ACCOUNTING WORKSHOP

A Model of Principles-Based vs. Rules-Based Standards

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A Model of Principles-Based vs. Rules-Based Standards.*

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Abstract

We develop a theoretical framework to examine the trade-offs between principles-based standards and rules-based standards. By relying on quantifiable evidence, a rules-based standard induces evidence management whereas by relying on the auditor’s professional judgement, a principles-based standard induces audit failure. Given evidence management and audit failure, we derive the optimal standard and show that it combines both principles-based and rules-based elements in a simple way: when the evidence is unfavorable, the standard prescribes an unfavorable treatment to a transaction but when the evidence is favorable, it leaves the treatment of the transaction to the auditor’s professional judgement.

We demonstrate that the standard relies more on the auditor’s professional judgement as (1) the effectiveness of the regulatory enforcement increases, (2) the cost of evidence management increases, and (3) the cost of undue optimism errors decreases and/or the cost of false alarm errors increases.

*Preliminary and Incomplete.
1 Introduction

Whether accounting standards should be principles-based rather than rules-based is an unresolved issue. The general belief is that, by relying on bright-line and quantifiable evidence, rules-based standards “often provide a vehicle for circumventing the intention of the standard” (SEC, 2003). However, others have argued that, by relying on auditors’ professional judgement, principles-based standards require auditors to be competent and independent. For example, Robert Herz, former chairman of the Financial Accounting Standards Board (FASB), has cited the recent events in the U.S. as “evidence that preparers and auditors cannot be trusted to properly exercise professional judgement with objectivity and courage” (Herz, 2003). Furthermore, principles-based standards “may present enforcement difficulties because they provide little guidance or structure for exercising professional judgment by preparers and auditors” (SEC, 2003). Despite its practical importance, theoretical guidance on this debate is surprisingly sparse. Perhaps this is because accountants lack a theoretical framework allowing a comparison of the relative merits of principles-based vs. rules-based standards. Our objective in this paper is to provide such a framework. We develop an economic model and show that the trade-off between principles-based versus rules-based standards is far from being one-sided. Given this trade-off, we ask the following questions: How should both principles-based and rules-based elements be optimally combined? What form does the optimal standard take? What are the properties of the optimal standard? How do these properties depend on various features of the firm’s environment?

Our model consists of a firm’s manager, an auditor, a standard setter, and the firm’s investors. The standard setter chooses an accounting standard that stipulates how quantifiable evidence and/or auditor professional judgement should be used to assess the economic substance of a transaction. We capture two frictions in our environment. Namely, for a given standard, the firm’s manager can improve the chance
of receiving a favorable treatment for a transaction by tampering with the implementation of the standard in two ways. First, after receiving initial evidence about the transaction, the manager may engage in costly evidence management that boosts the evidence disclosed to the auditor. Second, the manager may compromise the auditor’s professional judgement resulting in audit failure and the auditor’s willingness to accommodate the manager’s wishes depends on the enforcement level of the standard. The standard, potential evidence management and audit failure jointly determine the report. Upon observing the report, investors decide whether or not to invest in the firm.

To motivate our model, consider a revenue recognition standard. The standard aims to measure the economic substance of a transaction, i.e., whether a firm has transferred the majority of the risks and rewards associated with its products to its customers. To assess the economic substance, various types of evidence can be collected. Certain types of evidence, such as product shipment, are easier to quantify, require little professional judgement, and can therefore be written in a rule. However, other types of evidence, such as measuring the firm’s remaining obligations to its customers, are much harder to quantify, require more professional judgement, and hence cannot be incorporated in a rule. We refer to a standard that relies exclusively on quantifiable evidence as a rules-based standard. An example could be “revenue is recognized if product has been delivered to customers.” For such a rules-based standard, by engaging in channel stuffing, the manager may recognize revenue without changing the economic substance of the transaction. Conversely, we refer to a standard that leaves the recognition of revenue entirely to the auditor’s professional judgement as a principles-based standard. An example could be “revenue is recognized if in the auditor’s opinion the majority of the risk has been transferred to customers.” For such a principles-based standard, by implicitly promising future audit engagements to the auditor, the manager may also recognize revenue by impairing the auditor’s professional judgement.
We focus on the design of standards under the frictions of evidence management and audit failure. A standard determines a transaction’s treatment. In general, a standard could mix the principles-based and rules-based elements in any arbitrary manner so that the space of possible standards can be very large. For example, a standard can prescribe favorable and unfavorable treatment for sufficiently good and bad evidence, respectively, but for intermediate evidence, leaves the treatment to the auditors’ professional judgement. For another example, a standard can prescribe favorable treatment for intermediate evidence while leave it to the auditor’s professional judgement for evidence that lies in the left and right tails.

