“Idiosyncratic Risk, Systematic Risk, and Firm Welfare”

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April 2012

First Draft: March 2012

Abstract: We study the motivation of an entrepreneur to provide disclosure that works primarily to reduce investors’ assessment of the firm’s idiosyncratic risk. We refer to this disclosure as “idiosyncratic disclosure.” We assume that the firm’s cash flows are determined by both idiosyncratic and systematic events. The model shows that when the entrepreneur faces a capital market where competition to acquire firm’s shares is imperfect, he trades off the reduction in investors’ assessment of the firm’s idiosyncratic risk against the reduction in liquidity as a result of increased competitiveness among investors who trade in the firm’s shares. In other words, a reduction in idiosyncratic risk yields countervailing effects. This result contrasts with the one in Lambert, Leuz, and Verrecchia (2007) that shows that a firm benefits unambiguously from disclosure that reduces investors’ assessment of systematic risk: in effect, “systematic disclosure.”

We thank seminar participants at UCLA and, specifically, Judson Caskey for insightful comments.
1 Introduction

One of the contributing factors that firms take into consideration when deciding about their disclosure policy is the risk premium that investors demand for holding a firm’s shares. The risk premium itself is mainly determined by the total amount of risk that investors face and the risk bearing capacity of the respective market or, in other words, of a firm’s shareholder base. However, in much of the literature on firms’ disclosure choices in perfect competition settings such as the Capital Asset Pricing Model (CAPM), the risk bearing capacity of firms’ shareholder base is fixed. As a perfect competition setting requires that there be a very large number of investors who are willing to provide capital, this assumption seems natural. This implication notwithstanding, there may only be a few firms in a few instances whose access to capital markets approximates a perfect competition setting. When a capital market is illiquid or imperfect, the assumption that investors are willing to provide capital independent of a firm’s disclosure choice is not as straightforward. For example, an implication of perfectly competitive capital markets and perfect diversification in investors’ portfolios is that the level of systematic risk alone should affect the cost of funding. When, instead, the number of investors is limited, idiosyncratic risk has the potential to affect both the cost to the firm to acquire funding and the general willingness of investors to provide capital.

The goal of our paper is to develop a model that captures the above mentioned effects and investigates the motivation of firm to provide disclosure that works primarily to reduce investors’ assessments of the firm’s idiosyncratic risk. Henceforth we employ the expressions “idiosyncratic disclosure” to describe disclosure that reduces investors’ assessment of the
firm’s idiosyncratic risk, and “systematic disclosure” to describe disclosure that reduces investors’ assessment of the covariance of the firm’s cash flow with the market cash flow (i.e., systematic risk). Here we study the level of idiosyncratic disclosure that maximizes an entrepreneur’s welfare in the presence of a fixed level of systematic risk. There are two reasons why we couch the problem along these lines, and, in particular, treat idiosyncratic disclosure as endogenous and systematic risk as fixed, or at least very “sticky” (i.e., not susceptible to change). First, a firm’s systematic risk results from the collective investment and disclosure choices of many firms within an industry or economy, and is thus may be difficult to associate with the actions of a single firm. As such, a firm’s level of systematic risk may not be a choice variable for a firm. For example, investment choices and disclosure practices within an industry or economy, or as the result of an economy’s Generally Accepted Accounting Principles (GAAP), may largely govern levels of systematic risk. Second, and as discussed in Lambert, Leuz, and Verrecchia (2007), a firm benefits unambiguously from disclosure that reduces investors’ assessment of systematic risk - which leaves little to say about this issue. Alternatively, we focus on idiosyncratic disclosure as a choice variable because we show that disclosure that reduces idiosyncratic risk can have countervailing effects on the welfare of the entrepreneur and/or firm.

Broadly stated, these countervailing forces operate as follows. On the one hand, increased idiosyncratic disclosure reduces investors’ assessment of idiosyncratic risk and thereby works to lower the discount that investors apply to a firm’s expected cash flow. This discount is central to notions of cost of capital (see, e.g., Lambert et al., 2007). On the other hand, increased idiosyncratic disclosure reduces the incentive for investors to provide capital to the firm, which makes trade in shares of the firm less liquid. In imperfect competition settings,
lower liquidity adversely affects the discount investors apply to the firm’s expected cash flow and by implication cost of capital. Increased idiosyncratic disclosure reduces the incentive to become aware of an investment because less idiosyncratic risk induces those investors who trade in the firm’s shares to trade more aggressively. This, in turn, increases the degree of competition among investors, which decreases their expected utility; decreased expected utility leads to fewer investors manifesting a willingness to provide capital to the firm in the first place. When investors decline to trade, they make the market for the investment less liquid, and a reduction in liquidity increases the discount investors apply to the investment’s cash flow. In short, in determining the optimal level of idiosyncratic disclosure, the entrepreneur has to trade off the reduction in investors’ assessment of the firm’s idiosyncratic risk against the reduction in liquidity as a result of increased competitiveness among its investors.

To showcase these countervailing forces, we introduce a parsimonious model of a market economy. The economy has three distinctive features. First, the economy is populated by $M$ investors, where $M$ is large. Second, the economy offers three investment opportunities: shares in a firm, shares in the market portfolio, and a risk-free asset. Distinguishing idiosyncratic risk from systematic disclosure necessitates having multiple investment opportunities, and so along with shares in the firm we include the market portfolio as an alternative investment vehicle. Third, to incorporate effects of the entrepreneur’s disclosure decision on the availability of capital, we employ a structure similar to that in Merton (1987) and Allen and Gale (1994) where, before trading in a firm’s shares, an investor has to decide whether to enter the market at a fixed cost or eschew trade. In other words, we assume that all $M$ investors can trade in the market portfolio and the risk-free asset, but an investor can only trade in the firm’s shares if he absorbs a cost $C$. Thus, only a subset $N$ of the $M$
investors will be able to trade the firm’s shares. We determine the subset \( N \) as follows. As all investors have the opportunity to trade in the firm’s shares, in equilibrium it must be the case that each investor is indifferent between being able to trade in the firm or not. Thus, in equilibrium the expected utility of an investor who can trade in the firm’s shares must be equivalent to the expected utility of an investor who cannot.

Merton (1987) suggests that it is generally naive to believe that investors include in their personal portfolios the universe of equity securities in the economy. Rather, each investor is only likely to include in his portfolio those securities about which he has some knowledge. Merton bases his argument on the observation that “portfolios held by actual investors (both individual and institutional) contain only a small fraction of the thousands of traded securities available,” (see p. 488). There are prominent examples of situations in which the number of investors is limited and investors have to expend resources to trade in the shares of a specific firm. First, rule 144A of the Securities Act of 1933 describes that certain securities can only be sold to “qualified institutional buyers” (QIB) and financial statements do not have to be provided to the buyers. These QIBs include large institutional investors with at least $100 million in investable assets. Thus, the shareholder base is limited, there are requirements that the investors have to meet, and the investors face a cost to become informed about the specific firm. Livingston and Zhou (2002) finds that debt issued under rule 144A has higher yields (after adjusting for risk) than publicly issued bonds and suggests lack of liquidity as a potential reason for this finding. Second, firms that trade their shares on the Pink Sheets (or, OTC Pink) face an imperfectly competitive market in which the demand by one trader significantly affects the price of the firm’s shares. While investors do not bear any costs of investing in any specific firm, the Pink Sheets gained negative publicity
when incidences of “pump and dump” schemes became public (the value of a company is talked up publicly and all shares are sold at artificially high prices afterwards).\footnote{See, for example, Dunham (2007) on the use of mass emails in “pump and dump” schemes. Bushee and Leuz (2007) provide evidence that an increase in mandatory disclosure regulation increased liquidity in OTC markets. However, some firms decided not to comply with the disclosure rules and instead traded their shares in the Pink Sheets. Ang, Shtauber, and Tetlock (2010) finds that illiquidity premiums are higher in OTC market than in listed markets.} Therefore, before investing in a firm, investors will invest some time to ensure that the firm they are interested in is not involved in fraudulent activities. Finally, firms that raise private equity usually have a limited number of investment companies that provide the funding. Thus, imperfect competition among these investment companies is apparent. Additionally, these companies expend significant resources to be able to judge the risk and return resulting from the investment in a specific firm.

