Earnings Manipulation and the Cost of Capital

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Abstract

The widespread use of accounting information by investors and financial analysts to help value stocks creates an incentive for managers to manipulate earnings in an attempt to influence short-term stock price performance. This paper examines the role of earnings management in affecting a firm’s cost of capital. Using an agency model with multiple firms whose cash flows are correlated, we demonstrate that the extent of earnings manipulation varies across the business cycle. Managers are more inclined to engage in manipulation during periods of economic expansion. Because of this dependence on the state of the economy, earnings manipulation influences a firm’s cost of capital despite the forces of diversification. In particular, we find that manipulation reduces the correlation between the firms’ cash flows and thus lowers the risk premium required by investors.

JEL classification: G12, M41

Keywords: Earnings management; Information manipulation; Cost of capital

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1 Introduction

The last few years have witnessed a number of corporate scandals that have created the public perception that accounting information provided in a corporate culture fixated on stock price performance cannot be trusted. While media attention has focused on a few high-profile cases of fraudulent accounting schemes (e.g., at Enron and WorldCom), recent empirical studies suggest that the practice of earnings management is prevalent among publicly traded companies. The findings indicate that firms manage earnings to influence stock market perceptions, to increase management’s compensation, to reduce the likelihood of violating lending agreements, and to avoid regulatory intervention.

In this article, we investigate the role of earnings management in affecting a firm’s cost of capital. Given the importance of a firm’s cost of capital for a variety of corporate decisions (from determining the hurdle rate for investment projects to influencing the composition of the firm’s capital structure), it is surprising that the link between cost of capital and earnings management has received little formal scrutiny. To date, the theoretical literature has primarily focused on identifying conditions under which earnings manipulation emerges in a single-firm setting. While this literature has provided many useful insights, its applicability to cost of capital issues is limited. In a single-firm setting, firm-specific risk is priced, because there are no alternative securities that would allow investors to diversify away idiosyncratic risk. It is unclear, however, to what extent accounting information reduces non-diversifiable risks in a multi-asset economy.

In this paper, we take up this task, and present a simple but rigorous model of earnings manipulation with multiple firms whose cash flows are correlated. Important features of our model are risk averse investors, myopic managers, and resource costs of manipulation. Managers are concerned about short-term stock prices because of their compensation contract.

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1 According to the General Accounting Office, about 10 percent of all listed companies in the US announced at least one earnings restatement between January 1997 and June 2002 (see GAO (2002)).

2 See, e.g., the review of the earnings management literature by Healy and Wahlen (1999).
The use of accounting information by investors to value stocks therefore creates an incentive for managers to manipulate earnings. We define earnings manipulation as any action that enables low-value firms to report the same earnings as high-value firms. Such action is costly for managers. The effort involved in manipulating earnings reports and the chance of being caught and punished represent a disutility to managers. In addition, earnings manipulation reduces the value of the firm, because resources are diverted from more productive endeavors. This diverted-resource cost lowers the manager’s compensation payments.

Our analysis leads to two main findings. First, we demonstrate that when managers care about short-term stock prices, the extent of earnings manipulation depends on the state of the economy. In particular, we show that managers of low-value firms are more inclined to engage in manipulation when the stock market is booming. The intuition for this result is as follows. When business conditions are good, most firms have high earnings. Investors thus correctly believe that few firms have an incentive to manipulate their accounting statements. This means that reported earnings have a great effect on a firm’s stock price. A favorable report leads to a significantly higher price than an unfavorable one. However, this is exactly when the incentives for a manager of a low-value firm to issue an upwardly biased report are highest. In bad times, on the other hand, incentives are low, because investors expect a large number of firms to manipulate their earnings and, hence, put less emphasis on the observed reports. Our model therefore predicts the most severe manipulation attempts to occur when the economy is performing well. This prediction seems to be consistent with recent corporate scandals. Many firms (including Enron, WorldCom, and Global Crossing) were found guilty of fraudulent accounting during the economic boom of the late 1990s.

Our second result shows that earnings management influences a firm’s cost of capital, despite the forces of diversification. We find that in large economies, manipulation can significantly lower the risk premium that investors require to hold a stock in equilibrium by

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3This includes cases of fraudulent accounting in which firms get auditors to approve statements that are inconsistent with accounting standards, as well as cases in which firms take actions within accepted accounting and legal standards to improve their accounting performance.
reducing the stock’s beta.\textsuperscript{4} This result is driven by the more frequent occurrence of manipulation attempts in periods of economic expansion and crucially depends on the negative effect of manipulation on firm value. If managers follow different strategies across the business cycle, the diverted-resource cost of manipulation will have an impact on the correlation structure of the firms’ cash flows. Assuming that managers are less likely to manage earnings in bad times, earnings manipulation leads to a smaller reduction in firm value when the economy is performing poorly and the marginal utility of consumption is high. This reduces a stock’s required return in equilibrium. Viewing this result from the perspective of the firm, a firm with relatively stronger incentives to engage in manipulation during a boom, compared to its incentives during a recession, thus faces a lower cost of equity capital. We want to emphasize that this result does not rely on the existence of naive investors who underestimate the extent of manipulation. Rather, it is derived under the assumption that all investors are perfectly rational and correctly anticipate the extent of manipulation in equilibrium.

Our model highlights the importance of disclosure requirements for the manager’s manipulation decision. Common intuition suggests that forcing firms to disclose more information to the public reduces the incidence of manipulation. Our analysis shows that this conclusion is not always correct. All else equal, more informative accounting disclosures (in the absence of manipulation) increase the expected quality of a firm that reports high earnings, and, hence, lead to a higher stock price. This, in turn, increases the manager’s incentives to engage in manipulation when earnings are low. Thus, our results suggest that stricter disclosure requirements can be counterproductive in terms of reducing the extent of manipulation. Unless disclosure laws make manipulation more costly for managers, more disclosure may actually lead to more manipulation. This adds a new perspective to the debate on how disclosure regulation can be used to curb earnings manipulation.

Prior theoretical work has primarily focused on managerial incentives to manage earnings

\textsuperscript{4}We want to point out, however, that, controlling for beta, there is no cost of capital effect in the cross section.
in a single-firm setting. Dye (1988), Evans and Sridhar (1996), and Demski (1998) present models in which the manager is, by assumption, unable to communicate relevant private information to shareholders. In these models, the revelation principle is not applicable and the optimal managerial compensation contract encourages earnings management. Bar-Gill and Bebchuk (2003) develop a model in which firms misreport their performance in order to obtain better terms when raising funds for new investments. Goldman and Slezak (2006) consider a variation of the principal-agent model where the agent can take a costly but unobservable action to manipulate disclosed information. They show that the optimal managerial compensation contract balances incentives to exert effort against incentives to commit fraud. However, none of these papers examines the effect of changing economic conditions on the manager’s incentives to engage in manipulation, and thus do not shed any light on the question of how earnings management affects a firm’s cost of capital. In this respect, our approach is closer to the work of Povel, Singh, and Winton (2007), who study fraud and monitoring decisions in a setting with multiple firms that seek financing from outside investors. Their results show that a firm’s decision to commit fraud depends on the investors’ beliefs about the state of the economy. Firms are more likely to manipulate their financial reports in relatively good times (as measured by the average quality of firms seeking financing). However, Povel, Singh, and Winton do not analyze how the firms’ manipulation decisions affect security prices.

