Strategic Informed Trades, Diversification, and Expected Returns*

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Abstract

We develop a model showing that diversification and competition among uninformed traders eliminates idiosyncratic effects from expected returns, regardless of the nature of competition among informed traders. First, we characterize a noisy rational expectations equilibrium in a setting with a risk neutral monopolist informed trader and a continuum of price-taking risk averse uninformed traders. Working from a factor structure of payoffs for multiple assets and taking the large economy limit, we show that controlling for sensitivity of asset payoffs to systematic risks (betas), there are no cross-sectional effects of asymmetric information on expected returns. This result complements results of previous models with perfectly competitive informed traders. Second, we generalize these findings by showing that the absence of arbitrage, mild conditions on the covariance matrix of payoffs, and full diversification by uninformed traders yield similar results. Importantly, this implies that, for firms traded by diversified investors, reporting choices affecting firm-specific information asymmetries have no ‘denominator’ effects on firm value after controlling for betas. Hence, we predict that firm-specific accounting choices affect firm value only via ‘numerator’ (i.e., cash flow) effects.
1 Introduction

Studies by Lambert, Leuz, and Verrecchia (2007) and Hughes, Liu, and Liu (2007) show that asymmetric information affects expected returns only through systematic risk premiums in large economies comprised solely of perfectly competitive price-takers. We address the robustness of this result to imperfect competition by modeling a market with a single monopolist informed trader and a continuum of uninformed. Our principal finding is that, controlling for betas in the large economy limit, there are no cross-sectional effects of asymmetric information on expected returns, irrespective of imperfect competition in the exploitation of that information. We then generalize the setting following Chamberlain (1983) and Chamberlain and Rothschild (1983) by first showing that a factor model follows from no arbitrage and very mild restrictions on the structure of payoffs, and then characterizing the powerful impact of diversification and competition by uninformed traders in ensuring that only systematic risks are priced in equilibrium.

These results imply that reporting choices affecting firm-specific information only impact firm value through betas. As a consequence, we predict that, controlling for betas, value maximizing firms will use ‘numerator’ (i.e., cash flow) effects to evaluate reporting choices affecting firm-specific information. For example, Gao and Verrecchia (2012) illustrate the distinction between denominator and numerator effects of information employed in resolving moral hazard. While idiosyncratic risk increases the risk premium paid to managers as part of a performance-based compensation package (a numerator effect), the expected return to investors

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1 When we speak of expected returns, we refer to expected return conditioned only on price. Privately informed traders may, and often do, expect returns in excess of the compensation for systematic risk.
2 Accordingly, they reinforce the recommendation from theoretical studies in competitive settings for caution in interpreting findings of empirical studies that attribute a cost of capital effect to properties of firm-specific disclosures (e.g., Botosan, Plumlee, and Xie 2004; Francis, LaFond, Olsson, and Schipper 2004).
3 Also, see Ou-Yang (2005).
(denominator effect) depends entirely on systematic risk.\textsuperscript{4} Better firm-specific information reduces premiums required to induce managerial effort, while better economy-wide information reduces systematic risk and, hence, expected return.\textsuperscript{5}

We are not the first to consider the effects of imperfect competition among informed traders on cost of capital. Lambert, Leuz, and Verrecchia (2012) show that imperfect competition among traders can reduce the information content of prices relative to perfect competition. Imperfectly competitive traders curb the aggressiveness of their trades in anticipation of the inferences uninformed traders will draw from prices.\textsuperscript{6} As a consequence, less information becomes impounded in price in a setting with a finite number of strategic traders. They show that imperfect competition among privately informed traders increases expected returns relative to perfect competition, but do not examine whether this leads to a firm-specific component to expected returns. As we discuss below, imperfect competition does not create a firm-specific component of expected returns but, instead, increases systematic risk premiums. This occurs because imperfect competition in exploiting an information advantage reduces the informed trader’s absorption of noise traders’ demands, leaving risk-averse uninformed traders with greater exposure to systematic risk for which they require a premium.

Stepping back to reflect upon the intuition for our results on what drives expected returns,

\textsuperscript{4} Christensen, Feltham, and Wu (2002) characterize a similar role for idiosyncratic risk in resolving moral hazard. Their interest is in the appropriate hurdle rate for assessing managerial performance in decentralized settings. Both models distinguish between the total cost of an investment, which includes the costs of compensating managers, and the required (expected) return on the investment.

\textsuperscript{5} In some cases, concerns over cash flow effects can provide incentives to reduce the information provided by the accounting system. For example, Caskey and Hughes (2012) show that conservative accounting measures, which provide distorted information, have beneficial cash flow effects because they mitigate asset substitution problems in levered firms. In their case, more informative (unbiased) accounting reports can reduce firm value.

\textsuperscript{6} Large, imperfectly competitive traders also take into account that their absorption of liquidity diminishes risk premiums. Heinle and Verrecchia (2012) provide a stark example of this. In their model, investors incur a cost to trade in a restricted market for shares in an entrepreneur’s firm. These investors have symmetric information and therefore earn no gains from private information. Imperfect competition among the investors determines the risk premium. Investors balance their incentive to buy the premium-bearing shares against the fact that their trades reduce the premium.
in the large economy limit uninformed traders hold fully diversified portfolios. Each firm comprises an infinitesimal fraction of the market and, therefore, represents an infinitesimal fraction of the typical portfolio. Because of this, prices depend on diversified uninformed traders’ demands and, therefore, include risk premiums only for non-diversifiable risk. In other words, expected returns depend only on betas applied to systematic risk premiums. The informed trader’s information may impact expected returns directly by partially revealing information about systematic risk, which reduces risk premiums. Also, an indirect impact on expected returns arises because informed traders absorb less of the noise trades, which results in higher risk premiums.

Elaborating on our model, the informed trader considers the impact of her demands on prices in a market that also includes noise traders and uninformed, price-taking traders. Facing no competition, the informed trader extracts monopoly profits, exhibiting the maximum impact of imperfect competition on expected returns. While risk neutrality maximizes the informed trader’s incentive for information-based trades, it also positions her to capture a larger portion of the systematic risk premium. This incentive to capture the systematic risk premium by providing liquidity to noise traders serves as an additional brake on information-based demands. The incentive to exploit an information advantage inhibits the informed trader’s ability to also

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7 In principle, uninformed investors could have some undiversified holdings so long as they do not all ‘un-diversify’ in the same way.
8 At the margin, if an uninformed trader were to perceive that an individual security is mispriced, he could initiate small trades in that security without affecting his or her overall risk exposure due to local risk-neutrality. Because all uninformed will perceive the security is mispriced, as well, they would also adjust their demands. In equilibrium, competition among uninformed traders dissipates distortions from full diversification.
9 Our analysis focuses on a risk-neutral informed trader in order to highlight the impact of imperfect competition on trade. In an appendix, we show that a risk-averse informed trader yields qualitatively similar results so long as risk aversion is not too large. The similarity stems from the fact that the uninformed traders drive the unconditional expected returns.
10 For example, in a setting with no private information where both the informed trader and the uninformed traders observe price (i.e., places limit orders as in Kyle 1989), the risk-neutral trader absorbs half of the liquidity demand. In a setting where the risk-neutral trader places market orders, she bases her risk premium extraction on the expected demands of noise traders.
capture the systematic risk premium.

