Cost of Capital in Imperfect Competition Settings

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Abstract
This paper analyzes the role of information in pricing and cost of capital in security markets characterized by imperfect competition among investors. Imperfect competition results in markets being less than perfectly liquid. Our analysis demonstrates that the interaction between illiquid markets and asymmetric information gives rise to a role for information in cost of capital that is absent in perfect competition settings. Our results are relevant to a large empirical literature that examines the relation between various information attributes and the cost of capital.

Keywords Cost of capital, imperfect competition, information asymmetry
1 Introduction

This paper analyzes the role of information in pricing and cost of capital in security markets characterized by imperfect competition: that is, in markets that are less than perfectly liquid. Standard setters and policy makers often claim that high-quality information and a level playing field are essential to the efficient allocation of capital in the economy. For example, Robert Herz, chairman of the Financial Accounting Standards Board states: “It’s about lowering the cost of capital, lowering the cost of preparation, and lowering the cost of using it,” (see Wild, 2004). Similarly, Arthur Leavitt, former chairman of the Securities and Exchange Commission, argues that “high quality accounting standards ... improve liquidity [and] reduce capital costs,” (see Leavitt, 1997).

Yet information issues are largely absent in conventional models of asset pricing and cost of capital. For example, in a Capital Asset Pricing Model (CAPM) framework, all investors are presumed to have homogeneous information, so issues that arise from information asymmetry are precluded from occurring. Moreover, only non-diversifiable risk is priced, and the relevant non-diversifiable risk is the covariance of the firm’s cash flows with the market. Therefore, the only way information can affect cost of capital is through its impact on this covariance. In noisy rational expectations models (e.g., Grossman and Stiglitz, 1980; Hellwig, 1980; Diamond and Verrecchia, 1981), heterogeneous information plays a prominent role, and the aggregation of this information through price is an important part of the analysis. The literature finds, however, that only the average precision of investors’ information matrices (e.g, the inverse of their assessed covariance matrices) is relevant in deriving the equilibrium cost of capital (see, e.g., Lintner, 1969; Admati, 1985; and Lambert, Leuz, and Ver-
Information asymmetry across investors can affect cost of capital, but only through its effect on investors’ average precision. Controlling for average precision, the degree of information asymmetry across investors (e.g., the amount that investors’ information precisions differ from the average) does not affect cost of capital.

As discussed in Merton (1989) and O’Hara (2003), one feature that the above theoretical models have in common is that they assume market prices are based on perfect competition. That is, all investors act as price takers, and they can buy and sell any quantity at the market price: markets are perfectly liquid.\(^1\) In contrast, imperfect competition and asymmetric information are common features of market microstructure models going back to Kyle (1985) and Glosten and Milgrom (1985). Moreover, these models find that information asymmetry can affect market features such as bid-ask spreads.

Our model integrates imperfect competition into conventional asset-pricing models. We conduct our analysis using a noisy rational expectations framework for a market consisting of \( J \) risky assets and one risk-free asset. The multiple-firm dimension of the analysis is essential in that it allows us to distinguish between diversifiable and non-diversifiable risk. There are two types of investors: a more-informed type that has the ability to condition its demands on private information, and a less-informed type that has no information beyond that which it can glean from price.

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\(^1\) While securities markets for heavily followed, heavily traded firms are often held out as prime examples of settings that approximate perfect competition, there is growing empirical evidence that suggests that liquidity concerns can be important even for these cases (witness the recent market meltdown). For example, general asset pricing models such as Pastor and Stambaugh (2003) and Acharya and Pederson (2005) find that liquidity factors explain cross-sectional variation in expected returns not accounted for by conventional CAPM-betas. Other empirical papers that allude to liquidity risk include Chordia, Roll, and Subrahmanyam (2000) and Sadka (2006). Moreover, there are large numbers of firms with relatively light and infrequent trading volume, and firms where concerns about insider trading are large.
Kyle (1989), we model imperfect competition by assuming that investors understand that the magnitude of their demand for firm shares can affect the price at which their demand is fulfilled, and this introduces a cost to trading. Moreover, the degree to which the market is illiquid (or the magnitude of the cost of trading) is endogenously determined, not exogenously imposed. In particular, we analyze whether (and how) the information environment impacts the degree of market illiquidity, and therefore, the cost of capital. As is standard in the literature, we define cost of capital to be the extent to which investors discount firms’ share prices relative to the expected value of firms’ cash flows.

Our analysis demonstrates that the interaction between illiquid markets and asymmetric information gives rise to an additional role for information in cost of capital. Moreover, the degree of information asymmetry across investors can affect cost of capital even after controlling for the average precision of information. We demonstrate the interaction by introducing imperfect competition and information asymmetry separately into the model. In a setting of no information asymmetry and perfect competition, our model reduces to the standard CAPM. With homogeneous information but endogenous market illiquidity, we find that cost of capital can be decomposed into two components. The first component arises due to the covariance of a firm’s cash flows with the market, and thus is identical to the discount implicit in

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2 For example, the literature on block trades shows positive (negative) price effects arising from large purchases (sales) by investors; see Kraus and Stoll (1972), Holthausen, Leftwich, and Mayers (1990), Chan and Lakonishok (1995), and Keim and Madhavan (1997). While part of this effect is temporary and attributed to liquidity issues, there is also a permanent component that is attributed to the possible information-based motivation for the trade. In addition, a large literature has found positive price effects associated with index inclusion. See for example Harris and Gurel (1986), Shleifer (1986), Jain (1987), Dhillon and Johnson (1991), Beneish and Whaley (1996), Lynch and Mendenhall (1997), and Wurgler and Zhuravskaya (2002). These effects, which are generally found to be permanent, are attributed to the increased demand for shares of these firms by institutional investors whose investment policies lead them to mimic the composition of the index.

3 In contrast, other papers incorporate illiquidity by imposing exogenous trading costs in their analysis. See Acharya and Pederson (2005) for an example.
the CAPM. The second component is the additional discount in price that measures the extent to which the market is illiquid because competition is imperfect. Our analysis shows, however, that in equilibrium the discount that results from imperfect competition is exactly proportional to investors’ assessment of the covariance of firms’ cash flows. Therefore, the two effects together are empirically indistinguishable from a perfect competition setting with a proportionately higher premium for overall market risk. Thus, as with a model of perfect competition, there is no identifiable, separate risk factor in an imperfect competition setting with identically informed investors.

In contrast, in an imperfect competition setting where investors are asymmetrically informed, we find that cost of capital is not proportional to the covariance of cash flows. Market illiquidity affects cost of capital in two ways. First, it limits how aggressively informed investors trade based on their information. This reduces the informativeness of the information that less-informed investors can glean from price. This, in turn, lowers the precision of their information matrix, which lowers the average precision and therefore lowers cost of capital, ceteris paribus. In addition, by limiting the trading behavior of informed investors, market illiquidity lowers the weight applied to the precision of informed investors in determining the weighted average. Moreover, other characteristics of the information environment, including the degree of information asymmetry, also affect the degree of market illiquidity and cost of capital, even after controlling for the covariance of cash flows (or beta) of the firm. In short, we find a role for information asymmetry in cost of capital through its relation to the illiquidity of the market; this role does not exist in perfect competition settings.

Our results are relevant to a large empirical literature in accounting that exam-
ines the relation between various information attributes and the cost of capital.\footnote{For recent examples, see: Botosan, Plumlee, and Xie (2004); Francis, La Fond, Olsson, and Schipper (2004, 2005); Botosan and Plumlee (2007); Liu and Wysocki (2007); Cohen (2008); Core, Guay and Verdi (2008); Mohanram and Rajgopal (2008); Ng (2008); Ogneva (2008); Armstrong, Core, Taylor, Verrecchia, (2009); Ashbaugh-Skaife, Collins, Kinney, Lafond (2009); and Bhattacharya, Ecker, Olsson and Schipper (2009).} One impediment to empirical work in this area has been the absence of any guidance as to what constitutes “information risk” and when, or whether, it should be priced. For example, one view is that “information risk” per se is not priced, but that information variables can help better estimate the relevant parameters of conventional risk factors in finance. In this case, a significant coefficient on an information variable is interpreted as a variable that helps to provide a more precise estimate of, say, a forward-looking beta than can be extracted from historical returns data or other information.\footnote{This empirical literature dates back to Beaver, Kettler, Scholes (1970). See Barry and Brown (1984, 1985 ) and Lambert, Leuz, Verrecchia (2007) for theoretical analyses of the role of information in assessing beta.} An alternative view is that “information risk” is a risk factor separate from conventional risk factors such as beta. Asymmetric information risk is often cited as a proposed separate type of price risked. Consistent with the results in Lambert, et al. (2009), our analysis demonstrates the importance of distinguishing between the precision of investors’ information from the degree of information asymmetry across investors. Controlling for the level of average precision, the degree of information asymmetry has no effect on cost of capital in settings for which perfect competition is a good approximation. In more illiquid markets where perfect competition is less descriptive, however, information asymmetry can affect the liquidity of the market and therefore cost of capital.

