) is the agent’s utility when \( y_1 = y^j \) and \( y_2 = y^k \) for \( j, k \in \{H, L\} \).

An examination of the optimal contract reveals that it is independent of \( \delta \). That is, despite the fact that \( \delta \) impacts the distribution of earnings in each period, the earnings contingent compensation contract payments do not change with \( \delta \). The principal can therefore choose any \( \delta \) without having to alter the compensation contract offered to the manager. The key behind this finding is the reversal of any reporting choices made over the two time periods that we examine.

**Proposition 1** The optimal compensation contract is unaffected by the characteristics of the performance evaluation system as determined by \( \delta \).

**Proof of Proposition 1.** From the proof of Lemma 1, we have already established that

\[
\Omega = \frac{V}{(p^h - p^l)(\lambda^H + \lambda^L - 1)}.
\]

Thus, \( \Omega \) does not depend on \( \delta \).

We also have that

\[
\begin{align*}
P_1^h &= p^h (\lambda^H + \delta) + \left( 1 - p^h \right) \left( 1 - \lambda^L + \delta \right) \text{ and} \\
P_2^h &= p^h (\lambda^H - \delta) + \left( 1 - p^h \right) \left( 1 - \lambda^L - \delta \right).
\end{align*}
\]
We now examine how $\delta$ affects the optimal compensation payments:

$$
U_{HH} = \left(2 - P_1^h - P_2^h\right) \Omega + \bar{U} + 2V \\
= \{2 - \left[p^h (\lambda^H + \delta) + \left(1 - p^h\right) (1 - \lambda^L + \delta)\right] \\
- \left[p^h (\lambda^H - \delta) + \left(1 - p^h\right) (1 - \lambda^L - \delta)\right]\} \Omega + \bar{U} + 2V \\
= \left[-2p^h (\lambda^H + \lambda^L - 1) + 2\lambda^L\right] \Omega + \bar{U} + 2V.
$$

Thus, $U_{HH}$ does not depend on the $\delta$.

In a similar fashion we explore $U_{LL}$:

$$
U_{LL} = \left(-P_1^h - P_2^h\right) \Omega + \bar{U} + 2V \\
= \left[-p^h (\lambda^H + \lambda^L - 1) + 1 - \lambda^L + \delta\right] \\
- \left[p^h (\lambda^H + \lambda^L - 1) + 1 - \lambda^L - \delta\right] \Omega + \bar{U} + 2V \\
= \left[-2p^h (\lambda^H + \lambda^L - 1) - 2 + 2\lambda^L\right] \Omega + \bar{U} + 2V.
$$

Thus, $U_{LL}$ does not depend on $\delta$ either.

As we also know is that $\Omega_2^H = \Omega_2^L$ which means that $U_{HH} - U_{HL} = U_{LH} - U_{LL}$. It then follows that $U_{HL}$ and $U_{LH}$ are not affected by $\delta$ either. Thus, the optimal compensation contract is unaffected by $\delta$. ■

Beyond the independence of the accounting choice $\delta$, another key take-away from Proposition 1 is that the optimal contract here is "linear in utility space." While from a technical perspective this is very helpful for facilitating our analysis, this is interesting in its own right as well. With a risk-averse agent this immediately implies that the optimal contract in payoff (cash) space is strictly convex. Real world contracts generally appear to have convex features at their core. Typically, though, convexity of optimal agency contracts is specific to the specific preferences attributed to the agent. Not here - as long as the agent is risk averse, the optimal (cash) contract is convex.
4 Optimal Accounting Choices

From Proposition 1, the optimal compensation contract that ensures the manager supplies high productive effort in both periods is the same for all $\delta$s. Nevertheless, $\delta$s affect the distribution of accounting earnings. If we look at the effect of $\delta$ on earnings for each period in isolation, an $\delta > 0$ ($\delta < 0$) increases (decreases) the likelihood of high earnings in the first period, and decreases (increases) the likelihood of high earnings in the second period. However, a closer examination of the distribution of the aggregate earnings $(y_1 + y_2)$ reveals that increasing $\delta$ moves more probability mass on the "middle" outcomes, i.e., $(y_1^H, y_2^L)$ and $(y_1^L, y_2^H)$, and away from the $(y_1^H, y_2^H)$ and $(y_1^L, y_2^L)$ outcomes.

Lemma 2 The effect of $\delta$ on the distribution of the aggregate earnings $(y_1 + y_2)$ is to move probability mass to the $(y_1^H, y_2^L)$ and $(y_1^L, y_2^H)$ outcomes.

Proof of Lemma 2. The distribution of aggregate earnings over the two periods is as follows:

\[
\begin{align*}
pr(y_1^H, y_2^H | e_1^h, e_2^h, \delta) &= P_1^h P_2^h, \\
pr(y_1^H, y_2^L | e_1^h, e_2^h, \delta) &= P_1^h \left(1 - P_2^h \right), \\
pr(y_1^L, y_2^H | e_1^h, e_2^h, \delta) &= \left(1 - P_1^h \right) P_2^h, \\
pr(y_1^L, y_2^L | e_1^h, e_2^h, \delta) &= \left(1 - P_1^h \right) \left(1 - P_2^h \right).
\end{align*}
\]

It can then be easily verified that

\[
\begin{align*}
\frac{\partial \left(pr(y_1^H, y_2^H | e_1^h, e_2^h, \delta) \right)}{\partial \delta} &= -2\delta, \\
\frac{\partial \left(pr(y_1^H, y_2^L | e_1^h, e_2^h, \delta) \right)}{\partial \delta} &= 2\delta + 1, \\
\frac{\partial \left(pr(y_1^L, y_2^H | e_1^h, e_2^h, \delta) \right)}{\partial \delta} &= 2\delta - 1, \\
\frac{\partial \left(pr(y_1^L, y_2^L | e_1^h, e_2^h, \delta) \right)}{\partial \delta} &= -2\delta.
\end{align*}
\]

Thus, the effect of $\delta$ on the distribution of the aggregate earnings is to increase the probability mass on the $(y_1^H, y_2^L)$ and $(y_1^L, y_2^H)$ outcomes, and lower the probability mass on the $(y_1^H, y_2^H)$ and $(y_1^L, y_2^L)$ outcomes.
To this point, we have shown that the optimal compensation contract is independent of δ, and that increasing δ moves more probability mass to the middle outcomes. We also know that the principal will hold the manager to his or her reservation utility (the manager’s IR constraint is binding). These findings allow us to show that the principal has a preference over the distribution of accounting earnings and, perhaps surprising, that setting δ ≠ 0 results in lower expected compensation costs. In other words, it is in the principal’s best interests to introduce an additional bias to the performance evaluation system, despite the fact that any bias introduced in one period reverses during the following period.