The remainder of this paper is organized as follows. Section 2 presents the economic setting. Section 3 describes the equilibrium of the model and analyzes the manager’s manipulation decision and its effect on the firm’s stock price. Section 4 considers the impact of earnings manipulation on a firm’s cost of capital in a large economy. Section 5 provides a short summary and conclusion. All proofs are contained in the Appendix.
2 The Model

We consider an economy with $N$ all-equity financed firms indexed by $i = 1, \ldots, N$. The model takes place over times 0, 1, and 2. At time 0, the initial owners of a firm sell their shares to risk averse outside investors. Prior to trade at time 1, a certified third-party monitor issues a (potentially misleading) report concerning the future value of the firm. Based on this information, the market determines the intermediate stock price at which the manager sells her equity stake in the firm. At time 2, conclusive public information arrives and the firm is liquidated. Besides the firm’s shares, market participants can also invest in a riskless bond. The bond is in perfectly elastic supply and its interest rate is normalized to zero. The structure of the economy is common knowledge.

2.1 Firms and Managers

Each firm $i$ is controlled by a manager who owns $s^M_i$ shares of the firm. The remaining $s_i$ shares are issued to the public at time 0. Each share of firm $i$’s stock pays a liquidation value of $P_{2,i} = e_{1,i} + e_{2,i}$ at time 2, where $e_{t,i}$ is referred to as the firm’s economic earnings in period $t = 1, 2$. To focus on the manager’s incentives to manipulate earnings, we consider a simple setting in which earnings as well as informational variables are binary. In particular, we assume that, before any action is taken by the manager, the earnings distribution is described by the following factor structure:

$$e_{t,i} = \mu_i + \theta_{t,i} \sigma_i,$$

$$\theta_{t,i} = \begin{cases} 
+1, & \text{with probability } (1 + \beta_i F)/2, \\
-1, & \text{with probability } (1 - \beta_i F)/2, 
\end{cases} \quad (1)$$

where $\beta_i \in (0,1)$ measures the sensitivity of firm $i$’s expected payoff with respect to the systematic factor $F$. We assume that $F$ is equally likely to be $+1$ (“good state”) or $-1$ (“bad state”), and that the firms’ earnings are stochastically independent conditional on the factor.
realization, i.e., $\text{Cov}[e_{t,i}, e_{s,j} \mid F] = 0$ for all $s, t \in \{0, 1\}$ and $i \neq j$. As is standard in factor models, any comovement of firm values is thus captured by the common factor $F$. The values of $\mu_i$, $\sigma_i$, and $\beta_i$ are common knowledge, but the realizations of $\theta_{t,i}$ and $F$ are not revealed until time 2. The assumption that the same factor realization determines the firms’ earnings in both periods is only made for tractability and is not crucial to our results. Shares of the stock are infinitely divisible and are traded competitively in the stock market. The price of stock $i$ at time $t = 0, 1$ is denoted by $P_{t,i}$.

Before the stock market opens at time 1, a certified third-party monitor provides a (noisy) signal $r_i$ to the market concerning firm $i$’s terminal value. This signal, which we refer to as an earnings report, can take on one of two values, +1 or −1. Absent any managerial intervention, the report $r_i$ is correlated with firm $i$’s first-period earnings as follows:

$$
Pr(r_i = \theta_{1,i} \mid \theta_{1,i}, \text{no manipulation}) = (1 + \delta_i)/2, \quad \delta_i \in (0, 1).
$$

(2)

The parameter $\delta_i$ measures the quality of the auditor’s report. It represents various accounting standards and conventions in the economy as well as firm- and auditor-specific factors such as the auditor’s experience in the industry and the transparency of the firm’s operations.

Although the report is made by a third-party monitor, we assume that the manager can influence its outcome—for example, by hiding information from the auditor or by colluding with the auditor to issue a biased report. Such an effort increases the probability that a firm with low economic earnings generates a favorable signal:

$$
Pr(r_i = +1 \mid \theta_{1,i} = -1, \text{manipulation}) = (1 - \delta_i)/2 + \kappa_i(F)
$$

(3)

We assume that manipulation does not affect the earnings reports of firms with high first-period earnings. Allowing these firms to manipulate their reported earnings would not alter our basic conclusions, so long as low-payoff firms benefit more from manipulation than high-
payoff firms. For expository purposes, we refer to $\kappa_i(F)$ as the amount or the extent of earnings manipulation. We allow the manager to optimally choose the amount of manipulation after observing the factor realization $F$. This dependence on the state of the economy reflects the fact that the manager’s incentives to issue a biased report will depend on the investors’ beliefs about the firm’s earnings and, hence, on the general business conditions. Since $F$ is binary, the manager’s manipulation strategy is fully characterized by the vector $\kappa_i = (\kappa^+_i, \kappa^-_i)$, where $\kappa^+_i = \kappa_i(+1)$ and $\kappa^-_i = \kappa_i(-1)$.

Earnings manipulation is costly to the manager. Specifically, we assume that the manager’s private utility cost associated with the amount of manipulation $\kappa_i(F)$ is equal to $c_i \kappa^2_i(F)$. This cost includes both the effort involved in manipulating the auditor’s report and the chance that the manager is later caught and punished. The cost parameter $c_i$ is related to the legal environment in the economy, but may also depend on firm-specific factors such as the intensity of outside monitoring (by analysts, the media, etc.) that affect the manager’s opportunities to misreport the firm’s performance.

In addition, by diverting resources from more productive uses, earnings manipulation is costly, because it reduces the value of the manager’s equity stake. In particular, we assume that the actions taken by the manager to bias the auditor’s report carry an opportunity cost that lowers the terminal value of the firm as follows:

$$
Pr(\theta_{2,i} = +1 \mid F, \text{manipulation}) = \frac{1 + \beta_i F}{2} - \xi_i \kappa_i(F),
$$

where $\xi_i \geq 0$ denotes the marginal resource cost. This cost includes the opportunity cost of the time managers spend hiding information from auditors, the cost of bribing auditors,

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5One justification for assuming a convex cost function could be that both the detection probability and the penalty are increasing in the extent of the manipulation.

6This assumption seems reasonable considering the effort Congress has invested in devising new measures to combat earnings management. For instance, the Sarbanes–Oxley Act of 2002, which increased enforcement and introduced new penalties for fraudulent behavior, is the result of an on-going attempt by Congress to mitigate earnings management. Even NASDAQ has issued new guidelines requiring its listed firms to have financially literate audit committees. Thus, there seems to be a widespread perception that earnings management is detrimental to shareholders’ interests.
and the cost of making inefficient decisions based on inaccurate information. The following assumption ensures that the probability in (4) is positive for any $\kappa_i(F) < \delta_i$.

**Assumption 1** $\xi_i \leq (1 - \beta_i)/(2\delta_i)$.

Managers are assumed to be risk neutral. They own $s_i^M$ shares of their respective companies, which they are not allowed to sell prior to time 1. In addition, they are assumed to be cash-constrained and thus cannot purchase additional equity at time 0. We take $s_i^M$ as given and do not solve for the optimal compensation contract. Goldman and Slezak (2006) show how $s_i^M$ can be endogenized in a model with unobserved managerial effort. If managers were forced to hold their shares until the liquidation value $P_{2,i}$ is realized (which is, by assumption, not subject to manipulation), then no manipulation would occur. In most situations, however, such long-term incentive schemes are impractical. We therefore assume that managers sell their stake in the firm at time 1.

