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“A Measurement Approach to Binary Classifications and Thresholds”

By

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A Measurement Approach to
Binary Classifications and Thresholds

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Abstract

A threshold partitions evidence to a binary classification. This paper studies the design of an optimal threshold and explains one benefit of a binary classification (over the disclosure of evidence). In a model with managers’ opportunistic influence on evidence, a threshold affects not only the ex post use of information by stakeholders but also the ex ante earnings management by managers. The ex ante optimal threshold that balances these two effects is not ex post efficient, creating a time inconsistency problem. By suppressing ex post information, a binary classification serves as a commitment device to implement the ex ante optimal threshold and is overall more useful for stakeholders’ decisions than the disclosure of evidence.

JEL classification: M41, M49, G28, G38

Key Words: Binary Classifications, Thresholds, Earnings Management, Accounting Standard Setting
1 Introduction

Classifications and thresholds are common in accounting. A binary classification uses a threshold to partition raw evidence. For example, an expenditure is recognized as an asset if evidence suggests that a future controllable benefit is probable. Whether an expenditure is recognized as an asset or an expense is a binary classification, the raw evidence is about the probability of the future benefit, and the threshold is "probable." Similarly, a contingent liability is recognized if a future outflow associated with an ongoing law suit is more likely than not; revenue is recognized if the majority of the benefit and risk associated with the product has been transferred to customers. Dyed (2002) provides more examples.

The pervasive use of binary classifications in accounting seems puzzling. After all, by truncating the distribution of raw evidence, a binary classification presumably conveys less information than the disclosure of the entire distribution of raw evidence. In the example above, the probability of an expenditure’s future benefit is typically between 0 and 1, but the accounting rules recognize the entire expenditure as either an asset or an expense. The resulting accounting report is thus coarser than the raw evidence. Such information suppression entailed by a binary classification is obviously costly, but what are its possible benefits?1

Understanding the rationale for binary classifications has immediate implications for accounting standard setting. Binary classifications entail thresholds. When a binary classification is used, how is the optimal threshold determined? How does it respond to changes of the environment?

I analyze these questions in a model in which an accounting report provides information to the firm’s representative stakeholder. The efficiency of her decision decreases in the report’s classification errors. The model’s focus is on the design of an accounting rule that generates the report to maximize the efficiency of the stakeholder’s decision. Specially, I use a two-step representation to model the accounting measurement process, similar to Dye (2002) and Gao (2012). A transaction’s economic substance (or the state of nature) manifests itself in

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1One justification resorts to exogenous transaction costs that makes the full disclosure of raw evidence too costly. These costs include the direct cost of producing and disclosing more disaggregated data, and the indirect cost incurred by users in processing additional information. While these transaction costs are for real and can be substantial at times, this paper explores benefits of a binary classification from a more endogenous source.
evidence and an accounting rule converts the evidence to an accounting report. The main friction in the measurement process is that the manager has both the incentive and ability to engage in earnings management, activities that inflate the evidence stochastically without improving the economic substance. Such non-contractible earnings management is rational for the manager but inefficient for the firm as a whole.

In this setting, we compare two accounting rules: one prescribes a threshold to partition the evidence to a binary report (a classification rule) and the other prescribes disclosure of the evidence (a disclosure rule). When a classification rule is in place, a threshold has dual effects on the report’s classification errors. On one hand, for any given level of earnings management (ex post), a higher threshold reduces the report’s false positive error (classifying a bad state as good) but increases its false negative error (classifying a good state as bad) at the same time. The threshold that maximizes the stakeholder’s decision efficiency for given evidence is defined as the **ex post efficient threshold**. On the other hand, the threshold also affects the manager’s decision to engage in earnings management, which in turn influences the evidence generated and the report’s classification errors.

The threshold’s dual effects differ. The **ex post efficient threshold** does not deter ex ante earnings management optimally. For example, setting the threshold either infinitely high or infinitely low eliminates earnings management but also renders the resulting report useless. As a result, the **ex ante optimal threshold** that balances the dual effects is not ex post efficient. Ex post and ex ante refer to the timing of evidence generation.

That the ex ante optimal threshold is not ex post efficient raises the time inconsistency issue for its implementation and provides one benefit of the classification rule over the disclosure rule. Before the evidence is generated (ex ante), the stakeholder would like to commit to the ex ante optimal threshold in order to balance the thresholds’ dual effects on classification errors; after the evidence is generated (ex post), however, the manager’s earnings management decision has been made and the stakeholder with access to the raw evidence finds it incentive compatible to deviate. The stakeholder’s ex post deviation, rationally anticipated by the manager, unravels the ex ante deterrence on earnings management. By committing to coarsening the report, a classification rule reduces the stakeholder’s ability to negate ex post
and preserves the ex ante optimal balance of the threshold’s dual effects. In other words, compared with disclosure of the raw evidence ex post, a binary classification makes the report more useful to the stakeholder’s decision ex ante.

To help assess the actual approach to accounting standards setting used in practice, the model provides three observable differences between the ex ante optimal and ex post efficient approaches. First, binary classifications are dominated by disclosure of raw evidence under the ex post efficient approach, but not under the ex ante optimal approach. Second, the ex post likelihood ratio of the marginal firm is equal exactly to the relative decision costs of classification errors under the ex post efficient approach, but it is not the case under the ex ante optimal approach. For example, consider a special case with equal decision costs of classification errors. If ex post evidence suggests that the state is more likely to be good than bad, it is recognized as good under the ex post efficient approach, but not necessarily so under the ex ante optimal approach. Finally, the two thresholds respond differently to changes of the environment. For example, as the decision cost of the false positive error increases, the ex post efficient threshold always increases whereas the ex ante optimal threshold could decrease.

The main economic force in this paper, that controlling a manager’s ex ante incentive requires the inefficient use of information ex post, is a recurring theme in the agency literature. As such, this paper could be viewed as an application of the agency theory to accounting standard setting. This paper contributes to the literature by enhancing our understanding of classifications and thresholds, two prominent institutional features of accounting measurement. Other papers that apply the similar insight to explain different accounting measurement issues include, among others, Arya, Glover, and Sivaramakrishnan (1997), Dye and Sridhar (2004) and Kanodia, Singh, and Spero (2005).

This paper’s main antecedent is Dye (2002) who formulates a framework of accounting standard setting that features binary classifications and earnings management. Built on a similar framework, this paper provides a rationale for the pervasive use of binary classifications, which is assumed in Dye (2002). The extension is made possible by new modeling devices this paper explores. Another related paper on threshold choice is Fan and Zhang (2012). In their model, managers could exert private efforts to improve the precision of
evidence. They use the model to provide justification for conservatism.

The legal literature on the burden of proof also deals with issues of setting evidence thresholds (e.g., Kaplow (2011)). In that literature, the manipulation of evidence is often a lesser concern, partly because of the adversary adjudication system. In addition, the evidence threshold works exclusively through the ex ante incentive effect in Kaplow (2011), whereas in this paper, the threshold’s dual effects on ex ante incentive and on ex post efficiency of decision making are the key tension.

Finally, this paper relies on the same two-step representation of accounting measurement formalized in Gao (2012). The representation specifies three instruments a standard setter controls to influence the quality of accounting reports. First, what transaction characteristics (evidence) are admitted to a rule? Second, how much verification is required before transaction characteristics (evidence) are accepted? Finally, what is the threshold that partitions evidence to different classifications? Gao (2012) provides a general rationale for conservatism by focusing on the verification requirement, the second instrument. In this paper, I focus on the third instrument to study binary classifications and thresholds.

The rest of the paper proceeds as follows. Section 2 describes the model. In Section 3, I characterize the ex ante optimal threshold and the ex post efficient threshold in detail, articulate the time inconsistency problem, and explain the benefit of a binary classification over disclosure of raw evidence. Section 4 considers several extensions. Section 5 concludes.

2 The model

The baseline model has four dates, one representative firm consisting of a stakeholder and a manager, and the standard setter. The sequence of events is as follows.

1. At date 0, the standard setter chooses a threshold;

2. At date 1, the manager learns the state of nature and decides whether to engage in earnings management;

3. At date 2, evidence is generated and converted to a report according to the threshold.
Upon receiving the report the stakeholder makes a decision;

4. At date 3, payoffs are realized.

This timeline assumes the use of a classification rule. Later we add disclosure of evidence at date 2. We specify these events along two lines, the supply of accounting reports through the measurement process and the demand for accounting reports.