In fact, we find that the principal prefers "extreme aggressive" or "extreme conservative" reporting systems, i.e., δ* = 3 or δ* = δ, as this minimizes expected compensation costs. Moreover, the principal is indifferent between an "aggressive" or "conservative" reporting system. That is, the principal is indifferent between δ* = 0 and δ* = δ, provided that any bias introduced in one period reverses (stochastically) the following period. This suggests that the principal cares about the distribution of aggregate earnings, and not about how reporting choices affect earnings realizations in individual periods, provided of course that any biases that are introduced in the firm’s reporting system reverse over time.

**Proposition 2** The principal prefers an extreme reporting system and is indifferent between an "aggressive" or "conservative" system in the first period. Thus, δ* = 3 or δ* = δ. Such systems allow the principal to ensure that the manager supplies high productive effort in both periods at the lowest expected compensation cost.

**Proof of Proposition 2.** The manager’s expected utility is given as

\[
E(U) = P_1^h P_2^h U_{HH} + P_1^h (1 - P_2^h) U_{HL} + (1 - P_1^h) P_2^h U_{LH} + (1 - P_1^h)(1 - P_2^h) U_{LL}
\]

\[
= P_1^h P_2^h \left[ (2 - P_1^h - P_2^h) \Omega + \underline{U} + 2V \right]
\]

\[
+ P_1^h (1 - P_2^h) \left[ (1 - P_1^h - P_2^h) \Omega + \underline{U} + 2V \right]
\]

\[
+ (1 - P_1^h) P_2^h \left[ (1 - P_1^h - P_2^h) \Omega + \underline{U} + 2V \right]
\]

\[
+ (1 - P_1^h)(1 - P_2^h) \left[ (-P_1^h - P_2^h) \Omega + \underline{U} + 2V \right] - 2V,
\]
so that after some algebra

\[
\begin{align*}
\frac{dE(U)}{d\delta} &= \left( P_2^h - P_1^h \right) \left[ \left( 2 - P_1^h - P_2^h \right) \Omega + U + 2V \right] \\
&\quad + P_1^h P_2^h \left[ \left( 2 - P_1^h - P_2^h \right) \frac{d\Omega}{d\delta} \right] \\
&\quad + \left( 1 - P_2^h + P_1^h \right) \left[ \left( 1 - P_1^h - P_2^h \right) \Omega + U + 2V \right] \\
&\quad + P_1^h \left( 1 - P_2^h \right) \left[ \left( 1 - P_1^h - P_2^h \right) \frac{d\Omega}{d\delta} \right] \\
&\quad - \left( 1 + P_2^h - P_1^h \right) \left[ \left( 1 - P_1^h - P_2^h \right) \Omega + U + 2V \right] \\
&\quad + \left( 1 - P_1^h \right) P_2^h \left[ \left( 1 - P_1^h - P_2^h \right) \frac{d\Omega}{d\delta} \right] \\
&\quad - \left( P_1^h - P_2^h \right) \left[ \left( -P_1^h - P_2^h \right) \Omega + U + 2V \right] \\
&\quad + \left( 1 - P_1^h \right) \left( 1 - P_2^h \right) \left[ \left( -P_1^h - P_2^h \right) \frac{d\Omega}{d\delta} \right].
\end{align*}
\]

Note that for \( \delta = 0 \) we have \( P_1^h = P_2^h \). Substituting in \( \frac{dE(U)}{d\delta} \) and simplifying it can be easily verified that \( \frac{dE(U)}{d\delta} = 0 \) for \( \delta = 0 \).

The manager’s expected compensation is given as

\[
\begin{align*}
E(G) &= P_1^h P_2^h G \left[ \left( 2 - P_1^h - P_2^h \right) \Omega + U + 2V \right] \\
&\quad + P_1^h \left( 1 - P_2^h \right) G \left[ \left( 1 - P_1^h - P_2^h \right) \Omega + U + 2V \right] \\
&\quad + \left( 1 - P_1^h \right) P_2^h G \left[ \left( 1 - P_1^h - P_2^h \right) \Omega + U + 2V \right] \\
&\quad + \left( 1 - P_1^h \right) \left( 1 - P_2^h \right) G \left[ \left( -P_1^h - P_2^h \right) \Omega + U + 2V \right].
\end{align*}
\]
so that

\[
\frac{dE(G)}{d\delta} = \left( P_2^h - P_1^h \right) G_{HH} \\
+ P_1^h P_2^h G''_{HH} \times \left[ (2 - P_1^h - P_2^h) \frac{d\Omega}{d\delta} \right] \\
+ \left( 1 - P_2^h + P_1^h \right) G_{HL} \\
+ P_1^h (1 - P_2^h) G'_{HL} \times \left[ (1 - P_1^h - P_2^h) \frac{d\Omega}{d\delta} \right] \\
- \left( 1 + P_2^h - P_1^h \right) G_{LH} \\
+(1 - P_1^h) P_2^h G_{LH} \times \left[ (1 - P_1^h - P_2^h) \frac{d\Omega}{d\delta} \right] \\
- \left( P_1^h - P_2^h \right) G_{LL} \\
+(1 - P_1^h)(1 - P_2^h) G_{LL} \times \left[ (1 - P_1^h - P_2^h) \frac{d\Omega}{d\delta} \right],
\]

where \( G \) is the inverse of the agent’s utility function and

\[
G_{HH} = G \left[ (1 - P_1^h - P_2^h) \Omega + \bar{U} + 2V \right] \\
G_{HL} = G \left[ (1 - P_1^h - P_2^h) \Omega + \bar{U} + 2V \right] \\
G_{LH} = G \left[ (1 - P_1^h - P_2^h) \Omega + \bar{U} + 2V \right] \\
G_{LL} = G \left[ (1 - P_1^h - P_2^h) \Omega + \bar{U} + 2V \right].
\]