Our model generates several results. We first show that the optimal standard is fully characterized by an evidence threshold and combines both rules-based and principles-based elements in a very specific way: below the evidence threshold, the standard adopts a rules-based element that prescribes the unfavorable treatment; above the evidence threshold, the standard uses a principles-based element that leaves the treatment to the auditor’s professional judgement. In other words, a favorable treatment is granted only when both conditions are satisfied: the evidence is sufficiently positive and the auditor endorses the treatment.

The broad intuition for the shape of the optimal standard is as follows. The standard setter designs the optimal standard in order to minimize two types of measurement errors: false alarm errors whereby investors do not invest even though the economic substance of the transaction is good and undue optimism errors whereby investors invest even though the economic substance of the transaction is bad. Consider an arbitrary standard that prescribes favorable or unfavorable treatments or professional judgement for different intervals of quantifiable evidence. The optimal standard minimizes both types of errors via two modifications: (1) by moving all evidence intervals in which the unfavorable treatment is prescribed to the lower end of the evidence distribution and (2) by simultaneously imposing professional judgement for the upper end of
the evidence distribution. These two modifications reduce the standard’s measurement errors through two intertwined channels. First, the monotone likelihood ratio property (MLRP) property of the evidence distribution suggests that it could be beneficial to move the intervals with unfavorable treatment to the lower end where the bad types are more likely to concentrate. Second, since the auditor can utilize non-quantifiable information to reduce the measurement errors when their independence is not compromised, it could be beneficial to allow the auditor to exercise professional judgement on the upper end of the evidence distribution where the good types are more likely to concentrate.

The result on the shape of the optimal standard has several implications for the debate on rules-based versus principles-based standards. First, in an environment plagued by potential evidence management and audit failure, the optimal standard must necessarily contain both rules-based elements and principles-based elements. By relying on rules-based elements, the optimal standard alleviates the incidence of audit failure whereas by relying on principles-based elements, the optimal standard not only alleviates the incidence of evidence management but also incorporates the auditor’s professional judgement. Second, and perhaps more interestingly, the shape of the optimal standard illustrates how rules-based elements and principles-based elements should be efficiently combined. Namely, the optimal standard prescribes principles-based elements only if the evidence is sufficiently favorable. Otherwise, the standard ignores the auditor’s professional judgement and relies solely on the quantifiable evidence in granting the unfavorable treatment.

We view the higher hurdle for favorable treatment relative to unfavorable treatment to be consistent with how current accounting standards are applied in practice. Going back to our motivating example, the optimal standard implies that product delivery—a rules-based element—is only a necessary condition for revenue recognition. In practice, the auditor must still exercise professional judgement about the firm’s
remaining obligations—a principles-based element—in order to endorse revenue recognition. However, if product delivery has not occurred, then it is quite unlikely that the auditor would use her professional judgement to override the rule and grant a favorable treatment.

We next characterize the unique evidence threshold that characterize the optimal standard. The standard setter chooses the threshold to minimize the expected cost of measurement errors to investors. A lower evidence threshold means that the standard incorporates more principles-based elements. While more reliance on the auditor’s professional judgement enables the auditor to utilize non-quantifiable information in the recognition, it also alters the manager’s incentives to engage in evidence management. These two effects on measurement errors determine the optimal threshold.

Finally, we examine the equilibrium determinants of the optimal evidence threshold and demonstrate that it depends on several parameters of our model that capture various features of the firm’s environment. We show that the optimal standard relies more on the auditor’s professional judgement as (1) the effectiveness of regulatory enforcement of the standard (and therefore auditor independence) increases, (2) the cost of evidence management increases, (3) the manager’s incentive to prefer the favorable treatment decreases, and (4) the cost of undue optimism errors decreases and/or the cost of false alarm errors increases.

The intuition behind the above results are qualitatively similar so we only illustrate it for the result on regulatory enforcement. An increase in the effectiveness of the enforcement of the standard has two effects. First, it directly reduces the undue optimism error by diminishing the likelihood that the bad manager with favorable evidence obtains the favorable treatment from the auditor. Second, anticipating the reduced benefit of favorable evidence, the manager, in turn, reduces evidence management, which increases the false alarm error and simultaneously reduces the undue optimism error. Both effects compel the standard setter to lower the threshold to re-
store the balance so that better enforcement allows the standard to rely more on the auditor’s professional judgement confirming the argument made earlier that regulatory enforcement is an important ingredient for the implementation of principles-based standards.