Special cases provide additional insights. Specifically, we show the following: (i) As the variance of noise trades goes to zero, trading costs become infinitely large and the informed trader only takes infinitesimal positions. Accordingly, the informed trader captures none of the systematic risk premium leaving it to uninformed traders to meet the noise traders’ demands for which they require the full premium. (ii) As the variance of noise trades goes to infinity, prices lose their information content. In this case, there is no conflict between the informed trader’s incentives; acting as a monopolist in both respects, the risk-neutral informed trader reveals half of her information and absorbs half of the liquidity demands as the profit maximizing share of the systematic risk premium. Because the informed trader absorbs half of the noise trades, the systematic risk premium required by uninformed traders is half of what it was in the case where the variance of noise trades goes to zero; the same as the result when the informed trader has no private information.

Recent empirical studies by Armstrong, Core, Taylor, and Verrecchia (2011) and Akins, Ng, and Verdi (2012) show a positive association between imperfect competition among informed traders and cost of capital. Armstrong, et al. measure the degree of imperfection by the number of shareholders and various measures of information asymmetry including the adverse selection component of the bid-ask spread. The bid-ask spread can be interpreted as reflecting the marginal trading costs in our model. Akins, et al.’s (2012) study differs from Armstrong, et al. (2011) in the measure of imperfect competition – transient institutional ownership and a Herfindahl index of ownership concentration. Viewed from the context of our model, these results could be driven by a lack of full diversification or from the estimation of the idiosyncratic
portion of expected returns. In regard to diversification, an open question is whether the sets of firms and investors are sufficiently large to warrant approximation by a limiting economy of unbounded size.

We suggest that an important consideration in the design of future empirical inquiries regarding the effects of information asymmetries on expected returns is the extent of diversification by uninformed shareholders. This design element is present in Faccio, Marchica, and Mura (2011), who empirically find that firms with shares held by investors with well diversified portfolios care less about firm-specific risk taking. Additionally, controlling for betas, we suggest consideration of numerator, or cash flow, effects as drivers of cross-sectional differences in expected return for firms traded by well-diversified investors.

Briefly summarizing, prior studies show that information asymmetries have no effect on expected returns in perfectly competitive noisy rational expectations models, after controlling for betas (Hughes et al. 2007, Lambert et al. 2007). We show that this qualitative insight carries over to imperfectly competitive settings where a strategic monopolist informed trader, even one uninhibited by risk aversion, exploits an information advantage. Special cases illuminate how the presence or absence of disguise for informed trades provided by noise trades affects expected return required by uninformed traders for providing liquidity to noise traders.

Remaining sections are organized as follows: Section 2 lays out the principal model under the assumptions of CARA uninformed traders’ utility and normal distributions. Section 3 generalizes the marginality conditions of uninformed traders under mild restrictions on the structure of asset payoffs. Section 4 concludes with a brief summary. Appendix A provides proofs and Appendix B provides results for the case where the informed trader is risk-averse.

Narrow framing (e.g., Kahneman and Tversky 1984), whereby investors view stocks individually rather than in the context of a portfolio, could also explain these results.
2 Basic Model

2.1 Private Information on Asset Payoffs

In this section, we derive a linear equilibrium for security prices. We assume a payoff structure for $N$ assets as follows:

$$\mathbf{v} = \mathbf{\bar{v}} + \mathbf{Bf} + \mathbf{e},$$

(1)

where $\mathbf{\bar{v}}$ is an $N \times 1$ constant vector, $\mathbf{f}$ is a $K \times 1$ vector of mean zero, standard normal random variables, $\mathbf{B}$ is an $N \times K$ constant matrix, $\mathbf{e}$ is an $N \times 1$ vector of zero-mean normal random variables uncorrelated with $\mathbf{f}$ and with covariance matrix $\Sigma$, which we assume has bounded eigenvalues.\(^{12}\) This implies that $\mathbf{v}$ has the covariance matrix $\Sigma_v = \mathbf{B} \Sigma \mathbf{B}' + \Sigma$.\(^ {13}\) The supply of risky assets (noise trades) $\mathbf{x}$ is a $N \times 1$ vector of normally distributed random variables with mean $\mathbf{\bar{x}}$, covariance matrix $\Sigma_x$, and independent of $\mathbf{f}$ and $\mathbf{e}$. As we will show in section 3, there is very little loss in generality from assuming a factor model structure for asset payoffs.

Similar to Kyle (1989), we assume that a risk neutral informed trader has private information yielding the posterior belief that $\mathbf{v}$ has mean $\mathbf{\bar{v}}_i$ and covariance matrix $\Sigma_i = \Sigma_v - \Sigma_{\bar{v}}$. We assume that $\mathbf{\bar{v}}_i$ is joint normally distributed with $\mathbf{f}$, $\mathbf{e}$, and $\mathbf{x}$, is uncorrelated with $\mathbf{x}$, and that the informed trader’s posterior does not induce a conditional correlation between $\mathbf{f}$ and $\mathbf{e}$. The informed trader places limit orders $\mathbf{y}_n, n = 1, \ldots, N$ to maximize her expected payoff:

$$\mathbb{E}_i \left[ \sum_{n=1}^{N} y_n (v_n - p) \right] = \mathbb{E}_i \left[ y'(v - p) \right] = y' (\mathbf{\bar{v}}_i - \mathbb{E}_i[p]),$$

(2)

where, without loss of generality, we have normalized the risk-free rate to equal one and $y'p$ is

\(^{12}\) For example, if $\Sigma$ is diagonal with bounded elements, then it has bounded eigenvalues. Our assumption of bounded eigenvalues rules out cases where an unbounded number of firms have covariance with a given $\mathbf{e}_n$ for a given $n$. The $\mathbf{Bf}$ term is intended to include the cases where an unbounded number of firms share exposure to a given source of risk.

\(^{13}\) The assumption that $\mathbf{f}$ is a vector of standard normal random variables is without further loss of generality because the factors can be recast. For example, if the vector of factors is $\mathbf{g}$, with loadings $\mathbf{B}_g$ and covariance matrix $\Sigma_g$, then we can write a new factor $\mathbf{f} = \Sigma_{g}^{-1/2} \mathbf{g}$ with $\text{var}(\mathbf{f}) = \mathbf{I}$ and loadings $\mathbf{B} = \mathbf{B}_g \Sigma_{g}^{1/2}$ giving $\mathbf{Bf} = \mathbf{B}_g \Sigma_{g}^{1/2} \Sigma_g^{-1/2} \mathbf{g} = \mathbf{B}_g \mathbf{g}$. 


the opportunity cost of holding those shares.

As a monopolist with respect to trading on private information, the informed trader anticipates the impact of her demands on prices. We conjecture that a linear equilibrium exists in which price can be expressed as a function of excess demand \( y - (x - \bar{x}) \); i.e.,

\[
p = \mu + A(y - (x - \bar{x})),
\]

where the \( N \times 1 \) vector \( \mu \) and \( N \times N \) invertible matrix \( A \) are yet to be determined. Based on this conjecture, the informed trader can infer the liquidity demand \( x \). As we will show, this conjecture will be fulfilled in equilibrium. Substituting from (3) into the informed trader’s expected payoff (2) and taking the expectations gives the expected payoff:

\[
y' \left( \bar{v}_i - \left( \mu + A(y - (x - \bar{x})) \right) \right).
\]

The first-order conditions imply an optimal informed trader’s demand vector \( y \) of

\[
y = (A + A')^{-1} \left( \bar{v}_i - \mu + A(x - \bar{x}) \right),
\]

which yields an expected profit of \( y'A'y \).