It is then easily verified that \( \frac{dE(G)}{d\delta} = 0 \) for \( \delta = 0 \). Thus, \( \delta = 0 \) is a local minimum or maximum.

We then calculate \( \frac{d^2E(G)}{d(\delta)^2} \) and after some algebra we find :

\[
\frac{d^2E(G)}{d(\delta)^2} = -2 \left( G_{HH} - 2G_{HL} + G_{LL} \right) \\
+ P_1^h P_2^h \left[ 2 \frac{d^2\Omega}{d(\delta)^2} - 2P_1^h \frac{d^2\Omega}{d(\delta)^2} + 4 \frac{d\Omega}{d\delta} \right] \left( G'_{HH} - G'_{HL} \right) \\
+ 2P_1^h \left( 1 - P_2^h \right) G''_{HL} \left[ \frac{d\Omega}{d\delta} \right]^2 \\
- \left[ 4 \frac{d\Omega}{d\delta} - 2P_2^h \frac{d^2\Omega}{d(\delta)^2} \right] \left( 1 - 2P_1^h + P_1^h P_2^h \right) \left( G'_{HL} - G'_{LL} \right).
\]

(L1)  (L2)  (L3)  (L4)
Note that for $\delta = 0$ we have $P_1^h = P_2^h$, $\frac{d\Omega}{d\delta} = 0$, and $\frac{d^2\Omega}{d(\delta)^2} = 0$. It is then easily verified that for $\delta = 0$ each of the lines $L_2$, $L_3$, and $L_4$ in $\frac{d^2E(G)}{d(\delta)^2}$ are equal to zero. Thus, $\frac{d^2E(G)}{d(\delta)^2}$ simplifies to:

$$\frac{d^2E(G)}{d(\delta)^2} = -2(G_{HH} - 2G_{HL} + G_{LL})$$

which means that $\frac{d^2E(G)}{d(\delta)^2}$ has the same sign as $- [(G_{HH} - G_{HL}) - (G_{HL} - G_{LL})]$. Note that $G$ is a convex function (the inverse of the concave utility function), and $U_{HH} - U_{HL} = U_{LH} - U_{LL}$ at $\delta = 0$. Thus, $(G_{HH} - G_{HL}) > (G_{HL} - G_{LL})$ and therefore $\frac{d^2E(G)}{d(\delta)^2} < 0$. The latter combined with the fact that $\frac{dE(G)}{d(\delta)} = 0$ means that $\delta = 0$ is a global maximum. Thus, not only does the principal prefer a "biased" reporting system, it prefers an "extreme" reporting system, i.e., $\delta^* = \bar{\delta}$ or $\delta^* = \hat{\delta}$. Moreover, $\delta = \bar{\delta}$ or $\delta = \hat{\delta}$ has the same effect on the distribution of aggregate earnings; thus, the principal is indifferent between $\delta^* = \bar{\delta}$ or $\delta^* = \hat{\delta}$. 

We use Figure 2 to provide some insights into our findings so far.
The \((G, U)\) points depict the manager’s compensation and the utility that he derives from it, based on the optimal compensation contract. By Proposition 1 these points do not change as \(\delta\) changes. The horizontal line in the figure, \(\bar{U} + 2V\), shows the manager’s required expected utility level he derives from his compensation contract. We know that the principal will exactly satisfy the IR-constraint and thus hold the manager to this utility level. Accordingly, the expected compensation cost in equilibrium is determined by the convex combination of the \((G, U)\) points.

In Figure 2, \(A_1\) depicts this intersection when \(\delta = 0\). The issue now is how this point changes when \(\delta\) changes. By Lemma 2, increasing \(\delta\) moves more probability mass to the middle points. This means that the weight on point \((G_{HL}, U_{HL})\) increases, while less weight in put on the points \((G_{HH}, U_{HH})\) and \((G_{LL}, U_{LL})\). As indicated earlier, the principal always ensures that the manager receives his or her reservation utility. Moving more weight on the middle points means that we are moving along the \(\bar{U} + 2V\) line to the left of \(A_1\) towards the \((G_{HL}, U_{HL})\). Importantly, this means lower expected compensation costs (point \(A_2\) on the figure).

It is also important to realize that our results do not hinge on who controls the accounting process by determining \(\delta\). We find that the manager is actually indifferent over \(\delta\). If we therefore assume that an indifferent manager would set \(\delta\) to maximize the principal’s expected net payoff, decision rights over \(\delta\) do not matter. Thus, if control of the accounting process is delegated to the manager, he or she would set up an "extreme aggressive," \(\delta^* = \bar{\delta}\), or "extreme conservative," \(\delta^* = \hat{\delta}\), reporting system that would minimize expected compensation costs and maximize shareholder welfare. Our last proposition in this section makes this formal.

**Proposition 3** If control of the accounting process is delegated to the manager, he or she would choose \(\delta\) to maximize the principal’s expected net payoff.

**Proof of Proposition 3.** From the proof of Proposition 2, we know that the manager’s utility is unaffected by \(\delta\), \(\frac{dE(U)}{d\delta} = 0\). If we therefore assume a benevolent manager, he or she would set \(\delta\) to maximize the expected net payoff to the shareholders.