In Section 2 we study an imperfect competition setting where investors who are identically informed compete to hold shares in multiple firms. In Section 3 we extend our study to an imperfect competition setting where investors who are asymmetrically
informed compete to hold shares in multiple firms. In Section 4 we discuss the relation between information asymmetry and illiquidity, and how these factors interact in determining cost of capital. In a concluding section we summarize our results.

2 Identically informed investors

In this section we develop the imperfect competition feature of our model and analyze it in a setting of identically informed investors. We show that cost of capital can be expressed in terms of a covariance-of-cash-flows component, and an illiquidity component that is proportional to this covariance factor. We consider a one-period economy that consists of \( N \) identically informed investors, \( J \) firms, and a risk-free asset. The return on the risk-free asset has been normalized to zero and its price normalized to 1. Let \( \tilde{V} \) denote the \( J \times 1 \) vector of end-of-period firms’ cash flows, where the \( j \)-th element of the vector, \( \tilde{V}_j \), is the cash flow of the \( j \)-th firm. Let \( \tilde{P} \) denote the \( J \times 1 \) vector of beginning-of-period firms’ prices, or market values, where \( \tilde{P}_j \) is the price of the \( j \)-th firm. This implies that purchasing a single share of each firm yields an end-of-period return of \( (\tilde{V} - \tilde{P})^T \cdot 1 \), where 1 is a \( J \times 1 \) vector of 1’s and \( T \) denotes the transpose operator.

Our analysis of cost of capital is based on the imperfect competition setting of Kyle (1989). Imperfect competition settings are complex. In an attempt to grapple with this complexity, in this section we derive cost of capital in a series of steps for the case where investors are identically informed. The first step provides an expression

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6 Imperfect competition settings of the type we study require the participation of at least three investors (i.e., \( N \geq 3 \)) so as to eliminate the possibility of one investor, or a pair of investors, having too much monopoly power: see the discussion on p. 329 of Kyle (1989).

7 Henceforth we use a tilde (i.e., \( \sim \)) to distinguish a random variable from a fixed element, and put vectors and matrices in bold to distinguish them from scalars.

8 We summarize the notation we use in this section in Table 1 in the appendix.

9 Our model extends Kyle (1989) to multiple assets and focuses on cost of capital, an issue that plays no role in Kyle’s analysis.
for an investor’s demand for firms’ shares based on a belief that his demand affects
the price he pays to buy shares. The second step provides an expression for the
price vector based on a requirement that the total demand for firms’ shares equals
the supply of those shares. The price vector, in turn, yields and expression for cost
of capital; we introduce the expression for cost of capital in Proposition 1 before
proceeding to the final step. The final step reconciles the strategy an investor follows
to buy firms’ shares based on his demand for those shares (as established in the first
step) and price vector for those shares (as established in the second step).

An investor’s demand for firms’ shares. We represent an informed investor’s
demand for firms’ shares by the $J \times 1$ vector $D_I$. Following Kyle (1989), we char-
acterize imperfect competition as the self-sustaining belief by each investor that he
goes an upwardly-sloping price curve for firm shares. In particular, we assume each
investor believes that his demand is related to the price vector through the expression

$$P = p_0 + \Lambda \cdot D_I,$$

where $p_0$ is a $J \times 1$ intercept vector that incorporates all elements of the price vector
that are not related to an investor’s demand, and $\Lambda$ is a $J \times J$ matrix of coefficients.
In effect, each investor believes that prices result from a factor this is unrelated to his
demand, $p_0$, and a factor that is related to his demand through the matrix coefficient
$\Lambda$. The goal of our series of steps is to solve for $P$, $p_0$, $\Lambda$, and $D_I$, all of which
are endogenous variables that must be derived to achieve an expression for cost of
capital. As is standard in a model of imperfect competition, we interpret $\Lambda$ as the
degree of illiquidity associated with an individual investor’s demand. For example,
when the row/column elements of $\Lambda$ are small, an investor’s demand moves price less,
and thus the market for firm shares is more liquid with respect to demand; when the
row/column elements of $\Lambda$ are large, an investor’s demand moves price more, and thus the market is less liquid for firm shares.\textsuperscript{10}

We assume that each investor has a negative exponential utility function for an amount $w$ given by $- \exp \left(-\frac{w}{r_I}\right)$, where $r_I$ is the an investor’s constant absolute risk tolerance. When investors have negative exponential utility functions and random variables have multi-variate normal distributions, the certainty equivalent of each investor’s expected utility simplifies into the familiar expression of the expected value of his end-of-period wealth minus a term that is proportional to the variance of his wealth. Let $\Phi_I$ represent the information available to each identically informed investor. We assume that each investor believes that the vector of firms’ cash flows, $\tilde{\mathbf{V}}$, has a $J \times 1$ expected value of $E \left[ \tilde{\mathbf{V}} | \Phi_I \right]$ and a $J \times J$ covariance matrix of $\mathbf{Cov}_I$. In other words, each investor believes the expected cash flow of firm $j$ is $E \left[ \tilde{V}_j | \Phi_I \right]$, and assesses the covariance between the cash flows of firms $j$ and $k$ to be $\text{Cov}_I \left[ \tilde{V}_j, \tilde{V}_k \right]$, where $\text{Cov}_I \left[ \tilde{V}_j, \tilde{V}_k \right]$ is the $j$-th row, $k$-th column element of the matrix $\mathbf{Cov}_I$. Thus, based on his belief as to how his demand affects prices, an investor chooses $D_I$ to maximize the following objective function

\[
E \left[ \tilde{\mathbf{V}} | \Phi_I \right] - (p_0 + \Lambda D_I)^T D_I - \frac{1}{2 r_I} D_I^T \mathbf{Cov}_I D_I. \tag{2}
\]

Solving for $D_I$ yields

\[
D_I = \left( \frac{1}{r_I} \mathbf{Cov}_I + \Lambda^T \right)^{-1} \left( E \left[ \tilde{\mathbf{V}} | \Phi_I \right] - (p_0 + \Lambda D_I) \right) = \left( \frac{1}{r_I} \mathbf{Cov}_I + \Lambda^T \right)^{-1} \left( E \left[ \tilde{\mathbf{V}} | \Phi_I \right] - \mathbf{P} \right), \tag{3}
\]

\textsuperscript{10}We acknowledge, but nonetheless ignore, sources of illiquidity other than imperfect competition and asymmetric information. For example, we ignore inventory holding costs by specialists or an explicit market micro-structure setting where a specialist sets bid and ask prices. In models of the latter (e.g., Glosten and Milgrom, 1985), the specialist quotes prices for a transaction involving a single unit of the security, and then increases (decreases) it for the next transaction following a buy (sell). Therefore, a trader who buys multiple shares will face a total price that is increasing in his total demand, similar to our model.
where the last equality follows from the relation and $P = p_0 + \Lambda D_I$. Here, note that if every $j$-th row, $k$-th column element of $\Lambda$ is 0, eqn. (3) reduces to the standard expression for demand for firms’ shares in perfect competition settings.\(^{11}\) If every $j$-th row, $k$-th column element of $\Lambda$ is not 0, then higher (non-negative) row/column elements within $\Lambda$ lower an investor’s demand for shares \((\text{ceteris paribus})\). Intuitively, this is because the more an investor demands, the higher the price he pays, not just for the next share, but for all the shares he demands of that firm. This is easiest to see for the special case where the shares of only one firm is priced, or, equivalently, if the cash flows of all firms are uncorrelated and the $\Lambda$ matrix is diagonal: in this circumstance eqn. (3) reduces to $D_{Ij} = \left(\frac{1}{r_I} \text{Cov}_I [\tilde{V}_j, \tilde{V}_j] + \Lambda_j \right)^{-1} \left(\mathbb{E} [\tilde{V}_j | \Phi_I] - P_j \right)$, where $D_{Ij}$ is an investor’s demand for the shares of firm $j$, $\text{Cov}_I [\tilde{V}_j, \tilde{V}_j]$ is the variance of the cash flow of firm $j$, and $\Lambda_j$ is the $j$-th diagonal element of $\Lambda$. In this case, the investor’s demand is: 1) an increasing function of the extent to which he assesses the expected cash flow of the $j$-th firm to be higher than the price for that firm; 2) an increasing function of his risk tolerance; and 3) a decreasing function of his assessed variance of the $j$-th firm’s cash flow and his beliefs as to the impact of his demand on share price.