### 2.2 Investors

Our economy is populated by $M$ risk averse investors. For tractability, we assume that these investors are short-lived. An agent born at time $t = 0, 1$ can buy and sell securities at time $t$ and has to unwind her position at time $t + 1$. Investors born in the first period do not receive any information about the firms’ cash flow realizations before submitting their orders. Investors who enter the market in the second period, on the other hand, can base their asset demands on the firms’ earnings reports that are issued at time 1. However, they do not observe the extent of earnings manipulation.

All investors are assumed to have mean-variance preferences with an identical risk aversion coefficient of $\gamma$. They maximize their expected utility from end-of-period consumption given by:

$$E[U(W_{t+1} | \mathcal{F}_t)] = E[W_{t+1} | \mathcal{F}_t] - \frac{\gamma}{2} Var[W_{t+1} | \mathcal{F}_t], \quad t = 0, 1, \quad (5)$$

In Section 3.2 we will show that the equilibrium extent of manipulation $\kappa_i(F)$ never exceeds $\delta_i$. 

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where $F_t$ denotes the investors’ information set at time $t$ and $W_{t+1}$ denotes their wealth at time $t + 1$.

For simplicity, we assume that investors behave competitively. They take equilibrium prices as given even though their aggregate trades affect market prices. Such behavior can be justified by assuming that there is a large (to be precise, an infinite) number of investors, so that no single trader can influence the price. This assumption seems quite reasonable, as our analysis will, for the most part, focus on a large economy (in the sense that the number of firms goes to infinity). In fact, this assumption is necessary to obtain a finite market risk premium in equilibrium. As the number of firms in the economy increases without bound, each of a finite number of investors would have to bear an infinite amount of risk and would therefore demand an infinite risk premium. To rule out such an unrealistic scenario, we require that the economy’s risk-bearing capacity increase at the same rate as the total risk in the economy. More precisely, we assume that the number of investors expand at the same rate as the number of firms so that, in the limit, $\frac{M}{N}$ approaches a constant. Without loss of generality, we assume that this constant is unity. In the ensuing analysis, we use the term “large economy limit” to refer to the case where $M$ and $N$ approach infinity at the same rate.

3 Manipulation and Asset Prices in Equilibrium

In this section, we solve for the equilibrium of the economy defined above. We will focus on the relationship between earnings manipulation and asset prices in the large economy limit. The equilibrium concept we use is that of a Perfect Bayesian Equilibrium (PBE). Formally, a PBE of our economy is defined by a manipulation strategy $\kappa = (\kappa_1, \ldots, \kappa_N)$, by the market’s beliefs about $\kappa$, and by demand functions of time 0 and time 1 investors, such that: (i) manager $i$’s manipulation strategy maximizes her expected utility, for all $i = 1, \ldots, N$; (ii) for each price-taking investor, the trades specified by her demand function at a given date.

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*Similar restrictions have been adopted by Hughes, Liu, and Liu (2007) and Lambert, Leuz, and Verrecchia (2007).*
maximize her expected utility of consumption, subject to a budget constraint and available information; and (iii) the market’s beliefs about the managers’ behavior coincide with their actual behavior.

3.1 Investors’ Portfolio Problem and Asset Prices

Each investor chooses her portfolio of assets \( i = 1, \ldots, N \) to maximize her expected utility subject to her budget constraint. Let \( P_t \) denote the vector of asset prices, \( \mu_t = E[P_{t+1} | F_t] \) denote the vector of expected payoffs conditional on the information set \( F_t \), and \( \Omega_t = E[(P_{t+1} - E[P_{t+1} | F_t])(P_{t+1} - E[P_{t+1} | F_t])^T | F_t] \) denote the conditional covariance matrix of asset payoffs. Then, an investor born at time \( t \) faces the following optimization problem:

\[
\max_{x_t} x_t^T \mu_t + b_t - \frac{\gamma}{2} x_t^T \Omega_t x_t, \quad \text{subject to} \quad x_t^T P_t + b_t = W_t, \quad (6)
\]

where the vector \( x_t \) denotes the investor’s portfolio of risky assets, \( b_t \) denotes the amount invested in bonds, and \( W_t \) denotes her initial wealth. Maximizing this quadratic objective function with respect to \( x_t \) yields:

\[
x_t = \frac{1}{\gamma} \Omega_t^{-1} (\mu_t - P_t). \quad (7)
\]

If the payoff of asset \( i \) is independent of the systematic factor \( F \) (i.e., if \( \beta_i = 0 \)), it is straightforward to show that the investor’s demand for this asset is increasing in its expected payoff, and decreasing in its payoff variance. In the more general case when \( \beta_i > 0 \), the demand for asset \( i \) depends not only on the investor’s beliefs about asset \( i \)’s payoff, but also on her beliefs about the payoffs of other assets.

Imposing the market clearing condition that the aggregate demand from investors, which is given by \( Mx_t \), equals the supply \( s_t = (s_{t,1}, \ldots, s_{t,N})^T \), we obtain the following expression
for the prices of risky assets:

\[ P_t = \mu_t - \frac{\gamma}{M} \Omega_t s_t \]  

(8)

The price function in (8) depends on the investors’ beliefs about the assets’ risk and return. These beliefs are different for investors in the first and in the second period. We first consider the beliefs of investors who are born at time 1. These investors observe the firms’ earnings reports \( \{r_1, \ldots, r_N\} \) before making their investment decisions. As the number of firms goes to infinity, idiosyncratic shocks in these reports cancel out and we would therefore expect investors to perfectly learn the systematic factor in asset payoffs. The following lemma shows that a sufficient condition for \( F \) to be fully revealed is that the extent of manipulation when \( F = +1 \) exceeds the extent when \( F = -1 \). Intuitively, this condition, which will be shown to hold in equilibrium, ensures that the effect that the difference in the factor realization has on the “average” earnings report is not (exactly) offset by the effect of the difference in the extent of manipulation.

**Lemma 1** Suppose that \( \kappa_i^- < \kappa_i^+ < \delta_i \), for all \( i = 1, \ldots, N \). Then, in the large economy limit, the earnings reports provided by the third-party monitor fully reveal the common factor \( F \) in asset payoffs.

Assuming that the condition in Lemma 1 holds, the firms’ earnings reports eliminate any systematic risk associated with the factor \( F \). Thus, the conditional covariance of asset payoffs is zero and \( \Omega_1 \) becomes a diagonal matrix. This implies that second-period investors can diversify away any risk by holding a sufficiently large number of assets in their portfolios \(^{10}\) in the large economy limit, the required risk premium approaches zero and asset prices converge to the conditional expectation of the firms’ liquidation values \( P_2 \).

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\(^9\)Note that the supply of shares at time 1 includes the equity stake of the manager.

\(^{10}\)This is of course a consequence of our assumption that the realizations of the common factor \( F \) are perfectly correlated across periods (i.e., the same factor realization determines the earnings distribution in the first and the second period).
Lemma 2 For a given amount of earnings manipulation $\kappa_i(F)$, time 1 asset prices in the large economy limit are given by:

$$P_{1,i} = E[P_{2,i} \mid r_1, r_2, \ldots] = 2\mu_i + E[\theta_{1,i} + \theta_{2,i} \mid F, r_i] \sigma_i, \quad \text{for all } i = 1, 2, \ldots, \quad (9)$$

where:

$$E[\theta_{1,i} \mid F, r_i] = \frac{\beta_i F + \delta_i r_i - (1 - \beta_i) \kappa_i(F) r_i}{1 + \beta_i \delta_i F r_i + (1 - \beta_i) \kappa_i(F) r_i}, \quad (10)$$

$$E[\theta_{2,i} \mid F, r_i] = \beta_i F - 2\xi_i \kappa_i(F). \quad (11)$$

Not surprisingly, the price of asset $i$ is increasing in the factor realization $F$ and the reported earnings $r_i$, and decreasing in the resource cost $\xi_i$.