Our analysis to this point has been focusing on a two period setting. It is immediately clear, however, that absent any accrual choice akin to the one examined in the previous section, adding periods a production process of the type we employ would just "add \(\Omega s\)" to the optimal contract.
(measured in utiles) the same way going from one to two periods does here. The simple reason for this is that the optimal contract here always is a linear function in utility space of aggregate output. What may not be immediately clear here is whether our findings on the optimal use of accruals - the optimal accrual strategy - also extend to multi-period setting. Proposition 4 provides insights into that question.

**Proposition 4**  As the number of time periods grow from 2 to any \( n > 2 \), the accrual strategy that minimizes the variance of aggregate earnings is to replicate the two period strategy by make the maximum accruals in the first period, let it reverse in the subsequent period and then repeat. If \( n \) is odd a variance minimizing strategy has only \((n - 1)/2\) such repetitions and no accruals are made in the final period.

**Proof of Proposition 4.** We proceed by first deriving the variance minimizing strategies for a two period setting and then expand our analysis to a three and a four period setting. This allows us to generalize our findings to an \( n \)-period setting.

To facilitate the proof, let \( n \) denote the number of time periods under consideration and

\[
A = p^h(\lambda^h + \delta) + (1 - p^h)(1 - \lambda^l + \delta) \quad \text{(Income increasing probabilities)}
\]
\[
B = p^h(\lambda^h - \delta) + (1 - p^h)(1 - \lambda^l - \delta) \quad \text{(Income decreasing probabilities)}
\]
\[
C = p^h(\lambda^h) + (1 - p^h)(1 - \lambda^l) \quad \text{(Neutral probabilities)}
\]
\[
D = p^h(\lambda^h + 2\delta) + (1 - p^h)(1 - \lambda^l + 2\delta)
\]
\[
E = p^h(\lambda^h - 2\delta) + (1 - p^h)(1 - \lambda^l - 2\delta).
\]

Also, we use \( Var(\delta_1, \delta_2) \) to denote the variance of aggregate performance at the end of the 2\(^{nd}\) time period when the accrual strategy implemented by the manager resulted in increasing the probability of high performance in the first period by \( \delta_1 \) and in increasing the probability of high performance in the second period by \( \delta_2 \).

**Variance minimizing strategy for \( n = 2 \)**

We begin by calculating the variance of aggregate firm performance under either an income increasing or decreasing strategy \((\delta \neq 0)\) and then compare it to the variance under a neutral
strategy ($\delta = 0$). After some algebra, we find that:

\[
\begin{align*}
V_{\text{ar}}(\delta, \delta) &= A + B - A^2 - B^2 \quad \text{and} \\
V_{\text{ar}}(0, 0) &= 2C - 2C^2.
\end{align*}
\]

Thus,

\[
V_{\text{ar}}(\delta, \delta) - V_{\text{ar}}(0, 0) = -2\delta^2 < 0,
\]

which means that aggregate measured performance has lower variance under income decreasing or decreasing strategies relative to neutral strategies.

Moreover, it is easily verifiable that the variance is minimized for $\delta = \bar{\delta}$ or $\delta = \underline{\delta}$. In particular,

\[
\frac{\partial (V_{\text{ar}}(\delta, \delta))}{\partial \delta} = -4\delta \quad \text{and} \\
\frac{\partial^2 (V_{\text{ar}}(\delta, \delta))}{\partial (\delta)^2} = -4 < 0,
\]

which also means that $\delta = 0$ is a global maximum.

**Variance minimizing strategy for** $n = 3$

We have just shown that the variance of aggregate performance is minimized for $\delta = \bar{\delta}$ or $\delta = \underline{\delta}$. In particular, the variance is minimized if we have the maximum allowed income increasing accruals in one period and the maximum allowed income decreasing accruals in the other period. The order of the income increasing and decreasing accruals does not matter.

In $n = 3$, now, we show that the variance is minimized if we have the maximum allowable income increasing accruals in one period, the maximum allowable income decreasing accruals in another period, and no accruals (a neutral strategy) in the remaining period. Again, it should be noted that the order of accruals does not matter.

To show this formally, we examine the impact on variance if we divert from the optimal accrual strategy that we established when $n = 2$. In particular, we examine how variance changes when an income increasing or decreasing accrual in the "no-accruals" period changes the likelihood of high firm performance by $\varepsilon$, where $\varepsilon$ is some small positive or negative number. We begin by considering

---

\footnote{Recall that the variance of measured performance under income increasing or decreasing strategies are the same because of accrual reversals.}
the case where the "no-accruals" period is the last of the three periods. Of course, the accrual
reversal process requires that changing the likelihood of high firm performance in the last period
also changes the likelihood of high firm performance in the second time period. More specifically,
then, the following relationships can be easily verified:

\[ \text{Var} (\delta_1 = \tilde{\delta}, \delta_2 = 0) < \text{Var} (\delta_1 = \tilde{\delta} - \varepsilon, \delta_2 = \varepsilon) \text{ if } \varepsilon > 0 \text{ and} \]
\[ \text{Var} (\delta_1 = \tilde{\delta}, \delta_2 = 0) < \text{Var} (\delta_1 = \tilde{\delta} - \varepsilon, \delta_2 = \varepsilon) \text{ if } \varepsilon < 0. \]

Moreover, if we examine the variance of the first two period, we get the following:

\[ \text{Var} (\delta_1 = \tilde{\delta}, \delta_2 = \delta - \varepsilon) > \text{Var} (\delta_1 = \tilde{\delta}, \delta_2 = \delta) \text{ if } \varepsilon > 0 \text{ and} \]
\[ \text{Var} (\delta_1 = \tilde{\delta}, \delta_2 = \delta - \varepsilon) > \text{Var} (\delta_1 = \tilde{\delta}, \delta_2 = \delta) \text{ if } \varepsilon < 0. \]