As mentioned above, theoretical work on the desirability of rules-based versus principles-based standards is relatively sparse. Most auditing models assume that the auditor uses her information to issue a final report (see e.g., Dye, 1993, Schwartz, 1997, Lu and Sapra, 2009, Laux and Newman, 2010, and Deng et al., 2014). This assumption is akin to our purely principles-based standard in that the auditor’s exercise of professional judgement is not constrained by any rule. Our paper is also related to work that examines the rules-based element in accounting standards. Dye (2002) is the first to study threshold design in accounting standards. In Dye’s model and the subsequent literature (e.g., Fan and Zhang, 2012 Laux and Stocken, 2013, and Gao, 2014), the accounting standard is purely rules-based because the auditor’s professional judgement is absent. Our paper differs from the previous literature because we develop a framework to incorporate both principles-based and rules-based elements and examine how they should be optimally combined.

The rest of the paper is organized as follows. Section 2 describes the model. Section 3 provides the analysis. Section 4 concludes.

2 The Model

Our model consists of four dates and four players: a firm (i.e., its manager), an auditor, a standard setter, and the firm’s investors. The sequence of events is as follows.

1. At date 0, the standard setter chooses an accounting standard. A standard stipulates how observable evidence and/or auditor professional judgement should be used to measure the economic substance of the transaction.
2. At date 1, before observing the economic substance of the transaction, the manager privately learns about the initial evidence and decides whether to manipulate it at a cost.

3. At date 2, the auditor receives (potentially manipulated) evidence and observes additional circumstantial information about the economic substance of the transaction. The manager decides whether to make a side-payment to the auditor. The transaction is recognized according to the standard pre-specified at date 0.

4. At date 3, the firm’s investors make a decision upon observing the report. Payoffs are realized.

The economic substance of the transaction, denoted as $\omega \in \{G, B\}$, is either Good or Bad with probability $q_G$ and $q_B \equiv 1 - q_G$, respectively. The reporting process measures the transaction’s economic substance and produces a report $r \in \{g, b\}$. We refer to $r = g (b)$ as a favorable (unfavorable) report. We next describe the process that maps $\omega$ to $r$.

At date 1, nature privately determines the economic substance $\omega$. Before observing $\omega$, the firm’s manager privately observes the transaction’s initial evidence $t$, which is drawn from a differentiable density $f^\omega(t)$ over the real line $R$ with cumulative density $F^\omega(t)$. The density $f^\omega(t)$ satisfies the monotone likelihood ratio property (MLRP) in the sense that a high value of $t$ implies that the economic substance of the transaction is more likely to be good. After observing evidence $t$, we assume that the manager can boost it at a cost. More precisely, the manager can tamper with the evidence by adding $m$ to $t$ at a cost of $C(m; c)$ where $C_m > 0$ for $m > 0$, and $c$ is a cost parameter with $C_c > 0$. Thus, a higher value of $c$ indicates a higher cost of evidence management. Denote $t_m \equiv t + m$ as the evidence that is disclosed to the auditor. For ease of reference, we refer to $m$ or equivalently $t_m > t$ as evidence management.
Because $t_m(t) = t + m(t)$, choosing an amount of manipulation $m(t)$ is equivalent to choosing the level of evidence $t_m(t)$ presented to the auditor.

At date 2, the auditor and the manager obtain symmetric information about the economic substance $\omega$. For simplicity, we assume that they both perfectly observe $\omega$. However, unlike the evidence $t_m$, the auditor’s knowledge of the economic substance cannot be directly prescribed in a standard. Instead, it may only be indirectly incorporated in the recognition of the transaction if the standard allows the auditor to exercise professional judgement. When the standard requires the auditor to exercise professional judgement, the manager can influence the auditor’s independence by negotiating with the auditor in order to obtain favorable treatment for the transaction. We use the Nash bargaining solution and assume that the manager has all the bargaining power in the negotiation process. In particular, the manager can make a side payment $D$ to the auditor to induce the latter to grant the favorable treatment $g$.$^1$ If the manager is able to compromise the auditor’s independence, there is audit failure and the auditor suffers a loss of $\phi$.$^2$ However the ability of the manager to compromise the auditor’s independence depends on how well the standard is enforced. If the standard is properly enforced, the auditor uses her information appropriately in her attestation so that there is no audit failure. But when enforcement is ineffective, the manager can make any side payment $D$ to the auditor to compromise the latter’s independence, thereby causing audit failure. Denote the regulatory enforcement of the standard by the indicator variable $\mu$ where $\mu = 1(0)$ means that regulatory enforcement is (not) working properly. The probability that the regulatory enforcement works is $\tau \in (0,1)$, i.e., $\Pr(\mu = 1) = \tau$. We interpret $\tau$ as the strength of regulatory enforcement of the standard.

$^1$The side payment $D$ could capture rents that the manager shares with the auditor or could capture audit fees from future audit engagements.