2.1 The supply of accounting reports through accounting measurement

The state of nature, denoted as $\omega$, is either Good or Bad with probability $q_G$ and $q_B \equiv 1 - q_G$, respectively, i.e., $\omega \in \{G, B\}$. $\omega$, interpreted as a transaction’s economic substance, manifests itself in the transaction’s various characteristics. Denote $t$ as a sufficient statistic for all transaction characteristics with respect to $\omega$. We call $t \in R$ as evidence.

In the absence of the manager’s influence, the mapping from the state to the evidence is captured by the differentiable density and cumulative distribution functions $f^\omega(t)$ and $F^\omega(t)$, respectively. $f^\omega(t)$ satisfies the strict monotonic likelihood ratio property (MLRP). Thus, a higher $t$ is interpreted as strictly better news in the sense of Milgrom (1981).

However, the manager could engage in costly activities (earnings management) to improve evidence $t$ stochastically without improving the state. Specifically, if the manager engages in earnings management, $t$ is drawn from $f^M(t)$ instead of $f^\omega(t)$. $f^M$ is strictly better than $f^B$ but weakly worse than $f^G$ in the sense of MLRP. The manager’s private cost of earnings management is $c > 0$, with differentiable density and cumulative distribution functions $h(c)$ and $H(c)$, respectively. The manager learns both $\omega$ and $c$ at date 1 before earnings management. We examine different assumptions about the timing and technologies of earnings management in Section 4.

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2 One special case of this specification is to add a bias to a random signal. To see this, suppose both $f^B(t)$ and $f^M(t)$ are normally distributed with same variance $\sigma^2$ but different means $B$ and $M$ respectively, $M > B$. Without earnings management, manager $B$ receives evidence $t = B + \tilde{\varepsilon}$, with $\tilde{\varepsilon} \sim N(0, \sigma^2)$. Earnings management enables manager $B$ to receive $t = M + \tilde{\varepsilon}$ instead of $t = B + \tilde{\varepsilon}$. This is equivalent to firm $B$ adding a bias $(M - B)$ into its original signal, that is, $t = B + \tilde{\varepsilon} + (M - B) = M + \tilde{\varepsilon}$.

3 Earnings management could range from outright fabrication of evidence to sophisticated accounting-motivated transactions. In the case of fabricating evidence, $c$ includes the resources spent on falsification and cover-ups and the expected costs associated with its revelation. In the case of accounting-motivated transactions, $c$ could be fees paid to structure the transactions or the efficiency loss of deviating from the otherwise optimal transaction structures.
After evidence $t$ is generated at date 2, it is partitioned to a binary accounting report $r \in \{g, b\}$ according to a threshold $T: r = g$ if $t \geq T$ and $r = b$ if $t < T$. Later we consider the case in which $t$ is directly disclosed to the stakeholder.

This two-step representation, summarized below, relates report $r$ to state $\omega$ indirectly through evidence $t$. The manager can influence the first mapping from the state to the evidence, and the standard setter can design the second mapping from the evidence to the report. Therefore, the rule design interacts with earnings management. Specially, report $r$ can be informative about state $\omega$ but its informativeness is endogenous to the design of the threshold in this model.

\[
\begin{align*}
\omega & \xrightarrow{t} r \\
\text{managers’ influence} & \quad \text{rule design} \\
(\text{earnings management}) & \quad (\text{e.g., evidence threshold})
\end{align*}
\]

We close the model by specifying the demand for accounting report $r$.

### 2.2 The demand for accounting reports

At date 2, the stakeholder makes a binary decision, $a \in \{a_G, a_B\}$, to maximize the firm’s or her payoff. The optimal decision is assumed to be state-contingent. That is, had she known $\omega$, the stakeholder would prefer $a_\omega$ in state $\omega$. Denoting $S^\omega_{a_\omega}$ as the stakeholder’s payoff, which includes the manager’s, we have $L_G \equiv S^G_{a_G} - S^G_{a_B} > 0$ and $L_B \equiv S^B_{a_B} - S^B_{a_G} > 0$. $L_\omega$ is the cost of making wrong decision in state $\omega$. As a result, the stakeholder demands information.

The manager, however, does not always share the stakeholder’s interest. Instead, the manager prefers decision $a_G$ in both states.\(^4\) Denoting the manager’s payoff as $A^\omega_a$, we have $\delta_\omega \equiv A^\omega_{a_G} - A^\omega_{a_B} > 0$.

Finally, at date 0 the standard setter chooses threshold $T$ to maximize the ex ante firm value.\(^5\) Ex ante and ex post refer to the timing of earnings management or date 1. Whether

\(^4\) The alternative assumption that the manager prefers $a_B$ to $a_G$ in both states does not change the main results qualitatively. What is necessary is the existence of the misalignment of the manager’s interest, not the direction per se.

\(^5\) For the purpose of understanding the optimal design of accounting standards, I assume away the political
the manager’s private (direct) cost of earnings management should be included in the standard setter’s objective function has been controversial. Thus, I include it with a different weight $\lambda$, $\lambda \geq 0$. The ex ante firm value is thus defined as the sum of two components: $W(T) = \sum_{r,a,\omega} \Pr(r,\omega)S^\omega_{a(r)} - \lambda C$. $\sum_{r,a,\omega} \Pr(r,\omega)S^\omega_{a(r)}$ is the stakeholder’s payoff (including the manager’s) expected at date 0, $C$, whose endogenous expression is specified later, is the manager’s private cost of earnings management. All the results in the paper hold qualitatively for any $\lambda \geq 0$. We assume that the standard setter’s optimal threshold is interior in equilibrium. This assumption excludes the trivial case in which the report is too noisy to make a difference in the stakeholder’s decision.

An equilibrium consists of a triplet of decisions, the stakeholder’s decision, the manager’s earnings management decision, and the standard setter’s threshold decision, such that all the decisions maximize their respective objective functions and are consistent with each other (rational expectations).

3 The analysis of the model

3.1 The preliminary analysis

The model is solved backwards. At date 2, the stakeholder receives report $r$ and makes decision $a$. Conditional on report $r$, the differential of her expected payoffs with decision $a_G$ versus $a_B$ is

$$\Delta(r) = \sum_{\omega \in \{G,B\}} \Pr(\omega|r)S^\omega_{a_G} - \sum_{\omega \in \{G,B\}} \Pr(\omega|r)S^\omega_{a_B}$$

$$= \Pr(\omega = G|r)(L_G + L_B) - L_B.$$  (1)

It will be confirmed later that $\Pr(\omega = G|r = g) > \Pr(\omega = G|r = b)$, or $\Delta(g) > \Delta(b)$. The assumption that $r$ makes a difference in the stakeholder’s decision means $\Delta(g)\Delta(b) < 0$. economy issues of standards setting. Interested readers are referred to, for example, Bertomeu and Magee (2011).

This assumption is satisfied if $W(T^*) > \max\{W^{FB} - q_GL_G, W^{FB} - q_BL_B\}$. $W(T^*)$ is the firm value in equilibrium with an interior $T^*$ and $\max\{W^{FB} - q_GL_G, W^{FB} - q_BL_B\}$ is the firm value when a corner threshold $T$ is set or equivalently when non-contingent decisions are made.
Therefore, $\Delta(g) > 0 > \Delta(b)$ and the stakeholder’s optimal decision is $a^*(g) = a_G$ and $a^*(b) = a_B$. As a result, the decision-dependent payoffs can be rewritten as report-contingent, that is, $S^\omega_{a^*(r)} = S^\omega_r$ and $A^\omega_{a^*(r)} = A^\omega_r$.

At date 1, the manager decides on earnings management after learning about state $\omega$ and cost $c$. Anticipating the stakeholder’ decision rule at date 2, the manager endogenously prefers $r = g$ to $r = b$. If $\omega = G$, he does not engage in costly earnings management because it does not improve the evidence ($f^M$ is weakly worse than $f^G$). If $\omega = B$, he compares the expected benefit of earnings management with cost $c$. Without earnings management, the probability of receiving $r = g$ is $1 - F^B(T)$; with earnings management, the probability increases by $F^B - F^M$ to $1 - F^M(T)$. Therefore, the manager’s incremental expected benefit from earnings management is $\delta_B(F^B(T) - F^M(T))$. The manager engages in earnings management if and only if $\omega = B$ and $c \leq c^*(T)$, with $c^*(T)$ being determined by

$$\delta_B(F^B(T) - F^M(T)) = c^*(T). \quad (2)$$

At date 0, anticipating $a^*(r)$ and $c^*(T)$, the standard setter chooses threshold $T$. For a given $T$, the probability that the manager engages in earnings management is $q_B H(c^*(T))$. Thus, evidence $t$ is expected to be drawn from $f^G$ with probability $q_G$, from $f^M$ with probability $q_B H(c^*)$, and from $f^B$ with probability $q_B - q_B H(c^*)$.