The analysis is easily expanded to include cases where the "no-accruals" period is either the first or
the second period when \( n = 3 \). The results remain the same. Thus, any deviations from a strategy
that results in a period with income increasing accruals, a period with income decreasing accruals,
and a period where accruals have a no effect on the likelihood of high firm performance result in
higher variance.\(^8\)

**Variance minimizing strategy for** \( n = 4 \)

We know that for \( n = 2 \), the variance minimizing strategy is \( \delta = \tilde{\delta} \) or \( \delta = \underline{\delta} \). If \( n = 3 \), we know
that the variance minimizing strategy requires a period with \( \delta = \tilde{\delta} \), a period with \( \delta = \underline{\delta} \) and a period
with \( \delta = 0 \). If \( n = 4 \), however, the firm can implement the \( n = 2 \) strategy two times. This is the variance minimizing strategy. Based on our proof for \( n = 3 \), doing anything that would result in a
\( \delta < \tilde{\delta} \) or a \( \delta > \underline{\delta} \) would increase the overall variance.

**Variance minimizing strategy for** \( n > 4 \)

We can now derive the optimal variance minimizing strategy for \( n > 4 \). If \( n \) is an even number,
the optimal minimizing strategy is the \( n = 2 \) optimal strategy that repeats for every 2 periods.
If \( n \) is an odd number, the optimal minimizing strategy is one where we have one period where
accruals have no effect on the probabilities of high firm performance, and have full reversals for the

\(^8\)There are 12 possible strategies that result in such combination.
remaining \(n - 1\) periods. ■

As Proposition 4 focuses on variance, it admittedly does not offer a full guarantee that the (extreme accrual) strategy outlined also solves the principal’s problem in the \(n > 2\) case. Intuitively, reducing the variance of aggregate earnings and, thus, of the agent’s utility seems like a good thing. Unfortunately, variance is not generally a sufficient statistic for risk and what the principal really care about from a stewardship perspective is actually risk minimization for a given level of incentives. Still, as \(n\) grows large our binomial distribution will reasonably fast converge towards a normal in which case variance does become sufficient for risk. Moreover, as risk-reduction and variance in the \(n = 2\) case are achieved by the same choice, surely the same is the case for any even \(n\) no matter how small.

5 Valuation Implications

Having analyzed the preferred use of accruals as here defined from a stewardship perspective, in this section we explore the valuation implications of this extreme accrual reversal strategy. We do this by in turn exploring how some standard empirical measures of "earnings quality" are impacted by the accruals here and then contrasting those findings with results on the implications of accrual for how well earnings relate to the underlying fundamental value of a firm:

We start by exploring the impact of accruals on earnings response coefficients (\(ERCs\)). That is, we examine slope of the (linear) relation between "prices" and observed firm performance \((y)\). In our perfectly controlled world the only information about firm performance is that available through a firm’s financial reports. Accordingly, (changes in) market prices are entirely the result of realized accounting earnings and the level of accruals embedded therein. Under such conditions we have:

Proposition 5  The average price reaction to an earnings realization is independent of the accrual strategy.

Proof of Proposition 5.  We begin by calculating firm prices conditional on an earnings
realization: \( E[x|y, \text{ accrual strategy}] \). The unconditional expectation of firm output \( x \) is:

\[
E[x] = p^h x^H + \left(1 - p^h\right) x^L.
\]

For simplicity, we normalize \( E[x] \) to zero (e.g., by letting \( x^H = (1 - p^h) \) and \( x^L = -p^h \)). We then calculate the firm’s expected output conditional on observed performance and accrual strategy implemented by the firm. Let \( A \) and \( B \) denote an income increasing and decreasing strategy, respectively. Accordingly, then, we calculate the probabilities of \( x^H \) conditional on all possible combinations of observed performance and accrual strategies:

\[
\begin{align*}
\Pr[x^H|y^H, A] &= \frac{\left(\lambda^H + \delta\right) p^h}{\lambda^H p^h + (1 - \lambda^L) (1 - p^h) + \delta}, \\
\Pr[x^H|y^H, B] &= \frac{\left(\lambda^H - \delta\right) p^h}{\lambda^H p^h + (1 - \lambda^L) (1 - p^h) - \delta}, \\
\Pr[x^H|y^L, A] &= \frac{(1 - \lambda^H - \delta) p^h}{(1 - \lambda^H) p^h + \lambda^L (1 - p^h) - \delta} \text{ and} \\
\Pr[x^H|y^L, B] &= \frac{(1 - \lambda^H + \delta) p^h}{(1 - \lambda^H) p^h + \lambda^L (1 - p^h) + \delta}.
\end{align*}
\]

We can then calculate the firm’s expected output for all possible combinations of observed performance and accrual strategies. After some algebra, we get the following:

\[
\begin{align*}
E[x|y^H, A] &= \frac{p^h (1 - p^h) \left(\lambda^H + \lambda^L - 1\right)}{\lambda^H p^h + (1 - \lambda^L) (1 - p^h) + \delta}, \\
E[x|y^H, B] &= \frac{p^h (1 - p^h) \left(\lambda^H + \lambda^L - 1\right)}{\lambda^H p^h + (1 - \lambda^L) (1 - p^h) - \delta}, \\
E[x|y^L, A] &= -\frac{p^h (1 - p^h) \left(\lambda^H + \lambda^L - 1\right)}{(1 - \lambda^H) p^h + \lambda^L (1 - p^h) - \delta} \text{ and} \\
E[x|y^L, B] &= -\frac{p^h (1 - p^h) \left(\lambda^H + \lambda^L - 1\right)}{(1 - \lambda^H) p^h + \lambda^L (1 - p^h) + \delta}.
\end{align*}
\]

Also, from our previous analysis, the firm will allow for full reversal of accruals during any two
time periods. Thus, \( \Pr [A] = \Pr [B] = 0.5 \) (assuming an even number of time periods) and

\[
\begin{align*}
\Pr [y^H|A] &= p^h (\lambda^H + \delta) + (1 - p^h) (1 - \lambda^L + \delta) \\
\Pr [y^H|B] &= p^h (\lambda^H - \delta) + (1 - p^h) (1 - \lambda^L - \delta).
\end{align*}
\]