Next, we assume there are \( M \) price-taking, risk-averse and uninformed traders, each of whom trade to maximize expected utility as given by

\[
U(W_i) = -E[\exp\{-AW_i\}],
\]

subject to the following budget constraint:

\[
W_i = W_0 + d'(v - p),
\]

where \( W_0 \) is the trader’s initial wealth and \( d \) is an \( N \times 1 \) vector containing the number of shares traded. We assume that uninformed traders conjecture a linear pricing rule of the form (3) and that the informed trader follows a linear trading strategy

\[
y = q_0 + Q_v (\bar{v}_i - \bar{v}) + Q_x (x - \bar{x}),
\]

where \( q_0 \) is a \( N \times 1 \) vector and \( Q_v, Q_x \) are a \( N \times N \) invertible matrices. Under these conjectures, the uniformed traders can form an estimate \( s \) of the informed trader’s signal

\[
s = Q_v^{-1}(A^{-1}(p - \mu) - q_0) = \bar{v}_i - \bar{v} - Q_v^{-1}(I - Q_x)(x - \bar{x}),
\]
with the noise in $s$ having covariance matrix $\Sigma_s = Q_i^{-1}(I - Q_x)\Sigma_x(I - Q_x')(Q_i^{-1})'$.

Transforming the above problem of the representative uninformed trader into a mean-variance certainty equivalent and taking the first-order conditions results in the following aggregate uninformed traders’ demand:

$$Md = \frac{M}{A} \Sigma_{v,s}^{-1}(E[v | s] - p),$$

where the uninformed traders’ posterior expectation of asset payoffs and variance-covariance matrix are, respectively,

$$E[v | s] = \bar{v} + \Sigma_{\bar{v}}^{-1}(\Sigma_{v,v}^{-1} + \Sigma_{s,s}^{-1})^{-1}s = \bar{v} + (\Sigma_{v,v}^{-1} + \Sigma_{s,s}^{-1})^{-1}\Sigma_{s,s}^{-1}s,$$

and

$$\Sigma_{v,s} = BB' + \Sigma - (\Sigma_{v,v}^{-1} + \Sigma_{s,s}^{-1})^{-1}\Sigma_{v,v}^{-1}\Sigma_{v,v} = BB' + \Sigma - \Sigma_{v,v}^{-1}(\Sigma_{v,v} + \Sigma_{s,s})^{-1}\Sigma_{v,v}.$$  

Prices can be determined from market clearing:

$$x = y + Md.$$  

Substituting for the informed and uninformed traders’ demands from (5) and (10), respectively, in (13) and solving for $p$ in terms of model parameters up to $A$ leads to the following characterization:

**Proposition 1:**

Equilibrium prices are given by:

$$p = \bar{v} - A'\Sigma_{v,s}^{-1}(I + \frac{1}{A'}\Sigma_{v,s}^{-1})(\bar{x} + A(A + A')^{-1}(\bar{v} - \bar{v} - A'(x - \bar{x}))).$$

where the matrix $A$ solves the matrix cubic equation given in Appendix A.

We note that the presence of the risk-averse uninformed as providers of liquidity to noise traders enables the informed trader to extract a portion the systematic risk premium.

Accordingly, the informed trader’s demand includes two components, one to exploit private information and the other to extract the risk premium.
\[
y = (A + A')^{-1}(\bar{v} - \bar{v} + A(x - \bar{x})) + \frac{A}{M} \left( \frac{A}{M} \Sigma_{v|x} + A' \right)^{-1} \Sigma_{v|x} \bar{x},
\]  
(15)

which follows from substituting the equilibrium price parameter \( \mu \) into the informed trader’s demand (5), and simplifying. Both components are tempered by the informed trader’s awareness that larger trades have a greater adverse impact on price. Expression (15) implies that

\[ Q_v = (A + A')^{-1} \text{ and } Q_x = Q_v A \] so that the noise in the uninformed signal is:

\[ \Sigma_x = A' \Sigma_x A. \]  
(16)

Exploiting the market clearing condition (13) and price conjecture given by (3), we can express the matrix \( A \) as shown below:

\[ A = \frac{A}{M} \Sigma_{v|x} + \Sigma_{v|y} (\Sigma_{v|y} + \Sigma_x)^{-1} (A + A'). \]  
(17)

2.2 Perfect Information on Idiosyncratic Risks

If the informed trader privately holds perfect information on the idiosyncratic payoffs \( e \), then \( \Sigma_{v|y} = \Sigma \) and the uninformed traders’ posterior variance-covariance matrix \( \Sigma_{v|x} \) of asset payoffs, restated from (12), is:

\[ \Sigma_{v|x} = BB' + (\Sigma^{-1} + \Sigma_x^{-1}). \]  
(18)

This setting is useful since much of the confusion about the role of information asymmetries in relation to cost of capital pertains to firm-specific disclosures or their absence. It also contributes to the feasibility of solving for \( A \) in terms of exogenous parameters. In particular, if we further assume that there is one factor \( f \), idiosyncratic risk is iid \( (\Sigma = \sigma^2_e I) \), and noise trade is iid \( (\Sigma_x = \sigma^2_x I) \), and the informed knows idiosyncratic information \( e \), then we can fully characterize \( A \) as follows:
Proposition 2:

In the single-factor, iid case, there exists an equilibrium with

\[ A = aI + cbb', \quad a = \frac{A}{2M} \sigma_v^2 \left( 1 + \sqrt{1 + \left( \frac{2M}{\lambda \sigma_v \sigma_x} \right)^2} \right), \]  

where \( c \) solves the cubic equation given in Appendix A.

2.3 Special Cases

**Limiting case of risk neutral uninformed traders**

Assuming that \( \Lambda \) is positive definite, then from (17) we have the following expression for \( \Lambda \) as \( A \to 0 \):

\[ \Lambda = \Sigma_x^{-1/2} \left( \Sigma_x^{1/2} \Sigma_{\bar{v}} \Sigma_x^{1/2} \right)^{1/2} \Sigma_x^{-1/2}, \]  

which resembles the solution from Caballé and Krishnan (1994). We note that if there is a single asset, (20) reduces to:

\[ \lambda = \frac{\text{std}(\bar{v})}{\sigma_x} \]  

In effect, competitive risk neutral uninformed traders serve precisely the same function as the breakeven market maker in Kyle (1989).

**No private information**

If the informed trader has no private information \( \Sigma_{\bar{v}} = 0 \), we can use (17) and set \( \Sigma_{v|s} = \Sigma_v \) and to arrive at

\[ A = \frac{A}{M} \Sigma_v, \quad y = \frac{1}{2} x. \]  

Behaving as a monopolist, the risk-neutral trader maximizes profits from extracting the risk premium by setting demands at half of the liquidity trade. The informed trader cannot take all of the expected liquidity trade as that would eliminate the risk premium. Comparing to the risk premium portion of trade in (15), we see that the possession of private information alters the

14 Informed traders in Caballé and Krishnan (1994) submit market orders and their expression for \( A \) is equivalent to expression (20) multiplied by one half.
informed trader’s ability to share in risk bearing. In particular, the informed trader absorbs approximately half of the unexpected noise trades, as reflected by the term \((A + A')^{-1} A (x - \bar{x})\) in the informed demand (15).\(^{15}\) However, the term \(\frac{A}{M} \left( \frac{A}{M} \Sigma_{v} A' + A' \right)^{-1} \left( \Sigma_{v} x \right) = \left( I + \frac{M}{A} \Sigma_{v} A' \right)^{-1} \) that multiplies the expected liquidity demand \(\bar{x}\) in (15) differs from the \(\frac{1}{2} I\) matrix that multiplies \(x\) in (22).\(^{16}\) This makes clear the indirect effect on cost of capital of the tension created by the dual incentives facing the informed trader to exploit private information and capture the risk premium.