As an alternative to the imperfect competition setting, consider a perfectly competitive market. Here, one presumes that the market is sufficiently deep and liquid such that liquidity plays no role in disclosure choice: hence the result in Lambert, et al. (2007) that a firm benefits unambiguously from disclosure that reduces investors’ assessment of systematic risk. In other words, a feature that distinguishes idiosyncratic disclosure from systematic disclosure is a trade-off, in the case of the former, related to liquidity. Thereby, our analysis contributes to recent literature that has begun to re-examine the roles of disclosure and cost of capital with particular emphasis on whether the effect of disclosure works primarily to reduce idiosyncratic risk versus systematic risk. For example, Anilowski, Feng, and Skinner (2007) and Bonsall, Bozanic, and Fischer (2011) investigate the extent of macroeconomic information in management earnings guidance. Gao and Verrecchia (2012) study aspects of idiosyncratic versus systematic accounting information in the context of moral hazard. There seems to be
a growing awareness that in multi-firm economies, idiosyncratic disclosure/information has features that are different from systematic disclosure/information.

In the next section we describe our market setting in more detail. Section 3 offers empirical implications and Section 4 concludes.

2 Market Setting

We consider an economy where a risk neutral entrepreneur anticipates that he will have to raise capital in the future by selling shares of his firm. We treat the sale of firm shares as an exogenous shock that results from the entrepreneur’s need to satisfy some capital requirement. The capital requirement could arise from a need for: investment funds for positive net-present-value (NPV) projects; investment funds to acquire or invest in other firms; funds to satisfy outstanding debt obligations; funds for additional compensation to retain top managers; etc. A recent example of the type of exogenous shock that we describe is Facebook’s decision to execute an initial public offering (IPO) so venture capitalists (VCs) who initially funded Facebook can get some of their investment back.

The economy is populated by $M$ investors and offers three investment opportunities: 1) shares in the firm sold by the entrepreneur in response to the exogenous shock; 2) shares in the market portfolio; and 3) a risk-free asset. The return on the risk-free asset is normalized to zero and its price is normalized to 1. Each share of firm stock generates uncertain cash flow of $\tilde{V}_1 = \beta \tilde{x} + \tilde{y}$, and each share of the market portfolio generates uncertain cash flow of $\tilde{V}_2 = \tilde{x}$; eventually these cash flows are realized: let $V_1$ and $V_2$ represent their respective realizations.\(^2\) While the market portfolio’s (per-share) cash flow is given by $\tilde{x}$, the (per-share) cash

\(^2\) Henceforth we use a tilde (i.e., “~”) to distinguish a random variable from a fixed element.
cash flow of the firm is determined by the realizations of $\tilde{x}$ and $\tilde{y}$. Accordingly, we interpret $\tilde{y}$ as the idiosyncratic component of the firm’s cash flow and $\beta$ as the firm’s exposure to systematic risk.

A key assumption of our analysis is that while all $M$ investors know about the market portfolio and risk-free asset as investment opportunities, only a subset $N$ of the $M$ investors can trade in the firm’s shares. In his widely cited paper, Merton (1987) uses two criteria to characterize investors’ knowledge about a firm (see p. 488). First, an investor only includes an investment in his optimal portfolio “...if the investor knows about [the investment].” This condition rules out the possibility of an investor including in his portfolio an investment about which he has no knowledge. Second, all investors who are aware of an investment “have the same information about [the investment].” This condition rules out the possibility of information asymmetry in trade in the investment. In his paper, Merton discusses information cost structures that support these criteria. Merton’s notion of awareness/knowledgeability has a dimension that is reminiscent of standard information acquisition activities. An investor can acquire information about an investment opportunity that is valuable in assessing its cash flow; having acquired this information, the investor then includes the investment in his portfolio.

As mentioned above, we assume that all investors can trade in shares of the market portfolio. An investor can choose to trade in the firm’s shares by absorbing a cost of $C$ before the trade commences. As all investors have the opportunity to do so, in equilibrium it must be the case that before trade commences each investor is indifferent between trading in the firm’s shares or not. This “indifference condition” forms the basis for establishing the equilibrium number of investors who absorb $C$ (i.e., the equilibrium $N$).
The timeline for events in our economy is as follows. At \( t = 0 \) the entrepreneur (who holds \( S \) shares of the firm) chooses his firm’s disclosure policy, and at \( t = 1 \) investors decide whether to trade the firm’s shares (at a cost \( C \)) based on the disclosure profile chosen at \( t = 0 \). At \( t = 2 \), prompted by an exogenous shock, the entrepreneur sells some number of shares in his firm; those investors who can trade in the firm’s shares compete for those shares, while all investors trade in shares of the market portfolio. At \( t = 3 \) all cash flows are realized.

We assume all random variables are independent and have a normal distribution where the market cash flow \( \tilde{x} \) has mean \( \mu \) and precision \( \tau \), and the firm’s idiosyncratic cash flow \( \tilde{y} \) has mean \( \eta \) and precision \( \theta \). Thus, the \( N \) investors who become knowledgeable about the firm assess the vector of expected cash flows in the bivariate normal distribution of \( \tilde{V}_1 \) and \( \tilde{V}_2 \) to be

\[
E \left[ \tilde{V}_1 \right] = \beta \mu + \eta \quad \text{and} \quad E \left[ \tilde{V}_2 \right] = \mu, \tag{1}
\]

and the covariance matrix in the bivariate normal distribution of \( \tilde{V}_1 \) and \( \tilde{V}_2 \) to be

\[
\text{Cov} = \begin{pmatrix}
\beta^2 \frac{1}{\tau} + \frac{1}{\theta} & \beta \frac{1}{\tau} \\
\beta \frac{1}{\tau} & \frac{1}{\tau}
\end{pmatrix}. \tag{2}
\]

At \( t = 0 \) the entrepreneur discloses proprietary information that reduces uncertainty about idiosyncratic events that affect the firm’s cash flow. We represent the entrepreneur’s disclosure as the choice of the precision in the firm’s idiosyncratic cash flow, \( \theta \). For simplicity we assume that for every unit of precision \( \theta \), the entrepreneur has to invest \( c \theta \) units of capital (where \( c \) is a cost parameter).\(^3\) At \( t = 0 \) the entrepreneur chooses \( \theta \) in anticipation of the

\(^3\) For an interior solution to the optimal disclosure decision to exist, it is not necessary that the cost of disclosure be linear - just that it be increasing.
following events at \( t = 2 \): 1) the entrepreneur being required to sell an uncertain number of shares in the firm, which we represent by \( \tilde{Z}_1 \) (we discuss the distribution of \( \tilde{Z}_1 \) below); and 2) the prices of shares in the firm and market portfolio being \( \tilde{P}_1 \) and \( \tilde{P}_2 \), respectively. Taken together, this implies that the entrepreneur’s expected utility at \( t = 0 \), \( E[U_e] \), reduces to

\[
E[U_e] = E\left[ S \cdot \tilde{V}_1 + \tilde{Z}_1 \left( \tilde{P}_1 - \tilde{V}_1 \right) - c\theta \right]. \tag{3}
\]

This illustrates that the benefit of disclosure is a reduction in the discount that investors apply to the firm’s cash flow, \( E[\tilde{V}_1 - \tilde{P}_1] \). This discount is commonly interpreted as the firm’s cost of capital (e.g., Lambert et al., 2007). As the discount depends crucially on the firm’s equilibrium price, we derive this next.