The effect of the amount of earnings manipulation on $P_{1,i}$ is more complicated. On the one hand, a higher $\kappa_i(F)$ lowers the firm’s future cash flow because more resources are diverted away from productive uses. On the other hand, it leads, on average, to higher reported earnings. Thus, conditional on a favorable report ($r_i = +1$), the price $P_{1,i}$ is decreasing in the (expected) extent of manipulation. Investors take the extent of manipulation into account when forming their beliefs and adjust the expected asset payoff accordingly. In contrast, when the report indicates low earnings ($r_i = -1$), $P_{1,i}$ can actually increase in the extent of manipulation (for low values of $\xi_i$). The reason is that a higher amount of manipulation makes the report less informative, which means that an unfavorable report is more likely to be due to bad luck rather than to low fundamentals.

It is important to note that if investors have rational beliefs (in the sense that in equilibrium, the expected amount of manipulation coincides with the actual amount of manipulation), biased earnings reports have no effect on the investors’

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11 Note that without earnings manipulation, the covariance between the factor realization $F$ and the asset payoff $P_{2,i}$ is equal to $2\beta_i \sigma_i$, and is thus positive for all assets.

12 Consider the extreme case in which $\kappa_i(F) = \delta_i$. In this case the probability of a negative earnings report is independent of the true earnings, since $Pr(r_i = +1 \mid \theta_i = +1) = (1 + \delta_i)/2 = (1 - \delta_i)/2 + \kappa_i(F) = Pr(r_i = +1 \mid \theta_i = -1).$
expectation of $e_{1,i}$ in equilibrium. This does not mean, however, that managers have no incentive to manipulate earnings. Since the actual amount of manipulation is not observable, the investors’ expectation of $e_{1,i}$ is increasing in the actual amount of manipulation (holding fixed the expected amount of manipulation)\footnote{See our discussion in Section 3.2}.

We now discuss how earnings manipulation affects stock prices at time 0. Investors born in the first period do not receive any informative signal about the firms’ earnings before choosing their optimal portfolios. They do not observe the amount of earnings manipulation, but they can perfectly predict it in equilibrium. We can therefore determine the equilibrium prices by calculating the expected payoff vector $\mu_0$ and covariance matrix $\Omega_0$ based on the equilibrium amount of manipulation and substituting these expressions into the price function defined by equation (8).

**Lemma 3** In the large economy limit, asset prices at time 0 are given by:

$$P_{0,i} = 2\mu_i - \xi_i(\kappa_i^+ + \kappa_i^-)\sigma_i - (\beta_i - \xi_i(\kappa_i^+ - \kappa_i^-)/2)\sigma_i\eta, \text{ for all } i = 1, 2, \ldots,$$

where $\eta = \lim_{N \to \infty} \frac{4\gamma}{N} \sum_{j=1}^{N} \left( \beta_j - \xi_j(\kappa_j^+ - \kappa_j^-)/2 \right)\sigma_j s_j < \infty$.

The lemma shows that the equilibrium price at time 0 is equal to the expected terminal cash flow net of the resource cost minus a risk premium. In the large economy limit, the risk premium is finite and proportional to the asset’s systematic risk, which is characterized by $\beta_i\sigma_i$, if the firm’s earnings are reported truthfully or if earnings manipulation has no negative impact on the firm’s future cash flow. If the resource cost $\xi_i$ is positive, however, earnings manipulation can affect the firm’s risk premium. The reason is that the manager’s incentive to manipulate earnings depends on the state of the economy, as we will demonstrate in the next section.
3.2 Optimal Earnings Manipulation

Having solved the investors’ portfolio selection problem, we now turn to an analysis of the manager’s decision to manipulate earnings. This decision depends on the cost and benefit of manipulation. The former includes the manager’s direct utility cost \( c_i \kappa_i^2(F) \) as well as the indirect cost of diverted resources used to hide information from the auditor, which lowers the value of the manager’s equity stake in the firm. The latter depends on the difference in the investors’ response to positive and negative earnings reports as expressed by the difference in stock prices following a high and a low signal.

It is important to note that the manager takes the intermediate stock price \( P_{1,i} \) as given when choosing the optimal amount of manipulation. The reason is that while investors can predict the amount of manipulation in equilibrium, they do not observe the actual amount of manipulation \( \kappa_i(F) \). The price \( P_{1,i} \) is therefore determined by the investors’ beliefs about \( \kappa_i(F) \), which we denote by \( \hat{\kappa}_i(F) \), rather than by the actual \( \kappa_i(F) \). Assuming that the manager leaves the firm at time 1 and sells her equity stake of \( s_i^M \) shares at a price of \( P_{1,i} \), her utility maximizing manipulation strategy can be found by solving the following optimization problem:

\[
\max_{\kappa_i(F)} \quad s_i^M \mathbb{E}[P_{1,i}(r_i, F, \hat{\kappa}_i(F)) \mid \theta_{1,i} = -1, F] - c_i \kappa_i^2(F),
\]

where we have written the stock price \( P_{1,i} \) as a function of \( \hat{\kappa}_i(F) \) to emphasize its dependence on the amount of manipulation expected by investors. The manager seeks to maximize the expectation of the intermediate stock price conditional on the economic earnings being low. This is a consequence of our assumption that manipulation has no impact on the earnings reports of firms with high earnings. From the manager’s perspective, the distribution of \( P_{1,i} \) is characterized by the probability that the auditor issues a favorable report \( r_i = +1 \), conditional on the true earnings being low, which is given by \((1 - \delta_i)/2 + \kappa_i(F)\). Substituting this expression into (13), we derive the first-order condition for a maximum with respect to
\( \kappa_i(F) \) as:

\[
\kappa_i(F) = \frac{s^M}{2c_i} \left( P_{1,i}(r_i = +1, F, \hat{\kappa}_i(F)) - P_{1,i}(r_i = -1, F, \hat{\kappa}_i(F)) \right)
\]

(14)

It is easily verified that the extent of manipulation defined by (14) is the unique maximum, since (13) is strictly concave in \( \kappa_i(F) \).

In equilibrium, investors’ beliefs about the manager’s behavior must be correct. Thus, the amount of manipulation expected by investors has to coincide with the actual amount of manipulation. Substituting the price function specified in Lemma 2 into the first-order condition (14) and setting \( \hat{\kappa}_i(F) \) equal to \( \kappa_i(F) \) yields a cubic equation that defines the equilibrium amount of earnings manipulation. This equation has a unique root in the interval \([0,1]\). The following proposition characterizes the equilibrium amount of earnings manipulation.