**Limiting cases of zero and unbounded variance of noise trades**

The following proposition states the behavior of \(A\) as the noise trades become known \((\Sigma_x \to 0)\) or infinitely volatile \((\Sigma_x \to \infty)\):

**Proposition 3:**

The matrix \(A\) is of order \(\Sigma_x^{-\frac{1}{2}}\) as \(\Sigma_x \to 0\) (i.e., \(\Sigma_x^{\frac{1}{2}} A\) is nonzero and bounded as \(\Sigma_x^{\frac{1}{2}} \to 0\)) and approaches \(\frac{A}{M} \Sigma_x\) as \(\Sigma_x \to \infty\).

Proposition 3 implies that the price impact of trades becomes arbitrarily large as uncertainty about noise trades vanishes \((A \to \infty\) as \(\Sigma_x \to 0\)). As the variance of noise trades disappears, prices become increasingly revealing of the informed trader’s information. This diminishes her ability to profit and, hence, her incentive to trade. Perfect competition among uninformed traders, who draw inferences from prices, completely dissipates their expected profits and hence any influence of their trades. The informed trader’s demands become infinitesimally small, thereby absorbing virtually none of the noise traders’ liquidity demands. Hence, uninformed traders cover essentially all of the noise trades for which they require the full

\(^{15}\) The matrix \((A + A')^{-1} A = (I + A^{-1} A')^{-1}\) equals \((2I)^{-1}\) only if \(A\) is symmetric.

\(^{16}\) Using (17) to write \(\left( I + \frac{M}{A} \Sigma_{v} A' \right)^{-1} = \left( 2I + \frac{M}{A} \Sigma_{v} (A + A') (\Sigma_{\eta} + \Sigma_x) (\Sigma_{\eta} + \Sigma_x)^{-1} \right)^{-1}\), we note that \(\left( I + \frac{M}{A} \Sigma_{v} A' \right)^{-1}\) is not necessarily ‘smaller’ than \((2I)^{-1}\) because the matrix second matrix in the parentheses may not be symmetric.
premium on both idiosyncratic and systematic risks. Because \( A \to \infty \) but \( \Sigma_{vls} \) remains bounded as \( \Sigma_v \to 0 \), we have:

\[
E[v - p] = \frac{4}{M} \Sigma_{vls} \left( \frac{A}{M} \left( A' \right)^{-1} \Sigma_{vls} + I \right)^{-1} \bar{x} \to \frac{4}{M} \Sigma_{vls} \bar{x}.
\] (23)

Proposition 3 also implies that the uninformed investors’ signal from price becomes infinitely noisy as noise trade variance becomes unbounded (\( \Sigma_v \to \infty \)). This implies that \( \Sigma_{vls} \to \Sigma_v \) and, using (14) and (15), that price, informed trade, and expected returns approach the following respectively:

\[
p \to \frac{1}{2} \left( \bar{v} + \bar{v} - \frac{4}{M} \Sigma_v \bar{x} \right), \quad E[v - p] \to \frac{4}{2M} \Sigma_v \bar{x}, \quad y \to \frac{M}{2A} \Sigma_v^{-1} (\bar{v} - \bar{v}) + \frac{1}{2} \bar{x}.
\] (24)

In this case, the uninformed traders behave as if there is no informational impact on prices from demands of the informed trader. The informed trader takes half of the liquidity demands, \( \frac{1}{2} \bar{x} \), as in the no-information case (22). The information-based component of informed trades, \( \frac{4}{2A} \Sigma_v^{-1} (\bar{v} - \bar{v}) \), reflects the fact that taking large positions amplifies the systematic risk to be borne by uninformed traders for which they require a risk premium. This effect results in finite informed trader’s demands, notwithstanding infinite disguise for those demands in the form of an infinite variance of noise trades.

2.4 Large Economy Limit

We now take the large economy limit defined by \( N, M \to \infty \) with \( N / M \) approaching a constant which, without loss of generality, we assume to be one.\(^\text{17}\) The following characterizes expected returns (cost of capital):

\textbf{Proposition 4:}

\(^\text{17}\) The assumption that \( N / M \) approaches a constant means that both \( N \) and \( M \) grow at the same rate; otherwise, the ratio \( N / M \) approaches either zero or infinity. Hughes, Liu, and Liu (2007), Lambert, Leuz and Verrecchia (2007), and Ou-Yang (2005) employ similar assumptions.
In the limiting economy, the risk premium equals the following, and depends only on systematic risks:

\[
\mathbb{E}[\mathbf{v} - \mathbf{p}] = \frac{A}{M} \mathbf{B} \Sigma_{f|x} \mathbf{B}' \left( \mathbf{I} + \frac{A}{M} (A')^{-1} \Sigma_{v|x} \right)^{-1} \mathbf{x},
\]

(25)

where \( \mathbf{B} \) is the matrix of factor loadings and the vector of factor prices is:

\[
\frac{A}{M} \Sigma_{f|x} \mathbf{B}' \left( \mathbf{I} + \frac{A}{M} (A')^{-1} \Sigma_{v|x} \right)^{-1} \mathbf{x}.
\]

(26)

It is clear from (25) that information asymmetries only impact cost of capital through the factor risk premium (26). This implies the absence of cross-sectional predictions after controlling for betas.

Taking the large economy limits for the two extremes of the variance in noise trades, we first observe the expected returns in (23) and (24) approach the following, respectively:

\[
\mathbb{E}[\mathbf{v} - \mathbf{p}] \rightarrow \frac{A}{M} \mathbf{B} \Sigma_{f|x} \mathbf{B}' \mathbf{x} \text{ as } \Sigma_x \rightarrow 0 \quad \text{ and } \quad \mathbb{E}[\mathbf{v} - \mathbf{p}] \rightarrow \frac{A}{2M} \mathbf{B} \mathbf{B}' \text{ as } \Sigma_x \rightarrow \infty.
\]

(27)

As previously shown for finite assets, facing infinite trading costs as \( \Sigma_x \rightarrow 0 \), the informed trader’s demands vanish, leaving it to the uninformed traders to provide liquidity to the noise traders. The uninformed require a premium only for bearing systematic risk because they eliminate idiosyncratic risks through diversification. In particular, the market clearing condition implies that an uninformed trader’s holdings in individual firms, given by \( \mathbf{d} = \frac{1}{M} (\mathbf{x} - \mathbf{y}) \), approach zero as long as noise trades and informed trades are bounded.

In the case where \( \Sigma_x \rightarrow \infty \), the informed trader maximizes by taking the systematic risk premium for providing liquidity for one-half of the noise trades. The uninformed traders extract the premium on the remainder of the noise trades in the form of an expected return amounting to one-half of that in the case of the variance of noise trades equal to zero.

\[\text{Note: 18 Alternatively, we could view } \frac{A}{M} \Sigma_{f|x} \mathbf{B}' \left( \mathbf{I} + \frac{A}{M} (A')^{-1} \Sigma_{v|x} \right)^{-1} \mathbf{x} \text{ as the matrix of conditional factor loadings with conditional factor prices of } \frac{A}{M} \Sigma_{f|x} \mathbf{B}' \left( \mathbf{I} + \frac{A}{M} (A')^{-1} \Sigma_{v|x} \right)^{-1} \mathbf{x}.\]
If the informed trader possesses information only about idiosyncratic risks, then the uninformed investors’ posterior factor variance $\Sigma_{fij}$ equals the prior $I$ and the factor risk premium is:

$$\frac{\Delta M}{\Delta M} B' \left( I + \frac{\Delta M}{\Delta M} (A')^{-1} \Sigma_{eij} \right)^{-1} \bar{x}. \tag{28}$$

In this case, the informed trader’s activity has no bearing on what the uninformed learn about systematic risk, but there is still an effect on the factor risk premium. This effect arises because the informed trader’s incentive to limit revelation of private information about idiosyncratic risks mitigates absorbing the risk created by the liquidity demands of noise traders.