### 2.1 Market Prices

In order to provide a simple representation of the market process, we assume that the market for trade in the firm’s shares is imperfectly competitive while the market for trade in shares in the market portfolio is perfectly competitive. We motivate this assumption by appealing to the fact that the \( N \) investors who end up trading in shares in the firm are finite in number, whereas the \( M \) investors who trade in the market portfolio are sufficiently large in number as to approximate a perfectly competitive market.

Our representation of an imperfectly competitive market for trade in shares of the firm is motivated by Kyle (1989), but follows more closely the discussion in Lambert, Leuz, and Verrecchia (2012). Because imperfect competition settings are complex, we break the analysis up into a series of steps. The first step provides an expression for investors’ demand for shares in the firm and the market portfolio. The second step provides an expression for the prices of shares of the firm and market portfolio based on a requirement that the total demand for
shares equals the available supply. The final step reconciles the strategy investors follow to purchase shares based on their demand for those shares (as established in the first step) and prices for those shares (as established in the second step). Following Merton (1987), we refer to investors that are able to trade in the firm’s shares as “knowledgeable” and investors that cannot as “unknowledgeable”.

**A knowledgeable investor’s demand.** Knowledgeable investors form portfolios from shares in the firm, shares in the market portfolio, and the risk-free asset. Unknowledgeable investors form portfolios exclusively from shares in the market portfolio and the risk-free asset. Henceforth we distinguish knowledgeable investors from unknowledgeable investors using the subscripts “k” and “u,” respectively, and we distinguish the firm from the market portfolio using the subscripts “1” and “2,” respectively. To derive a knowledgeable investor’s demand, we assume that each knowledgeable investor conjectures that his demand for the shares in the firm, $D_{k1}$, is related to the per-share price of the firm, $\tilde{P}_1$, through the expression

$$\tilde{P}_1 = p_1 + \lambda D_{k1},$$

where $p_1$ is an intercept term and $\lambda$ is a coefficient. In effect, each investor believes that price results from a factor that is unrelated to his demand, $p_1$, and a factor that is related to his demand through the coefficient $\lambda$. As is standard in a model of imperfect competition, we interpret $\lambda$ as the degree of illiquidity associated with an individual investor’s demand. For example, when $\lambda$ is small, an investor’s demand moves price less, and thus the market for shares is more liquid with respect to demand. Finally, as the shares of the market portfolio are traded under the auspices of perfect competition, a knowledgeable trader conjectures that his demand for shares in the market portfolio, $D_{k2}$, does not affect the price he pays for
those shares.

We assume that each investor has a negative exponential utility function for an amount \( w \) given by \(- \exp(-w/r)\), where \( r \) is the investor’s constant absolute risk tolerance. Given the asserted linear structure and normal distributions, a knowledgeable investor chooses \( D_{k1} \) and \( D_{k2} \) to maximize his certainty equivalent

\[
CE_k = \left( E \left[ \bar{V}_1 \right] - P_1 \right) D_{k1} + \left( E \left[ \bar{V}_2 \right] - P_2 \right) D_{k2} - \frac{1}{2r} \left( \frac{1}{\theta} D_{k1}^2 + \frac{1}{\tau} (\beta D_{k1} + D_{k2})^2 \right). \tag{5}
\]

Substituting \( P_1 \) and maximizing over \( D_{k1} \) and \( D_{k2} \) yields

\[
D_{k1} = \frac{r\theta \eta - P_1 + \beta P_2}{1 + r\theta \lambda} \quad \text{and} \quad \tag{6}
\]

\[
D_{k2} = r\tau (\mu - P_2) - \beta D_{k1}. \tag{7}
\]

Eqn. (6) illustrates why idiosyncratic disclosure has countervailing effects in an imperfect competition setting. On the one hand, an increase in idiosyncratic precision, \( \theta \), increases an investor’s demand through the “\( r\theta \)” term in the numerator of eqn. (6). The “\( r\theta \)” term, which is the product of an investor’s risk tolerance and the precision of his information, represents the degree of confidence investors associate with their assessments of the firm’s uncertain cash flow. On the other hand, an increase in idiosyncratic precision decreases an investor’s demand through the “\( r\theta \lambda \)” term in the denominator. The “\( r\theta \lambda \)” term represents the effect of \( \theta \) on the level of competition among investors. In a perfect competition setting, \( \lambda = 0 \) and thus an increase in \( \theta \) has no effect on the level of competition. In other words, in a perfect competition setting there is no countervailing effect.

As for eqn. (7), the first term represents a knowledgeable investor’s demand for shares of the market portfolio, which are traded in a perfectly competitive market. The second
term adjusts an investor’s exposure to systematic risk as a result of trading in the firm’s shares. For example, if the market portfolio is priced at the expected value of its cash flow (i.e. \( P_2 = E[\hat{V}_2] = \mu \)), no investor who could only trade in the market portfolio would do so. If an investor trades in both shares, however, trade in shares in the firm will expose the investor to systematic risk. An investor will undo this exposure by trading in the market portfolio.

**An unknowledgeable investor’s demand.** Let \( D_{u2} \) denote an unknowledgeable investor’s demand in the market portfolio. Unknowledgeable investors can only trade in the market portfolio such that their certainty equivalent is given by

\[
CE_u = (\mu - P_2) D_{u2} - \frac{1}{2 \tau^2} \sigma_{u2}^2.
\]  

Maximizing the certainty equivalent over \( D_{u2} \) yields

\[
D_{u2} = \tau (\mu - P_2).
\]

As stated above, the demand of an unknowledgeable investor in the market portfolio equals the first term of a knowledgeable investor’s demand in the market portfolio. This is the case because all investors have the same risk tolerance and are equally knowledgeable about the market portfolio.