**Market clearing.** Having determined an investor’s demand for firms’ shares, our next step is to provide an expression for the price of those shares. Market clearing requires that the total demand for firms’ shares equals the supply of those shares. Let $\tilde{Z}$ represent the (random) supply vector of shares in the $J$ firms. We assume that $\tilde{Z}$ has a multi-variate normal distribution that is independent of the vector of cash flows, $\tilde{V}$, where $E [\tilde{Z}]$ represents $\tilde{Z}$’s $J \times 1$ vector of mean (or expected) firm shares, and $\text{Cov}_z$ represents $\tilde{Z}$’s $J \times J$ covariance matrix of firm shares. The realization of

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\(^{11}\)See, for example, the discussion in Sections 2.2 and 2.3 of Verrecchia (2001).
the \( j \)-th element of the vector \( \tilde{Z} = Z \) represents the number of shares firm \( j \) brings to the market at the beginning-of-the-period that investors compete to acquire. An alternative interpretation of \( \tilde{Z} \) is that it represents trades by market participants who do not act strategically (e.g., pure liquidity traders): our results do not depend on a specific interpretation of \( \tilde{Z} \). Expressed in equation form, market clearing requires that

\[
N \cdot D_I (\tilde{P}) - \tilde{Z} = 0.
\]

Substituting for \( D_I \) from eqn. (3) yields the following result:

\[
\tilde{P} = E \left[ \tilde{V} | \Phi_I \right] - \left( \frac{1}{r_I} \text{Cov}_I + \Lambda^T \right) \frac{\tilde{Z}}{N}.
\] (4)

Henceforth we describe the vector of firms’ shares, \( \tilde{P} \), as a random variable because it depends of the supply of those shares, \( \tilde{Z} \), which we assume to be random.\(^{12}\)

**Cost of capital.** Next, we employ the price vector in eqn. (4) to determine the appropriate expression for cost of capital. A standard definition of a firm’s cost of capital is the extent to which investors discount price at the beginning of the period (i.e., in expectation) relative to the expected value of the firm’s cash flow.\(^{13}\) Noting that the law of iterated expectations yields \( E \left[ E \left[ \tilde{V} | \Phi_I \right] \right] = E \left[ \tilde{V} \right] \), eqn. (4) implies the following expression for firms’ cost of capital.

**Proposition 1.** Cost of capital in an imperfect competition setting where \( N \) identical investors compete to hold firms’ shares is

\[
E \left[ \tilde{V} \right] - E \left[ \tilde{P} \right] = \left( \text{Cov}_I + r_I \Lambda^T \right) \cdot E \left[ \frac{\tilde{Z}}{N r_I} \right].
\]

\(^{12}\)In conventional models of asset pricing that are based on the assumption that investors are identically informed, such as the CAPM, there is no requirement that firms’ shares be random. Similarly, in this section where investors are identically informed, there is no requirement that firms’ shares be random. In Section 3, however, we introduce uninformed investors into the economy. There, as is standard in noisy rational expectations models of trade, we require that \( \tilde{Z} \) be random so as to thwart the possibility of prices fully revealing to uninformed investors the private information of informed investors.

\(^{13}\)See, for example, Diamond and Verrecchia (1991).
The salient feature of Proposition 1 is that it establishes that cost of capital in an imperfect competition setting has two components: a component that relies on the covariance of cash flows, $\text{Cov}_I$, and a component that relies on illiquidity, $\Lambda$. But as we show below, the illiquidity component is exactly proportional to the covariance of cash flows. Thus, as a practical matter when investors are identically informed cost of capital collapses into one factor (or one factor and the multiplier on that factor).

**An investor’s strategy.** To demonstrate that cost of capital collapses into one factor, one must solve for $\Lambda$. The solution to $\Lambda$ results from an investor’s strategy for buying firms’ shares. Specifically, in competing with other investors, we assume that each investor adopts a strategy

$$D_I = \alpha - \Gamma \cdot P,$$

where $\alpha$ is a $J \times 1$ intercept vector and $\Gamma$ is a $J \times J$ matrix of weights an investor places on $P$. First, note that for an investor’s strategy to be rational based on the computation of $D_I$ in eqn. (3), it must be the case that $\Gamma = \left(\frac{1}{r_I} \text{Cov}_I + \Lambda^T\right)^{-1}$ and $\alpha = \left(\frac{1}{r_I} \text{Cov}_I + \Lambda^T\right)^{-1} E[\tilde{V}|\Phi_I]$. Next, recall that market clearing requires $ND_I (\tilde{P}) - \tilde{Z} = 0$. Thus, substituting $D_I = \alpha - \Gamma \cdot P$ into the market-clearing expression and solving for the $\tilde{P}$ that clears the market yields

$$\tilde{P} = \Delta \left(N\alpha - \tilde{Z}\right),$$

where $\Delta$ is given by

$$\Delta = \frac{1}{N} \Gamma^{-1} = \frac{1}{N} \left(\frac{1}{r_I} \text{Cov}_I + \Lambda^T\right).$$

The matrix $\Delta$ represents the marginal impact on the price vector of an additional share brought to (or withdrawn from) the market by each firm. To reconcile the
market clearing condition $\tilde{P} = \Delta \left( N\alpha - \tilde{Z} \right)$ with the strategy that $D_I = \alpha - \Gamma \cdot P$, it must be the case that $\tilde{p}_0$ in the expression $\tilde{P} = \tilde{p}_0 + \Lambda D_I$ is of the form

$$\tilde{p}_0 = \Lambda \left( (N - 1)\alpha - \tilde{Z} \right),$$

where $\Lambda = \frac{1}{N-1} \Gamma^{-1} = \frac{1}{N-1} \left( \frac{1}{r_I} \text{Cov}_I + \Lambda^T \right)$. Thus, using the fact that the matrix $\text{Cov}_I$ is symmetric, the solution to $\Lambda^T$ must be $\Lambda^T = \frac{1}{N-2} \frac{1}{r_I} \text{Cov}_I$. Among other things, this implies that the off-diagonal elements of $\Lambda$ can be non-zero because they are proportional to the covariance of firms’ cash flows. For example, if the cash flows for firms $j$ and $k$ are positively correlated, then investors believe (and rightly so) that increasing their demand for shares of firm $j$ will increase the prices of both firms $j$ and $k$.

The solution to $\Lambda$ also implies that eqn. (4) can be re-expressed as

$$\tilde{P} = E[ \tilde{V} ] - \left( \frac{1}{r_I} \text{Cov}_I + \Lambda^T \right) \frac{\tilde{Z}}{N} = E[ \tilde{V} ] - \frac{N-1}{N-2} \text{Cov}_I \frac{\tilde{Z}}{N r_I}.$$

Summary. To summarize our results, we have solved for the endogenous variables $P$, $p_0$, $\Lambda$, and $D_I$ when investors are identically informed. In addition, the fact that $\Lambda^T = \frac{1}{N-2} \frac{1}{r_I} \text{Cov}_I$ implies that cost of capital can be expressed as follows.

**Proposition 2.** Cost of capital in an imperfect competition setting where $N$ identically informed investors compete to hold firms’ shares ultimately reduces to

$$E[ \tilde{V} ] - E[ \tilde{P} ] = \frac{N-1}{N-2} \text{Cov}_I \cdot E \left[ \tilde{Z} \frac{\tilde{Z}}{N r_I} \right].$$

The salient feature of Proposition 2 is that cost of capital reduces to an expression that is proportional to $\text{Cov}_I$. As a result, cost of capital in an imperfect competition setting collapses to the same factor that is priced in perfect competition settings, namely the covariance of a firm’s cash flow with the cash flows of other firms in the market. The only impact imperfect competition (or illiquidity) has on pricing is
that it affects the multiplier on the firm’s covariance. That is, imperfect competition increases the price of risk in the market as a whole (i.e., $\frac{N-1}{N-2}$ versus 1.0). Note that as investors become more numerous (i.e., as $N$ increases), liquidity’s role in the determination of cost of capital declines. This is intuitive, because as investors become more numerous, *ceteris paribus*, the market approaches a perfect competition setting.

Based on the expression for cost of capital in Proposition 2, we argue that there is no empirically distinguishable risk factor in an imperfect competition setting when investors are identically informed. That is, in settings with identically informed investors both perfection competition and imperfect competition result in the cost of capital being proportionate to the firms’ covariance with the market. While Proposition 2 does not preclude information having an effect on a firm’s cost of capital, it does imply that this effect must come through the covariance matrix $\text{Cov}_I$. In other words, information has no separate impact on cost of capital once one controls for the covariance matrix.