**Proposition 1** In the large economy limit, there exists a unique equilibrium. The amount of earnings manipulation satisfies

\[
0 < \kappa_i^- < \kappa_i^+ < \delta, \text{ for all } i = 1, 2, \ldots
\]

Proposition 1 reveals a number of important properties of the manager’s manipulation strategy. First, the extent of manipulation never exceeds \( \delta_i \), independent of the state of the economy. Thus, manipulation reduces the report’s correlation with the firm’s earnings, but the report remains somewhat informative in the sense that

\[
Pr(r_i = +1 \mid \theta_{1,i} = +1) = \frac{1 + \delta_i}{2} > \frac{1 - \delta_i}{2} + \kappa_i(F) = Pr(r_i = +1 \mid \theta_{1,i} = -1).
\]

This is true for any positive value of the cost parameter \( c_i \). The reason is that as \( \kappa_i(F) \) approaches \( \delta_i \), the benefit of manipulation in the form of a higher stock price after a positive earnings report decreases. In the limiting case, the signal \( r_i \) is pure noise and the difference in stock prices is zero, making the manager’s manipulation attempt useless.

The equilibrium amount of manipulation also depends on the state of the economy. It is higher in good times than in bad times. When business conditions are good (i.e., when \( F = +1 \)), a large proportion of firms have high earnings. Investors thus correctly believe that few firms have an incentive to manipulate their reports. This means that the signal \( r_i \) has
a great effect on a firm’s intermediate stock price. A positive signal leads to a significantly higher price $P_{t,i}$ than a negative signal. However, this is exactly when the incentives for a manager of a firm with low earnings to issue a positive report are highest. In bad times, on the other hand, incentives are low, because investors expect a large number of firms to manipulate their earnings and, hence, put less emphasis on the observed reports. Our model therefore predicts the most severe manipulation attempts to occur when the stock market is booming. This prediction seems to be consistent with recent corporate scandals. Many firms (including Enron, WorldCom, and Global Crossing) were found guilty of fraudulent accounting during the economic boom of the late 1990s.

To obtain a better understanding of what kind of firms are most susceptible to earnings manipulation, we now examine how various parameters affect the equilibrium amount of manipulation. The following proposition presents comparative static results with respect to the model primitives $\beta_i$, $\sigma_i$, $\delta_i$, $s_i^M$, $c_i$, and $\xi_i$.

**Proposition 2** In the large economy limit, the amount of earnings manipulation, $\kappa_i(F)$, is a decreasing function of the cost parameter $c_i$, and an increasing function of the manager’s equity stake $s_i^M$, the signal precision $\delta_i$, and the firm’s earnings variability $\sigma_i$. It does not depend on the marginal resource cost $\xi_i$. Furthermore, in the good state of the economy ($F = +1$), the amount of manipulation is non-monotonic in the firm’s systematic risk, as measured by $\beta_i$. It is increasing in $\beta_i$ for low values of $\beta_i$, and decreasing in $\beta_i$ for high values of $\beta_i$. In the bad state of the economy ($F = -1$), the amount of manipulation is always decreasing in $\beta_i$.

The intuition for the relationship between the equilibrium amount of manipulation and the cost parameter $c_i$ is straightforward. As the manager’s utility cost goes up, manipulation becomes more expensive, and the extent of manipulation decreases in equilibrium. This follows directly from equation (14). An increase in the manager’s equity stake $s_i^M$ has the opposite effect. The more shares the manager owns, the more she benefits from a high
stock price at time 1, and thus the higher her incentives are to engage in unobservable manipulation. An increase in the earnings variability $\sigma_i$ increases the difference between $P_{1,i}(r_i = +1)$ and $P_{1,i}(r_i = -1)$, the stock price after a positive and a negative report, again making manipulation more attractive.

A key factor in the determination of the manager’s optimal manipulation strategy is the signal precision $\delta_i$ in the absence of manipulation. A higher $\delta_i$ increases the probability that firm $i$ generates a positive (negative) signal if its earnings are high (low). All else equal, such a change improves (worsens) the expected quality of the firm when investors observe a favorable (unfavorable) report, and, hence, leads to a higher (lower) stock price at time 1. This, in turn, increases the manager’s incentives to engage in manipulation when earnings are low. Thus, our results suggest that stricter disclosure requirements can be counterproductive in terms of reducing the extent of manipulation. Unless disclosure laws make manipulation more costly for managers, more disclosure may actually lead to more manipulation. This adds a new perspective to the debate on how disclosure regulation can be used to curb earnings manipulation.

Interestingly, the diverted-resource cost has no effect on the equilibrium extent of manipulation. While a positive $\xi_i$ lowers the expected liquidation value of the firm and, hence, its stock price $P_{1,i}$, the magnitude of this reduction does not depend on the signal realization $r_i$, as shown in Lemma 2. From the manager’s perspective, the resource cost is a constant that is determined by the investors’ beliefs about the manager’s behavior rather than by her actual behavior. It therefore does not affect the manager’s incentives to engage in manipulation.

Finally, in the good state of the economy, our model predicts a non-monotonic relationship between the extent of manipulation and $\beta_i$, the systematic component of the firm’s earnings, peaking for high, but not too high, values of $\beta_i$. On the one hand, a high $\beta_i$ means that the firm’s earnings are highly correlated with the realization of the common factor $F$, making the signal $r_i$ less useful to investors. This would suggest that managers of high-$\beta$ firms are less inclined to engage in manipulation. On the other hand, however, a higher $\beta_i$ also increases
the likelihood that the firm has high earnings and therefore no need to manipulate the signal $r_i$ when $F = +1$. This makes investors trust the firm’s signal more, which, in turn, makes manipulation more attractive for bad firms. For low values of $\beta_i$, the latter effect dominates and the extent of manipulation increases in $\beta_i$, whereas for high values of $\beta_i$, the former effect dominates, reducing the extent of manipulation as $\beta_i$ increases. In contrast, in the bad state of the economy ($F = -1$), the second effect is reversed, because investors expect most firms to manipulate their earnings reports in bad times and thus put less weight on the observed signal $r_i$. In this case, both effects reduce the manager’s incentives to engage in manipulation and the extent of manipulation monotonically decreases in $\beta_i$.

It is important to note that the role of $\beta_i$ in our model slightly differs from that in a standard factor model, because $\beta_i$ represents the probability with which firm $i$’s earnings are determined by the common factor $F$, rather than by an idiosyncratic shock, and thus is restricted to the interval $[0, 1]$\textsuperscript{14} This means that $\beta_i$ specifies the firm’s systematic risk, holding fixed its total risk $\sigma_i^2$. Firms with a high $\beta_i$ are therefore firms with little idiosyncratic risk, and vice versa. Based on this alternative interpretation, the above result states that in bad times, firms with high levels of firm-specific risk are more inclined to manage their earnings. In good times, however, the relationship between idiosyncratic risk and earnings manipulation is non-monotonic. In this case, firms with moderate levels of firm-specific risk are the ones that benefit most from manipulation.

4 Manipulation and the Cost of Capital

Having established the equilibrium, we now turn to an analysis of how expected returns differ across stocks. We show that the extent of earnings manipulation affects the return investors require to hold an asset in equilibrium. From the firm’s perspective, this return represents

\textsuperscript{14}Note that $\theta_{1,i}$ can be written as $\omega_i F + (1 - \omega_i) \epsilon_i$, where $\omega_i$ is a random variable with possible values 0 and 1, and $\epsilon_i$ is equally likely to be +1 or −1. The random variables $\omega_i$ and $\epsilon_i$ are independent of each other and of the factor realization $F$. Then, $\beta_i$ is the probability that $\omega_i$ equals 1.
the cost of equity capital the firm faces when it decides to raise new funds. The two-period (dollar) return per share to holding asset $i$ is $P_{2,i} - P_{0,i}$. The expected return per share is thus $E[P_{2,i}] - P_{0,i}$, where the expectation is computed with respect to the investors’ time 0 information set. This return is equal to the expected return in the first period, because the firms’ earnings reports perfectly reveal the realization of the common factor $F$ at time 1 and investors bear no systematic risk in the second period. The following lemma describes this equilibrium risk premium in the large economy limit.