Based on this knowledge, we could write out the report’s classification errors as functions of threshold $T$ and earnings management $c^*$. There are two types of classification errors: the false negative error and the false positive error. The false negative error is the ex ante probability that the state is good but classified as bad:

$$Q_G(T) \equiv \Pr(\omega = G, r = b) = q_G F^G(T).$$

The false positive error is the ex ante the probability that the state is bad but classified
as good:

\[ Q_B(T; c^*(T)) \equiv \Pr(\omega = B, r = g) = q_B[H(c^*)(1 - F^M) + (1 - H(c^*))(1 - F^B)] \]

\[ = q_B(1 - F^B(T)) + q_B H(c^*)(F^B(T) - F^M(T)). \]

Given \( T \), earning management inflates evidence stochastically, increasing the false positive error by \( q_B H(c^*)(F^B - F^M) \). The manager’s direct cost of earnings management is \( C(c^*(T)) \equiv q_B \int_0^{c^*(T)} c \, dc \).

Collecting these terms, the ex ante firm value can be rewritten as

\[ W(T; c^*(T)) = W^{FB} - L_G Q_G(T) - L_B Q_B(T; c^*(T)) - \lambda C \tag{3} \]

\( W^{FB} \equiv q_G S^G_G + q_B S^B_B \) is the first-best firm value when decisions are perfectly matched to states. The actual firm value is short of the first-best because of the decision costs induced by the classification errors \( (L_G Q_G + L_B Q_B) \) and the direct cost of earnings management \( \lambda C \). Therefore, to understand how threshold \( T \) affects the firm value, we examine how the threshold affects classification errors and earnings management.

**Lemma 1**

1. Given any level of earnings management (ex post), a higher threshold increases the false negative error and reduces the false positive error;

2. A higher threshold first increases and then decreases earnings management;

3. Taking into account its effect on earnings management (ex ante), a higher threshold always increases the false negative error, but does not always reduce the false positive error.

Part 1 is the threshold’s direct effect on classification errors, a familiar trade-off in an ex post decision making framework. For any given level of earnings management (or after evidence \( t \) is produced), a higher threshold makes it more difficult to be recognized as good, which implies a higher false negative error \( (\frac{\partial Q_G(T)}{\partial T} = q_G f^G > 0) \) and a lower false positive error \( (\frac{\partial Q_B(T; c^*(T))}{\partial T} = -q_B(H f^M + (1 - H) f^B) < 0) \) at the same time.
Part 2 involves the threshold’s incentive effect on earnings management ex ante (before evidence is produced). It is proved by differentiating eqn. 2 with respect to \( T \):

\[
\frac{\partial c^*(T)}{\partial T} = \delta_B(f^B(T) - f^M(T)).
\]

(4)

Defining \( \hat{T} \) as \( \frac{f^M(T)}{f^B(T)} \bigg|_{T = \hat{T}} = 1 \). Strict MLRP implies \( f^B(T) - f^M(T) > 0 \) if and only if \( T < \hat{T} \). The intuition that raising the threshold could increase earnings management can be demonstrated clearly by looking at two extremes. On one hand, if the threshold is infinitely high, costly earnings management does not help to obtain \( r = g \) and thus does not occur. On the other hand, if the threshold is infinitely low, \( r = g \) is obtained without costly earnings management and thus no earnings management occurs either. Thus, whenever earnings management exists, it is not monotonic in \( T \).

Because earnings management increases the false positive error, the threshold has an indirect effect on classification errors via earning management.

Comparing the first two parts, part 3 of Lemma 1 states that the threshold’s dual effects on the classification errors differ both quantitatively and qualitatively. For example, consider its total effects on the false positive error:

\[
\frac{\partial Q_B(T; c^*(T))}{\partial T} = \frac{\partial Q_B(T; c^*(T))}{\partial T} + \frac{\partial Q_B(T; c^*)}{\partial c^*} \frac{\partial c^*(T)}{\partial T}.
\]

The two components, the direct effect of \( \frac{\partial Q_B(T; c^*(T))}{\partial T} \) and the indirect effect of \( \frac{\partial Q_B(T; c^*)}{\partial c^*} \frac{\partial c^*(T)}{\partial T} \), are not only different in magnitude but also can have different signs. The direct effect is always negative (part 1) but the indirect effect could be either positive or negative (part 2). Most results of the paper rely only on the fact that these two effects are not the same. The possibility of their different signs generates some interesting auxiliary results.

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7A more illustrative way to see the intuition of Part 2 is to examine the manager’s earnings management decision more closely. On one hand, with earnings management the probability that the manager of \( \omega = B \) receives \( r = g \) is \( 1 - F^M(T) \), which decreases in \( T \) at rate \( f^M \). For this effect a higher \( T \) reduces earnings management because it diminishes the benefit of receiving \( t \) from \( f^M \). On the other hand, without earnings management the probability that the manager of \( \omega = B \) receives \( r = g \) is \( 1 - F^B(T) \), which decreases in \( T \) at rate \( f^B \). For this second effect a higher \( T \) increases earnings management because it raises the "opportunity cost" of not engaging in earnings management. The net effect is thus determined by the rates of these two effects, or the sign of \( f^B(T) - f^M(T) \).
3.2 The ex ante optimal threshold

To determine the optimal threshold $T^*$, we differentiate the firm value $W(T; c^*(T))$ (eqn.3) with respect to $T$. The first order condition could be expressed as

$$- L_G \frac{\partial Q_G(T)}{\partial T} - L_B \frac{\partial Q_B(T; c^*)}{\partial T} - L_B \frac{\partial Q_B(T; c^*) \partial c^*(T)}{\partial T} - \lambda \frac{\partial C(c^*)}{\partial c^*} \frac{\partial c^*(T)}{\partial T} = 0. \quad (5)$$

It suggests that the ex ante optimal threshold $T^*$ is determined by balancing its dual effects on ex post classification errors and on ex ante earnings management. First, for given evidence $t$ or equivalently after earnings management $c^*$, the threshold affects the ex post classification errors, $Q_G$ and $Q_B$, which in turn affects the firm value (via the stakeholder’s decision). This channel is captured by the first two terms in eqn.5. Second, the threshold also influences the manager’s decision of earnings management before the evidence is generated, which in turn affects the classification errors and eventually the firm value. This channel is captured by the last two terms in eqn.5.

Collecting the above results, we characterize the equilibrium with the optimal classification rule.

**Proposition 1 (The Classification Equilibrium)** When the accounting rule prescribes a binary classification, in equilibrium,

1. the stakeholder chooses $a^*(r = g) = a_G$ and $a^*(r = b) = a_B$;
2. the manager engages in earnings management if and only if $\omega = B$ and $c \leq c^*(T^*)$. $c^*(T^*) = \delta_B(F^B(T^*) - F^M(T^*))$;
3. the optimal threshold set by the standard setter, $T^*$, satisfies the first order condition (eqn.5).

The proof is outlined in the previous subsection and detailed in Appendix. The firm value in this equilibrium can be calculated as $W(T^*; c^*(T^*))$.

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*With the general distribution functions, the firm value $W$ is not necessarily concave in $T$. As a result, the first order condition is not sufficient. None of the results in the paper, however, relies on the first order condition being sufficient.*
Before turning to the analysis of disclosing raw evidence at date 2, we discuss one important property of the ex ante optimal threshold $T^*$: the ex post likelihood ratio at $T^*$. By applying the Bayes rule and the first order condition for $T^*$ (eqn.5), we could write the ex post likelihood ratio as:

$$\frac{\Pr(\omega = G|t = T)}{\Pr(\omega = B|t = T)}\bigg|_{T=T^*} = \frac{L_B}{L_G} + I(T^*)$$

$$(6)$$

$I(T^*) \equiv \left(\frac{L_B}{L_G} \frac{\partial Q_B(T^*)}{\partial T} + \lambda \frac{\partial C(\omega^*)}{\partial T} \right) \frac{\partial c^*(T)}{\partial T} \bigg|_{T=T^*}$ is not equal to 0 in general (except at one point of $\frac{\partial c^*(T)}{\partial T} \big|_{T=T^*} = 0$ or $T^* = \hat{T}$, which has zero measure).

**Corollary 1** When the threshold is chosen to maximize the ex ante firm value, the ex post likelihood ratio of the marginal firm, the firm with $t = T^*$, differs from $\frac{L_B}{L_G}$ in general.