3 Asymmetrically informed investors

In this section, we incorporate information asymmetry into the analysis. As in Section 2, we consider a one-period economy with $J$ firms and a risk-free asset whose price is normalized to 1. Let $\tilde{V}$ denote the $J \times 1$ vector of end-of-period firms’ cash flows, where the $j$-th element of the vector, $\tilde{V}_j$, is the cash flow of the $j$-th firm. Let $E[\tilde{V}]$ denote the *a priori* vector of expected values of firms’ cash flows, $\text{Cov}_v$ and $\Pi_v$ the *a priori* covariance and precision matrices for firms’ cash flows, respectively. Also as in Section 2, let $\tilde{P}$ denote the $J \times 1$ vector of beginning-of-period firms’ prices, or
market values, where \( \tilde{P}_j \) is the price of the \( j \)-th firm.\(^{14}\) [Insert Table 2 here.]

We incorporate information asymmetry into the analysis by introducing \( M \) uninformed investors to go along with the \( N \) informed investors whom we introduced earlier.\(^ {15}\) To distinguish between the two types of investors, henceforth we subscript parameters and activities associated with informed and uninformed investors by \( I \) and \( U \), respectively. We assume that each investor has a negative exponential utility function with constant absolute risk tolerance \( r_t \), where \( t \) distinguishes an investor’s type, i.e., \( t \in \{ I, U \} \). Let \( \Phi_t, t \in \{ I, U \} \), represent the information available to an investor of type \( t \). Along with prices, each informed investor observes the same \( J \times 1 \) vector of private information \( \tilde{X} = \tilde{V} + \tilde{\epsilon} \), where \( \tilde{\epsilon} \) is a \( J \times 1 \) vector of “error terms” whose expected value is \( 0 \) and whose precision matrix is \( \Pi_\epsilon \). In short, \( \Phi_I = \{ \tilde{P} = P, \tilde{X} = X \} \). Henceforth let \( \Pi_t \) denote the \( J \times J \) posterior precision matrix an investor of type \( t \in \{ I, U \} \) associates with cash flows. When an informed investor observes \( \tilde{X} = \tilde{V} + \tilde{\epsilon} \), the precision of his beliefs about firms’ cash flows is \( \Pi_I = \Pi_v + \Pi_\epsilon \). Moreover, a straightforward application of Bayes’ Theorem implies that the expected value an informed investor assigns cash flows based on his information is \( E [ \tilde{V} | \Phi_I ] = \Pi_I^{-1} [ E [ \tilde{V} ] + \Pi_\epsilon (X - E [ \tilde{V} ])] \).

**An informed investor’s demand.** As in Section 2, an informed investor believes that his demand affects the price vector. Thus, as before we characterize an informed investor’s demand as

\[
D_I = \left( \frac{1}{r_I} \text{Cov}_I + \Lambda^T \right)^{-1} \left( E [ \tilde{V} | \Phi_I ] - P \right).
\]

\(^ {14} \)We summarize the notation we use in this section in Table 2.\(^ {15} \)We discuss assumptions about \( N \) and \( M \) as the analysis unfolds.
example, let $\beta$ denote the sensitivity of the informed investor’s demand as a function of his information; here, $\beta$ is defined as

$$
\beta = \frac{\partial}{\partial X} D_I = \left( \frac{1}{r_I} \text{Cov}_I + \Lambda^T \right)^{-1} \Pi_I^{-1} \Pi_\epsilon. 
$$

(6)

An uninformed investor’s demand. Uninformed investors have no private information. Nonetheless, as is standard in any rational expectations setting, uninformed investors glean some of the informed investors’ private information about firms’ cash flows by conditioning their expectations on price.\(^{16}\) With this in mind, let $\Phi_U = \{ \hat{P} = P \}$ represent the information available to uninformed investors. We denote uninformed investors’ beliefs about the covariance of cash flows as $\text{Cov}_U$, and the precision of their beliefs by $\Pi_U$. We express the precision of their posteriors as $\Pi_U = \Pi_v + \Pi_\delta$, where $\Pi_\delta$ is the precision of the information that uninformed investors glean from price. The precision of the information that uninformed investors glean from price is endogenous; we discuss it in more detail below.\(^{17}\)

So as to make the analysis as facile as possible, we assume that the uninformed investors are price takers.\(^{18}\) Specifically, we operationalize this by assuming that the number of uninformed investors is very large (i.e., $M$ is countably infinite), which means that each uninformed investor is “small” or atomistic in relation to the number of uninformed investors in total. Thus, an uninformed investor believes that his demand has no effect on prices, and in equilibrium this belief is sustained. Based on his belief that his demand has no effect on prices, an uninformed investor takes the

\(^{16}\)See, for example, Grossman and Stiglitz (1980), Hellwig (1980), and Diamond and Verrecchia (1981).

\(^{17}\)In the Appendix we are specific as to how uninformed investors rely on price to infer informed investors’ private information about cash flows.

\(^{18}\)Uninformed investors need not be price takers: competition among uninformed investors could also be imperfect. An analysis along these lines is considerably more cumbersome, however, because here each investor-type faces a different illiquidity matrix (i.e., $\Lambda_I$ versus $\Lambda_U$). All of the qualitative insights discussed in the paper carry over to this setting.
price vector as a given and chooses $D_U$ to maximize the following objective function

$$E \left[ \tilde{V} | \Phi_U \right] - P]^{T} D_U - \frac{1}{2r_U} D_U^T \text{Cov}_U D_U,$$

(7)

where $\text{Cov}_U$ represents an uninformed investor’s beliefs about the covariance of firms’ cash flows. Solving for $D_U$ yields

$$D_U = r_U \text{Cov}_U^{-1} \left( E \left[ \tilde{V} | \Phi_U \right] - P \right) = r_U \Pi_U \left( E \left[ \tilde{V} | \Phi_U \right] - P \right).$$

(8)

**Market clearing.** Market clearing requires that the total demand for firms’ shares equals the supply of those shares. Recall that $\tilde{Z}$ represents the (random) supply vector of shares in the $J$ firms. In our asymmetric information economy, market clearing requires that

$$N \cdot D_I (\tilde{P}, \tilde{X}) + M \cdot D_U (\tilde{P}) - \tilde{Z} = 0.$$

So as to ensure that the large number of uninformed investors (i.e., $M$ is countably infinite) does not “swamp” or overwhelm the finite number of informed investors in the market-clearing condition, we assume that an uninformed investor’s tolerance for risk is sufficiently small such that in the limit as $M$ gets large $M \cdot r_U$ converges to an arbitrary (positive) constant, $\rho$: that is,

$$\lim_{M \to \infty} M \cdot r_U \to \rho.$$

Note that $\rho = Mr_U$ represents the aggregate risk tolerance of the uninformed investors *in toto*. One advantage of our approach in dealing with informed and uninformed investors is that the size of the parameter $\rho$ captures the significance of each type. For example, $\rho$ small is tantamount to an economy where competition is primarily imperfect, whereas $\rho$ large is tantamount to an economy where competition is approximately perfect. We regard the appropriate value of $\rho$ for any specific economy as largely an empirical issue.
An equilibrium solution. To characterize the equilibrium to our asymmetric information setting, we must solve for the two endogenous variables: the element of illiquidity that informed investors face, \( \Lambda \), and precision of the information uninformed investors glean from price, \( \Pi_\delta \). Using techniques similar to those in Section 2, we show that these two variables must satisfy the following set of equations: see the Appendix for proofs to all results in this section.

**Proposition 3.** In a setting where information is asymmetric and competition is imperfect, informed investors’ illiquidity matrix, \( \Lambda \), and precision matrix of information conveyed by price to uninformed investors, \( \Pi_\delta \), will satisfy the following two equations:

\[
\rho \Lambda \Pi_U \Lambda + \Lambda \left( (N-2)I + \frac{\rho}{r_I} \Pi_U \text{Cov}_I \right) - \frac{1}{r_I} \text{Cov}_I = 0 \quad \text{and} \quad (9)
\]

\[
\Pi_\delta - \left( \Pi_\delta^{-1} + (N\beta)^{-1} \text{Cov}_z \left( (N\beta)^{-1} \right)^T \right)^{-1} = 0, \quad (10)
\]

where \( I \) is the \( J \times J \) identity matrix.