**Proposition 3** In the large economy limit, the expected return per share for stock $i$ is given by:

$$E[P_{2,i} - P_{0,i}] = (\beta_i - \xi_i (\kappa_i^+ - \kappa_i^-)/2) \sigma_i \eta,$$

where $\eta$ is defined in Lemma 3.

Proposition 3 has several interesting implications. First, it shows that in the large economy limit, the risk premium depends only on the covariance between the stock’s payoff and the return on the market portfolio, or, equivalently, the systematic factor $F$. This result is not surprising, because in the limiting case as the number of firms goes to infinity, all investors hold perfectly diversified portfolios and do not demand any compensation for idiosyncratic risk.

In the absence of manipulation, a firm’s systematic risk is characterized by its $\beta$. If the marginal resource cost $\xi_i$ is nonzero, the systematic risk is also affected by the manager’s manipulation strategy. Interestingly, it is the difference in manipulation intensities $\Delta \kappa_i = \kappa_i^+ - \kappa_i^-$, rather than the level of manipulation, that determines the risk premium. To obtain a better understanding of this result, suppose that the manager chooses the same extent of manipulation in good and in bad times. In this case, by diverting resources from more productive uses, earnings manipulation reduces the value of the firm, but this reduction is independent of the state of the economy, and, hence, it does not change the correlation.
between the payoff $P_{2,i}$ and the factor $F$. This means that as long as $\kappa_i^+ = \kappa_i^-$, earnings manipulation affects the price level of a stock, but not its expected return. In contrast, if the manager follows different strategies in good and in bad times, the resource cost of manipulation does have an impact on the correlation structure of cash flows. Assuming that $\kappa_i^+ > \kappa_i^-$, earnings manipulation leads to a greater reduction in firm value when $F = +1$. Thus, the stock’s payoff is comparatively larger when the economy is performing poorly and the marginal utility of consumption is high. This reduces the stock’s required return in equilibrium. The magnitude of this effect depends linearly on the difference between the diverted-resource cost in good and in bad times, $\xi_i \Delta \kappa_i$. The higher this cost difference, the greater the reduction in the covariance between $P_{2,i}$ and $F$, and thus the lower the risk premium of the stock.

We conclude this section by observing that the effect of manipulation on the correlation of stock returns is not a reflection of the correlation of reported earnings. The covariance between the firm’s stock price $P_{2,i}$ and the systematic factor $F$ crucially depends on the difference in the diverted-resource cost of manipulation across different states of the economy, whereas the covariance between the signal $r_i$ and $F$ is primarily driven by the average amount of manipulation across states.

**Lemma 4** The covariance between firm $i$’s earnings report $r_i$ and the common factor $F$ is given by:

$$
\text{Cov}[r_i, F] = \beta_i (\delta_i - (\kappa_i^+ + \kappa_i^-)/2) + (\kappa_i^+ - \kappa_i^-)/2.
$$

(16)

Lemma 4 shows that, holding the difference in manipulation intensities across states $\Delta \kappa_i$ fixed, an increase in the extent of earnings manipulation reduces the covariance between the firm’s report and the systematic factor, even if the diverted-resource cost of manipulation is zero.\footnote{Note that there is also no indirect effect of $\xi_i$ on the covariance between $r_i$ and $F$ through the equilibrium extent of manipulation, because both $\kappa_i^+$ and $\kappa_i^-$ are independent of the resource cost (see Proposition 2).} In the limiting case as $c_i$ goes to zero, both $\kappa_i^+$ and $\kappa_i^-$ approach $\delta$ and the distribution
of the signal \( r_i \) becomes independent of \( F \). In contrast, Proposition 3 implies that the covariance between the stock price and the systematic factor does not depend on the extent of manipulation, as long as \( \xi_i \Delta \kappa_i \) remains unchanged. Thus, a change in the correlation of reported earnings does not necessarily translate into a change in systematic risk. The explanation for this result is that rational investors are not fooled by manipulated reports. They have the same information as managers about the state of the economy and can perfectly predict the extent of manipulation in equilibrium. Any effect of manipulation on reported earnings can therefore be undone by investors. The following proposition formalizes this result and shows that the correlation in intermediate stock prices only reflects the correlation in fundamentals.

**Proposition 4** The covariance between the intermediate stock price \( P_{1,i} \) and the systematic factor \( F \) is equal to the covariance between the firm’s liquidation value \( P_{2,i} \) and \( F \); it does not reflect changes in the covariance between the earnings report \( r_i \) and \( F \), as long as \( \Delta \kappa_i \) remains unchanged.

We want to point out, however, that the above result relies heavily on the fact that investors are rational and have complete information about the state of the economy. If (some) investors systematically underestimate the extent of manipulation (either because they do not have rational expectations or because they do not possess enough information about the realization of the systematic factor and its relation to the manager’s behavior), the result in Proposition 4 will no longer obtain, and an increase in the correlation of earnings reports across firms will lead to an increase in the correlation of stock prices, and will consequently raise the required risk premium.
5 Conclusion

This paper develops an agency model that allows us to explore the relationship between earnings manipulation and a firm’s cost of capital in a multi-asset economy. We assume that managers are concerned about short-term stock prices because of their compensation contract. The use of accounting information by investors to value stocks therefore creates an incentive for managers to manipulate earnings reports.

Our analysis reveals that the extent of earnings manipulation is related to the business cycle. In particular, we demonstrate that managers of low-earnings firms have stronger incentives to report high earnings when the stock market is booming. The reason for this result is that, in good times, investors expect most firms to do well and thus to have little incentive to manipulate their accounting statements. This means that reported earnings have a great effect on a firm’s stock price. A favorable report leads to a significantly higher price than an unfavorable report. However, this is exactly when managers of low-value firms benefit most from reporting upwardly biased earnings. In bad times, on the other hand, incentives are low, because investors correctly anticipate a large number of fraudulent reports and, hence, put little weight on earnings announcements. Our model therefore predicts a higher incidence of earnings manipulation during periods of economic expansion. This prediction seems to be consistent with recent corporate scandals. Many firms (including Enron, WorldCom, and Global Crossing) were found guilty of fraudulent accounting during the stock market boom of the late 1990s.

The dependence of the manager’s manipulation decision on the state of the economy has important implications for the risk premium that market participants demand. Our results show that earnings management can significantly reduce the required return of a stock despite the forces of diversification, because it lowers the correlation between the firm’s cash flows and the market return. Viewing this result from the perspective of the firm, a firm with relatively stronger incentives to engage in manipulation during a stock market boom, compared to its
incentives during a recession, thus faces a lower cost of capital.