Corollary 1 means that the binary classification created by $T^*$ does not utilize ex post evidence efficiently. To illustrate, consider the special case of equal costs of classification errors, i.e., $\frac{L_B}{L_G} = 1$. In this case, Corollary 1 states that a firm can be recognized as bad even if ex post evidence $t$ suggests that it is more likely to be good than bad. This property is the key intuition behind the rationale for binary classifications we study later.

Its intuition lies in the threshold’s dual effects. The threshold affects not only the use of evidence ex post but also the generation of evidence ex ante via earnings management. The ex ante optimal threshold, which balances the dual effects, thus does not utilize ex post evidence efficiently. This intuition is borne out in the proof because $I(T^*) \neq 0$ is caused by $\frac{\partial c^*(T)}{\partial T} \big|_{T=T^*} \neq 0$.

### 3.3 The implementation of $T^*$, the time inconsistency problem, and the benefit of a binary classification

This section explains one benefit of a binary classification by showing two results. First, disclosure of evidence $t$ at date 2 induces the stakeholder and the manager to deviate from their actions prescribed in the classification equilibrium in Proposition 1. Second, the deviation lowers the ex ante firm value. Because this time inconsistency problem is caused by
the disclosure of evidence at date 2, the binary classification rule, by suppressing ex post probabilistic details of the evidence, has an advantage over the disclosure rule.

First, with disclosure of evidence \( t \) at date 2, the stakeholder has incentive to deviate from the classification equilibrium. To see this, consider her decision after receiving both \( t \) and \( r \) at date 2, assuming that the standard setter and the manager have followed their respective equilibrium decisions \((T^*, c^*(T^*))\). The stakeholder follows the same decision making process as in the baseline model except that her information set is expanded from \( r \) to \( t \). Because \( t \) conveys strictly more information than \( r \), \( r \) is redundant for her decision making and thus omitted from her information set. Conditional on \( t \), the differential of her expected payoffs with decision \( a_G \) versus \( a_B \) is:

\[
\Delta(t) = \sum_{\omega \in \{G, B\}} \Pr(\omega | t) S^\omega_{a_G} - \sum_{\omega \in \{G, B\}} \Pr(\omega | t) S^\omega_{a_B} = \Pr(\omega = G | t)(L_G + L_B) - L_B \tag{7}
\]

At date 2, the stakeholder treats the manager’s earnings management decision as given because it was made at date 1. For any given level of earnings management \( c^* \), \( \Delta(t) \) increases monotonically in \( t \) by MLRP. Thus, the stakeholder’s optimal decision follows a cut-off rule: choose \( a_G \) if and only if \( t \geq t_0 \) with \( \Delta(t_0) = 0 \). By Corollary 1, we know that \( \Delta(t = T^*) \neq 0 \) in general. Therefore, \( t_0 \neq T^* \) and the stakeholder deviates from her equilibrium action \( a^*(r) \). For example, consider the case \( t_0 < T^* \). For \( t \in (t_0, T^*) \), the equilibrium action for the stakeholder is \( a_B \) (because \( r = b \)) but she prefers \( a_G \) because of \( \Delta(t) > \Delta(t_0) = 0 \). Anticipating the stakeholder’s deviation, the manager adjusts his decision at date 1 because his payoff depends on the stakeholder’s decision, not on the report per se. As a result, the classification equilibrium in Proposition 1 unravels.

**Proposition 2 (The Disclosure Equilibrium)** If the accounting rule prescribes disclosure of evidence \( t \) at date 2, in equilibrium,

1. the stakeholder chooses \( a^*(t \geq t_0^*) = a_G \) and \( a^*(t < t_0^*) = a_B \). \( t_0^* \) is uniquely determined

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9By the Bayes rule, \( \Pr(\omega = G | t; c^*) = \frac{q_G f^G + q_B (1-H(c^*))f^B}{q_G f^G + q_B (1-H(c^*))f^B + (1-H(c^*))f^M} = \frac{q_G f^G f^G + (1-H(c^*))f^B f^G}{q_G f^G f^G + (1-H(c^*))f^B f^G + (1-H(c^*))f^M f^G} \). For any given \( c^* \), both \( \frac{f^M}{f^G} \) and \( \frac{f^B}{f^G} \) decrease in \( t \) by MLRP. Thus, \( \Pr(\omega = G | t; c^*) \) increases in \( t \).
by \( \frac{\Pr(\omega = G|t)}{\Pr(\omega = B|t)} | t = t_0 = \frac{L_B}{L_G}. \)

2. the manager engages in earnings management if and only if \( \omega = B \) and \( c \leq c^*(t_0^*). \)

\[ c^*(t_0^*) = \delta_B(F^B(t_0^*) - F^M(t_0^*)). \]

3. the standard setter’s choice of threshold \( T \) is irrelevant.

Proposition 2 confirms that, with disclosure of evidence \( t \) at date 2, both the stakeholder and the manager deviate from their actions prescribed in the classification equilibrium in Proposition 1. Furthermore, part 3 confirms the puzzle of the pervasive use of binary classifications: the problem of designing the threshold and the manager’s incentive to manipulate the report are both induced by the use of a binary classification rule in the first place. If instead disclosure of evidence \( t \) is made at date 2, the stakeholder seems to receive strictly more information, the manager’s incentive to manipulate accounting report \( r \) disappears, and the standard setter’s choice of threshold becomes redundant. Thus, one might be led to conclude that we have created our own problems by insisting on using a binary classification rule that suppresses ex post information and that the solution is to switch from the classification rule to the disclosure rule. However, the next proposition shows that, despite its problems, the classification equilibrium in Proposition 1 generates a higher ex ante firm value than the disclosure equilibrium in Proposition 2.

**Proposition 3** The ex ante firm value in the classification equilibrium is higher than that in the disclosure equilibrium. It is strictly so if \( T^* \neq t_0^* \).

Proposition 3, together with Proposition 2, completes the time inconsistency problem. The stakeholder prefers the classification equilibrium at date 0 but finds it incentive compatible to deviate to the disclosure equilibrium at date 2. Because the ex post deviation is made possible by the access to evidence \( t \), a binary classification mitigates the time inconsistency problem exactly because of its feature of suppressing probabilistic details of the evidence ex post. We first prove Proposition 3 and then provide more intuition. The proof utilizes the following lemma.
Lemma 2 The same ex ante firm value in the disclosure equilibrium can be achieved with the following accounting rule: sets $T = t_0^*$ at date 0 and provides no disclosure of evidence at date 2.

Lemma 2 is verified by noting two things. First, the ex ante firm value is determined ultimately by the decisions of the stakeholder and the manager. Second, these decisions under the proposed accounting rule are the same as in the disclosure equilibrium. Specifically, with $T = t_0^*$ and no disclosure, the stakeholder chooses $a_G$ if and only if $r = g$, which is equivalent to $t \geq T = t_0^*$. This is the same decision rule as in the disclosure equilibrium. Moreover, the manager’s earnings management decision is the same as in the disclosure equilibrium because it takes $t_0^*$ as its sole input.

Lemma 2 transforms the disclosure equilibrium to an equivalent classification equilibrium with a different threshold of $\tilde{T} = t_0^*$. With Lemma 2, Proposition 3 becomes self-evident. In a classification equilibrium, $\tilde{T}$ is a feasible choice for the standard setter but not chosen at date 0. Thus, by revealed preference, $T^*$ generates a higher ex ante firm value than $\tilde{T}$ does, which proves Proposition 3.

To provide more intuition for the nature of the time inconsistency problem and thus the rationale for a binary classification, we compare the two classification equilibria with $\tilde{T}$ and $T^*$, respectively.

Corollary 2 Comparing two threshold choices $\tilde{T}$ and $T^*$ in their respective equilibria,

1. for the same level of earnings management $c^*$, the stakeholder’s decision is more efficient under $\tilde{T}$ than under $T^*$;

2. earnings management is higher under $\tilde{T}$ than under $T^*$.

Part 1 rephrases the previous result that the stakeholder would like to deviate from the ex ante optimal threshold $T^*$ to the ex post efficient cut-off $t_0^*$ at date 2. We label $\tilde{T}$ (or equivalently $t_0^*$) as the ex post efficient threshold because it maximizes the efficiency of the stakeholder’s decision ex post (by Lemma 2). That is, $\tilde{T}$ can be rewritten as

$$\tilde{T} \equiv \arg\max_{t_0} \quad W^{FB} - Q_G(t_0)L_G - Q_B(t_0; c^*(t_0^*))L_B$$
$c^*(t_0)$ is treated as given by the stakeholder at date 2. Part 2 of Corollary 2 reveals the cost of the ex post deviation from $T^*$ to $\bar{T}$: it exacerbates ex ante earnings management.