2.5 Information on Systematic Risks

The dependence of large-economy expected returns solely on systematic risks begs the question of how prices reflect information about systematic risks. Within the context of our model’s pure strategy, linear equilibria, informed traders with information on systematic risks will only take non-infinitesimal positions in the systematic factor (mimicking) portfolios if the unexpected noises also include such positions.\(^{19}\) This occurs because the same forces that remove idiosyncratic risk from the factor portfolios also remove idiosyncratic trades. Formally, if the unexpected noise trades do not involve factor portfolios, then uninformed traders can perfectly infer the informed trades, creating a situation similar to our example of zero-noise-trade-variance in the finite economy.\(^{20}\) Just as in that example, the absence of noise trades in factor portfolios creates unbounded costs of trading those portfolios, which causes the informed trader to take infinitesimal positions.

The necessity of noise trades in the factor portfolios becomes quite apparent if we follow

\[^{19}\] The $K$ factor-mimicking portfolios are given by $(B' B)^{-1} B' \nu$, which approach $f$ as the number of assets $N \to \infty$.

\[^{20}\] Specifically, the noise demand for the factor portfolios is $x_f = (B' B)^{-1} B' x$ where $\text{var}(x_f) \to 0$ as $N \to \infty$ if the unexpected noise trades, $x - \bar{x}$, exclude the factor portfolios.
Van Nieuwerburgh and Veldkamp (2010) by decomposing the assets into their principal components (hereafter ‘factors’). They assume that trade and information pertains directly on the portfolios associated with each factor of the covariance matrix $\Sigma_v$ and that the informed trader’s information pertains to distinct factor.$^{21}$ Denote the $k^{th}$ factor’s variance by $\theta_k$. In other words, we can write the prior variance as $\Sigma_v = T\Theta T'$ with $\theta_1 > \theta_2 > \cdots > \theta_N$. Traders can form portfolios to isolate the risk represented by a given factor. For example, the portfolio associated with the largest eigenvalue $\theta_1$ (analogous to a normalized factor portfolio) is the linear combination $\theta_1^{-1/2}t_1^\top v$ of assets with $\text{var}(\theta_1^{-1/2}t_1^\top v) = \sum_{k=1}^N \theta_1^{-1} \theta_k t_k^\top t_k t_1^\top t_1 = 1$.

If the informed trader has information directly on the factors, we can write her posterior covariance matrix as $\text{var}_i(v) = T(\Theta - \xi_t)T'$, which is the prior variance $\Sigma_v$ less the variance of her posterior mean, $\Sigma_{v_i} = T\Theta_i T' = \sum_{k=1}^N \theta_{ki} t_k^\top t_k$ with variance $0 < \theta_{ki} < \theta_k$. In this case, condition (17) for $A$ can be written as follows each $k = \{1, \ldots, N\}$ where $\sigma_{ik}^2$ denotes the variance of noise trades for the $k^{th}$ portfolio:

$$\lambda_k = \frac{\Lambda}{M} \left( \theta_k - \frac{\sigma_{ik}^2}{\theta_{ki} + \lambda_k^2 \sigma_{ik}^2} \right) + \frac{2 \theta_k \lambda_k}{\theta_{ki} + \lambda_k^2 \sigma_{ik}^2} \Rightarrow \lambda_k^3 \frac{\sigma_{ik}^2}{\theta_{ki}} - \lambda_k^2 \frac{\theta_k}{\theta_{ki}} - \lambda_k^2 - \frac{\Lambda}{M} \frac{\theta_k}{\theta_{ki}} = 0 \quad (29)$$

As we discuss in the next section, a systematic risk corresponds to an eigenvalue that becomes unbounded as the economy grows $(N,M \to \infty)$.

Consistent with our assumption of $K$ systematic factors, we assume that $K$ eigenvalues become unbounded as $N,M \to \infty$. There are two cases to consider, depending on the extent to which unexpected noise trades impact the factor portfolios. If unexpected noise trades in the factor portfolios grow with the economy, then each of the terms in the cubic (29) are bounded,
implying that \( \lambda_k \) is bounded, as well. If unexpected noise trades do not grow with the economy, then \( \lambda_k \) must be of order \( N^{1/2} \), analogous to the finite-security case where \( A \) is of order \( \Sigma_x^{-1/2} \) as \( \Sigma_x \to 0 \) in Proposition 3. Specifically, we can write (29) and assign orders of magnitude as follows, where \( O(1) \) denotes terms that are bounded:

\[
\begin{align*}
\frac{\lambda_k}{O(N^{1/2})} - \frac{\sigma_{x_k}^2}{O(1)} - A \frac{\sigma_{x_k}^2}{O(1)} \frac{\theta_k}{\theta_{ii}} - \frac{\lambda_k}{O(N^{1/2})} - A \frac{\theta_k - \theta_{ii}}{M} &= 0.
\end{align*}
\]

(30)

As the economy grows (\( N, M \to \infty \)), by fully diversifying, the uninformed traders constructively remove the effect of unexpected noise trades on the factor portfolios. This increases the informed traders’ trading costs so that \( \lambda_k \) becomes unbounded at a rate \( N^{1/2} \) analogous to the finite-economy example with zero variance of noise trades.

The idiosyncratic risks, \( k > K \) with \( \theta_k \) bounded as \( N, M \to \infty \), have the following pricing parameter \( \lambda_k \) if the variance of noise trades remains bounded:

\[
\lambda_k^3 \frac{\sigma_{x_k}^2}{\theta_{ii}} - A \frac{\sigma_{x_k}^2}{\theta_{ii}} \lambda_k^2 - \lambda_k - A \frac{\theta_k - \theta_{ii}}{M} \Rightarrow \lambda_k = \sqrt{\frac{\theta_{ii}}{\sigma_{x_k}^2}},
\]

(31)

which is the same as for risk-neutral traders. This means that, in essence, fully-diversified traders are risk-neutral with respect to the idiosyncratic risks that comprise an infinitesimal portion of their overall portfolio risk.22

3 General Setting

We depart from the structure on preferences and payoffs in the previous section by assuming that uninformed traders’ utility \( u(\cdot) \) is monotone increasing in wealth and concave. Recasting the representative uninformed trader’s objective function for a finite number of assets, conditional on whatever information can be inferred from price, we have expected utility of:

---

22 The case where the noise trade variance for an idiosyncratic risk \( k \) grows with the economy is pathological. In this case \( \lambda_k \to 0 \) and both the informed and uninformed investors take arbitrarily large positions in the idiosyncratic risk in order to exploit a sort of bubble created by the large noise trader demands relative to the small payoff risk.
E\left[ u(d'(v - p)) \mid p \right], \quad (32)

which gives the following first-order condition expressed in terms of a pricing kernel with a stochastic discount factor $\xi$:

$$p = E[\xi v \mid p], \quad \xi \equiv \frac{u'(d'(v - p))}{E[u'(d'(v - p)) \mid p]}, \quad (33)$$

Chamberlain (1983) and Chamberlain and Rothschild (1983) show that under no-arbitrage and mild restrictions on the structure of asset payoffs, the conditional variance-covariance matrix $\Sigma_{v|p}$ of the uninformed trader’s payoffs, $v - p$, will have an approximate $K$-factor structure:

$$BB' + R, \quad (34)$$

where $B$ is now a conditional (on price) $N \times K$ covariance matrix for which the $nk^{th}$ element is the covariance of the payoff of asset $n$ and factor $k$. The $K$ factors can be normalized to have a mean zero and variance of one. The following proposition lends a useful perspective on when the restrictions described in the appendix are met including a formal definition of no arbitrage:

**Proposition 5:**

*If there is no arbitrage and the equally weighted market portfolio has positive, but bounded variance, then $\Sigma_{v|p}$ has an approximate factor structure.*

Given $\Sigma_{v|p}$ has an approximate factor structure, the set of possible portfolios (i.e., linear combinations of net asset payoffs) can be separated into a set of fully diversified portfolios and a set of undiversified portfolios. A portfolio $d$ of an uninformed trader is fully diversified if its weights on individual assets become infinitesimally small in the large economy limit:

$$\lim_{N \to \infty} \sum_{n=1}^{N} d_{n}^2 = 0 \quad (35)$$

Because price is known to the trader, the conditional variance of $v - p$ equals the conditional variance of $v$.\footnote{23}
The approximate $K$-factor structure ensures that any portfolio with payoff $d'(v - p)$ can be decomposed into a fully diversified portfolio that has only systematic (factor) risk due to correlation with the $K$ factors, and an undiversified portfolio that has only idiosyncratic risk and is uncorrelated with both the $K$ factors and the payoff of the fully diversified portfolios. The decomposition follows from a projection theorem similar to a least squares regression in which a dependent variable is decomposable into a predictable component and an orthogonal residual.

Returning to the uninformed trader’s first-order conditions, it must be the case that expected returns arise from the covariance of payoffs with the trader’s marginal utility:


(36)

If the uninformed trader holds a diversified portfolio in equilibrium as we expand the number of assets and uninformed investors $(N, M \to \infty)$, then the stochastic discount factor only includes the effect of systematic risk on marginal utility.\(^{24}\)

**Proposition 6:**

*If the conditions of Proposition 5 are satisfied and the equilibrium portfolio of uninformed traders is fully diversified, then the uninformed trader’s payoff can be written as a linear combination of $K$ diversified portfolios that span systematic risks (i.e., $d'(v - p) = \beta' f$) and prices can be written as:

$$p = E[v | p] + Bp_f + E[\xi E[e | f, p] | p], \quad \xi = \frac{u'(\beta' f)}{E[u'(\beta' f) | p]}$$

(37)

where $e$ is a vector of idiosyncratic risks orthogonal to $f$. Moreover, if $e$ is mean-independent of $f$, then price can be written as

$$p = E[v | p] + Bp_f$$

(38)

Our last proposition identifies sufficient conditions for uninformed traders to take fully

\(^{24}\) Similar to the previous section, the number of assets $N$ and investors $M$ must expand at the same rate.
Proposition 7:

If noisy supply \( x \) and informed trader’s demand \( y \) are bounded with probability 1, then the uninformed traders’ equilibrium portfolio is fully diversified.

It follows from the above propositions that our results that utilize CARA utility and normal distributions in a factor model setting carry over to far more general settings. In particular, after controlling for betas, there are no cross-sectional effects on cost of capital of imperfect competition with respect to the exploitation of private information. The ability of price-taking, competitive uninformed traders to fully diversify across a large number of assets in a pure exchange economy ensures that only systematic risks are priced.

4 Conclusion

In this study, we show that diversification and competition of uninformed traders removes the effect of idiosyncratic risks from expected returns, even in settings with imperfect competition among informed traders. We extend earlier theoretical studies on expected returns effects of information asymmetries in pure exchange economies with perfectly competitive informed traders to settings with a monopolist informed trader. We conduct our principal analysis in a standard noisy rational expectations modeling context wherein we assume a factor structure for asset payoffs, CARA utility by a continuum of uninformed traders, and normal distributions. We introduce imperfect competition in the form of a risk neutral monopolist informed trader with private information. Our results show that in the large economy limit, imperfect competition only affects expected returns through its impact on systematic risk premiums; i.e., controlling for betas, asymmetric information about firm-specific risks leads to no cross-sectional differences in expected returns.
In addition to exploiting private information, the informed trader may also provide liquidity to noise traders in order to capture part of the systematic risk premium. This reduces the risk premium required by uninformed traders. In the special case of infinite variance of noise trades, the informed trader extracts the risk premium on half of the noise trades. At the other extreme of no variance of noise trades, trading costs become infinitely large causing the informed trader’s demands to become infinitesimally small. In this case, the uninformed traders require the same risk premium as in the case where there is no informed trader.

We further show that under no arbitrage and very mild restrictions on the structure of asset payoffs as assumed in Chamberlain (1983) and Chamberlain and Rothschild (1983), if perfectly competitive uninformed traders are fully diversified then only systematic risks affects expected returns. Moreover, if noise traders’ and informed traders’ demands are bounded with probability one, then uninformed traders will hold fully diversified positions in equilibrium. These results support the robustness of our predictions regarding the impact of imperfect competition on expected returns in large economies where uninformed traders are able to fully diversify.
Bibliography


Appendix A

Proof of Proposition 1

The market clearing condition, combined with informed demand (5) and uninformed demand (10) imply the following price

\[ p = \mathbb{E}[v \mid s] + \frac{A'}{M} \Sigma_{vis} (y - x). \]  

(A1)

In equilibrium, the informed demand is as follows, which gives expressions for the uninformed conjecture (8) of the informed trading strategy:

\[ y = -(A + A')^{-1}(\bar{v} - \mu) + (A + A')^{-1}(\bar{v}_i - \bar{v}) + (A + A')^{-1}A(x - \bar{x}). \]  

(A2)

Expression (A2) implies that \( Q_v \) is symmetric and invertible, yielding an expression for the variance matrix of noise in the uninformed traders’ signal:

\[ \Sigma_y = A' \Sigma_x A. \]  

(A3)

Using the expression for \( Q \), we can write the uninformed expectation as:

\[ \mathbb{E}[v \mid s] = \bar{v} - \left( \Sigma_{\bar{v}}^{-1} + \Sigma_s^{-1} \right)^{-1} \Sigma_s^{-1} (\bar{v} - \mu) + \left( \Sigma_{\bar{v}}^{-1} + \Sigma_s^{-1} \right)^{-1} \Sigma_s^{-1} (A + A')(y - (x - \bar{x})). \]  

(A4)

Substituting from (A4) into the price (A1) and rearranging yields:

\[ p = \bar{v} - \frac{A'}{M} \Sigma_{vis} \bar{x} - \Sigma_{\bar{v}} \left( \Sigma_{\bar{v}}^{-1} + \Sigma_s^{-1} \right)^{-1} (\bar{v} - \mu) + \left( \frac{A'}{M} \Sigma_{vis} + \Sigma_{\bar{v}} \left( \Sigma_{\bar{v}}^{-1} + \Sigma_s^{-1} \right) \right)(y - (x - \bar{x})). \]  

(A5)

Solving relation (A5) for \( \mu \) gives the following, which uses a substitution from the relation (A5) for \( A \):

\[ \mu = \bar{v} - \frac{A'}{M} (A + A') \left( \frac{A'}{M} \Sigma_{vis} + A' \right)^{-1} \Sigma_{vis} \bar{x}. \]  

(A6)

Taking the relation for \( A \) in (A5) and rearranging yields the following equilibrium condition:

\[ A' \Sigma_x A \Sigma_{\bar{v}}^{-1} A - \frac{A'}{M} A' \Sigma_x A \Sigma_{\bar{v}}^{-1} \Sigma_v - A' - \frac{A'}{M} (\Sigma_v - \Sigma_{\bar{v}}) = 0. \]  

(A7)

Substituting from (A7) and the informed strategy (A2) into the conjectured price (3) and
simplifying yields expression (14). Substituting from (A6) into the informed trade (A2) gives informed trade:

\[ y = \frac{A}{M} \left( \frac{A}{M} \Sigma v_{ijs} + A' \right)^{-1} \Sigma v_{ijs} \bar{x} + (A + A')^{-1} \left( \bar{v}_i - \bar{v} + A(x - \bar{x}) \right). \]  

Substituting from (A6) and (A8) into price (A5) and rearranging gives (14).