6 Appendix

Proof of Lemma 1

For a given extent of manipulation $\kappa_i(F)$, the probability distribution of the signal $r_i$, conditional on the factor realization $F$, can be summarized by:

\[
Pr(r_i = +1 \mid F) = Pr(\theta_{1,i} = +1 \mid F) Pr(r_i = +1 \mid \theta_{1,i} = +1)
+ Pr(\theta_{1,i} = -1 \mid F) Pr(r_i = +1 \mid \theta_{1,i} = -1)
\]

\[= \left(1 + \kappa_i(F) + \beta_i(\delta_i - \kappa_i(F))\right)F/2 \tag{17}\]

\[
\]

Thus, for $\kappa_i^- < \kappa_i^+ < \delta_i$, we have:

\[
E[r_i \mid F = +1] = \kappa_i^+ + \beta_i(\delta_i - \kappa_i^+) > \kappa_i^- - \beta_i(\delta_i - \kappa_i^-) = E[r_i \mid F = -1], \tag{19}\]

for all $i = 1, \ldots, N$. Let $\bar{m}_N(F) = \frac{1}{N} \sum_{i=1}^{N} E[r_i \mid F]$. It follows from (19) that $\bar{m}_N(+1) > \bar{m}_N(-1)$, for all $N = 1, 2, \ldots$. Now define $\bar{r}_N(F) = \frac{1}{N} \sum_{i=1}^{N} r_i$. Since the signals $r_1, r_2, \ldots$ are uniformly bounded and stochastically independent conditional on the factor realization $F$, the law of large numbers holds, and $\bar{r}_N(F)$ converges to $\bar{m}_N(F)$ almost surely as $N$ goes to infinity.\footnote{See, e.g., Feller (1968), chapter X.5.} Thus, $\lim_{N \to \infty} \bar{r}_N(+1) > \lim_{N \to \infty} \bar{r}_N(-1)$. This proves that $\kappa_i^- < \kappa_i^+ < \delta_i$ is a sufficient condition for the signals to fully reveal the factor realization $F$ in the large economy limit.
Proof of Lemma \[2\]

In the large economy limit, the firms’ earnings reports fully reveal the realization of the systematic factor \( F \) (Lemma \[1\]). Thus, the conditional covariance of asset payoffs is zero and the price function defined by equation (8) implies that:

\[
P_{1,i} = E[P_{2,i} | r_1, r_2, \ldots] - \lim_{N \to \infty} \frac{\gamma}{N} \text{Var}[P_{2,i} | r_1, r_2, \ldots] (s_i + s_i^M) \tag{20}
\]

\[
= 2\mu_i + E[\theta_{1,i} + \theta_{2,i} | F, r_i] \sigma_i \tag{21}
\]

Note that conditioning on \( \{r_1, r_2, \ldots\} \) is equivalent to conditioning on \( \{F, r_i\} \), since, conditional on \( F \), the signals \( \{r_1, \ldots, r_{i-1}, r_{i+1}, \ldots\} \) are stochastically independent of \( P_{2,i} \). The expressions for the conditional expectations of \( \theta_{1,i} \) and \( \theta_{2,i} \) in Lemma \[2\] follow directly from the following conditional probabilities:

\[
Pr(\theta_{1,i} = +1 | F, r_i) = \frac{Pr(\theta_{1,i} = +1 | F) Pr(\tilde{r}_i = r_i | \theta_{1,i} = +1)}{Pr(\tilde{r}_i = r_i | F)} \tag{22}
\]

\[
= \frac{(1 + \beta_i F)/2 (1 + \delta_i r_i)/2}{(1 + \beta_i \delta_i F r_i + (1 - \beta_i F) \kappa_i(F) r_i)/2} \tag{23}
\]

\[
Pr(\theta_{2,i} = +1 | F, r_i) = (1 + \beta_i F)^2 / 2 - \xi_i \kappa_i(F), \tag{24}
\]

where we have used the symbol \( \tilde{r}_i \) to explicitly differentiate the random variable \( \tilde{r}_i \) from its realization \( r_i \). \[17\]

Proof of Lemma \[3\]

In the large economy limit, it follows from the price function defined by equation (8) that:

\[
P_{0,i} = E[P_{1,i}] - \lim_{N \to \infty} \frac{\gamma}{N} \sum_{j=1}^{N} \text{Cov}[P_{1,i}, P_{1,j}] s_j \tag{25}
\]

\[17\]If there is no danger of confusion, we use the same symbol to denote a random variable and its realization.
Since $F$ is equally likely to be $+1$ or $-1$, the probability that $\theta_{1,i} = +1$ is equal to $\frac{1}{2}$. The probability that $\theta_{2,i} = +1$ depends on the manager’s manipulation strategy and, hence, on the factor realization $F$:

$$Pr(\theta_{2,i} = +1) = Pr(F = +1) Pr(\theta_{2,i} = +1 \mid F = +1)$$

$$+ Pr(F = -1) Pr(\theta_{2,i} = +1 \mid F = -1)$$

$$= \left(\frac{1}{2}\right)\left((1 + \beta_i)/2 - \xi_i \kappa_i^+\right) + \left(\frac{1}{2}\right)\left((1 - \beta_i)/2 - \xi_i \kappa_i^-\right)$$

$$= \frac{1}{2}(1 - \xi_i(\kappa_i^+ + \kappa_i^-))$$

(26)

Thus, the expected value of $P_{1,i}$ is equal to:

$$E[P_{1,i}] = 2\mu_i + E[\theta_{1,i} + \theta_{2,i}] \sigma_i = 2\mu_i - \xi_i(\kappa_i^+ + \kappa_i^-) \sigma_i$$

(29)

In order to calculate the covariance of $P_{1,i}$ and $P_{1,j}$, we use the fact that:

$$Cov[P_{2,i}, P_{2,j}] = E[Cov[P_{2,i}, P_{2,j} \mid r_1, r_2, \ldots]] + Cov[E[P_{2,i} \mid r_1, r_2, \ldots], E[P_{2,j} \mid r_1, r_2, \ldots]]$$

(30)

Since $P_{1,i} = E[P_{2,i} \mid r_1, r_2, \ldots]$ (Lemma 2) and $Cov[P_{2,i}, P_{2,j} \mid r_1, r_2, \ldots] = 0$ (Lemma 1), we know that $Cov[P_{1,i}, P_{1,j}] = Cov[P_{2,i}, P_{2,j}]$. Tedious but straightforward calculations show that the covariance of $P_{2,i}$ and $P_{2,j}$ is given by:

$$Cov[P_{2,i}, P_{2,j}] = (2\beta_i - \xi_i(\kappa_i^+ - \kappa_i^-))(2\beta_j - \xi_j(\kappa_j^+ - \kappa_j^-))\sigma_i\sigma_j$$

(31)

Substituting the expression for the expected value and the covariance into equation (25) yields the price function specified in Lemma 3.
Proof of Proposition 1

Substituting the price function specified in Lemma 2 into the first-order condition (14) and setting the amount of manipulation expected by investors \( \hat{\kappa}_i(F) \) equal to the actual amount of manipulation \( \kappa_i(F) \) yields the following cubic equation:

\[
G(\kappa_i(F), F) \equiv g_0 + g_1 \kappa_i(F) + g_2 \kappa_i^2(F) + g_3 \kappa_i^3(F) = 0
\] (32)

where:

\[
g_0 = -(1 - \beta_i^2) \delta_i \sigma_i s_i^M
\] (33)

\[
g_1 = (1 - \beta_i^2) \sigma_i s_i^M + (1 - \beta_i^2 \delta_i^2) c_i
\] (34)

\[
g_2 = -2 \beta_i F (1 - \beta_i F) \delta_i c_i
\] (35)

\[
g_3 = -(1 - \beta_i F)^2 c_i
\] (36)

First, note that \( g_0 < 0, g_1 > 0, \) and \( g_3 < 0. \) Thus, independent of the sign of \( g_2 \) (i.e., independent of the factor realization \( F \)), there are always two changes of signs in the sequence of coefficients. Hence, by Descartes' rule of sign, the polynomial \( G(\cdot) \) has at most two positive roots. Furthermore, since \( G(0, +1) = G(0, -1) = g_0 < 0 \) and \( G(\delta_i, +1) = G(\delta_i, -1) = (1 - \delta_i^2) \delta_i c_i > 0, \) it follows that there is a unique positive root in the interval \( (0, \delta_i) \).