$^2$For simplicity, we do not explicitly model $\phi$ but treat as exogenous. It captures reputation damages and legal liability costs from audit failure.
An accounting standard $S$ is defined as a mapping $S(t_m) \rightarrow \{b, g, p\}$ that stipulates one of the three possible treatments for each (potentially manipulated) observable evidence $t_m$. The mapping $S(t_m) = b$ (or $g$) means that the firm with evidence $t_m$ receives the treatment $b$ (or $g$). This is the rules-based element in a standard. It conditions the transaction’s treatment on accounting evidence in a pre-specified manner and leaves no room for the auditor’s professional judgement. On the other hand, the mapping $S(t_m) = p$ is the principles-based element of the standard. It means that the treatment for the transaction with evidence $t_m$ is left to the auditor’s professional judgement. Stated differently, $S(t_m) = p$ implies that the auditor is not constrained by the evidence $t_m$ and can utilize her professional judgement to determine the treatment.

According to this definition, the space of possible standards is large because a standard can be an arbitrary combination of rules-based and principles-based elements. Examples 1 and 2 list two possible standards.

**Example 1** A hybrid standard is as follows:

$$
S(t_m) = \begin{cases} 
g & \text{if } t_m > T_2, 
p & \text{if } t_m \in [T_1, T_2], 
b & \text{if } t_m < T_1.
\end{cases}
$$

**Example 2** A rules-based standard is a binary classification such that

$$
S(t_m) = \begin{cases} 
g & \text{if } t_m > T, 
b & \text{if } t_m \leq T.
\end{cases}
$$

The first example presents a standard that combines both accounting evidence and auditor professional judgement in a specific manner. The standard shown in the example prescribes favorable and unfavorable reports for sufficiently good and bad evidence, respectively. For intermediate evidence, it allows the auditor to exercise
professional judgement. The second example is a purely rules-based standard in that it specifies a treatment for any possible evidence, leaving no room for the auditor to exercise professional judgement.\footnote{Dye (2002) confines attention to such a binary classification standard and analyzes how the manager’s evidence management affects the design of the optimal standard within its class.}

The report is useful for the decision making of investors who don’t directly observe the economic substance $\omega$. Upon observing report $r$, the investors make a binary decision $a \in \{0,1\}$ whose payoff depends on the transaction’s economic substance in the following manner:

\[
\begin{array}{|c|c|c|}
\hline
& \omega = G & \omega = B \\
\hline
a = 1 & 0 & -L_B \\
\hline
a = 0 & -L_G & 0 \\
\hline
\end{array}
\]

In words, if the action is not matched to the state, the investor suffers a loss $L_\omega$ in state $\omega$. Thus, the investors benefit from the report’s informativeness. We will focus on a non-trivial decision rule of $a(g) = 1$ and $a(b) = 0$ by the investors. That is, the report is informative enough to sway the investors’ decision.\footnote{Other decision rules either are equivalent ($a(b) = 1$ and $a(g) = 0$) or render the report irrelevant for the decision making.} We will focus on the region in which this decision rule is optimal.

The manager’s preference, however, is not fully aligned with the investors. While the investors would like to take the correct action, the manager prefers that the investors always take a higher action. In particular, the manager’s payoff is given by $u(a, \omega) = \delta a$ where the parameter $\delta > 0$ captures the degree of the conflict of interest between the manager and the investor.\footnote{The alternative assumption that the manager prefers $a = 0$ to $a = 1$ in both states does not change the main results qualitatively. What is necessary is the existence, not the direction, of the misalignment of interest.} This misalignment of interest creates incentives for the manager to engage in evidence management and to influence the auditor.

We assume that $\delta > D$ so that if the standard is not enforced, the manager indeed has
incentives to influence the auditor.

At date 0 the standard setter chooses a standard to minimize the investors’ loss function:

$$L = q_G L_G \Pr(r = b|\omega = G) + q_B L_B \Pr(r = g|\omega = B)$$  \hspace{1cm} (1)$$

This objective function is consistent with FASB’s goal of providing information useful for investors’ decision-making. We call $L$ the social cost. $\Pr(r = b|\omega = G)$ is the false alarm error and $\Pr(r = g|\omega = B)$ is the undue optimism error. Both are endogenously determined later. Note that neither the side payment $D$ nor the direct cost of evidence management $C$ enter the social cost: $D$ is a transfer from the manager to the auditor while the cost $c$ is a private cost borne by the manager and thus not a concern to the standard setter.

Given the accounting standard, at date 1, the manager chooses evidence management $m$ in order to maximize her expected payoffs net of the expected costs of evidence management and net of any expected transfers to the auditor at date 2. At date 2, the auditor makes her attestation decision in order to minimize her expected litigation risks.