**Market clearing.** Having determined investors’ demand for shares in the firm and market portfolio, our next step is to provide an expression for the prices of the firm and market portfolio. Market clearing requires that the total demand for an asset’s shares equals the available supply. We assume that the number of shares that the entrepreneur sells in his firm, \( \tilde{Z}_1 \), and the supply in shares of the market portfolio, \( \tilde{Z}_2 \), are both uncertain until \( t = 2 \). As for the distribution of \( \tilde{Z}_1 \) and \( \tilde{Z}_2 \), we assume that: 1) \( \tilde{Z}_1 \) and \( \tilde{Z}_2 \) have a bivariate normal
distribution that is independent of all other random variables including each other (i.e., the bivariate is the product of two univariate normal distributions); 2) \( E[\tilde{Z}_1] \) and \( E[\tilde{Z}_2] \) represent their respective means; and 3) \( \sigma_{z_1}^2 \) and \( \sigma_{z_2}^2 \) represent their respective variances. Thus, market clearing requires that

\[
\tilde{Z}_1 = N \cdot D_{k1} \quad \text{and} \quad \tilde{Z}_2 = N \cdot D_{k2} + (M - N) \cdot D_{u2},
\]

Substituting the demand of knowledgeable and unknowledgeable investors from eqns. (6), (7), and (9) yields the result

\[
P_1 = \eta + \beta \mu - \beta \frac{\tilde{Z}_1 \beta + \tilde{Z}_2}{M r \tau} - \tilde{Z}_1 \frac{(1 + r \theta \lambda)}{N r \theta} \quad \text{and} \quad (12)
\]

\[
P_2 = \mu - \frac{\tilde{Z}_1 \beta + \tilde{Z}_2}{M r \tau} \quad \text{and} \quad (13)
\]

Henceforth we describe the share prices \( \tilde{P}_1 \) and \( \tilde{P}_2 \) as random variables because they depend on the supply of shares, \( \tilde{Z}_1 \) and \( \tilde{Z}_2 \), and this supply is uncertain until \( t = 2 \).

**An investor’s strategy.** Next we solve for \( \lambda \). The solution to \( \lambda \) results from knowledgeable investors’ trading strategies for purchasing shares in the firm (remember that in order to trade in shares in the firm an investor has to become knowledgeable). Specifically, in competing with other investors we assume that a knowledgeable investor adopts the strategies

\[
D_{k1} = \psi_1 - \gamma P_1 + \phi_1 P_2 \quad \text{and} \quad D_{k2} = \psi_2 + \phi_1 P_1 - \phi_2 P_2,
\]

where \( \psi_1 \) and \( \psi_2 \) are intercept terms and \( \gamma, \phi_1, \) and \( \phi_2 \) are the weights a knowledgeable investor places on \( P_1 \) and \( P_2 \). First, note that for an investor’s strategy to be rational based
on the computation of $D_{k1}$ and $D_{k2}$ in eqns. (6) and (7) it must be the case that

$$\gamma = \frac{r\theta}{1+r\theta \lambda}, \quad \phi_1 = \gamma \beta, \quad \text{and} \quad \phi_2 = r\tau + \beta^2 \gamma.$$ 

This implies that the intercept terms must be

$$\psi_1 = \gamma \eta \quad \text{and} \quad \psi_2 = r\tau \mu - \beta \eta \gamma.$$ 

Thus, substituting $D_{k1} = \psi_1 - \gamma P_1 + \phi_1 P_2$ into the market-clearing condition for the firm and solving for the $P_1$ that clears the market yields

$$P_1 = \Delta \left( N \left( \psi_1 + \phi_1 \tilde{P}_2 \right) - \tilde{Z}_1 \right),$$

where $\Delta$ is given by

$$\Delta = \frac{1}{\gamma N} = \frac{r\theta}{N(1+r\theta \lambda)}.$$ 

The variable $\Delta$ represents the marginal impact on the price of the firm of an additional share brought to (or withdrawn from) the market. To reconcile the firm’s market clearing condition $P_1 = \Delta \left( N \left( \psi_1 + \phi_1 \tilde{P}_2 \right) - \tilde{Z}_1 \right)$ with the strategy that $D_{k1} = \psi_1 - \gamma P_1 + \phi_1 P_2$, it must be the case that $p_1$ in the expression $\tilde{P}_1 = p_1 + \lambda D_{k1}$ is of the form

$$p_1 = \lambda \left( (N-1) \left( \psi_1 + \phi_1 \tilde{P}_2 \right) - \tilde{Z}_1 \right)$$

and thus

$$\tilde{P}_1 = \frac{\lambda}{(1+\lambda \gamma)} \left( N \left( \psi_1 + \phi_1 \tilde{P}_2 \right) - \tilde{Z}_1 \right).$$

This implies that

$$\lambda = \frac{1}{r\theta (N-2)}.$$ 

4 For $\lambda$ to be well defined, it must be the case that $N \geq 3$ so as to eliminate the possibility of one investor, or a pair of investors, having too much monopoly power in trade in the firm’s shares; see the discussion on p. 329 of Kyle (1989).
The solution to $\lambda$ allows us to re-express prices in eqns. (12) and (13) as follows.

**Proposition 1** The (per-share) prices of the firm and market portfolio, where the $N$ knowledgeable investors trade in shares in the former and all $M$ investors trade in shares of the latter, reduce to

\begin{align*}
P_1 &= \eta + \beta \mu - \frac{N - 1}{r \theta (N - 2)} \frac{\tilde{Z}_1}{N} - \frac{\beta \tilde{Z}_1 + \tilde{Z}_2}{M} \quad \text{and} \\
P_2 &= \mu - \frac{\beta \tilde{Z}_1 + \tilde{Z}_2}{Mr\tau},
\end{align*}

respectively.

Note that when the prices are as stated in Proposition 1, investors’ demand functions are

\begin{align*}
D_{k1} &= \frac{\tilde{Z}_1}{N}, \\
D_{k2} &= \frac{\tilde{Z}_2}{M} - \beta \tilde{Z}_1 \left( \frac{1}{N} - \frac{1}{M} \right) \quad \text{and} \\
D_{u2} &= \frac{\beta \tilde{Z}_1 + \tilde{Z}_2}{M}.
\end{align*}

In other words, in equilibrium each knowledgeable investor holds $1/N$-th of the firm’s shares issued by the entrepreneur, $\tilde{Z}_1$. Furthermore, if all $M$ investors become knowledgeable about the firm and thus trade in the firm’s shares (i.e., $N = M$) then each investor holds $1/M$-th of the total supply vector $(\tilde{Z}_1, \tilde{Z}_2)$. If $N < M$, however, then each knowledgeable investor holds a smaller fraction of the shares of the market portfolio to control for the systematic risk he already bears through holding the firm’s individual shares. This, in turn, implies that each unknowledgeable investor holds relatively more shares of the market portfolio.

As noted above, because the entrepreneur has to sell shares in his firm, he is motivated to minimize the discount that investors apply to the firm’s cash flow. This implies the following corollary to Proposition 1.
**Corollary 1.** In an economy where \( N \) knowledgeable investors trade in shares of the firm and all \( M \) investors trade in shares of the market portfolio, the discount reduces to

\[
E \left[ \tilde{V}_1 - \tilde{P}_1 \right] = \frac{(N - 1)}{r \theta N (N - 2)} E \left[ \tilde{Z}_1 \right] + \frac{\beta}{r \tau M} \left( \beta E \left[ \tilde{Z}_1 \right] + E \left[ \tilde{Z}_2 \right] \right).
\]

A salient feature of Corollary 1 is that the discount decreases as either \( N \) or \( M \) increases. The discount decreases as \( M \) rises because investors that trade in the firm’s shares (knowledgeable investors) are better able to absorb their share of the firm’s systematic risk. Furthermore, *ceteris paribus*, any reduction in risk (i.e., an increase in \( \theta \) or, provided that \( \beta > 0 \), an increase in \( \tau \) or a decrease in \( \beta \)) also reduces the discount. In the next section, however, we discuss why treating the number of investors that are willing to trade in the firm’s shares as endogenous provides a countervailing force that results in greater idiosyncratic risk increasing the number of investors. We refer to the total number of investors that trade in the firm’s shares as its shareholder base.