**Proof of Proposition 2**

We conjecture that the price impact matrix has the following form:

\[ A = aI + cbb'. \]  

Substituting (A9), \( \Sigma = \sigma^2 c I \), and \( \Sigma_x = \sigma^2 x I \) into the right-hand-side of (A7) and rearranging gives:

\[ A = \sigma^2 c a^2 \left( a - \frac{d}{M} \sigma^2 c \right) I + \sigma^2 c \left( c^3 + c^2 + \left( c + \frac{\sigma^2}{\sigma^2 (b')^2} \right) \right) c - c_0 bb', \]  

where:

\[ c_0 = \frac{A}{M} \frac{\sigma^2 + a^2 \sigma^2}{(b')^2} > 0, \quad c_2 = \frac{3a - \frac{d}{M} \sigma^2}{(b')^2}, \quad c_1 = \frac{\sigma^2 a \left( 3a - \frac{4}{(b')^2} \right)}{4(b')^2 \sigma^2}. \]  

Solving (A10) for \( a \) has three solutions. One is \( a = 0 \), which would render \( A \) noninvertible and is thus invalid. The other two are positive and negative solutions to a quadratic, where the positive solution, along with a positive \( c \), ensures that the informed investors’ second-order condition is satisfied, giving the solution for \( a \) specified in (19). Given a solution for \( a \), \( c \) solves the following cubic equation:

\[ 0 = c^3 + c^2 c_2 + c_1 c - c_0. \]  

The right-hand-side of (A12) is negative at \( c = 0 \) and approaches positive infinity as \( c \) approaches positive infinity, so there exists a \( c > 0 \) that solves (A12).

**Proof of Proposition 3**

Using (12) and (16), we write can the equilibrium condition (17) for \( A \) as follows where
\[ C = A' \Sigma_x^{1/2}; \]

\[ A(A + A')^{-1} = \frac{A}{M} \left( \Sigma_v - \Sigma_{\eta} (\Sigma_{\eta} + CC')^{-1} \Sigma_{\eta} \right)(A + A')^{-1} + \Sigma_{\eta} (\Sigma_{\eta} + CC')^{-1}. \] (A13)

The left-hand-side of (A13) is bounded, so the right-hand-side must be bounded, as well. If \( C \to \infty \), then (A13) must satisfy:

\[ A(A + A')^{-1} = \frac{A}{M} \Sigma_v (A + A')^{-1} \Rightarrow A = \frac{A}{M} \Sigma_v. \] (A14)

If \( C \to 0 \), then:

\[ A(A + A')^{-1} = \frac{A}{M} \left( \Sigma_v - \Sigma_{\eta} \right)(A + A')^{-1} + I \Rightarrow A' = -\frac{A}{M} \left( \Sigma_v - \Sigma_{\eta} \right), \] (A15)

but that violates the informed trader’s second-order condition because \( \Sigma_v - \Sigma_{\eta} \) is positive definite.

For the case of known noise trades \((\Sigma_x \to 0)\), note that, because \( C = A' \Sigma_x^{1/2} \) cannot approach zero, \( A \) must become unbounded at a rate of at least \( \Sigma_x^{-1/2} \). If \( A \to \infty \) at a faster rate than \( \Sigma_x^{-1/2} \), then \( C \to \infty \) and (A14) states that \( A \to \frac{A}{M} \Sigma_v \), but that contradicts \( A \to \infty \). Thus, it must be the case that \( A \to \infty \) at a rate of \( \Sigma_x^{-1/2} \).

For the case of unbounded noise trades \((\Sigma_x \to \infty)\), if \( C \) is bounded as \( \Sigma_x \to \infty \), then it must be that \( A \to 0 \). Multiplying (A13) by \( A + A' \) and taking limits as \( A \to 0 \) gives:

\[ 0 = \frac{A}{M} \left( \Sigma_v - \Sigma_{\eta} (\Sigma_{\eta} + CC')^{-1} \Sigma_{\eta} \right), \] (A16)

which is a contradiction since the matrix on the right-hand-side of (A16) is positive definite.

Thus, from (A14), it must be that \( A \to \frac{A}{M} \Sigma_v \) and \( C = (A + A') \Sigma_x^{1/2} \to \infty \).

Proof of Proposition 4

From (14), we have the unconditional expected return:

\[ \text{E}[v - p] = \frac{A}{M} \Sigma_{\eta \eta} \left( I + \frac{A}{M} (A')^{-1} \Sigma_{\eta \eta} \right)^{-1} \bar{x}. \] (A17)

First, note that the matrix that multiplies \( \bar{x} \) is bounded:
\[
\left( I + \frac{A}{M}(A')^{-1}\Sigma_{v_is} \right)^{-1} = \left( A' + \frac{A}{M}\Sigma_{v_is} \right)^{-1} A' \prec I \iff \frac{A}{M}\Sigma_{v_is} > 0. \quad (A18)
\]

We can write \( \Sigma_{v_is} = B\Sigma_{f_p} B' + \Sigma_{e_{is}} \). Because \( \Sigma_{e_{is}} \prec \Sigma \) and \( \Sigma \) has bounded eigenvalues, then

\[
\frac{A}{M}\Sigma_{v_is} = \frac{A}{M}B\Sigma_{f_p} B' \text{ as } N, M \rightarrow \infty. \text{ This implies (25).} \quad \blacksquare
\]

**Notation for general setting**

In an exchange economy with \( N \) securities, denote portfolio weights by the \( N \)-vector \( d_r \) with payoff \( r_N = d_r'v \) with \( \lim_{N \rightarrow \infty} r_N = r \), portfolio prices \( p_{rN} = d_r'p \rightarrow p_r \) and portfolio expected payoffs \( \overline{v}_{rN} = d_r' \overline{v} \rightarrow \overline{v}_r \).

**Condition A: No arbitrage**

1) There are no riskless, costless portfolios with nonzero payoffs. Formally, if \( \text{var}(r_N) \rightarrow 0 \) and \( p_{rN} \rightarrow 0 \), then \( \overline{v}_{rN} \rightarrow 0 \).

2) There are no riskless portfolios with positive cost and non-positive payoffs. Formally, if \( \text{var}(r_N) \rightarrow 0 \) and \( p_{rN} \rightarrow p_r \neq 0 \), then \( \overline{v}_r \neq 0 \) and \( \overline{v}_r \propto p_r \).

If Condition A1 were violated by some limit portfolio \( \hat{r} \), then every investor would gain by adding long (short) positions in \( \hat{r} \) if \( \overline{v}_r > 0 \) \( (\overline{v}_r < 0) \). If Condition A2 were violated by some limit portfolio \( \hat{r} \) with \( p_r > 0 \) \( (p_r < 0) \) and \( \overline{v}_r \leq 0 \) \( (\overline{v}_r \geq 0) \), then every investor would gain by adding short (long) positions in \( \hat{r} \) without incurring any future obligations.

**Condition B: Covariance matrix restrictions**

For finite \( N \), denote the uninformed posterior covariance matrix of \( v_n \) by \( \Sigma_{v_ipN} \) and denote the \( k \textsuperscript{th} \) largest eigenvalue of \( \Sigma_{v_ipN} \) by \( \delta_{KN} \).