An equilibrium of our model consists of the following decisions: the manager’s evidence management decision $m$ and side-payment decision $D$, the standard setter’s choice of standard $S(t_m)$, and the auditor’s reporting decision such that each player’s decisions maximize his/her respective objective function and all decisions are consistent with each other in the sense of rational expectations. Note that we do not include the investors in the definition of the equilibrium. We have deliberately simplified their strategies and treat them as passive players in order to focus exclusively on the roles of the manager, the standard setter, and the auditor.

To fix ideas, we return to our motivating example. Consider a firm that engages in a sales transaction in which it transfers goods to a buyer. The transaction’s economic
substance $\omega$ is whether the firm has transferred the risks and rewards associated with
the goods to the buyer. To measure $\omega$, a revenue recognition standard could incorporate
observable accounting evidence $t_m$ such as product delivery or allow the auditor to use
her professional judgement in determining whether risks and rewards associated with
the goods have been transferred to the buyer. By engaging in costly channel stuffing $m$,
the manager can expedite product delivery $t_m$ without changing the economic substance
of the transaction. By implicitly promising future engagements worth $D$, the manager
can also influence the auditor’s judgement.

3 Analysis

3.1 Shape of the Optimal Standard

We first identify the optimal standard and then characterize its properties. We start by
formalizing the standard space over which we search for the optimal one. Recall that
a standard is a menu that stipulates one of the three possible treatments ($b, g$ or $p$) for
each (potentially manipulated) evidence $t_m$. Formalizing the standard as a partition
over $t_m$ is helpful in structuring the discussion. The definition below formalizes the
idea.

**Definition 1** A standard $S$ can be characterized by intervals (i.e., a partition) of the
reported evidence $t_m$ that satisfy (i) $S(t_m) = S(t'_m) \forall t_m, t'_m \in I_n$ and (ii) $S(t_m) \neq S(t'_m) \forall t_m \in I_n, t'_m \in I_{n+1}$.

In words, a standard does two things at once. First, it specifies a partition over
the space of reported evidence $t_m$. Second, it assigns the same treatment ($b, g$ or $p$)
for all evidence $t_m$ that lie in the same interval, while it assigns different treatment to
adjacent intervals.
Two factors create challenges for the search of the optimal standard. First, it is clear from the definition that there are, in theory, infinitely many possible standards to consider because one can partition the support of the evidence $t_m$ arbitrarily. Second, the presence of the manager’s evidence management creates another layer of complication when we compare different standards. While the infinite support of the standard space is easy to see from the definition above, we use the following example to demonstrate the difficulty brought by the manager’s evidence management.

Consider two standards $S$ and $S'$ shown in Figure 1: both standards give the same treatment to all evidence $t_m$ except for those in the interval $I_N$. To be concrete, the example assumes that, if the evidence $t_m$ lies in $I_N$, $S$ issues $r = g$ for sure while $S'$ requires, in addition, the auditor’s endorsement before issuing $r = g$. In the absence of the manager’s evidence management, comparing the efficiency of these two standards is straightforward because it can be restricted to comparing the measurement errors in interval $I_N$ alone. In this particular example, standard $S'$ strictly dominates standard $S$. The auditor has perfect information and reduces the “bad” firm’s chance of receiving $r = g$ over the interval $I_N$.

Once we take into account evidence management, however, comparing the efficiency of $S$ and $S'$ is complicated because it cannot be restricted to comparing the measurement errors in interval $I_N$ alone any longer. Replacing $S(I_N) = g$ with $S'(I_N) = p$, a change of treatment in one interval only, alters evidence management incentive for all managers whose preliminary evidence $t$ is below the interval $I_N$. Managers with pre-
liminary evidence $t \in \bigcup_{n=1}^{N-1} I_n$ may choose not to manipulate evidence $t$ to the interval $I_N$ even though they would have done it had the standard been $S$. Therefore, in order to compare the efficiency between the two standards $S$ and $S'$, we need to keep track of the manager’s evidence management activity for all evidence $t \in \bigcup_{n=1}^{N-1} I_n$ even if the standard only changes its treatment over the interval $I_N$. Stated differently, a local change of the standard has “global” effect on the manager’s evidence management. It is this global nature of induced changes in evidence management that complicates the comparison of any two standards and the search for the optimal standard.

Despite the difficulties discussed above, the proposition below shows that it is without loss of generality to confine attention to a particular type of standard that takes a surprisingly simple form.