### 2.2 Shareholder base

Our next goal is to determine \( N \), the number of investors who choose to trade in the firm’s shares at \( t = 1 \) before trade actually commences. Investors who choose to do so absorb a cost \( C \). To determine \( N \), recall from Proposition 1 that

\[
P_1 = \eta + \beta \mu - \frac{(N - 1)}{r \theta (N - 2)} \frac{\tilde{Z}_1}{N} - \frac{\beta \tilde{Z}_1 + \tilde{Z}_2}{M} \quad \text{and} \quad \text{(19)}
\]

\[
P_2 = \mu - \frac{\beta \tilde{Z}_1 + \tilde{Z}_2}{Mr \tau}, \quad \text{(20)}
\]

and each knowledgeable investor’s demand is

\[
D_{k1} = \frac{\tilde{Z}_1}{N} \quad \text{(21)}
\]

\[
D_{k2} = \frac{\tilde{Z}_2}{M} - \beta \tilde{Z}_1 \left( \frac{1}{N} - \frac{1}{M} \right). \quad \text{(22)}
\]

16
Because an investor must absorb a cost \( C \) to become knowledgeable and denoting \( \tilde{X} = \tilde{x} - \mu \) and \( \tilde{Y} = \tilde{y} - \eta \), a knowledgeable investor’s expected utility is

\[
E[U_k] = E \left[ -\exp \left( \frac{-1}{r} \left( \tilde{Y} + \frac{(N - 1) \tilde{Z}_1}{r \theta N (N - 2)} \tilde{Z}_1 + \left( \tilde{X} + \frac{\beta \tilde{Z}_1 + \tilde{Z}_2}{r \tau M} \right) \frac{\beta \tilde{Z}_1 + \tilde{Z}_2}{M} - C \right) \right) \right].
\]  

(23)

Given the asserted structure, integrating over all random variables yields

\[
E[U_k] = \frac{r^4 \tau M^2 \theta N (N - 2)}{\sigma^2_1 (r^2 \tau M^2 + \sigma^2_2) + r^2 \theta N (N - 2) (r^2 \tau M^2 + \beta^2 \sigma^2_1 + \sigma^2_2)} \exp \left( \frac{C}{r} \right) \times \exp \left( \frac{-(M^2 + \sigma^2_2) \left( E \left[ \tilde{Z}_1 \right] \right)^2 + \sigma^2_1 \left( E \left[ \tilde{Z}_2 \right] \right)^2 + r^2 \theta N (N - 2) \left( \beta E \left[ \tilde{Z}_1 \right] + E \left[ \tilde{Z}_2 \right] \right)^2}{2 \left( r^2 \tau M^2 + \sigma^2_1 \left( M^2 + \sigma^2_2 \right) + r^2 \theta N (N - 2) \left( r^2 + \beta^2 \sigma^2_1 + \sigma^2_2 \right) \right)} \right). \tag{24}
\]

If an investor elects not to become knowledgeable and thus only trades in the market portfolio, his expected utility is

\[
E[U_u] = E \left[ -\exp \left( - \left( \tilde{X} + \frac{\beta \tilde{Z}_1 + \tilde{Z}_2}{r \tau M} \right) \frac{\beta \tilde{Z}_1 + \tilde{Z}_2}{M} \right) \right].
\]  

(25)

Again, integrating over all random variables yields

\[
E[U_u] = -\sqrt{\frac{r^2 \tau M^2}{r^2 \tau M^2 + \beta^2 \sigma^2_1 + \sigma^2_2}} \exp \left( \frac{1}{2} \left( \beta E \left[ \tilde{Z}_1 \right] + E \left[ \tilde{Z}_2 \right] \right)^2}{r^2 \tau M^2 + \beta^2 \sigma^2_1 + \sigma^2_2} \right). \tag{26}
\]

This shows that an unknowledgeable investor’s expected utility depends on the expected supply of shares in the firm, \( E \left[ \tilde{Z}_1 \right] \), despite the fact that he is unable to trade in these shares. The reason for this is that, as can be seen in \( D_{k2} \), \( E \left[ \tilde{Z}_1 \right] \) affects the extent to which knowledgeable investors trade in the market portfolio through \( \beta \), the correlation of cash flows, and thus affects the amount of risk an unknowledgeable investor holds in equilibrium.

As an aside, a common interpretation of the expected supply of shares in the firm and market portfolio is that it represents the total amount of risk the economy is expected to bear. For example, as expected supplies increase, the economy is expected to bear more risk.
2.3 The Equilibrium

All investors have the opportunity to trade in the firm’s shares, which implies that in equilibrium each investor has to be indifferent between doing so or not. In other words, the following condition has to hold

\[ E[U_k] = E[U_u]. \tag{27} \]

From eqns. (24) and (26) it is obvious that the equilibrium condition will be fairly complex. In order to facilitate the analysis (and without loss of generality) we assume that the expected supply of shares in the market portfolio equals zero (i.e., \( E[\bar{Z}_2] = 0 \)). This implies that in equilibrium the following condition has to hold

\[ F(N) = 0, \tag{28} \]

where

\[
F(N) = \exp \left[ \frac{2C}{r} - \frac{(\sigma_{z1}^2 + M^2 r^2 \tau)^2}{\sigma_{z1}^2 (M^2 r^2 \tau + \sigma_{z2}^2) + r^2 \theta N (N - 2) (M^2 r^2 \tau + \beta^2 \sigma_{z1}^2 + \sigma_{z2}^2)} \right]
- \frac{\sigma_{z1}^2 (M^2 r^2 \tau + \sigma_{z2}^2)}{r^2 \theta N (N - 2) (M^2 r^2 \tau + \beta^2 \sigma_{z1}^2 + \sigma_{z2}^2)} - 1.
\]

Lemma 1 shows that there exists a unique solution to the “knowledgeability” game outlined above.

**Lemma 1.** Assuming that \( M \) is sufficiently large, there exists a unique \( N \) that solves eqn. (28).

The number of knowledgeable investors in equilibrium is determined by the exogenous parameters of the model and the entrepreneur’s choice of idiosyncratic disclosure, \( \theta \). For example, an increase in the expected supply of shares implies that, ex ante, an investor expects to
hold more shares and thus to absorb more risk. As investors are risk averse, however, they demand a premium for holding this risk; this leads to the difference between the expected value of cash flow and price, as provided in Corollary 1. This premium is sufficiently high such that knowledgeable investors prefer a higher expected supply of shares.

Different from most theory-based papers that investigate firms’ disclosure choices, we assume that shares in the firm are traded in an imperfect competition setting. Corollary 2 shows that a unique aspect of this setting is that knowledgeable investors prefer trading in a market with high idiosyncratic risk.

**Corollary 2.** If there is imperfect competition in the market for the firm’s shares, the firm’s shareholder base increases (decreases) as the amount of idiosyncratic risk increases (systematic risk increases). If there is perfect competition in the market for the firm’s shares, the firm’s shareholder base decreases as either the amount of idiosyncratic risk or systematic risk increases.