The intuition of both results goes back to the thresholds’ dual effects. For the purpose of decision making at date 2, $\bar{T}$ is the ex post efficient threshold. For the purpose of curtailing earnings management at date 1, $T = \infty$ or $T = -\infty$ completely eliminates earnings management. The ex ante optimal threshold $T^*$ balances these two effects and thus differs from either benchmark. By moving away from $\bar{T}$, $T^*$ reduces the ex post efficiency of the stakeholder’s decision but at the same time provides better incentive regarding earnings management. The first order condition for $T^*$ (eqn.5) makes the optimal trade-off of these two effects.

3.4 The ex ante versus ex post approaches to accounting standards setting

The difference between $\bar{T}$ and $T^*$ makes it important to assess which approach, the ex ante optimal approach or the ex post efficient approach, prevails in practice. To assist this task, we summarize and contrast three observable predictions about the properties of $\bar{T}$ and $T^*$ in this subsection.

First, a binary classification can only be justified under the ex ante optimal approach. Under the ex post efficient approach, it is always (weakly) optimal to replace a binary classification with disclosure of raw evidence. Thus, the pervasive use of binary classifications in accounting is indicative of the use of the ex ante optimal approach.

Second, the ex post likelihood ratio for the marginal firm at $\bar{T}$ is solely determined by the relative costs of classification errors (part 1 of Proposition 2), but its counterpart at $T^*$ is not (Corollary 1). Consider again the special case of $L_B = L_G$. If the ex post evidence suggests that the probability of the occurrence of the good state is larger than 50%, it is recognized as good under the ex post efficient approach, but not necessarily so under the ex ante optimal approach. Most revenue recognition rules seem to require that ex post likelihood ratios be much higher than 50%. More empirical investigations are required to understand whether this excess can be fully explained by the ratio of $\frac{L_B}{L_G}$. If not, then the ex ante optimal approach might be responsible.

Finally, we examine the equilibrium responses of $\bar{T}$ and $T^*$ as parameters in the model.
Corollary 3 In their respective equilibria, $\bar{T}$ and $T^*$ have the following properties:

1. As $L_G$ increases, both $\bar{T}$ and $T^*$ decrease;

2. As $L_B$ increases, $\bar{T}$ increases but $T^*$ may decrease;

3. As $\delta_B$ increases, $\bar{T}$ increases if and only if $\frac{\partial c^*(T)}{\partial T}\bigg|_{T=\bar{T}} < 0$ but $T^*$ increases if and only if $(2h + c^*h')\frac{\partial c^*(T)}{\partial T}\bigg|_{T=T^*} < 0$.

Corollary 3 states that the policy responses under the two approaches are qualitatively different. Thus, intuition gleaned from the ex post efficient approach provides limited guidance for setting the ex ante optimal threshold.

The intuition of Corollary 3 lies again in the threshold’s dual effects. Recall that part 1 of Lemma 1 suggests that a higher threshold increases the false negative error and reduces the false positive error ex post (after evidence is generated). Thus, $\bar{T}$ increases as the false negative error becomes less costly (a lower $L_G$) or the false positive error becomes more costly (a higher $L_B$). In contrast, part 3 of Lemma 1 suggests that a higher threshold does not always reduce the false positive error ex ante (before evidence is generated). As a result, $T^*$ may decrease as $L_B$ increases. Similarly, the ex ante optimal approach indicates a more fine-tuned response to an increase in the benefit of earnings management $\delta_B$. In particular, the shape of the cost function of earnings management, $h$ and $h'$, shows up in the equilibrium response of $T^*$, but not of $\bar{T}$. This attests to the cause of the difference under the two approaches because $h$ and $h'$ affects the marginal behavior of earnings management.

3.5 Discussion

To compare actual accounting standards with the predictions of a model, it is necessary to provide a theoretical representation of the actual accounting standards we observe. Of course, any such formalization is bound to be incomplete due to the enormous complexity of accounting standards in practice. With this caveat, we discuss the following representation as an explorative attempt.
As discussed in the introduction, one recurring theme in many accounting standards seems to involve the following consideration: given the evidence, does the probability that the good state has occurred exceed some threshold level \( p \in [0, 1] \)? For example, an expenditure is recognized as an asset if the evidence suggests that it is probable that the expenditure will generate a future benefit controlled by the firm. In this case, the state is whether the expenditure is "good" (will generate a future benefit or \( \omega = G \)) or "bad" (will not generate a future benefit or \( \omega = B \)). The evidence is \( t \). "Probable" implies a requisite threshold level \( p \). Thus, the above recognition rule may be formalized as follows:

\[
    r(t) = g \quad \text{if and only if} \quad \Pr(\omega = G|t) \geq p.
\]

This formulation is equivalent to the binary classification rule in the model. That is, the choice of \( T \) can be replicated by the choice of \( p \). To see this, we rewrite \( \Pr(\omega = G|t) \geq p \) in terms of ex post likelihood ratio \( \frac{\Pr(\omega = G|t)}{\Pr(\omega = B|t)} \geq \frac{p}{1-p} \). Because \( \frac{\Pr(\omega = G|t)}{\Pr(\omega = B|t)} \) is increasing in \( t \), the marginal firm at a threshold \( T \) satisfies \( \frac{\Pr(\omega = G|t = T)}{\Pr(\omega = B|t = T)} = \frac{p}{1-p} \). Thus, there is a one-to-one correspondence between \( T \) and \( p \). Raising \( T \) implies a higher \( p \). This re-formulation of the classification rule helps interpret some results in the literature.

First, in the three examples discussed in the introduction, the thresholds in terms of \( p \) are "probable", "more likely than not" and "the majority," respectively. These terms imply that \( p \) is interpreted as 50\% (or \( \frac{p}{1-p} = 50\% \)) in practice.

Second, since Demski (1973) and Demski (1974), a large literature has pointed out that the value of information depends on the specifics of the economic problem that demands the information. In our setting, the costs of classification errors are \( L_B \) and \( L_G \), respectively. Thus, \( p \) should be determined by the ratio of \( \frac{L_B}{L_G} = \frac{p}{1-p} \), not be constant at 50\%. This is equivalent to setting \( T \) at \( T^{**} \) such that \( \frac{\Pr(\omega = G|t = T^{**})}{\Pr(\omega = B|t = T^{**})} = \frac{p}{1-p} = \frac{L_B}{L_G} \). Thus, this literature implies \( T^{**} = \tilde{T} \).

Finally, this paper suggests that when we explicitly model the manager’s influence on evidence, the optimal \( p \) is set at \( \frac{L_B}{L_G} + I(T^{**}) = \frac{p}{1-p} \). It takes into account not only \( \frac{L_B}{L_G} \) but also the incentive factor \( I(T^{**}) \equiv \left[ \frac{L_B \partial Q_B(T, c^*)}{L_G \partial Q_G(T, c^*)} + \lambda \frac{\partial C_c(T, c^*)}{\partial T} \right] \partial c^*(T)|_{T=T^{**}} \). Equivalently, \( T^{**} = T^* \).
which differs from $\tilde{T}$.

## 4 Extensions

In the baseline model, it is the presence of the threshold’s influence on earnings management that generates the difference between the ex ante optimal and ex post efficient thresholds, which in turn provides the benefit of the classification rule over the disclosure rule. To make the interplay most transparent, the baseline model has used a particular specification of earnings management in a two-state, two-decision framework. This section considers four extensions to the baseline model. The first three, regarding the micro-foundation of the stakeholder’s decision, different timing and technologies of earnings management, and a continuous decision space, do not change the main result that the classification rule generates a higher ex ante firm value than the disclosure rule. The last one, which considers a continuous decision space, makes the dominance of the classification rule conditional on parameters.

### 4.1 Endogenous incentive for earnings management

The managers’ incentive for earnings management has been captured exogenously as $\delta_\omega$. One benefit of using a reduced-form $\delta_\omega$ is that it focuses us on the design of optimal accounting standards that generate accounting reports, as opposed to the design of the optimal use of given accounting reports, as a direct response to managers’ opportunistic earnings management. Another benefit is that it makes the model’s results applicable to a broader setting characterized by $\delta_\omega$ and $L_\omega$. In this subsection, we provide a "micro-foundation" for $\delta_\omega$ and $L_\omega$ by modeling explicitly the stakeholder’s decision in a capital market setting.

The timeline is the same as before. The stakeholder is specified as investors and the manager as the entrepreneur. At date 2 after report $r$ is provided, the entrepreneur sells his project to investors at price $A_r$ for life-cycle reasons. Investors’ decision $a$ is whether to finance the project or not.