It is easily seen from the first-order condition in (14) that the equilibrium extent of manipulation cannot exceed \( \delta_i. \) If \( \kappa_i(F) \) equals \( \delta_i, \) the signal \( r_i \) is pure noise (i.e., \( Pr(r_i = +1 \mid \theta_{1,i} = +1) = Pr(r_i = +1 \mid \theta_{1,i} = -1) \)) and the difference in stock prices on the right-hand side of equation (14) is zero. If \( \kappa_i(F) > \delta_i, \) the difference in prices actually becomes negative, which means that the first-order condition cannot hold.

Thus, in order to prove Proposition 1, we are left to show that \( \kappa_i^+ > \kappa_i^- \). This follows immediately from the fact that \( G(\kappa_i, +1) - G(\kappa_i, -1) = -4 \beta_i (\delta_i - \kappa_i) \kappa_i^2 c_i, \) which is negative.
for all $\kappa_i \in (0, \delta_i)$. Since $G(0, \cdot) < 0$ and $G(\delta_i, \cdot) > 0$, this implies that the unique positive
root of $G(\kappa_i, +1)$ has to exceed the root of $G(\kappa_i, -1)$.

**Proof of Proposition 2**

In order to derive the comparative static results in Proposition 2 it is useful to rewrite the
first-order condition for the optimal amount of manipulation as follows:

$$H(F) \equiv \frac{s_i^M}{2c_i} \Delta P_{1,i}(F) - \kappa_i(F) = 0, \quad (37)$$

where:

$$\Delta P_{1,i}(F) = P_{1,i}(r_i = +1, F) - P_{1,i}(r_i = -1, F)$$

$$= \frac{2(1 - \beta^2)(\delta_i - \kappa_i(F))\sigma_i}{(1 + \kappa_i(F) + \beta_i F(\delta_i - \kappa_i(F))) \left(1 - \kappa_i(F) - \beta_i F(\delta_i - \kappa_i(F))\right)}. \quad (39)$$

First, note that the function $H(F)$, which characterizes the equilibrium amount of manip-
ulation, does not depend on the marginal resource cost $\xi_i$. Furthermore, it immediately
follows from (37) that $\partial H(F)/\partial s_i^M > 0$ and $\partial H(F)/\partial c_i < 0$. Some straightforward algebra
also shows that $\partial \Delta P_{1,i}(F)/\partial \delta_i > 0$, $\partial \Delta P_{1,i}(F)/\partial \sigma_i > 0$, and $\partial \Delta P_{1,i}(F)/\partial \kappa_i(F) < 0$, which
implies that $\partial H(F)/\partial \delta_i > 0$, $\partial H(F)/\partial \sigma_i > 0$, and $\partial H(F)/\partial \kappa_i(F) < 0$. Using the Implicit
Function Theorem, we therefore have:

\[
\frac{d\kappa_i(F)}{ds_i^M} = -\frac{\partial H(F)}{\partial s_i^M} / \frac{\partial H(F)}{\partial \kappa_i(F)} > 0 \quad (40)
\]

\[
\frac{d\kappa_i(F)}{dc_i} = -\frac{\partial H(F)}{\partial c_i} / \frac{\partial H(F)}{\partial \kappa_i(F)} < 0 \quad (41)
\]

\[
\frac{d\kappa_i(F)}{d\delta_i} = -\frac{\partial H(F)}{\partial \delta_i} / \frac{\partial H(F)}{\partial \kappa_i(F)} > 0 \quad (42)
\]

\[
\frac{d\kappa_i(F)}{d\sigma_i} = -\frac{\partial H(F)}{\partial \sigma_i} / \frac{\partial H(F)}{\partial \kappa_i(F)} > 0 \quad (43)
\]

The effect of \(\beta_i\) on the equilibrium amount of manipulation depends on the state of the economy. If \(F = +1\), we have:

\[
\frac{\partial \Delta P_{1,i}(+1)}{\partial \beta_i} \bigg|_{\beta_i=0} = \frac{4\kappa_i^+ (\delta_i - \kappa_i^+)^2 \sigma_i}{(1 - (\kappa_i^+)^2)^2} > 0, \quad (44)
\]

\[
\frac{\partial \Delta P_{1,i}(+1)}{\partial \beta_i} \bigg|_{\beta_i=1} = \frac{-4(\delta_i - \kappa_i^+) \sigma_i}{1 - \delta_i^2} < 0. \quad (45)
\]

Thus, \(\kappa_i^+\) is increasing in \(\beta_i\) for values of \(\beta_i\) close to zero, and decreasing in \(\beta_i\) for values of \(\beta_i\) close to one. In the bad state of the economy \((F = -1)\), we have \(\partial \Delta P_{1,i}(-1)/\partial \beta_i < 0\) for all \(\beta_i \in [0, 1]\), which proves that \(\kappa_i^-\) is decreasing in \(\beta_i\).

**Proof of Proposition 3**

This result follows immediately from the price function \(P_{0,i}\) specified in Lemma 3 and the expected asset payoff \(E[P_{2,i}] = E[P_{1,i}]\), which is given by equation (29).
Proof of Lemma 4

Since $F$ is equally likely to be $+1$ or $-1$, the covariance between $r_i$ and $F$ is given by:

\[
\text{Cov}[r_i, F] = E[r_i F] \tag{46}
\]

\[
= E[E[r_i | F] F] \tag{47}
\]

\[
= E[(\kappa_i(F) + \beta_i(\delta_i - \kappa_i(F)) F)] \tag{48}
\]

\[
= (\kappa_i^+ - \kappa_i^-)/2 + \beta_i(\delta_i - (\kappa_i^+ + \kappa_i^-)/2), \tag{49}
\]

where the third equality follows from the conditional probability $Pr(r_i = +1 | F)$ given in equation (18).

Proof of Proposition 4

The first part of Proposition 4 follows immediately from the fact that:

\[
\text{Cov}[P_{2,i}, F] = \text{Cov}[E[P_{2,i} | F, r_i], F] = \text{Cov}[P_{1,i}, F]. \tag{50}
\]

In order to prove the second part of the proposition, recall that $Pr(\theta_{1,i} = +1 | F) = (1 + \beta_i F)/2$ and that $Pr(\theta_{2,i} = +1 | F) = (1 + \beta_i F)/2 - \xi_i \kappa_i(F)$. Thus, we have:

\[
\text{Cov}[P_{2,i}, F] = E[E[P_{2,i} | F] F] \tag{51}
\]

\[
= E[(2\mu_i + E[\theta_{1,i} + \theta_{2,i} | F] \sigma_i) F] \tag{52}
\]

\[
= E[2(\mu_i + (\beta_i F - \xi_i \kappa_i(F)) \sigma_i) F] \tag{53}
\]

\[
= (2\beta_i - \xi_i(\kappa_i^+ - \kappa_i^-)) \sigma_i. \tag{54}
\]

This shows that the covariance between $P_{1,i}$ and $F$ only depends on the difference between $\kappa_i^+$ and $\kappa_i^-$.  

References