These two equations, and the two unknowns \( \Lambda \) and \( \Pi_\delta \), characterize the equilibrium. The first equation is a quadratic in \( \Lambda \), the informed investor’s illiquidity matrix. But this equation also depends on the endogenous variable \( \Pi_\delta \), which is a component of \( \Pi_U \). Similarly, the second equation specifies the precision of the information uninformed investors glean from price; here, the endogenous variable \( \Lambda \) is part of the definition of the term \( \beta \) on the left-hand-side. Recall that \( \beta \) represents the sensitivity of an informed investor’s demand to his private information. In principle, one could solve the second equation to determine an expression for \( \Pi_\delta \) as a function of \( \Lambda \), and this could be substituted into the first equation to solve for \( \Lambda \). Because of the non-linearity of the equations, however, one cannot obtain a closed-form solution for \( \Lambda \) and \( \Pi_\delta \). Nevertheless, we can use these equations to develop insights into the
properties of the equilibrium.

In particular, Proposition 3 provides conditions where the market approaches a perfect competition setting, in which case \( \Lambda \) approaches 0; it also provides conditions where \( \Lambda \) is non-zero.

**Corollary 1.** Informed investors’ illiquidity matrix, \( \Lambda \), approaches 0 as either \( N \) or \( \rho \) approaches infinity; otherwise \( \Lambda \neq 0 \).

Corollary 1 establishes that when the number of informed investors in the market becomes large, each informed investor becomes sufficiently atomistic so as to behave as a price taker. This is analogous to the results in Section 2 when \( N \) gets large. Alternatively, if the aggregate risk tolerance of uninformed investors becomes large (i.e., \( \rho = Mr_U \) becomes large), uninformed investors dominate the market. As a result, they provide sufficient liquidity to absorb the trading activities of informed investors, and thus informed investors behave as price-takers. Absent these two conditions, the market is less than perfectly liquid. We discuss the determinants of illiquidity later.

**Cost of capital.** Next we turn to the cost of capital. We use the investors’ demands as characterized by eqns. (5) and (8), along with market clearing conditions to derive the cost of capital.

**Proposition 4.** Cost of capital in an imperfect competition setting with asymmetrically informed investors reduces to

\[
E[\tilde{V}] - E[\tilde{P}] = \left( \frac{(I + r_I \Pi_I \Lambda^T)^{-1} Nr_I \Pi_I + Mr_U \Pi_U}{Nr_I + Mr_U} \right)^{-1} E\left[ \frac{\tilde{Z}}{Nr_I + Mr_U} \right].
\]

Proposition 4 demonstrates that cost of capital can be expressed as function of the precision matrix of each type of investor, their aggregate risk tolerances, and the illiquidity matrix that informed investors face. Note that as \( Mr_U \) approaches 0, Proposition 4 is identical to Proposition 2, the characterization of cost of capital.
when investors are identically informed. Alternatively, note that if $\Lambda = 0$ (i.e., perfect liquidity), cost of capital reduces to

$$E \left[ \tilde{V} \right] - E \left[ \tilde{P} \right] = \left( \frac{N_{rI} \Pi_I + M_{rU} \Pi_U}{N_{rI} + M_{rU}} \right)^{-1} E \left[ \tilde{Z} \right] \left( \frac{1}{N_{rI} + M_{rU}} \right);$$

here, cost of capital depends solely on the inverse of the average precision matrix of information across investors, where the average precision is defined as

$$\Pi_{avg} = \frac{N_{rI} \Pi_I + M_{rU} \Pi_U}{N_{rI} + M_{rU}}.$$

In other words, $\Pi_{avg}$ represents the precision matrix of each investor type weighted by the aggregate risk tolerance of that type, i.e., $N_{rI}$ and $M_{rU}$, respectively. This implies that under perfect competition, the degree to which each investor-type’s precision differs from the average has no effect on the cost of capital once one controls for the average. Stated somewhat differently, under perfect competition the degree of information asymmetry across the two investor types has no additional explanatory power in explaining the behavior of cost of capital once one controls for the average (as in Lambert et al., 2009).

In contrast, when the illiquidity matrix $\Lambda$ is not 0, Proposition 4 indicates that illiquidity can affect cost of capital in two ways. First, illiquidity affects the average precision of investors in the economy. In particular, the greater the degree of illiquidity faced by informed investors, the less aggressively they trade on the basis of their private information, and the less uninformed investors learn by conditioning their expectations on prices. Therefore, ceteris paribus, greater market illiquidity lowers the precision of information held by the less informed type.\(^{19}\) This lowers investors’ average precision of information, and thus raises the cost of capital. Moreover, this

\(^{19}\)Although beyond the scope of our analysis, if the precision of the information available to informed investors was a choice variable, it is likely they would acquire less information as the market became more illiquid. This would further reduce the average precision of information across investors.
effect occurs even if the illiquidity matrix is exactly proportionate to the informed investor’s covariance matrix. Hence, market illiquidity has a more substantive impact on prices than in Section 2.

Second, when $\Lambda \neq 0$, the “weighting” scheme applied to investors in the cost of capital equation is more complex; it is not the simple weighted average of the precisions of the two investor types. While the weight assigned the precision of the uninformed type continues to be the aggregate risk tolerance of that type, $Mr_U$, illiquidity reduces the weight assigned to the precision of the informed type. Specifically, the weight assigned to the precision of informed investors is now \( (I + r_I \Pi_I \Lambda^T)^{-1} Nr_I \); one can think of the term \( (I + r_I \Pi_I \Lambda^T)^{-1} \) as a “discount” factor arising from market illiquidity. This reduction arises because informed investors have to curb the aggressiveness of their demands because of market illiquidity. Thus, informed investors’ private information does not get reflected in price as fully as the precision of their information (and their aggregate risk tolerance) would suggest. Because the informed type has greater precision, and this precision is discounted because of market illiquidity, this also suggests that market illiquidity leads to higher cost of capital, ceteris paribus. In this circumstance, the discount reflects the element of adverse selection in the economy. In other words, adverse selection results from the interaction between imperfect competition and asymmetric information.

In contrast to a setting where investors are identically informed (see Section 2), it is less likely that comparative static results on the cost of capital can be unambiguously signed in an asymmetric information, imperfect competition setting. This is because here there are now two endogenous factors that affect cost of capital. A change in

\[20\] Because \( (I + r_I \Pi_I \Lambda^T)^{-1} \) is a matrix, the term “discount” factor in not strictly appropriate. When restricted to a one-firm economy, however, this collapses to a scalar that is strictly less than 1.0. This is consistent with the intuition that the weight informed investors receive in the formation of price is reduced.
an exogenous variable that increases both investors’ average precision and market liquidity (decreases illiquidity) is likely to have an unambiguous effect. A change in an exogenous variable that increases investors’ average precision but decreases liquidity, however, is likely to have countervailing effects on cost of capital; more structure would be necessary to sort out the sign of the net effect.

4 Information asymmetry and illiquidity

Recall that in a market characterized by perfect competition, once one controls for average precision information asymmetry has no additional explanatory power in explaining the behavior of cost of capital. That said, observe that the precisions of the two investor-types are not equally weighted in Proposition 4. This suggests that the degree of information asymmetry in the imperfect competition economy can affect the cost of capital through its effect on market illiquidity. Intuitively, an increase in how far each type’s precision deviates from the average precision will lead to a lower liquidity-adjusted weighted average, and a higher cost of capital. The occurs because the precision of the more informed type gets less weight in determining cost of capital. To properly analyze this conjecture, it is important that any comparative statics analysis hold constant investors’ average precision of information. This is not a trivial exercise, because the average precision and degree of information asymmetry are both endogenous variables, and thus changes in parameters will affect them simultaneously. For example, increasing the precision of information for the informed type increases both average precision and information asymmetry. On the other hand, increasing the precision of the information of the uninformed type will increase average precision but decrease information asymmetry.

To make the distinction between average precision and information asymmetry
more transparent, we recast the equation for cost of capital as follows:

\[
E[\tilde{V}] - E[\tilde{P}] = \left( \Pi_{\text{avg}} - \frac{r_I \Pi_I \Lambda^T (I + r_I \Pi_I \Lambda^T)^{-1} N r_I \Pi_I}{N r_I + M r_U} \right)^{-1} E \left[ \frac{\tilde{Z}}{N r_I + M r_U} \right].
\]

In this specification, cost of capital can be expressed as a function of the average precision of investors’ information, \( \Pi_{\text{avg}} \), minus a term that reflects the discount that is attributable to the interaction between information asymmetry and illiquidity. In particular, holding the average precision constant, an increase in the precision of the information of informed investors, \( \Pi_I \), is a proxy for the degree of information asymmetry in the economy. That is, a higher value of \( \Pi_I \) has to be offset by a lower value of \( \Pi_U \) to preserve the average precision in the economy. This proxy for information asymmetry is then multiplied by a “fractional” amount, \( r_I \Pi_I \Lambda^T (I + r_I \Pi_I \Lambda^T)^{-1} \), that depends on both the degree of illiquidity and the precision of informed investors’ information. When there is no illiquidity, i.e., \( \Lambda = 0 \), both the fractional term and the reduction relative to the average precision are zero. Examining the equation above, any combination of parameter changes that holds the average precision constant and weakly increases the degree of information asymmetry (as proxied by the precision matrix \( \Pi_I \)) and the illiquidity matrix \( \Lambda \), while strictly increasing at least one, is sufficient to increase the cost of capital. That is, holding the illiquidity matrix \( \Lambda \) “constant,” a higher precision for informed investors (e.g., more information asymmetry) will increase this discount and increase cost of capital. Similarly, holding the precision of the informed investors’ information constant, an increase in the degree of illiquidity will also increase this discount and increase the cost of capital. Of course, these are sufficient conditions, not necessary ones.