We can now calculate the \( ERC \) as

\[
ERC = 0.5 \left\{ \Pr [y^H|A] \left[ E [x|y^H, A] - E [x] \right] + \Pr [y^H|B] \left[ E [x|y^H, B] - E [x] \right] + \Pr [y^L|A] \left[ |E [x|y^L, A]| - E [x] \right] + \Pr [y^L|B] \left[ |E [x|y^L, B]| - E [x] \right] \right\}
\]

which after some algebra simplifies to the following:

\[
ERC = 2p^h(1 - p^h) (\lambda^H + \lambda^L - 1).
\]

and thus not a function of \( \delta \). ■

The reason that the \( ERC \)s (as per Proposition 5) are unaffected by accrual strategies is of course again because accruals fully reverse here. Stronger market reactions when a given level of earnings is realized in spite of the direction of the accruals are offset by weaker reactions when accruals are favorable towards the same earnings level. On average the reaction is here exactly the same no matter what the accrual strategy is.

Importantly, as we will show next, the same "accrual irrelevance" result does not hold for other common empirical measures of "earnings quality" or "value relevance" however. Consider for example first the implications of either aggressive or conservative accrual strategies on \( R^2 \)s obtained from regressing market prices on observed performance.

**Proposition 6** The \( R^2 \) from a regression of market prices on observed accounting performance is strictly decreasing in \( |\delta| \).

**Proof of Proposition 6.** We calculate \( R^2 \) under a neutral strategy (\( \delta = 0 \)) for a single time period and compare that to \( R^2 \) under either an income increasing or decreasing strategy (\( \delta \neq 0 \)) again for a single time period. We use \( \hat{x} \) to denoted the fitted values of price changes in an OLS
regression of price changes on earnings. Then, notice first that under a neutral strategy $R^2 = 1$.

More specifically,

$$R^2 = \frac{\sum_{i=1}^{2} \sum_{j=1}^{2} (\langle \hat{x}|y^j, i \rangle - E[x])^2}{\sum_{i=1}^{2} \sum_{j=1}^{2} (E\[x\]|y^j, i] - E[x])^2} = 1$$

where $i \in \{A, B\}$, $j \in \{H, L\}$ and $E[x]$ is normalized to zero.

If $\delta \neq 0$, $R^2$ is equal to the following:

$$R^2 = \frac{\sum_{i=1}^{2} \sum_{j=1}^{2} (E\[x]\|y^j, i] - E[x])^2}{\sum_{i=1}^{2} \sum_{j=1}^{2} (E\[x\]|y^j, i] - E[x])^2} = \frac{\sum_{j=1}^{2} (E\[x\]|y^j, \delta = 0] - E[x])^2}{\sum_{i=1}^{2} \sum_{j=1}^{2} (E\[x\]|y^j, i] - E[x])^2}$$

The fitted values of price values $x$ are the same as the expected values of $x$ conditional on the realization of $y^j$’s for $\delta = 0$. From our earlier proof on ERCs, the fitted values do not depend on $\delta$. The denominator on the other hand is a function of $\delta$. It can then be verified that the denominator is an increasing function of $\delta$, which means that the $R^2$ decreases as $\delta$ moves from zero.

While we have yet to determine the impact of accruals on the relation between accounting earnings and the underlying fundamental value of the firm an important message here is that the ERC and the $R^2$ measures do not lead to the same inference about earnings quality and accruals. In our perfectly controlled study where financial statements are the only source of information about firms combined the two suggest an adverse implication of non-neutral accruals. In a real-world regression where the potential for correlated omitted variables always linger it is easy to imagine that the two measures because of the different way they respond to accruals could lead to opposing conclusions.

For completeness and in an attempt to obtain additional insight into the implications of non-neutral accrual strategies on empirical measures of earnings quality we finally examine the correlation coefficient between the firm’s earnings ($y$) and prices ($E\[x]\|y, accrual\text{ strategy}]$) before we turn our attention to the implications of non-neutral accrual strategies for the relation between fundamentals of a firm and its accounting earnings. The analysis of correlation here focuses on a
single period setting.

**Proposition 7** The correlation coefficient $\rho$ between market prices and observed accounting performance are strictly decreasing in $|\delta|$.

**Proof of Proposition 7.** The correlation coefficient $\rho$ between $E[x|y, accrual strategy]$ and $y$ for $n = 1$ is:

$$
\rho = \frac{\text{cov}(E[x|y, accrual strategy], y)}{\sigma_x \sigma_y}
= \frac{E[(E[x|y, accrual strategy] - E(x))(y - E(y))]}{\sqrt{E((E[x|y, accrual strategy])^2) - E^2(x)\sqrt{E(y^2) - E^2(y)}}}
$$

where the accrual strategy can be either income increasing or income decreasing, i.e., \textit{accrual strategy} $\in \{A, B\}$. Also note that we have normalized $E(x)$ and $E(y)$ to zero. Thus,

$$
\rho = \frac{E[(E[x|y, accrual strategy])(y)]}{\sqrt{E((E[x|y, accrual strategy])^2)\sqrt{E(y^2)}}}
$$

We proceed by examining $\rho$ in smaller pieces. We start with the numerator:

$$
E[(E[x|y, accrual strategy])(y)] = 
\sum_{y \in Y} \sum_{\text{strategy} \in \{A, B\}} \text{pr}(y|\text{strategy}) \text{pr}(\text{strategy}) E(x|y, \text{strategy}) E(y)
$$

which after some algebra simplifies to:

$$
E[(E[x|y, accrual strategy])(y)] = p^h(1 - p^h)(\lambda^H + \lambda^L - 1)
$$

which is not a function of $\delta$.