Merton (1987) assumes that knowledgeable investors behave as price takers and, thus, all shares are traded in a perfectly competitive market. Corollary 2 shows that the nature of the competition has an effect on how idiosyncratic and systematic disclosure change the investors’ decision to become knowledgeable (or, trade in the firm’s shares). This further implies that when maximizing his welfare, the entrepreneur has to take into account two countervailing forces: the force that reduces the idiosyncratic risk that investors associate with the firm’s cash flow directly reduces the discount (as shown above), but it also decreases the number investors, thereby increasing the discount.
The entrepreneur’s maximization problem is given by

\[
\max_\theta E[U_e] = E\left[SV_1 + \tilde{Z}_1\left(\tilde{P}_1 - V_1\right)\right] - c\theta
\]

s.t. \[
\tilde{P}_1 = \eta + \beta\mu - \frac{(N-1)}{r\theta(N-2)} \tilde{Z}_1 - \frac{\beta}{r\tau} \tilde{Z}_1 + \tilde{Z}_2,
\]

\[
\tilde{P}_2 = \mu - \frac{\beta\tilde{Z}_1 + \tilde{Z}_2}{Mr\tau}
\]

and \[E[U_i] = E[U_u].\]

Substituting \(\tilde{P}_1\) into the entrepreneur’s expected utility yields

\[
E[U_e] = S(\eta + \beta\mu) - \left(\frac{(N-1)}{r\theta N(N-2)} + \frac{\beta^2}{Mr\tau}\right) \left(\left(E[\tilde{Z}_1]\right)^2 + \sigma_{\tilde{Z}_1}^2\right) - c\theta. \tag{29}
\]

From eqn. (29) it is easy to see that any cost function that is increasing in the extent of idiosyncratic disclosure ensures an interior solution. When maximizing his welfare, the entrepreneur takes into consideration the “knowledgeability” equilibrium that will later emerge on the capital market.\(^5\) Maximizing eqn. (29) subject to condition (28) yields

\[
\theta^2 = \frac{\left(E[\tilde{Z}_1]\right)^2 + \sigma_{\tilde{Z}_1}^2}{2rc(N-1)}.
\tag{30}
\]

Holding everything else constant, the higher investors’ risk tolerance or the more numerous the number of knowledgeable investors, the lower the level of idiosyncratic disclosure. Furthermore, the lower the cost of disclosure, the higher the expected number of shares sold, and/or the higher the uncertainty about the shares sold, the higher the level of idiosyncratic disclosure.

The equilibrium to our “knowledgeability” game is defined by the two equilibrium conditions (28) and (30). In the next section we investigate empirical implications that arise from this equilibrium.

\(^5\) Technically, the entrepreneur will base his disclosure decision on the expected number of knowledgeable investors, say \(\hat{N}\), which has to equal the actual number of knowledgeable investors in equilibrium.
3 Empirical Implications

Our analysis allows for several exogenous parameters to influence the entrepreneur’s optimal level of idiosyncratic disclosure and the number of investors who become knowledgeable about the firm, $N$. Henceforth we refer to the number of investors who become knowledgeable about the firm as the firm’s “shareholder base.” In order to provide a structured discussion of the effect of these different parameters, we assign each to one of two groups: Market characteristics and firm characteristics. Accordingly, we provide two corollaries that investigate the comparative static results.

Corollary 3 investigates the effect of changes in market characteristics on the optimal level of idiosyncratic disclosure and shareholder base. Specifically, Corollary 3 shows the effects of changes in systematic risk, $\tau$; the uncertainty associated with the supply in shares in the market portfolio, $\sigma_{x^2}$; the total number of investors, $M$, the cost of becoming knowledgeable about the firm, $C$, and investors’ risk tolerance, $r$.

**Corollary 3 - Market Characteristics.** (a) A firm’s optimal level of idiosyncratic disclosure increases (shareholder base decreases) as the level of systematic risk and uncertainty about the supply in shares in the market portfolio increases. (b) A firm’s optimal level of idiosyncratic disclosure decreases (shareholder base increases) as the total number of investors increases, and increases (decreases) as investors’ cost of becoming knowledgeable increases. (c) The firm’s optimal level of idiosyncratic disclosure and shareholder base are both ambiguous with respect to an increase in investors’ risk aversion.

From Corollary 3 it is apparent that changes in exogenous parameters have countervailing effects on the firm’s optimal level of idiosyncratic disclosure and shareholder base. This
derives from Corollary 2, which shows that a knowledgeable investor’s expected utility increases as the level of idiosyncratic risk increases. Corollary 3 then shows that this effect on shareholder base dominates the (potentially) countervailing effects that arise from changes in exogenous parameters. This highlights again the fact that the entrepreneur trades off the decrease in the discount that arises from the decrease in risk with the increase that arises from reducing the number of knowledgeable investors. Corollary 3 (a) indicates that systematic and idiosyncratic risk are substitutes. In other words, the greater the market’s systematic risk, the greater the entrepreneur’s incentives to provide idiosyncratic disclosure about his firm. This relation is driven entirely by the fact that we treat knowldegeability as endogenous.

Alternatively, from Corollary 1 it is obvious that idiosyncratic and systematic risk are separable in the discount applied to the firm’s expected cash flow such that higher systematic risk does not alter the marginal benefit of reducing idiosyncratic risk. However, Corollary 2 shows that an increase in systematic risk decreases the firm’s shareholder base; the entrepreneur counters the resulting increase in the discount in price by increasing idiosyncratic disclosure (thereby further decreasing the firm’s shareholder base). The same reasoning applies to changes in uncertainty about the supply in the market portfolio, $\sigma_{zz}^2$.

Corollary 3 (b) together with the discount in Corollary 1 again shows how the endogenous determination of shareholder base changes the firm’s optimal level of idiosyncratic disclosure. From Corollary 1 it is straightforward that the total number of investors and (obviously) the cost of becoming knowledgeable do not change the marginal benefit of decreasing the idiosyncratic risk investors associate with the firm’s cash flow. More investors $M$ allow knowledgeable investors to better diversify systematic risk associated with the firm’s cash flow.
In addition, however, more investors will choose to become knowledgeable about the firm, which allows the entrepreneur to decrease the level of idiosyncratic disclosure. On the other hand, a higher cost of knowledgeability $C$ prevents investors from becoming knowledgeable such that the level of idiosyncratic disclosure increases. Finally, while it is straightforward from the discount to see that the direct effect of an increase in risk tolerance is a decrease in idiosyncratic disclosure, Corollary 3 (b) shows that there may be a countervailing effect on shareholder base. As we assume that all investors have the same risk tolerance, increasing $r$ affects the expected utility of both knowledgeable and unknowledgeable investors. Depending on the exact parameter values, this can increase or decrease shareholder base and Corollary 3 (b) shows that there are situations in which shareholder base decreases so much that it is optimal for the firm to increase idiosyncratic disclosure.

Corollary 4 identifies the effects on the firm’s idiosyncratic disclosure and the firm’s shareholder base following changes in the firm specific characteristics covered in the model: the firm’s exposure to market risk through $\beta$; the firm’s cost of disclosure $c$; the number of shares the entrepreneur expects to sell, $E\left[\tilde{Z}_1\right]$; and the uncertainty about the entrepreneur’s shock, $\sigma_{1z}^2$.

**Corollary 4 - Firm Characteristics.** (a) A firm’s optimal level of idiosyncratic disclosure increases (shareholder base decreases) as the firm’s exposure to market risk increases, and decreases (increases) as the firm’s cost of disclosure increases. (b) The optimal level of idiosyncratic disclosure and shareholder base are both ambiguous with respect to an increase in the entrepreneur’s expected shock. (c) The optimal level of idiosyncratic disclosure increases as uncertainty about the entrepreneur’s shock increases. For a sufficiently high expected shock, shareholder base decreases as uncertainty about the shock increases; for suffi-
ciently low expected shock, the shareholder base first increases as uncertainty about the shock increases and then decreases.