**Additional structure.** We would like to illustrate our observations above numerically. A problem, however, arises from the fact that holding the average precision
constant while changing the degree of information asymmetry requires changing at least two exogenous variables simultaneously (while also calibrating the magnitude of these changes), and this is computationally challenging. Thus, we impose more structure on the model. Specifically, henceforth we assume that the precision in the error terms in investors information about those cash flows, $\Pi_\varepsilon$, and the covariance of the supply of firms’ shares, $\text{Cov}_z$, are proportionate to the a priori covariance of cash flows, $\text{Cov}_v$, through the relations $\Pi_\varepsilon = \sigma_\varepsilon^{-2} \text{Cov}_v^{-1}$, and $\text{Cov}_z = \sigma_z^2 \text{Cov}_v^{-1}$, where $\sigma_\varepsilon^2$, and $\sigma_z^2$ are arbitrary, positive parameters. Note that this imposed structure is without loss of generality in a single-firm economy because in a single-firm economy matrices become scalars, and scalars are always proportionate to other scalars.

With this structure and some tedious calculations, one can show that eqn. (9) in Proposition 4 reduces to the requirement that $\Lambda = \lambda \text{Cov}_v$, where $\lambda$ solves the following 4th-order polynomial

$$\sigma_\varepsilon^2 r_I^3 \rho \left( 1 + \sigma_\varepsilon^2 \right)^3 \lambda^4 + r_I^2 \sigma_z^2 \left( 1 + \sigma_\varepsilon^2 \right)^2 \left( r_I \left( 1 + \sigma_\varepsilon^2 \right) (N - 2) + 3 \rho \sigma_\varepsilon^2 \right) \lambda^3$$

$$+ r_I \left( 1 + \sigma_\varepsilon^2 \right) \left( \left( 3 \sigma_z^2 \sigma_\varepsilon^4 + N^2 r_I^2 \left( 1 + \sigma_\varepsilon^2 \right) \rho + r_I \sigma_z^2 \sigma_\varepsilon^2 \left( 1 + \sigma_\varepsilon^2 \right) \left( 2N - 5 \right) \right) \right) \lambda^2$$

$$+ \sigma_\varepsilon^2 \left( \left( N^2 r_I^2 \left( 1 + \sigma_\varepsilon^2 \right) + \sigma_z^2 \sigma_\varepsilon^4 \right) \rho + r_I \left( 1 + \sigma_\varepsilon^2 \right) \left( \sigma_z^2 \sigma_\varepsilon^2 \left( N - 4 \right) + N^2 r_I^2 \left( N - 2 \right) \right) \right) \lambda$$

$$- \sigma_\varepsilon^4 \left( \sigma_z^2 \sigma_\varepsilon^2 + N^2 r_I^2 \right) = 0. \quad (11)$$

All the exogenous variables in eqn. (11) are positive, so casual inspection suggests that a sufficient condition for $\lambda$ to have a unique solution is that there are at least four informed investors in the economy: that is, $N \geq 4$. Having solved for $\lambda$, eqn. (10) in Proposition 4 reduces to the following requirement for $\Pi_\delta$:

$$\Pi_\delta = \frac{N^2 r_I^2}{r_I^2 \sigma_\varepsilon^2 \left( 1 + \sigma_\varepsilon^2 \right)^2 \lambda^2 + 2 r_I \sigma_z^2 \sigma_\varepsilon^2 \left( 1 + \sigma_\varepsilon^2 \right) \lambda + \sigma_z^2 \left( N^2 r_I^2 + \sigma_\varepsilon^2 \sigma_z^2 \right)} \text{Cov}_v^{-1}.$$
Moreover, cost of capital can now be written as.

\[
E[\tilde{V}] - E[\tilde{P}] = \left( \Pi_{avg} - \frac{\lambda r_I(1 + \sigma^{-2}_\varepsilon)}{1 + \lambda r_I(1 + \sigma^{-2}_\varepsilon)} \frac{N r_I(1 + \sigma^{-2}_\varepsilon) \Pi_v}{N r_I + M r_U} \right)^{-1} E \left[ \frac{\tilde{Z}}{N r_I + M r_U} \right].
\]

**Illustrating the effect of information asymmetry.** In order to increase the degree of information asymmetry while holding the average precision constant, we simultaneously change the precision of informed investors’ private information (through the parameter \(\sigma^{-2}_\varepsilon\)) and the variance of the liquidity shock (through the parameter \(\sigma^2_z\)). Increasing the precision of private information increases the precision of information available to informed investors. The increase in variance of the liquidity shock injects more noise into the information conveyed by price; this, in turn, ensures that the precision of the uninformed investors’ information is reduced. As a result, the degree of information asymmetry between the two types is increased. We calibrate these two changes so as to hold the average precision of the two types constant. We examine the effect of these changes on the illiquidity factor \(\lambda\) and on the cost of capital.

Consider an economy with two firms, \(J = 2\), and four informed investors, \(N = 4\), who have a risk tolerance of 1 (i.e., \(r_I = 1\)). Let the aggregate risk tolerance of the uninformed investors converge to 1 as their number becomes large (i.e., \(\lim_{M \to \infty} M r_U = \rho \to 1\)). We assume that the *a priori* the covariance of firms’ cash flows is \(\text{Cov}_v = \begin{pmatrix} 1 & .1 \\ .1 & 1 \end{pmatrix}\). We begin with the assumptions that \(\sigma^2_\varepsilon = 1\) and \(\sigma^2_z = 1\). These assumptions imply that the precision of the information available to informed investors is \(\Pi_I = \Pi_v + \Pi_\varepsilon = 2 \cdot \text{Cov}^{-1}_v\) and in equilibrium \(\lambda = 0.15409\), and thus the precision of the information available to uninformed investors is \(\Pi_U = \Pi_v + \Pi_\delta = 1.9034 \cdot \text{Cov}^{-1}_v\) and \(\Lambda = 0.15409 \cdot \text{Cov}_v\). Here, investors’ average precision computes
to \( \Pi_{avg} = 1.9807 \cdot \text{Cov}^{-1}_v \). In addition, firms’ cost of capital computes to

\[
0.12471 \cdot \text{Cov}_v \cdot E[\tilde{Z}] = \begin{pmatrix}
0.12471 & 1.2471 \times 10^{-2} \\
1.2471 \times 10^{-2} & 0.12471
\end{pmatrix} \cdot E[\tilde{Z}].
\]

Now consider a circumstance where an informed investor acquires more private information, but the average precision of information remains at the same level. For example, suppose \( \sigma^2_\epsilon \) falls from \( \sigma^2_\epsilon = 1 \) to \( \sigma^2_\epsilon = 0.9 \), which implies that the precision of an informed investor’s private information increases: specifically, \( \Pi_I = 2.1111 \cdot \text{Cov}^{-1}_v \). In order for average precision of information to remain at the same level, we now require the variance of the liquidity shock to increase such that \( \sigma^2_z \) rises to \( \sigma^2_z = 14.034 \). This causes \( \lambda \) to increase to 0.16183 and the precision of the information an uninformed investor gleans from the price vector drops to \( \Pi_U = 1.459 \cdot \text{Cov}^{-1}_v \). Here, investors’ average precision remains at the same level: \( \Pi_{avg} = 1.9807 \cdot \text{Cov}^{-1}_v \).