We now move on to the denominator and start with the variance of prices:

$$
E \left( (E[x|y, accrual strategy] - E(x))^2 \right) = 
\sum_{y \in Y} \sum_{\text{strategy} \in \{A, B\}} \text{pr}(y|\text{strategy}) \text{pr}(\text{strategy}) (E(x|y, \text{strategy}))^2
$$
After some algebra we find that

\[
E \left( (E [x|y, accrual \ strategy] - E (x))^2 \right)
= \frac{(-1 + p^h)^2(\lambda^H + \lambda^L - 1)^2 (p^h)^2 \left[ p^h (\lambda^H - 1) + \lambda^L (p^h - 1) + 1 \right]}{[p^h (\lambda^H - 1) + \lambda^L (p^h - 1) - \delta + 1)(p^h (\lambda^H - 1) + \lambda^L (p^h - 1) + \delta + 1)]}
- \frac{(-1 + p^h)^2(\lambda^H + \lambda^L - 1)^2 (p^h)^2 \left[ p^h (\lambda^H - 1) + \lambda^L (p^h - 1) \right]}{[p^h (\lambda^H - 1) + \lambda^L (p^h - 1) + \delta)(p^h (\lambda^H - 1) + \lambda^L (p^h - 1) - \delta)]}
\]

We then proceed by differentiating the variance with respect to \( \delta \). After some algebra we find:

\[
\frac{\partial}{\partial \delta} \left( E \left( (E [x|y, accrual \ strategy] - E (x))^2 \right) \right)
= 2 \frac{(-1 + p^h)^2(\lambda^H + \lambda^L - 1)^2 (p^h)^2 \delta \left[ p^h (\lambda^H - 1) + p^h (\lambda^L - 1) + 1 \right]}{(p^h (\lambda^H - 1) + \lambda^L (p^h - 1) - \delta + 1)^2(p^h (\lambda^H - 1) + \lambda^L (p^h - 1) + \delta + 1)^2}
- 2 \frac{(-1 + p^h)^2(\lambda^H + \lambda^L - 1)^2 p^h \delta \left[ p^h (\lambda^H - 1) + \lambda^L (p^h - 1) \right]}{(p^h (\lambda^H - 1) + \lambda^L (p^h - 1) - \delta)^2(p^h (\lambda^H - 1) + \lambda^L (p^h - 1) + \delta)^2}
\]

which can easily be verified that it is positive. Thus, the variance of prices is an increasing function of \( \delta \).

Lastly, we need to examine how the variance of \( y \) changes with \( \delta \). After some algebra we find:

\[
E (y - E (y))^2
= \sum_{y \in Y} \sum_{strategy \in \{A,B\}} pr (y|strategy) pr (strategy) y^2
= - \left[ p^h (\lambda^H - 1) + \lambda^L (p^h - 1) + 1 \right] \left[ p^h (\lambda^H - 1) + \lambda^L (p^h - 1) \right]
\]

which is not a function of \( \delta \).

Thus, \( \delta \) affects \( \rho \) only though its effect on the variance of prices. Therefore, \( \rho \) is a decreasing function of \( \delta \).

Perhaps not too surprisingly, the correlation coefficient turns out to send the same negative message about non-neutral accruals as does the \( R^2 \); they appear to be bad from a value relevance perspective. After all, the type of accruals the principal here is compelled to introduce are some that introduces volatility into the way earnings numbers are interpreted. Moreover, the reason
the principal prefer the non-neutral use of accruals is purely to enhance the stewardship value of earnings and it seems to be a commonly held belief that valuation and stewardship are conflicting roles that have to be traded off - improving one damages the other. Our results so far do little to challenge belief. Indeed, when viewed in this light one might actually argue that the more surprising result actually is the indifference here of the ERC.

Before one can conclude that principals and/or agents concerned with enhancing the stewardship qualities of accounting earnings do so at the expense of the ability of outsiders to value the firm, we need to examine the implications of non-neutral accruals on the relation between firm fundamentals and accounting earnings. After all, the real question is how earnings relate to fundamentals - not how earnings relate to stock prices and measures of the latter relation would be of far more limited interest if one could get archival data on the former. The benefit of our modeling approach is that we get to assess the real relation of interest. We do this by examining the role of non-neutral accruals in determining the correlation coefficient between the fundamentals of the firm, $x$, and the history of observed accounting performances, $y$.

**Proposition 8** The correlation coefficient $\rho_{x,y}$ between aggregate realized economic earnings, $x$, and aggregate observed accounting performance over some horizon ($n > 1$) is strictly increasing in $|\delta|$.

**Proof of Proposition 8.** We examine the correlation coefficient $\rho$ between firm output $x$ and observed performance $y$ for two consecutive time periods. The correlation coefficient between $x$ and $y$ is defined as:

$$\rho_{x,y} = \frac{\text{cov}(x,y)}{\sigma_x \sigma_y} = \frac{E[(x - E(x)) (y - E(y))]}{\sqrt{E(x^2)} \sqrt{E(y^2)} - E^2(x) \sqrt{E(y^2)} - E^2(y)}.$$

We begin by examining how the numerator is affected by $\delta$. To simplify the analysis we normalize the expected value of $y$ to zero. In other words, for the purpose of this proof we treat the earnings realizations simply as the unexpected component of earnings. Accordingly, we assume that $y^H = (p^L \lambda^H + (1 - p^L) \lambda^L)$ and $y^L = -[p^L \lambda^H + (1 - p^L) (1 - \lambda^L)]$. In a similar fashion, we normalize $E[x]$ to zero by letting $x^H = (1 - p^H)$ and $x^L = -p^H$ (it is easily verified that $E[x] = 0$.
for $n = 1$ and for $n = 2$. After some algebra, it can then also be verified that

$$
E [(x - E(x)) (y - E(y))] = (1 + p^h)p^h \\
(4(p^h)^2 \lambda^H - 4(p^h)^2 + 4(p^h)^2 \lambda^L - 2p^h\lambda^H + 4p^h - 6p^h\lambda^L + 1 - 2\lambda^H)
$$

which is not a function of $\delta$. Also notice that the standard deviation of firm output $x$ is not a function of $\delta$. Moreover, we have already established that the variance of measured performance $y$ is a function of $\delta$ and decreases for income increasing or decreasing strategies. Thus, income increasing or decreasing accrual strategies ($\delta \neq 0$) result in higher correlation coefficients between firm performance and observed performance.