Similar to the finding in Corollary 3 (a), Corollary 4 (a) shows that systematic risk inherent in the firm’s cash flow (as given by the correlation coefficient $\beta$) acts as a substitute for idiosyncratic risk. Furthermore, when the cost of disclosure $c$ increases, the entrepreneur provides less idiosyncratic disclosure, which works to increase the shareholder base in Corollary 2.

Corollary 4 (b) is more subtle: with a given shareholder base the entrepreneur will increase disclosure if he expects to have to sell more shares. This result is obvious from the entrepreneur’s expected utility in eqn. (29): the higher $E \left[ \bar{Z}_1 \right]$, the more important it is to reduce the discount in price. On the other hand, for a fixed level of disclosure the shareholder base increases in the expected number of shares that will be sold. As $E \left[ \bar{Z}_1 \right]$ increases the discount in price, it is more profitable for investors to trade in shares of the firm.

In an equilibrium where both the firm’s idiosyncratic disclosure and the shareholder base are determined endogenously, it is possible for either effect to dominate the other such that idiosyncratic disclosure and shareholder base can increase or decrease in the entrepreneur’s expected shock.

Finally, Corollary 4 (c) shows that the firm’s idiosyncratic disclosure increases as uncertainty about the entrepreneur’s shock increases. Again, from the entrepreneur’s expected utility in eqn. (29), it is more important to reduce the discount in price as $\sigma_{1z}^2$ increases. Effectively, because the price of shares in his firm decreases as the number of shares the entrepreneur has to sell increases, the entrepreneur becomes risk averse with respect to his shock. While for a given level of disclosure shareholder base can increase or decrease as $\sigma_{1z}^2$
increases, the entrepreneur’s direct interest in decreasing the discount in price dominates. However, the effect of an increase in disclosure dominates the described ambiguity only when the discount in price is very pronounced (i.e., for a high $E\left[Z_1\right]$). When this is not the case, investors profit from uncertainty about the shock (again because price is low for a high supply of shares) but, ex ante, dislike the uncertainty about their final endowment. Corollary 4 (c) shows that the first effect dominates for low values of $\sigma^2_{1z}$ and the second effect dominates for high values of $\sigma^2_{1z}$.

4 Conclusion

We study an entrepreneur’s motivation to reduce the idiosyncratic risk investors associate with the firm’s cash flow by increasing costly disclosure. While we assume that the entrepreneur is risk neutral, he benefits from reducing idiosyncratic risk through his need to raise capital by selling shares of the firm to risk-averse investors. In determining the optimal level of idiosyncratic disclosure, we show that countervailing forces exist that result in the entrepreneur trading off the reduction in investors’ assessment of the firm’s idiosyncratic risk against the reduction in liquidity as a result of increased competitiveness among investors who know about the firm. Alternatively, the firm benefits unambiguously from a reduction of the investors’ assessment of systematic risk. This suggests that in studies of the relation between the risk/return profile of firms and their welfare (or cost of capital), some care should be taken to distinguish between activities that serve primarily to reduce investors’ assessments of idiosyncratic risk versus activities that serve primarily to reduce investors’ assessments of systematic risk.
5 References


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6 Appendix

**Lemma 1:** To prove that there exists a unique equilibrium we will show that the equilibrium condition is strictly negative for $N = 2$, is strictly positive for $N \to \infty$ and is monotonically increasing in $N$. The equilibrium condition developed in the text can equivalently be stated as

$$F_2 (N) = 0,$$

where

$$F_2 (N) = N (N - 2) \exp \left[ \frac{2C}{r} - \frac{(\sigma_{z_1}^2 + M^2 r^2 \tau)^2}{r^2 \theta (M^2 r^2 \tau + \beta^2 \sigma_{z_1}^2 + \sigma_{z_2}^2)} \left( E \left[ \tilde{Z}_1 \right] \right)^2 \right] - \frac{\sigma_{z_1}^2 (M^2 r^2 \tau + \sigma_{z_2}^2)}{r^2 \theta (M^2 r^2 \tau + \beta^2 \sigma_{z_1}^2 + \sigma_{z_2}^2)} - N (N - 2).$$

Clearly, at $N = 2$ it is the case that $F_2 (N = 2) = - \frac{\sigma_{z_1}^2 (M^2 r^2 \tau + \sigma_{z_2}^2)}{r^2 \theta (M^2 r^2 \tau + \beta^2 \sigma_{z_1}^2 + \sigma_{z_2}^2)} < 0$. Furthermore, the limit of $F_2 (N)$ is equal to $\lim_{N \to \infty} F_2 (N) = \lim_{N \to \infty} N (N - 2) \left( \exp \left[ \frac{2C}{r} \right] - 1 \right) > 0$ for
which shows that in our setting

\[ \frac{\partial F_2(N)}{\partial N} = \frac{2(N-1)(M^2r^2\tau + \sigma_{z_{2}}^2)}{(M^2r^2\tau + \beta^2\sigma_{z_{1}}^2 + \sigma_{z_{2}}^2)} \times \left( \frac{\sigma_{z_{1}}^2}{r^2\theta N(N-2)} + \frac{(\sigma_{z_{2}}^2 + M^2r^2\tau)(E[\tilde{Z}_1])^2}{\sigma_{z_{1}}^2(M^2r^2\tau + \sigma_{z_{2}}^2) + r^2\theta N(N-2)(M^2r^2\tau + \beta^2\sigma_{z_{1}}^2 + \sigma_{z_{2}}^2)} \right) \]

which is strictly positive for \( N > 2 \).

**Corollary 2:** By the implicit function theorem, for any exogenous parameter \( y \) we can write

\[ \frac{dN}{dy} = -\frac{\frac{\partial F_2(N)}{\partial y}}{\frac{\partial F_2(N)}{\partial N}}. \]

The respective partial derivatives are given by

\[ \frac{\partial F_2(N)}{\partial \theta} = \frac{(M^2r^2\tau + \sigma_{z_{2}}^2)}{r^2\theta^2(M^2r^2\tau + \beta^2\sigma_{z_{1}}^2 + \sigma_{z_{2}}^2)} \left( \frac{r^2\theta N(N-2)(\sigma_{z_{2}}^2 + M^2r^2\tau)(E[\tilde{Z}_1])^2}{\sigma_{z_{1}}^2(M^2r^2\tau + \sigma_{z_{2}}^2) + r^2\theta N(N-2)(M^2r^2\tau + \beta^2\sigma_{z_{1}}^2 + \sigma_{z_{2}}^2)} + \sigma_{z_{1}}^2 \right) \]

and

\[ \frac{\partial F_2(N)}{\partial \tau} = -\frac{M^2\beta^2\sigma_{z_{1}}^2(M^2r^2\tau + \sigma_{z_{2}}^2)N^2(N-2)(E[\tilde{Z}_1])^2}{(M^2r^2\tau + \beta^2\sigma_{z_{1}}^2 + \sigma_{z_{2}}^2)(\sigma_{z_{1}}^2(M^2r^2\tau + \sigma_{z_{2}}^2) + r^2\theta N(N-2)(M^2r^2\tau + \beta^2\sigma_{z_{1}}^2 + \sigma_{z_{2}}^2))} \]

\[ -\frac{M^2\sigma_{z_{1}}^2\beta^2(M^2r^2\tau + \sigma_{z_{2}}^2 + \sigma_{z_{1}}^2(M^2r^2\tau + \beta^2\sigma_{z_{1}}^2 + \sigma_{z_{2}}^2))}{\theta(M^2r^2\tau + \beta^2\sigma_{z_{1}}^2 + \sigma_{z_{2}}^2)^3}, \]

which shows that in our setting \( \frac{dN}{dy} < 0 \) and that \( \frac{dN}{d\tau} > 0 \).