The project requires an initial investment $I$ and its payout at date 3 depends on its quality $\omega$. If $\omega = G$, it pays out $Y$ with probability $\gamma$ and 0 with probability $1 - \gamma$. If $\omega = B$, it pays
out 0 with probability 1. We assume that \( \gamma Y > I > q_G \gamma Y \). \( \gamma Y > I \) means that financing the project \((a_G)\) is a positive NPV decision if and only if \( \omega = G \). The convenient assumption of \( I > q_G \gamma Y \) means that it is optimal not to invest in the project without information. The payoffs to investors are thus determined by both state \( \omega \) and action \( a : S^G_{a_G} = \gamma Y - I, S^B_{a_B} = -I \), and \( S^G_{a_B} = S^B_{a_G} = 0 \). It is confirmed that \( L_G \equiv S^G_{a_G} - S^G_{a_B} = \gamma Y - I > 0 \) and \( L_B \equiv S^B_{a_B} - S^B_{a_G} = I > 0 \), as assumed in the baseline model. \( L_G \) is the cost of not financing a good project and \( L_B \) is the cost of financing a bad project.

The entrepreneur’s payoff \( A_r \) is endogenously determined. As standard, we assume that investors face competitive capital market, expect zero surplus, and price the project accordingly. As before, we assume that the accounting report is sufficiently informative to influence investors’ decision in equilibrium, that is, \( \Pr(\omega = G | r = g; c^{**}) \gamma Y > I > \Pr(\omega = G | r = b; c^{**}) \gamma Y \). \( c^{**} \) is the equilibrium level of earnings management. Thus, investors finance the project if and only if \( r = g \) and the equilibrium prices are determined as

\[
A_g = \Pr(\omega = G | r = g; c^{**}) \gamma Y - I,
\]

\[
A_b = \max\{0, \Pr(\omega = G | r = b; c^{**}) \gamma Y - I\} = 0.
\]

\( A_r \) does not depend directly on \( \omega \). Anticipating \( A_r \), the entrepreneur prefers \( r = g \) to \( r = b \) because \( \delta_B(T; c^{**}) = A_g - A_b > 0 \). Thus, the incentive for earnings management \( \delta_B \) is endogenized from the interactions in the capital market. The rest of the model’s setup is the same as in the baseline model.

This new setting preserves the threshold’s ex ante incentive effect. Specifically, the entrepreneur engages in earnings management in equilibrium if and only if \( \omega = B \) and his cost \( c \leq c^{**}(T) \),

\[
\delta_B(T; c^{**}(T))(F^B(T) - F^M(T)) = c^{**}(T).
\] (8)

Differentiating the equation with respect to \( T \), we have

\[
\frac{\delta_B(f^B_f^M)}{f^M} + \left( \frac{\partial \delta_B}{\partial T} + \frac{\partial \delta_B}{\partial c^{**}} \frac{\partial c^{**}}{\partial T} \right)(F^B - F^M) = \frac{\partial c^{**}}{\partial T}.
\]
The threshold affects earnings management through two channels. The first channel, \( \delta_B(f^B(T) - f^M(T)) \), is the same as in the baseline model. The threshold directly affects earnings management because it changes the classification of evidence the entrepreneur attempts to influence. The new channel, \( (\frac{\partial \delta_B}{\partial T} + \frac{\partial \delta_B}{\partial c^*} \frac{\partial c^*(T)}{\partial T})(F^B - F^M) \), represents an "indirect" effect of the threshold on earnings management. The threshold changes the information content of report \( r \), which affects investors’ pricing decision \( A_r \), which in turn influence the entrepreneur’s incentive for earnings management. This channel can be signed unambiguously as positive.\(^\text{10}\) Thus, the net effect of the threshold on earnings management in equilibrium, \( \frac{\partial c^*(T)}{\partial T} \), could be rewritten as

\[
\frac{\partial c^*(T)}{\partial T}(1 - \frac{\partial \delta_B}{\partial c^*}(F^B - F^M)) = \delta_B(f^B - f^M) + \frac{\partial \delta_B}{\partial T}(F^B - F^M). \tag{9}
\]

The main results about the difference between the ex ante optima threshold \( T^* \) and the ex post efficient threshold \( \tilde{T} \) are preserved because the threshold’s ex ante incentive effect is still present (\( \frac{\partial c^*(T)}{\partial T} \neq 0 \) in eqn.9). For example, the new first order condition for the ex ante optimal threshold \( T^{**} \) is similar to that for \( T^* \) in eqn.5:

\[
-L_G \frac{\partial Q_G(T)}{\partial T} - L_B \frac{\partial Q_B(T; c^{**}(T))}{\partial T} - L_B \frac{\partial Q_B(T; c^{**})}{\partial c^{**}} \frac{\partial c^{**}(T)}{\partial T} - \lambda \frac{\partial C(c^{**})}{\partial c^{**}} \frac{\partial c^{**}(T)}{\partial T} = 0.
\]

The first two terms capture the threshold’s effect on ex post decision efficiency and the last two reflect the threshold’s effect on ex ante incentive for earning management. Compared with eqn.5, the only difference is that \( \frac{\partial c^*(T)}{\partial T} \) is more complicated with the indirect term described above.

4.2 Different specifications of earnings management

This section considers the different timings and technologies of earnings management. First, we consider different timings of earnings management. Managers could influence transaction characteristics at different stages of a transaction. In the baseline model, managers makes

\(^{10}\) These two channels help understand one important difference between this model and Dye (2002). Loosely speaking, the threshold affects earnings management through the direct channel in my model while through the indirect channel in Dye (2002).
the choice after $\omega$ is privately observed but before $t$ materializes. We consider two other possible timings for earnings management. The case in which the manager makes the choice before observing state $\omega$ is simple. To simplify the analysis in this case we set $f^M = f^G$. The cost of earnings management is still $c$ and the benefit is still to receive $t$ from $f^M$ instead of $f^B$ in case of $\omega = B$. The only change is that the manager spends $c$ in advance, which becomes a waste in case of $\omega = G$. Thus, the condition for earnings management is the same as in eqn.2 except that $q_B$, the probability of $\omega = B$, is added to the left hand side. No results are qualitatively affected by this scaling of $c^e$.

The more complicated case is when the manager makes the decision after observing $t$, as in Dye (2002). If $t > T$, earnings management is not necessary. If $t < T$, the benefit of inflating $t$ to $T$ depends on the manager’s knowledge about the state and the realization of $t$. For simplicity, we assume that $\delta = \delta_B = \delta_G$, as in the case of the capital market setting in Section 4.1. We also assume that it costs the manager a unit cost of $c$ to add $T - t$ to $t$. Thus, the manager engages in earnings management if and only if $t \in (\bar{t}(T), T)$, with $\bar{t}(T)$ being determined by $\delta = (T - \bar{t})c$. The ex ante probability of earning management (expected at date 0) is

$$
\tau(T) = \int_{\bar{t}(T)}^{T} (q_G f^G(t) + q_B f^B(t)) dt = q_G (F^G(T) - F^G(\bar{t}(T))) + q_B (F^B(T) - F^B(\bar{t}(T))).
$$

$$
\frac{\partial \tau(T)}{\partial T} = q_G (f^G(T) - f^G(\bar{t})) + q_B (f^B(T) - f^B(\bar{t})).
$$

The threshold’s ex ante incentive effect is still preserved because $\frac{\partial \tau(T)}{\partial T} \neq 0$ for general distributions of $f^G$ and $f^B$.

Second, we consider a different technology of earnings management. In the baseline model, managers either engage in earnings management or not. We consider another commonly used modeling device of earnings management in which the earnings management choice is continuous. Instead of deciding whether to engage in earnings management, the manager can increase the probability of receiving $t$ from $f^M$, instead of from $f^\omega$, by $\beta$ at cost $K(\beta)$, $\beta \in [0, 1]$. $K(\beta)$ has the standard properties of a cost function: it is increasing and convex.
with \( K(0) = K'(0) = 0 \), and \( K'(1) \) is sufficiently large. For example, the standard quadratic cost function \( K(β) = \frac{kβ^2}{2} \) with \( k \) properly restricted satisfies these assumptions.

This new setting preserves the threshold’s non-monotonic ex ante incentive function. For any given \( T \), the manager of \( \omega = B \) chooses \( β \) to maximize \( \beta((1 - F^M(T))A^B_g + F^M(T)A^B_b) + (1 - \beta)((1 - F^B(T))A^B_g + F^B(T)A^B_b) \). Recall \( δ_B = A^B_g - A^B_b \). The first order condition for \( β \) is

\[
δ_B(F^M(T) - F^B(T)) = K'(β^*(T)).
\]

**Lemma 3** A higher threshold first increases then decreases earnings management. Specifically,

\[
\frac{∂β^*(T)}{∂T} = \frac{δ_B}{K'}(f^B(T) - f^M(T)).
\]

Thus, the threshold’s ex ante incentive effect is qualitatively the same as in the baseline model. As a result, the main results in the baseline model can be replicated with little modification.