**Proposition 1** The optimal standard is fully characterized by a unique threshold $T$ such that

$$S(t_m) = \begin{cases} p & \text{if } t_m > T \\ b & \text{if } t_m \leq T. \end{cases}$$

Under the optimal standard, a favorable treatment is granted only when the evidence is sufficiently positive and the auditor endorses the treatment. Otherwise, the transaction receives an unfavorable treatment. In other words, when the evidence is weak, a strict rule is applied but when the evidence is strong, the auditor can exercise professional judgement. While the proof of the proposition is somewhat abstract given the level of generality it deals with, the basic idea can be summarized as follows. For any given standard $S$, we can always construct a new standard $S'$ in the form described in Proposition 1, and, by judiciously choosing the threshold $T$ of the new standard $S'$, the newly constructed standard (at least weakly) lowers the social loss $L$ from the level under the original standard $S$. To do so, we first calculate the implied false alarm error and undue optimism error for any given standard $S$, taking into account the manager’s
evidence management. Second, we construct a new standard $S'$ that takes the form of Proposition 1. Because the false alarm and undue optimism errors under the new standard $S'$ are functions of the threshold $T$, we can choose $T$ so that the false alarm errors under the two standards ($S$ and $S'$) are the same. Finally, we show that the MLRP property of the evidence distribution implies that the undue optimism error is at least weakly lower under our constructed standard $S'$ than $S$.

**Proof of Proposition 1.** The proof is by construction in three steps. First, for an arbitrary standard $S$, we identify its false alarm error and undue optimism error. Second, we construct a new standard $S'$ that takes the form claimed in Proposition 1. The false alarm and undue optimism error of the new standard $S'$ are characterized by a threshold $T$. We choose the threshold $T$ such that the false alarm error under the two standards are same. Finally, we show that the undue optimism error is at least weakly lower under the new standard $S'$ than under the original standard $S$, which completes the proof.

Consider a simple standard $S$ that does not prescribe $r = b$ to any interval of reported evidence, i.e., $S(t_m) \neq b$ for any $t_m$. Now construct a new standard $S'$ that takes the form of Proposition 1 and set $T = -\infty$. It is easy to see that constructed (pure principle-based) standard is weakly better than the original standard $S$ because there is no false alarm error under either standard, and the undue optimism error is at least lower under the new standard.

Now consider a more general standard $S$ that prescribes $r = b$ to some non-trivial interval(s) of reported evidence. Denote by $I_b(S)$ the set of *reported* evidence for which the standard $S$ prescribes $b$, that is:

$$I_b(S) \doteq \{t_m | S(t_m) = b\}. \quad (2)$$


Because the manager can manipulate evidence, the reported evidence $t_m$ can be different from the un-manipulated evidence $t$, which is governed by the exogenous distribution $f^G(\cdot)$ or $f^B(\cdot)$. Because the manager’s incentive of manipulating evidence is standard-dependent, we write the reported evidence $t_m = t_m(t, S)$ to emphasizes its dependence on the un-manipulated evidence $t$ and the standard $S$. Denote by $\hat{I}_b(S)$ the set of un-manipulated evidence $t$ that, after manager’s manipulation, lies in the set $I_b(S)$ defined in (2).

$$\hat{I}_b(S) = \{ t | t_m(t, S) \in I_b(S) \}.$$  

(3)

In words, $\hat{I}_b(S)$ is the set of evidence that the manager chooses to accept $r = b$ by default without even trying to engage in evidence management. Therefore, we can calculate the ex ante probability for a “good” type manager to receive $r = b$ (i.e., false alarm) under the standard $S$ as follows:

$$m^{False}(S) = q_G \int_{\hat{I}_b(S)} f^G(t) \, dt,$$  

(4)

Similarly, denote by $m^{Undue}(S)$ the ex ante probability that a “bad” type manager receives $r = g$ (i.e., undue optimism) under $S$. Note that $m^{Undue}(S)$ satisfies the following:

$$m^{Undue}(S) \geq q_B [1 - \int_{\hat{I}_b(S)} f^B(t) \, dt] (1 - \tau),$$  

(5)

where the inequality holds as a strict one except when all reported evidence $t_m$ outside the set $I_b(S)$ are subject to professional judgement (as opposed to receiving $r = g$ by default).

Having calculated the measurement errors under the original standard $S$, we turn to the second step: we construct a new standard $S'$ that takes the form described in the proposition. Given a new standard $S'$ that is characterized by a threshold $T$, we show in Proposition 2 that there exists a unique threshold $\hat{T}$ (with $\hat{T} < T$) such that
the manager manipulates evidence upwards if and only if \( t \in [\hat{T}, T] \). In the language of (3), the set of evidence that the manager accepts \( r = b \) without even even trying to manipulate evidence is

\[
\hat{J}_b(S') = \{t| t < \hat{T}\}.
\]

One can therefore calculate the ex-ante false alarm error under the new standard \( S' \) as

\[
m_{False}(S') = q_G \int_{t<\hat{T}} f^G(t) \, dt,
\]

and the ex-ante undue optimism probability as

\[
m_{Undue}(S') = q_B [1 - \int_{t<\hat{T}} f^B(t) \, dt] (1 - \tau).
\]  

Note that, in designing the new standard \( S' \), we can pick the cut-off \( T \) so that false alarm error \( m_{False}(S') \) equals the false alarm error in the original standard \( S \) (calculated in (4)). That is, we pick \( T \) such that

\[
\int_{t<\hat{T}} f^G(t) \, dt = \int_{\hat{I}_b(S)} f^G(t) \, dt.
\]

We can do so because \( m_{False}(S') \) is strictly monotonic in \( \hat{T} \) and that there is a one-to-one relation between \( \hat{T} \) and \( T \) (proved in Proposition 2).