Merton (1987) assumes that investors behave as price takers. In our model this translates to setting \( \lambda = 0 \). Maximizing a knowledgeable and an unknowledgeable investor’s expected utility over \((D_{k1}, D_{k2})\) and \(D_{k2}\), respectively, yields

\[ D_{k1} = r\theta(\eta + \beta\mu - P_1) - r\theta\beta(\mu - P_2), \]

\[ D_{k2} = r(\tau + \beta^2\theta)(\mu - P_2) - \beta r\theta(\eta + \beta\mu - P_1), \]

and

\[ D_{k2} = r\tau(\mu - P_2). \]
Prices are then given by

\[ P_1 = \eta + \beta \mu - \frac{1}{r} \left( \frac{\tilde{Z}_1}{N\theta} + \frac{\beta \tilde{Z}_2 + \beta^2 \tilde{Z}_1}{M\tau} \right) \] 

and

\[ P_2 = \mu - \beta \tilde{Z}_1 + \tilde{Z}_2. \]

Substituting the above demand orders as well as prices and integrating over all random variables yields following expected utilities for a knowledgeable and an unknowledgeable investor respectively:

\[
E[U_k] = -\sqrt{\frac{r^2\tau M^2 r^2\theta N^2}{r^2\theta N^2 (r^2\tau M^2 + \sigma_{z2}^2) + \sigma_{z1}^2 (r^2\tau M^2 + \sigma_{z2}^2 + \beta^2 r^2\theta N^2)}} \times \exp \left[ \frac{C}{r} - \frac{1}{2} \frac{r^2\theta N^2 (r^2\tau M^2 + \sigma_{z2}^2) + \sigma_{z1}^2 (r^2\tau M^2 + \sigma_{z2}^2 + \beta^2 r^2\theta N^2)}{(r^2\tau M^2 + \sigma_{z2}^2 + \beta^2 r^2\theta N^2)(E[Z_1])^2} \right]
\]

and

\[
E[U_u] = -\sqrt{\frac{M^2 r^2 \theta}{\beta^2 \sigma_{z1}^2 + \sigma_{z2}^2 + M^2 r^2 \theta}} \times \exp \left[ -\frac{1}{2} \frac{\beta^2 (E[Z_1])^2}{\beta^2 \sigma_{z1}^2 + \sigma_{z2}^2 + M^2 r^2 \theta} \right]
\]

Thus, in equilibrium

\[ H(N) = 0 \]

where

\[
H(N) = r^2\theta N^2 \exp \left[ \frac{2C}{r} - \frac{(\sigma_{z2}^2 + M^2 r^2 \theta)^2 (E[Z_1])^2}{(r^2\theta N^2 (r^2\tau M^2 + \sigma_{z2}^2) + \sigma_{z1}^2 (r^2\tau M^2 + \sigma_{z2}^2 + \beta^2 r^2\theta N^2)) (\beta^2 \sigma_{z1}^2 + \sigma_{z2}^2 + M^2 r^2 \theta)} \right]
\]

By the implicit function theorem, for any exogenous parameter \( y \) we can write

\[
\frac{dN}{dy} = -\frac{\partial H(N)}{\partial y} \div \frac{\partial H(N)}{\partial N}
\]
Following partial derivatives prove the claim

\[
\frac{\partial H(N)}{\partial N} = \frac{2 (r^2 \tau M^2 + \sigma_z^2)}{N \left( \beta^2 \sigma_{\sigma_{11}}^2 + \sigma_{\sigma_{22}}^2 + M^2 \tau^2 \right)} \left( \sigma_{\sigma_{22}}^2 + \frac{(\sigma_{\sigma_{22}}^2 + M^2 \tau^2 \theta) N^2 \theta}{(r^2 \theta N^2 (r^2 \tau M^2 + \sigma_{\sigma_{22}}^2) + \sigma_{\sigma_{22}}^2 (r^2 \tau M^2 + \sigma_{\sigma_{22}}^2 + \beta^2 \tau^2 \theta N^2))} \right),
\]

\[
\frac{\partial H(N)}{\partial \theta} = \frac{\sigma_{\sigma_{22}}^2 (2 \theta \tau M^2 N^2 \tau^4 + \tau M^2 r^2 \sigma_{\sigma_{22}}^2)}{\theta (\beta^2 \sigma_{\sigma_{11}}^2 + \sigma_{\sigma_{22}}^2 + M^2 \tau^2)} \left( \sigma_{\sigma_{22}}^2 + \sigma_{\sigma_{22}}^2 \theta \left( E \left[ Z_1 \right] \right)^2 \right) \left( \sigma_{\sigma_{22}}^2 (r^2 \tau M^2 + \sigma_{\sigma_{22}}^2) + \sigma_{\sigma_{22}}^2 (r^2 \tau M^2 + \sigma_{\sigma_{22}}^2 + \beta^2 \tau^2 \theta N^2) \right),
\]

\[
\frac{\partial H(N)}{\partial \tau} = \frac{\sigma_{\sigma_{22}}^2 M^2 \tau^2 \beta^2}{(\beta^2 \sigma_{\sigma_{11}}^2 + \sigma_{\sigma_{22}}^2 + M^2 \tau^2)} \left( \sigma_{\sigma_{22}}^2 + M^2 \tau^2 \theta \right) \frac{2 N^2 \tau^4 \tau M^2 \sigma_{\sigma_{22}}^2 + 2 \theta N^2 \tau^2 \sigma_{\sigma_{22}}^2 + \sigma_{\sigma_{22}}^2 \sigma_{\sigma_{22}}^2}{(r^2 \theta N^2 (r^2 \tau M^2 + \sigma_{\sigma_{22}}^2) + \sigma_{\sigma_{22}}^2 (r^2 \tau M^2 + \sigma_{\sigma_{22}}^2 + \beta^2 \tau^2 \theta N^2)) \left( \beta^2 \sigma_{\sigma_{22}}^2 + \sigma_{\sigma_{22}}^2 + M^2 \tau^2 \theta \right)^2}.
\]

Corollaries 3 and 4:

While \( F_2 (N) = 0 \) is the equilibrium condition that describes the number of investors that become knowledgeable, eqn. (30) provides an equilibrium condition that describes the entrepreneur’s choice of idiosyncratic disclosure. To prove the comparative static results in Corollary 3 and Corollary 4 we restate eqn. (30) as

\[
G(\theta) = 0, \text{ where } G(\theta) = \frac{\left( E \left[ Z_1 \right] \right)^2 + \sigma_{\sigma_{22}}^2}{2 rc(N - 1)} - \theta^2.
\]

For any parameter \( y \), we can take the total derivatives of \( F_2 (N) \) and \( G(\theta) \). Rearranging terms yields

\[
\frac{d\theta}{dy} = \frac{\partial F_2(N)}{\partial N} \frac{\partial G(N)}{\partial y} - \frac{\partial F_2(N)}{\partial \theta} \frac{\partial G(N)}{\partial y},
\]

and

\[
\frac{dN}{dy} = \frac{\partial F_2(N)}{\partial \theta} \frac{\partial G(N)}{\partial y} - \frac{\partial F_2(N)}{\partial \theta} \frac{\partial G(N)}{\partial y}.
\]

This shows that the denominator for all comparative statics is the same. Taking the respective derivatives proves the claims in Corollary 3 and Corollary 4.