Finally, we consider the possibility of "good earnings management." In the baseline model, the assumption of \( f^M \) being inferior to \( f^G \) in the sense of MLRP leads to the result that managers engage in earnings management only in the bad state. As a result, it is unambiguous that earnings management increases classification errors and reduces the efficiency of the stakeholder’s decision. We relax this assumption and considers the case in which there is room for managers to improve the evidence in the good state as well. As a result, earnings management has ambiguous effects on the report’s informativeness and the efficiency of the stakeholder’s decision.

Suppose at date 1 after \( ω \) is observed, if a manager of \( ω \) engages in earnings management, \( t \) is drawn from \( f^M_ω(t) \) instead of \( f^ω(t) \). The baseline model is the special case in which \( f^{MB} = f^{MG} = f^M \) with \( f^M \) being strictly better than \( f^B \) but weakly worse than \( f^G \) in the sense of MLRP. In this extension, we assume that \( f^{MB} \) is strictly better than \( f^B \) and \( f^{MG} \) is strictly better than \( f^G \) in the sense of MLRP. Moreover, the private cost of earnings manager is \( c \), with differentiable density and cumulative distribution functions \( h_ω(c) \) and \( H_ω(c) \), respectively. In this new setting, earnings management by a manager of \( ω \) is characterized by \( c_ω^* \):
\[ \delta_\omega(F^\omega(T) - F^{M_\omega}(T)) = c^* \]

The false positive error \( Q_B(T; c_B^*(T)) \) is the same as in the baseline model but the false negative error becomes parallel to \( Q_B(T; c_B^*(T)) \):

\[ Q_G(T; c_G^*(T)) = q_G[H_GF^{M_G} + (1 - H_G)F^G] = q_GF^G - q_GH_G(c_G^*)(F^G - F^{M_G}). \]

Earnings management increases the false positive error in the bad state but reduces the false negative error in the good state. In this sense, earnings management could be good: it improves the report’s informativeness in the good state. The threshold’s ex ante incentive effect is captured similarly by

\[ \frac{\partial c_G^*(T)}{\partial T} = \delta_\omega(F^\omega(T) - F^{M_\omega}(T)). \]

The additional effect of \( \frac{\partial c_G^*(T)}{\partial T} \) does not change the presence of the threshold’s ex ante incentive effect on earnings management. As a result, the main result that the ex ante optimal threshold \( (T^*) \) differs from the ex post efficient threshold \( (\tilde{T}) \) is preserved, even though the levels of \( T^* \) and \( \tilde{T} \) will change.

### 4.3 Continuous state space

In the baseline model, the state space is binary. This assumption is not crucial in generating the benefit for a binary classification. As long as the decision is binary, \( i.e \), \( a \in \{a_G, a_B\} \), a continuous space of \( \omega \in \Omega \) could be partitioned into two sets, depending on whether \( a_G \) or \( a_B \) is preferred by the stakeholder. That is, we could redefine \( G \equiv \{\omega|S^G_{a_G} \geq S^G_{a_B}\} \), and \( B \equiv \Omega \setminus G \). Earnings management and the firm value can be aggregated similarly.

### 4.4 Continuous decision space

The baseline model uses a binary decision space. It is clear, from the proof of Proposition 3, that the unconditional part of the result relies critically on this binary decision space.
and thus is by no means general. The binary decision space is used because the model intends to provide one benefit of a binary classification. It is not designed to conduct a comprehensive comparison of disclosure and binary classifications, partly because the cost of binary classifications in terms of suppressing information (or the benefit of disclosure in terms of providing more information) is relatively better understood and orthogonal to the threshold’s incentive effect on earnings management. Thus, while Proposition 3 shows that the classification equilibrium dominates the disclosure equilibrium unconditionally, its main point is about the possibility part of the dominance, not the unconditional part.

In this extension, we consider a continuous space for the stakeholder’s decision. The main implication is that Lemma 2 is no longer valid. As a result, the trade-off of the ex post decision efficiency effect and the ex ante incentive effect characterized in Corollary 2 could swing in favor of disclosure and the comparison will be conditional on parameters. However, the same economic force that gives rise to the benefit of a binary classification, i.e., the threshold’s ex ante incentive effect, is preserved in the modified setting.

Suppose the stakeholder chooses decision \( a \in [0, \bar{a}] \). Her payoff from decision \( a \) and state \( \omega \) is \( S_\omega^a = S(a)I_\omega - a \). \( I_{\omega=G} = 1 \) and \( I_{\omega=B} = 0 \). \( S(a) \) is increasing, concave, and satisfies the boundary conditions that guarantee \( a^* \) to be interior (i.e., \( S(0) = S'(\bar{a}) = 0, S'(0) = \infty \)). The manager’s payoff, \( A_\omega^a \), is strictly increasing in \( a \). With this setting, we could re-characterize both the classification equilibrium and the disclosure equilibrium. While a general comparison of the two equilibria is not tractable, examples with further specifications of \( S(a), A_\omega^a \) and \( F_s \) are available in which Proposition 3 is either preserved or reversed.

5 Conclusion

This paper develops a model of accounting measurement to study the optimal design of thresholds that create binary classifications. It provides a benefit of binary classifications that might explain partially the pervasive use of binary classifications in accounting. When managers could engage in earnings management to influence evidence, the evidence threshold takes on dual functions. On one hand, before the evidence is produced, the threshold
influences managers’ ex ante decisions to engage in earnings management, which in turn affects the ex post classification errors. On the other hand, after the evidence is produced, the threshold directly affects the classification errors of the report. These dual effects differ quantitatively and qualitatively from each other. As a result, the ex ante optimal threshold that balances the dual functions is not ex post efficient, giving arise to the time inconsistency problem. While the stakeholder would like to commit to a binary classification created by the ex ante optimal threshold, she finds it incentive compatible to deviate ex post when she has access to raw evidence. By suppressing ex post information, a binary classification serves as a commitment device to implement the ex ante optimal solution. In other words, by committing to coarsening the report, a binary classification makes the report more informative overall in equilibrium because of its improved deterrence of earnings management.

To help assess accounting standards in practice, the model provides three observable differences between the ex ante optimal approach and the ex post efficient approach to accounting standard setting. First, binary classifications are redundant under the ex post efficient approach but could have value under the ex ante optimal approach. Second, the likelihood ratio of the marginal firm is determined solely by the relative costs of measurement errors under the ex post optimal approach, but it is not the case under the ex ante optimal approach. Finally, the policy responses under the two approaches are different. For example, as the cost of the false positive error increases, the equilibrium threshold increases under the ex post efficient approach but may decrease under the ex ante optimal approach.

This paper belongs to the growing literature that opens the black box of accounting measurement in order to explain its institutional features. In particular, the paper relies on a two-step representation of accounting measurement. First, a transaction’s economic substance manifests itself in various characteristics of the transaction. This correlation between the transaction’s economic substance and its characteristics is not only noisy but also vulnerable to managers’ influence. Second, an accounting rule is a mapping from transaction characteristics to an accounting report. The standard setter controls at least three instruments to influence the accounting report: the set of transaction characteristics admissible to a rule, the verification requirement for the admitted transaction characteristics, and the threshold
above which an accounting treatment is accorded. This paper focuses on the design of the last instrument, the evidence of threshold. Gao (2012) studies the optimal design of the second instrument (verification requirement) and provides a general rationale for conservatism. The future research could examine other instruments or the interactions among the instruments to help understand other prominent institutional features of accounting measurement.

6 Appendix

Proof. of Lemma 1: Part 1 is proved by \( \frac{\partial Q_G(T)}{\partial T} = q_G f^G > 0 \) and \( \frac{\partial Q_B(T,c^*(T))}{\partial T} = -q_B(H f^M + (1 - H) f^B) < 0 \). Part 2 is proved by \( \frac{\partial c^*(T)}{\partial T} = \delta_B(f^B(T) - f^M(T)) \). Due to strict MLRP, \( \frac{\partial c^*(T)}{\partial T} \) is positive if and only if \( T < \hat{T} \) (recall that \( \hat{T} \) is uniquely determined by \( \frac{f^M(T)}{f^B(T)}|_{T=\hat{T}} = 1 \)).