In the last step, we show that

\[
\int_{t<\hat{T}} f^B(t) \, dt \geq \int_{\hat{I}_b(S)} f^B(t) \, dt.
\]

That is, the “low” type manager is more likely to accept \( r = b \) without even trying to manipulate evidence under the new standard \( S' \) constructed above than under the original standard \( S \). Note that, once we prove the inequality (8), we can show the
following:

\[
m^{Undue}(S') = q_B[1 - \int_{t<\hat{T}} f^B(t) \, dt](1 - \tau) 
\leq q_B[1 - \int_{\hat{I}_b(S)} f^B(t) \, dt](1 - \tau) 
\leq m^{Undue}(S),
\]

where the first inequality is by (8) and the second inequality is by (5), which then completes the proof the proposition because the false alarm errors satisfy \( m^{False}(S) = m^{False}(S') \) by construction (recall 7). Therefore, the remaining of the proof aims to prove (8).

More notation is required to prove (8) because, in order to calculate the right hand side of the of the inequality, we need to keep track of each of the interval(s) over which the original standard \( S \) prescribes \( S(t_m) = b \). Consider the general case in which \( r = b \) is prescribed to \( N \) intervals, denoted as \([b^n, B^n]\) for \( n = 1, 2, 3, ..., N \). As we argued above, there exists a de-facto threshold \( \hat{T}^n \in [b^n, B^n] \) in each of these interval such that the manager will choose to manipulate evidence if and only if \( t \in [\hat{T}^n, B^n] \). Therefore, we can express (2) and (3) as \( I_b(S) \equiv \bigcup_{n=1}^{N} [b^n, B^n] \) and \( \hat{I}_b(S) \equiv \bigcup_{n=1}^{N} [b^n, \hat{T}^n] \), respectively. We construct a sequence of \( N + 1 \) points \(-s_0, s_1, ..., s_N - \) to divide the interval \((-\infty, \hat{T})\) of the new standard \( S' \) into \( N \) subintervals. The sequence of points \( s_n \) is chosen to satisfies the following properties:

\[
\int_{s_{n-1}}^{s_n} f^G(t) \, dt = \int_{b^n}^{\hat{T}^n} f^G(t) \, dt, \quad \forall n = 1, 2, ..., N,
\]

with \( s_{n-1} < s_n \) and the boundary condition that \( s_0 = -\infty \), and \( s_N = \hat{T} \). The sequence of \( s_n \) exists and is unique because we choose \( \hat{T} \) to match the false alarm errors between the two standards, i.e., \( \sum_{n=1}^{N} \int_{s_{n-1}}^{s_n} f^G(t) \, dt = \sum_{n=1}^{N} \int_{b^n}^{\hat{T}^n} f^G(t) \, dt \).
Having divided the interval \((-\infty, \hat{T})\) in the new standard \(S'\) into \(N\) subintervals, we prove the inequality (8) by shows that the claim holds for each subinterval as follows:

\[
\int_{s_{n-1}}^{s_n} f^B(t) \, dt \geq \int_{b_n}^{\hat{T}_n} f^B(t) \, dt, \quad \text{for any } n.
\] (10)

Condition 10 follows from MLRP, which requires \(f^G(t)/f^B(t)\) increase in \(t\). Note that we can write the MLRP requirement equivalently as following

\[
f^G(t) = f^B(t) a(t),
\]

where \(a(t)\) is an strictly increasing function of \(t\).\(^\text{6}\) Suppose by contradiction that condition (10) fails, that is \(\int_{s_{n-1}}^{s_n} f^B(t) \, dt < \int_{b_n}^{\hat{T}_n} f^B(t) \, dt\). Then we can show that, for any \(n = 1, \ldots, N\),

\[
\int_{b_n}^{\hat{T}_n} f^G(t) \, dt = \int_{b_n}^{\hat{T}_n} a(t) f^B(t) \, dt
\]

\[
> a(b_n) \int_{b_n}^{\hat{T}_n} f^B(t) \, dt
\]

\[
\geq a(s_n) \int_{b_n}^{\hat{T}_n} f^B(t) \, dt
\]

\[
> a(s_n) \int_{s_{n-1}}^{s_n} f^B(t) \, dt
\]

\[
> \int_{s_{n-1}}^{s_n} a(t) f^B(t) \, dt = \int_{s_{n-1}}^{s_n} f^G(t) \, dt,
\]

which contradicts condition (9). Hence, condition (10) holds. The second from the last inequality makes use of the (contradiction) assumption, and the first and the last inequality makes use of the fact \(a(t)\) is strictly increasing. This completes the proof of