Despite the fact that the average precision does not change, the increase in information asymmetry between the two investor types manifests in greater illiquidity: specifically, \( \lambda \) increases from 0.15409 to 0.16183. This, in turn, results in higher cost of capital: here, firms’ cost of capital computes to

\[
\begin{pmatrix}
0.12898 & 1.2898 \times 10^{-2} \\
1.2898 \times 10^{-2} & 0.12898
\end{pmatrix} \cdot E[\tilde{Z}].
\]

This demonstrates that in our imperfect competition setting cost of capital increases as information asymmetry increases, despite the fact that average precision remains unchanged. Thus, we find a role for information asymmetry in cost of capital through its effect on market illiquidity - this role does not exist in perfect competition settings.
5 Conclusion

This paper examines the role of information in a multi-asset economy populated by risk averse investors who have rational expectations but different information. The novel feature of our analysis is that we consider imperfect competition settings; that is, settings where investors understand that the magnitude of their demand for firm shares can affect the price at which their demand is fulfilled. In an imperfect competition setting, markets are not perfectly liquid and thus there are costs to trading. The cost is endogenously determined as part of the equilibrium, and it occurs through an increase in price that must be paid when investors wish to buy more shares. Similarly, investors understand that attempts to sell more shares will lower the price they receive. As Kyle (1989) discusses, imperfect competition resolves the “schizophrenia” manifest in perfect competition settings, where no single investor believes his demand influences price but yet in equilibrium prices do reflect aggregate demand.

Our primary result is to demonstrate the importance of the interaction between imperfect competition and asymmetric information. With homogeneous information and perfect competition, the CAPM applies. In this circumstance, the effect of information on cost of capital must occur through the assessment of the covariance between a firm’s cash flow and the market cash flow. There are no separate “information risk” factors that affect cost of capital. Incorporating imperfect competition into a model with symmetric information adds a new factor to a firm’s cost of capital. In particular, prices are lower (and cost of capital is higher) because investors reduce their demand for shares for fear of driving up the price. We show that in equilibrium, however, that the illiquidity discount is exactly proportional to the covariance matrix
of cash flows. Hence, price is proportional to the covariance matrix of cash flows and thus there is no empirically identifiable, separate, information-risk factor that influences prices.

When investors are asymmetrically informed, market prices depend on how diverse information gets aggregated. When markets are perfectly competitive, investors’ demands are proportional to their assessed precision matrix of cash flows. Thus, once again cost of capital is proportional to a covariance with the market, this time based on the aggregate of their precision matrices. With regard to pricing, only the aggregated average precision matrix is relevant; the deviation of investors’ precisions from this average precision is irrelevant. In particular, neither information asymmetry nor any other information-risk factor affects cost of capital once one controls for the aggregate precision matrix (or associated covariance matrix). When markets are imperfectly competitive, however, one cannot reduce the illiquidity discount to an expression that is proportional to the covariance of cash flows. The degree of market illiquidity influences the amount of information that is reflected in prices; this, in turn, reduces the average precision of investors’ information and thus raises the cost of capital. Moreover, the degree of information asymmetry in the economy influences the amount of market illiquidity, which also raises the cost of capital. Therefore, even after controlling for average precision, the interaction between imperfect competition and asymmetric information can have an impact on cost of capital.
<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$N$</td>
<td>number of investors</td>
</tr>
<tr>
<td>$r_I$</td>
<td>an investor’s risk tolerance</td>
</tr>
<tr>
<td>$\tilde{V}$</td>
<td>vector of firm cash flows</td>
</tr>
<tr>
<td>$\tilde{P}$</td>
<td>vector of firm share prices</td>
</tr>
<tr>
<td>$\tilde{Z}$</td>
<td>vector of supply of firms’ shares</td>
</tr>
<tr>
<td>$\text{Cov}_I$</td>
<td>covariance matrix of investors’ beliefs about firms’ cash flows</td>
</tr>
<tr>
<td>$D_I$</td>
<td>vector of an individual investor’s demand for firms’ shares</td>
</tr>
<tr>
<td>$\tilde{p}_0$</td>
<td>vector in $\tilde{P}$ unrelated to an investor’s demand</td>
</tr>
<tr>
<td>$\Lambda$</td>
<td>vector of individual investor’s degree of illiquidity</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>vector of coefficients in an individual investor’s demand for firms’ shares</td>
</tr>
<tr>
<td>$\Gamma$</td>
<td>matrix that measures the extent to which an investor’s demand moves prices</td>
</tr>
<tr>
<td>$\Delta$</td>
<td>marginal impact on price of share demand</td>
</tr>
</tbody>
</table>
Table 2 - Notation, Section 3

\( N, M \) number of informed and uninformed investors, respectively

\( r_I, r_U \) risk tolerances of informed and uninformed investors, respectively

\( \Phi_I = \{ \tilde{P} = P, \tilde{X} = X \} \) information available to informed investors

\( \Phi_U = \{ \tilde{P} = P \} \) information available to uninformed investors

\( \rho \) ratio of \( r_I \) to \( M \) as \( M \) becomes large

\( \tilde{V} \) vector of firm cash flows

\( \tilde{P} \) vector of firm share prices

\( \tilde{Z} \) vector of supply of firms’ shares

\( \text{Cov}_v \) a priori covariance matrix for firms’ cash flows

\( \Pi_I = \Pi_v + \Pi_\xi \) precision matrix of informed investors’ beliefs about firms’ cash flows

\( D_I \) vector of an informed investor’s demand for firms’ shares

\( \beta \) the sensitivity of an informed investor’s demand to private information

\( \tilde{p}_0 \) vector in \( \tilde{P} \) unrelated to an informed investor’s demand

\( \Lambda \) vector of an informed investor’s degree of illiquidity

\( \alpha \) vector of coefficients in an informed investor’s demand for firms’ shares

\( \Gamma \) matrix that measures the extent to which an informed investor’s demand moves prices

\( \text{Cov}_U \) covariance matrix of uninformed investors’ beliefs about firms’ cash flows

\( \Pi_U = \Pi_v + \Pi_\delta \) precision matrix of uninformed investors’ beliefs about firms’ cash flows

\( D_U \) vector of an uninformed investor’s demand for firms’ shares

\( \Delta \) marginal impact on price of share demand

\( \Pi_{\text{avg}} \) investors’ average precision matrix
Appendix

Proof of Proposition 3. Similar to the discussion in Section 2, in competing with other investors each informed investor adopts the strategy

\[ D_I(\tilde{P}, \tilde{X}) = \alpha + \beta \cdot X - \Gamma \cdot P, \]  

(A1)

where \( \alpha \) is a \( J \times 1 \) vector of intercept terms, \( \beta \) is a \( J \times J \) matrix of weights an investor places on the realization of his information \( \tilde{X} = X \), and \( \Gamma \) is a \( J \times J \) matrix of weights an investor places on \( P \). For this strategy to be rational based on the computation of \( D_I \) in eqn. (5), it must be the case that

\[ \Gamma = \left( \frac{1}{r_I} \text{Cov}_I + \Lambda^T \right)^{-1}. \]  

(A2)

In addition, substituting an informed investor’s conditional expectation into his demand function in eqn. (A1), it must also be the case that

\[ \alpha = \Gamma [\Pi_v + \Pi_\varepsilon]^{-1} \Pi_\varepsilon E[\tilde{V}] \text{ and } \beta = \Gamma [\Pi_v + \Pi_\varepsilon]^{-1} \Pi_\varepsilon. \]  

(A3)

Market clearing requires that the total demand for firms’ shares equals the supply of those shares. Recall that \( \tilde{Z} \) represents the (random) supply vector of shares in the \( J \) firms. In our asymmetric economy, market clearing requires that

\[ N \cdot D_I(\tilde{P}, \tilde{X}) + M \cdot D_U(\tilde{P}) - \tilde{Z} = 0. \]

Substituting for \( D_I \) and \( D_U \) from eqns. (A1) and eqn. (8), we express market clearing as

\[ N (\alpha + \beta X - \Gamma P) + MrU\Pi_U \left( E[\tilde{V} | \Phi_U] - P \right) - \tilde{Z} = 0. \]

This allows the price vector to be expressed as

\[ \tilde{P} = \Delta \left( N\alpha + N\beta X + \rho\Pi_U E[\tilde{V} | \Phi_U] - \tilde{Z} \right), \]  

(A4)
where $\Delta$ is given by

$$\Delta = (N\Gamma + \rho \Pi_U)^{-1}.$$ 

Equilibrium conditions rely on the solutions to $\Lambda$ and $\Pi_U$. We discuss the solution to $\Lambda$ first and then turn our attention to the solution to $\Pi_U$.