1) In the limiting economy, there are \( K \prec \infty \) unbounded eigenvalues: \( \sup_{N} \delta_{KN} = \infty \) and \( \sup_{N} \delta_{K+1,N} = \delta_{K+1} < \infty \).
2) In the limiting economy, there are no redundant securities: \( \inf_{N} \delta_{NN} = \delta_{\infty} > 0 \).

Chamberlain and Rothschild (1983) and Chamberlain (1983) prove the following:

**Lemma A1:**

If Condition A holds and if Condition B holds with respect to the uninformed information set, then:

\[
\Sigma_{v|pN} = B_{N}B'_{N} + R_{N},
\]

where the \( nk \) element of the \( N \times K \) matrix \( B_{N} \) is \( \text{cov}_{d}(v_{n}, f_{k}) \) and \( \{ R_{N} \} \) is a sequence of positive semidefinite matrices with bounded eigenvalues. The set \( P_{1} \) of fully-diversified portfolios, from the perspective of uninformed investors, is spanned by the \( K \) orthonormal portfolios \( f_{1}, \ldots, f_{K} \).

The set \( P_{2} \) of undiversified portfolios is orthogonal to \( P_{1} \) and any portfolio payoff \( r \) can be decomposed as \( r = r_{1} + r_{2} \) with \( r_{1} \in P_{1} \) and \( r_{2} \in P_{2} \) with \( \text{var}(r | p) = \text{var}(r_{1} | p) + \text{var}(r_{2} | p) \).

**Proof of Proposition 5**

Denote the payoff from the equally-weighted market portfolio by \( r_{ew} \) and diagonalize the matrix \( \Sigma_{v|pN} = \text{var}(v_{N} | p_{N}) \) as \( T_{N}D_{N}T'_{N} \):

\[
\text{var}(r_{ew} | p) = \lim_{N \to \infty} \text{var}_{d} \left( \frac{1}{N} \sum_{n} v_{n} \right) = \lim_{N \to \infty} \frac{1}{N^{2}} \text{tr} \sigma_{dN} t = \lim_{N \to \infty} \sum_{k=1}^{N} \delta_{k} (t'_{k} t_{k})^{2}. \]

(A20)

Because the eigenvectors are orthonormal so that \( \sum_{k=1}^{N} t_{k}^{2} = t'_{k} t_{k} = 1 \), it must be that the elements of \( t_{k} \) are of order \( N^{-1/2} \). This implies that \( t'_{k} t_{k} \sim N^{1/2} \) and \( \frac{1}{N^{2}} (t'_{k} t_{k})^{2} \sim N^{-1} \). Thus, any summand in (A20) corresponding to an eigenvalue of order less than \( N \) approaches zero as \( N \to \infty \) and for any eigenvalue of order \( N, \delta_{k} \sim N \), the summand \( \frac{1}{N^{2}} \delta_{k} (t'_{k} t_{k})^{2} \sim 1 \). If there are \( K \) such unbounded eigenvalues, then the sum is of order \( K, \text{var}(r_{ew} | p) \sim K \). Thus, if there are no

---

\( \delta_{\infty} = O(y_{N}) \) to denote that \( x_{N}/y_{N} \) is bounded, \( x_{N} = o(y_{N}) \) denotes that \( x_{N} / y_{N} \to 0 \), and \( x_{N} \sim y_{N} \) denote that both \( x_{N}/y_{N} \) and \( y_{N}/x_{N} \) are bounded and thus increase at roughly the same rate as \( N \to \infty \).
unbounded eigenvalues, then \( \text{var}(r_{ew} \mid p) \to 0 \) while if there are infinitely many, then 
\( \text{var}(r_{ew} \mid p) \to \infty. \)

If the equally-weighted portfolio has positive but bounded variance, then Condition B holds. In conjunction with the assumption that the no-arbitrage Condition A holds, this implies that \( \Sigma_{v/p} \) has an approximate K-factor structure as implied by Lemma A.

**Proof of Proposition 6**

If the assumptions of Proposition 3 are satisfied, the well-diversified portfolio can be written as a linear combination \( \beta'f \) of the \( K \) portfolios that span the set \( P_1 \) of well-diversified portfolios. In other words, \( d'(v - p) = \beta'f \). We can also divide \( v \) into the component correlated with \( f \) and an orthogonal component \( e \):

\[
v = E[v \mid p] + Bf + e.
\]

From the equilibrium condition (33), we then have:

\[
p = E\left[ \frac{u'(\beta f)}{E[u(\beta f)']} v \right] = E[v \mid p] + B E[\xi f \mid p] + E[\xi E[e \mid f, p] \mid p].
\]

The pricing condition (33) holds for all securities, and therefore it must hold for the spanning portfolios \( f \), so that the price vector \( p_f \) of the \( K \) spanning portfolios must be \( p_f = E[\xi f \mid p] \), giving:

\[
p = E[v \mid p] + Bp_f + E[\xi E[e \mid f, p] \mid p].
\]

Expression (38) follows after putting \( E[e \mid f, p] = 0 \) for the case where \( e \) is mean-independent of \( f \), given the conditioning information in prices \( p \).

**Proof of Proposition 7**

The market clearing condition given \( M \) uninformed investors is:

\[
x = Md + y \Rightarrow \quad d = \frac{1}{M} (x - y) \to 0 \quad \text{as} \quad M \to \infty,
\]

where the limits hold if \( x \) and \( y \) are bounded.
Appendix B

This appendix provides expressions for the case where the informed trader has CARA utility with risk aversion coefficient $A_i$. The informed trader’s objective function (in terms of certainty equivalents), trade, and expected utility are the following where $\Sigma_i = \Sigma_v - \Sigma_i$:

Objective: $y'(\bar{v}_i - \mu - A_i(y - (x - \bar{x}))) - \frac{A_i}{2} y'S_iy$,

Trade: $y = (A + A_i + A_i\Sigma_i)^{-1}(\bar{v}_i - \mu + A(x - \bar{x}))$, (B1)

Expected profit: $y'(A' + \frac{A_i}{2} \Sigma_i) y$.

The uninformed traders’ demands are unchanged from what is given in expressions (10) through (12) in the manuscript. Applying the market clearing condition and equating coefficients in the conjectures of price and informed demand gives:

$$
p = \bar{v} - \frac{A}{M} \Sigma_{s\ell s} \left( I + \frac{A}{M} (A' + A_i \Sigma_i)^{-1} \Sigma_{s\ell s} \right)^{-1} \bar{x} + A_i (A' + A_i \Sigma_i)^{-1} \left( \bar{v}_i - \bar{v} + A_i(x - \bar{x}) \right),
$$

(B2)

$$
y = \frac{A}{M} \Sigma_{s\ell s} + (A' + A_i \Sigma_i)^{-1} \Sigma_{s\ell s} \bar{x},
$$

where the matrix $A$ solves:

$$(A' + A_i \Sigma_i) \Sigma_x (A + A_i \Sigma_i)^{-1} (A + A_i \Sigma_i)$$

$$- \frac{A}{M} (A' + A_i \Sigma_i) \Sigma_x (A + A_i \Sigma_i)^{-1} \Sigma_v - (A' + A_i \Sigma_i) - \frac{A}{M} (\Sigma_v - \Sigma_{\ell}) = 0. \quad (B3)$$

The cubic (B3) is identical to (A7) except that it is solved by $A + A_i \Sigma_i$ instead of $A$. The remaining results and expressions are likewise qualitatively similar those shown in the manuscript for a risk-neutral informed trader. Risk-aversion reduces the informed trader’s willingness to absorb liquidity demands, which further increases expected returns. In the large economy limit, this increases factor prices but does not alter the result that expected returns depend only on systematic risk. As a potential caveat, even if the matrix that solves (B3) is positive definite, $A$ may not be positive definite if $A_i$ is too large.