The basic idea behind this last Proposition is that when assessing the quality of an accounting system, one need to take into account all the information it has produced. Firms being traded publicly typically have accumulated value over some period of time and investors typically have multiple financial reports at their disposal when evaluating and attempting to value the firm. Our result says that if one was to sum up past accounting earning, that number is more correlated with the actual value created over that horizon if a non-neutral accrual strategy is employed. This suggests that the accounting system actually becomes better from a valuation perspective when non-neutral accruals are introduced. What is uplifting then is the suggestion that there is no real trade-off between stewardship and valuation usefulness of accrual accounting. What is grounds for concern is that standard measures of value relevance do not capture this at best and are potentially misleading otherwise. At least not in our setting.

### 6 Empirical Connections

Information produced by accounting systems competes in general with information from a variety of other sources. An often heard criticism of accounting information is that it is not as timely or as informative. Indeed, there is empirical evidence, albeit mixed, suggesting that the usefulness of accounting information has been deteriorating over time. On the one hand, for example, Lev (1989) documents a weak relationship between accounting earnings and stock returns. In particular, he finds that $ERCs$ are unstable over time and $R^2s$ are relatively small.
In a similar spirit, Lev and Zarowin (1999) examine the association between stock returns and accounting earnings and find that $R^2$s and ERCs have been decreasing over time. The authors find similar results, although less pronounced, when they examine the association between returns and cash flows. Ball and Shivakumar (2008) also argue that earnings announcements provide only a modest amount of information to the market. They find that quarterly earnings announcements are associated with only one or two percent of total yearly price volatility.

On the other hand, Collins, Maydew, and Weiss (1997) find that the value relevance of accounting earnings has declined, but the value relevance of book values has increased. They argue that the combined value relevance of earnings and book values has increased slightly over time. Similarly, Landsman and Maydew (2002) also find evidence that the information content of quarterly earnings, as measured by abnormal trading volume and return volatility around earnings announcements, has increased over time. Francis and Schipper (1999) also find that the relationship between earnings and returns ($R^2$s) has been deteriorating over time, but the value relevance of balance sheet and book value information has been increasing.

Our analysis provides a potential explanation for the relatively weak relationship between accounting earnings and stock returns. That is, we show that accrual reversals have a negative effect on $R^2$s. Of course, in our model there are no other competing information sources and we cannot therefore talk about the informativeness of accounting earnings in the presence of other information. However, our analysis suggests that even in a world where accounting earnings does not compete with any other information sources, $R^2$s decrease as a result of accrual reversals (Proposition 6). In addition, we also show (Proposition 7) that the correlation coefficient between accounting earnings and the firm’s fundamentals for a single period decreases because of accrual reversals.

Most importantly, though, we find that the usefulness of accounting information does not necessarily decrease because of accrual reversals. As we show in Proposition 8, the correlation coefficient between the fundamentals of the firm and aggregate realized accounting performance is increasing in $|\delta|$. The usefulness of accounting earnings improves because of accruals, but only if we focus on aggregate performance instead of a firm’s performance in a particular time period. This can potentially help explain empirical evidence that the value relevance of book values has been increasing (Collins et al., 1999; Francis and Schipper, 1999). Our findings may thus also provide an additional explanation for Campell and Shiller’s (1988) findings that long moving averages of
7 Conclusions

In this paper we seek to gain insight into desirable properties of accrual accounting constructs. In particular we are interested in understanding the implications of using the (reversing) accrual mechanism both for the stewardship and the valuation uses of accounting earnings. To that end we study a n-period model in which the distribution of (equilibrium) economic earnings is the same in each period. We then ask whether it would be better to have a noisy and potentially biased accounting system that also have identically distributed earnings for these periods or to use the accrual mechanism aggressively to move expected accounting earnings forward to the earlier period. Alternatively, to be conservative by moving expected accounting income from the earlier to the later period.

We show a number of things in the process. First, we show that the optimal (accounting-based) incentive contract is robust to the accrual choice - it is always the same. Further that the agent is completely indifferent about the choice of accruals. Accordingly, the choice can be delegated to the agent who presumable given his own indifference can be persuaded to do what is best for the owners. What is important to realize is that the owners, despite the robustness of the optimal contract and the indifference of the agent actually do care about how accruals are used. Specifically, from a stewardship perspective they strictly prefer either aggressive or conservative accounting to the neutral approach as either result in the minimum compensation expense for any given level of sustained effort.

The result highlights a key trade-off rarely if ever recognized in accounting. That while reversing (extreme) accruals may make each period’s earnings less representative of the underlying economic reality of that specific period, the auto-correlation introduced actually may make the aggregate (retained) earnings distribution less volatile. And for the control problem the aggregate earnings over the relevant employment horizon is key - the less volatile the more useful it is for stewardship purposes. As we proceed to show, the same is true when one considers the relation between accounting reports and the underlying value of the firm. Specifically, the reduced volatility increases the correlation of aggregate earnings over a given horizon with the economic value created over the
same period of time.

The final part of our analysis is dedicated to explore the implications the optimal accrual strategy from a stewardship vantage point has for the valuation properties of accounting reports. We split the analysis into two broad questions: what is the impact on the actual relation between accounting and real economic earnings and what is the impact on standard empirical measures of that relation. The good news is that in our model there is no conflict or trade-off between stewardship and valuation implications of accruals here - the strategy that is best from a stewardship perspective is also the one that makes aggregate earning most correlated with the true value of the firm. The bad news is that standard empirical measures at best fail to convey this positive message.
References (incomplete)