To reconcile the market-clearing condition in eqn. (A4) with an informed investor’s strategy, it must be the case that $\tilde{p}_0$ in the expression $\tilde{P} = \tilde{p}_0 + \Lambda D$ is of the form

$$\tilde{p}_0 = \Lambda \left( (N-1)\alpha + (N-1)\beta X + \rho \Pi_U E \left[ \tilde{V} | \Phi_U \right] - \tilde{Z} \right),$$

and thus

$$\tilde{P} = \Lambda \left( N\alpha + N\beta X + \rho \Pi_U E \left[ \tilde{V} | \Phi_U \right] - \tilde{Z} \right) - \Lambda \Gamma \tilde{P}. \quad (A5)$$

Eqn. (A5) implies

$$\tilde{P} = (I + \Lambda \Gamma)^{-1} \Lambda \left( N\alpha + N\beta X + \rho \Pi_U E \left[ \tilde{V} | \Phi_U \right] - \tilde{Z} \right). \quad (A6)$$

But eqn. (A6) implies the following identity:

$$(I + \Lambda \Gamma)^{-1} \Lambda = \Delta = (N\Gamma + \rho \Pi_U)^{-1},$$

or

$$\Lambda (N\Gamma + \rho \Pi_U) = I + \Lambda \Gamma. \quad (A7)$$

Because $\Gamma = \left( \frac{1}{r_I} \text{Cov}_I + \Lambda^T \right)^{-1}$, we can express eqn. (A7) as

$$\Lambda \left( N \cdot I + \rho \Pi_U \left( \frac{1}{r_I} \text{Cov}_I + \Lambda^T \right) \right) = \left( \frac{1}{r_I} \text{Cov}_I + \Lambda^T \right) + \Lambda.$$

The fact that $\Pi_U$ and $\text{Cov}_I$ are both symmetric matrices yields the following quadratic matrix equation that solves for $\Lambda$:

$$\rho \Lambda \Pi_U \Lambda + \Lambda \left( N - 2 + \rho \frac{\Pi_U}{r_I} \text{Cov}_I \right) - \frac{1}{r_I} \text{Cov}_I = 0.$$
To derive the uninformed investors’ beliefs, recall that the market clearing condition implies

\[ \tilde{P} = \Delta \left( N\alpha + N\beta X + \rho \Pi_U E[\tilde{V}|\Phi_U] - \tilde{Z} \right). \]

As is standard in a rational expectations economy (see, e.g., Grossman and Stiglitz, 1980; Hellwig, 1980; Diamond and Verrecchia, 1981), we assume that each uninformed trader can manipulate the market clearing price \( \tilde{P} \) to obtain information about \( \tilde{V} \) through the statistic \( \tilde{Q} \), where

\[ \tilde{Q} = (N\Delta\beta)^{-1} \left[ \tilde{P} - \Delta \left( N\alpha + \rho \Pi_U E[\tilde{V}|\Phi_U] \right) \right] = \tilde{X} - (N\beta)^{-1} \tilde{Z} = \tilde{V} + \tilde{\delta}, \]

where \( \tilde{\delta} = \tilde{\epsilon} - (N\beta)^{-1} \tilde{Z} \). The statistic \( \tilde{Q} \) measures an informed investor’s private information, \( \tilde{X} \), with error; this makes \( \tilde{Q} \) a noisier measure of the cash flow vector \( \tilde{V} \) than \( \tilde{X} \). Stated somewhat differently, an uninformed investor’s error about the realization of the cash flow vector \( \tilde{V} = V \) is the sum of two terms: the error in \( \tilde{X} \) plus the additional error in \( \tilde{Q} \). Henceforth let \( \Pi_\delta \) represent the precision matrix for the covariance matrix \( \tilde{Q} \).

Conditional on the realization of the price vector \( \tilde{P} = P \) and manipulating this vector to yield \( \tilde{Q} = Q \), an uninformed investor’s posterior beliefs about the vector of cash flows \( \tilde{V} \) is that it has an expected value of

\[ E[\tilde{V}|\tilde{Q} = Q] = E[\tilde{V}] + (\Pi_v + \Pi_\delta)^{-1} \Pi_\delta \left[ Q - E[\tilde{Q}] \right]; \]

in addition, an uninformed investor associates a precision of

\[ \Pi_U = \Pi_v + \Pi_\delta \]

to those beliefs. Thus, we have to solve for \( \Pi_\delta \) to determine \( \Pi_U \). Note, however, that from eqns. (A2) and (A3), we know that \( \Gamma = \left( \frac{1}{\sigma_l} \text{Cov}_I + A^T \right)^{-1} \) and \( \beta = \)
$\Gamma [\Pi_\nu + \Pi_\epsilon]^{-1} \Pi_\epsilon$. Thus, the precision matrix for $\tilde{Q}, \Pi_\delta$, must equal

$$\Pi_\delta = \left( \Pi_\epsilon^{-1} + (N\beta)^{-1} \text{Cov}_z \left[ (N\beta)^{-1} \right]^T \right)^{-1}.$$ 

Q.E.D.

**Proof Of Corollary 1.** Dividing eqn. (9) by $N$ yields

$$\frac{1}{N^2} \rho \Lambda \Pi_U \Lambda + \Lambda \left( 1 - \frac{2}{N} \rho + \frac{1}{r_I} \Pi_U \text{Cov}_I \right) - \frac{1}{N r_I} \text{Cov}_I = 0.$$ 

Taking the limit as $N$ approaches infinity implies $\Lambda = 0$. Similarly, dividing eqn. (9) by $\rho$ yields

$$\Lambda \Pi_U \Lambda + \Lambda \left( \frac{N - 2}{\rho} + \frac{1}{r_I} \Pi_U \text{Cov}_I \right) - \frac{1}{\rho r_I} \text{Cov}_I = 0.$$ 

Taking the limit as $\rho$ approaches infinity yields $\Lambda \Pi_U \Lambda + \Lambda \left( \frac{1}{r_I} \Pi_U \text{Cov}_I \right) = 0$, which implies $\Lambda = 0$. Finally, suppose $N$ and $\rho$ are both finite. Then if $\Lambda = 0$, eqn. (9) becomes $-\frac{1}{r_I} \text{Cov}_I = 0$, which is a contradiction. Q.E.D

**Proof Of Proposition 4.** Market clearing implies

$$N \cdot D_I \left( \tilde{P}, \tilde{X} \right) + M \cdot D_U \left( \tilde{P} \right) - \tilde{Z} = 0.$$ 

Substituting for $D_I$ and $D_U$ implies

$$N \left( \frac{1}{r_I} \text{Cov}_I + \Lambda^T \right)^{-1} \left( E \left[ \tilde{V} | \Phi_I \right] - \tilde{P} \right) + M \cdot r_U \Pi_U \left( E \left[ \tilde{V} | \Phi_U \right] - \tilde{P} \right) - \tilde{Z} = 0.$$ 

Re-arranging terms yields

$$N \left( \frac{1}{r_I} \text{Cov}_I + \Lambda^T \right)^{-1} E \left[ \tilde{V} | \Phi_I \right] + M r_U \Pi_U E \left[ \tilde{V} | \Phi_U \right] - \left[ N \left( \frac{1}{r_I} \text{Cov}_I + \Lambda^T \right)^{-1} + M r_U \Pi_U \right] \tilde{P} - \tilde{Z} = 0.$$ 

Solving for $\tilde{P}$ results in

$$\tilde{P} = \left[ N \left( \frac{1}{r_I} \text{Cov}_I + \Lambda^T \right)^{-1} + M r_U \Pi_U \right]^{-1} \times \left[ N \left( \frac{1}{r_I} \text{Cov}_I + \Lambda^T \right)^{-1} E \left[ \tilde{V} | \Phi_I \right] + M r_U \Pi_U E \left[ \tilde{V} | \Phi_U \right] - \tilde{Z} \right].$$ 

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Taking expected values and using the law of iterated expectations implies \( E \left[ E \left[ \tilde{V} | \Phi_U \right] \right] = E \left[ V \right] \), or

\[
\tilde{P} = E \left[ \tilde{V} \right] - \left[ N \left( \frac{1}{r_I} \text{Cov}_I + \Lambda^T \right)^{-1} + M_{r_U} \Pi_U \right]^{-1} E \left[ \tilde{Z} \right]
\]

\[
= E \left[ \tilde{V} \right] - \left[ \frac{N \left( \frac{1}{r_I} \text{Cov}_I + \Lambda^T \right)^{-1} + M_{r_U} \Pi_U}{N_{r_I} + M_{r_U}} \right]^{-1} E \left[ \frac{\tilde{Z}}{N_{r_I} + M_{r_U}} \right].
\]

Finally we substitute \( \left( \frac{1}{r_I} \text{Cov}_I + \Lambda^T \right)^{-1} = \left( I + r_I \Pi_I \Lambda^T \right)^{-1} r_I \Pi_I \) to get

\[
\tilde{P} = E \left[ \tilde{V} \right] - \left[ \frac{\left( I + r_I \Pi_I \Lambda^T \right)^{-1} N_{r_I} \Pi_I + M_{r_U} \Pi_U}{N_{r_I} + M_{r_U}} \right]^{-1} E \left[ \frac{\tilde{Z}}{N_{r_I} + M_{r_U}} \right].
\]

Q.E.D.
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