For the first part of Part 3, \( \frac{dQ_G(T)}{dT} = \frac{\partial Q_G(T)}{\partial T} > 0 \). For the second part, \( \frac{dQ_B(T,c^*(T))}{dT} = \frac{\partial Q_B(T,c^*(T))}{\partial T} + \frac{\partial Q_B(T,c^*)}{\partial c} \frac{\partial c^*(T)}{dT} = -q_B(H f^M + (1 - H) f^B) + q_B h(F^B - F^M) \frac{\partial c^*(T)}{dT} \). Thus, \( \frac{dQ_B(T,c^*(T))}{dT} < 0 \) if and only if \( H f^M + (1 - H) f^B > h(F^B - F^M) \frac{\partial c^*(T)}{dT} \). 

Proof. of Proposition 1: We verify the equilibrium according to its definition. First, it is optimal for the stakeholder to choose \( a^*(r = g) = a_G \) and \( a^*(r = b) = a_B \). At date 2 when the stakeholder makes her decision, she takes \( c^*(T^*) \) and \( T^* \) as given. For given \( c^* \), she interprets \( r = g \) as better news than \( r = b \) because \( \Pr(\omega = G| r = g) = \frac{q_G(1 - F^G)}{q_G(1 - F^G) + q_B(H(1 - F^M) + (1 - H)(1 - F^B))} > q_G \) and similarly \( \Pr(\omega = G| r = b) < q_G \). The first inequality relies on \( \frac{1 - F^M}{1 - F^G} < 1 \) and \( \frac{1 - F^B}{1 - F^G} < 1 \), as implied by MLRP. \( \Pr(\omega = G| r = g) > \Pr(\omega = G| r = b) \) implies \( \Delta(g) > \Delta(b) \). \( \Delta(r) \) is defined in eqn.1. In addition, by our assumption that in equilibrium \( r \) makes a difference in the stakeholder’s decision, that is, \( \Delta(g) \Delta(b) < 0 \), we have \( \Delta(g) > 0 > \Delta(b) \) or \( a^*(r = g) = a_G \) and \( a^*(r = b) = a_B \). 

Second, anticipating \( a^*(r = g) = a_G \) and \( a^*(r = b) = a_B \), the manager prefers \( r = g \) to \( r = b \). His decision on earnings management is thus determined by eqn.2 as analyzed in the text.

Finally, anticipating \( a^*(r) \) and \( c^*(T) \), the standard setter’s optimal choice of \( T \) has to satisfy the first order condition (eqn.5). 

Proof. of Corollary 1 is given in the text.
Proof. of Proposition 2: We verify the equilibrium according to its definition. We start with the stakeholder’s decision. We have shown in the text that the stakeholder’s optimal decision is a cut-off rule because $\Delta(t)$ is increasing in $t$. The cut-off is set at $\Delta(t)|_{t=t_0^*} = 0$, which is equivalent to $\frac{\Pr(\omega=G(t)|t=t_0^*)}{\Pr(\omega=B|t)} = \frac{LB}{LG}$. Second, anticipating $a^*(t \geq t_0^*) = a_G$ and $a^*(t < t_0^*) = a_B$, the manager cares about the cut-off $t_0^*$ rather than the official threshold $T$. His decision on earnings management is the same as in Proposition 1 except that $T^*$ is replaced by $t_0^*$. Finally, because neither $a^*(t)$ nor $c^*(t_0^*)$ is a function of $T$, the standard setter’s choice of $T$ is irrelevant. 

Proof. of Proposition 3 and Lemma 2 are given in the text. 

Proof. of Corollary 2: We recover the stakeholder’s decision problem at date 2. At date 2, she treats the level of earnings management $c^*$ as given. Her expected payoff for following the cut-off rule $t_0$ is $\Pr(t \geq t_0) \sum_\omega \Pr(\omega|t \geq t_0) S_G^\omega + \Pr(t < t_0) \sum_\omega \Pr(\omega|t < t_0) S_B^\omega = W^{FB} - Q_G(t_0)L_G - Q_B(t_0; c^*(t_0^*))L_B$. Note that $c^*(t_0^*)$ is treated as given and not a function of the stakeholder’s choice $t_0$. The first order condition can be rewritten as $\frac{\Pr(\omega=G(t)|t=t_0^*)}{\Pr(\omega=B|t)}|_{t=t_0^*} = \frac{LB}{LG}$. Thus, by the definition of $\tilde{T}$ and $T^*$, the stakeholder’s decision is more efficient under $\tilde{T}$ than under $T^*$. Note that whether the cost of earnings management, $C = q_B \lambda \int_0^{c^*(t_0^*)} hc \; dc$, is added to the stakeholder’s objective function does not affect the optimal solution because $c^*(t_0^*)$ is treated as given. Hence, part of Corollary 2.

To prove part 2, we write out the difference of the ex ante firm value in two equilibria:

$$\rho \equiv W(T^*; c^*(T^*)) - W(\tilde{T}; c^*(\tilde{T})) = W(T^*; c^*(T^*)) - W(T^*; c^*(\tilde{T})) + W(T^*; c^*(\tilde{T})) - W(\tilde{T}; c^*(\tilde{T}))$$

Suppose $c^*(\tilde{T}) \leq c^*(T^*)$. Because $\frac{\partial W(T;c^*)}{\partial c^*} = -q_B h(F_B - F^M) L_B - q_B \lambda c^* < 0$, $W(T^*; c^*(T^*)) - W(T^*; c^*(\tilde{T})) \leq 0$. Moreover, $W(T^*; c^*(\tilde{T})) - W(\tilde{T}; c^*(\tilde{T})) < 0$ from part 1. Therefore, we have $\rho < 0$, which contradicts Proposition 3. Thus, it must be $c^*(\tilde{T}) > c^*(T^*)$. 

Proof. of Corollary 3: The comparative statics are obtained by the implicit function theorem. Rewrite the first order condition for $T^*$ as $g(T, L_G, L_B, \delta_B) = -L_G \frac{\partial Q_G(T)}{\partial T} - L_B \frac{\partial Q_B(T; c^*)}{\partial T} - L_B \frac{\partial Q_B(T; c^*)}{\partial c^*} \frac{\partial c^*(T)}{\partial T} - \lambda \frac{\partial c^*(c^*)}{\partial c^*} \frac{\partial c^*(T)}{\partial T}$ and $g(T^*, L_G, L_B, \delta_B) = 0$. For an optimal $T^*$, the second
order condition $\frac{\partial \theta}{\partial T}|_{T=T^*} < 0$ must be satisfied. Therefore,

$$\frac{\partial T^*}{\partial L_G} = -\frac{\partial \theta}{\partial L_G}|_{T=T^*} = -\frac{q_G f^G}{\theta}|_{T=T^*} < 0,$$

$$\frac{\partial T^*}{\partial L_B} = \frac{\partial Q_B(T, c^*(T))}{\partial T} \frac{\partial c^*(T)}{\partial \delta_B} \frac{\partial \theta}{\partial T}|_{T=T^*} = -q_B \left( H f^M + (1 - H) f^B \right) - h(F^B - F^M) \frac{\partial c^*(T)}{\partial T}|_{T=T^*}.$$

Thus, $\frac{\partial T^*}{\partial L_B} > 0$ if and only $H f^M + (1 - H) f^B > h(F^B - F^M) \frac{\partial c^*(T)}{\partial T}|_{T=T^*}.$

$$\frac{\partial T^*}{\partial \theta} = -\frac{\partial \theta}{\partial \theta}|_{T=T^*} = \frac{\partial Q_B(T, c^*(T))}{\partial \theta} \frac{\partial c^*(T)}{\partial \delta_B} \frac{\partial \theta}{\partial T}|_{T=T^*} \frac{\partial \theta}{\partial \theta} \frac{\partial \theta}{\partial T}|_{T=T^*} \frac{\partial \theta}{\partial \theta} \frac{\partial \theta}{\partial T}|_{T=T^*} = \frac{1}{\partial \theta} q_B c^*(T) \left( L_B + \lambda \delta_B \right) \frac{\partial c^*(T)}{\partial T}|_{T=T^*}.$$

Thus, $\frac{\partial T^*}{\partial \theta} > 0$ if and only $(2h + c^* h') \frac{\partial c^*(T)}{\partial T}|_{T=T^*} < 0.$

The comparative statics for $\bar{T}$ can be conducted similarly by writing the first order condition as $\frac{\partial \theta}{\partial T}|_{T=T^*} = \frac{\partial \theta}{\partial T}|_{T=T^*} - \frac{\partial \theta}{\partial T}|_{T=T^*} = 0.$

Proof. of Lemma 3 is explained in the text.

References