\(^\text{6}\)It is without loss of generality to consider the case where \(s_n \leq b_n\) for all \(n\). If otherwise, we can take out the overlapping interval \([b_n, s_n]\) as it does not affect the ranking of the two standard \(S\) and \(S'\).
the proposition. ■

3.1.1 Implications of the Optimal Standard

The result that the optimal standard must necessarily contain both rules-based elements and principles-based elements is not surprising. In our second-best environment, the rules-based element ignores the auditor’s circumstantial information and induces evidence management, while the principles-based element induces audit failure. By relying on rules-based elements, the standard setter tackles the incidence of audit failure whereas by relying on principles-based elements, the standard setter not only tackles the incidence of evidence management but also utilizes the auditor’s professional judgement. Second and, perhaps more interestingly, the shape of the optimal standard illustrates how rules-based elements and principles-based elements should be efficiently combined. Namely, the optimal standard prescribes principles-based elements only if the evidence is sufficiently favorable. Otherwise, the transaction is purely rules-based and receives an unfavorable treatment.

We believe that the higher hurdle for favorable treatment relative to unfavorable treatment is consistent with how current accounting standards are applied in practice. This result implies that, unlike negative evidence, when faced with positive evidence, the auditor should not just rubber-stamp the rules and automatically grant favorable treatment. Rather, the auditor should utilize her professional judgement and see if it concurs with the favorable evidence. In addition to our motivating example explained in the introduction, lease accounting under US GAAP provides another example. Consider a firm that wants to obtain off-balance sheet financing (a favorable treatment) for a leasing transaction by treating the transaction as an operating lease rather than a capital lease. Under current US GAAP, the auditor uses four criteria to determine whether the firm bears all or most of the risks and rewards of owning the leased asset. However, structuring the transaction so that it does not satisfy any of the four leasing
criteria is not sufficient for classification as an operating lease. The auditor should still use her professional judgement to ensure that the firm does not bear most of the risks and rewards of owning the asset. But if the transaction satisfies even one of the criteria for capital lease treatment, then it is less likely that the auditor would use her professional judgement to override the rule and grant favorable treatment.

Having established in Proposition 1 that the optimal standard is fully specified by a unique threshold $T$, we next characterize the equilibrium threshold. This, in turn, allows us to investigate the properties of the equilibrium threshold in order to better understand when a standard is more likely to be principles-based.

### 3.2 Equilibrium

At $t = 2$ after the transaction’s economic substance $\omega$ is revealed, the unfavorable report $b$ is issued if $t_m < T$. Otherwise, the appropriate treatment for the transaction depends upon the negotiation between the manager and the auditor. Since the manager has all the bargaining power, depending on the effectiveness of enforcement regime, he may be able to make a take-it-or-leave-it side payment $D = \phi$ to the auditor. The equilibrium side payment is thus a function of both auditor’s information $\omega$ and regulatory enforcement regime $\mu$, denoted as $D^*(\omega, \mu)$. Since the good firm doesn’t have any audit failure risk, the manager of the good firm sets $D^*(G, \mu) = 0$ for $\mu \in \{0, 1\}$ and receives the favorable treatment. There are two cases for the manager of the bad firm. First, if the enforcement of the standard is effective, the manager has to set $D^*(B, 1) = 0$ so that the auditor gives the unfavorable treatment. Second, if the enforcement of the standard is ineffective, the manager optimally chooses $D^*(B, 0) = \phi$ to solicit the favorable report from the auditor. In sum, the manager’s side payment and the auditor’s reporting decisions depend on the observable evidence $t_m$, the regulatory
enforcement regime \(\mu\), and the economic substance \(\omega\) of the transaction as follows:

\[
D^*(G, \mu) = D^*(B, 1) = 0, \quad D^*(B, 0) = \phi
\]

\[
r^*(t_m < T, \mu, \omega) = r^*(t_m \geq T, \mu = 1, B) = b,
\]

\[
r^*(t_m \geq T, \mu = 1, G) = r^*(t_m \geq T, \mu = 0, \omega) = g.
\]

Furthermore, because the manager has all the bargaining power in negotiating with the auditor, the auditor’s expected payoffs is always zero regardless of \(t_m, \mu, \) and \(\omega\).

At \(t = 1\) after observing the initial evidence \(t\) (but before \(\omega\) is revealed) the manager decides on evidence management. Since the optimal standard is binary in nature, evidence management that doesn’t get evidence \(t_m\) to \(T\) or that pushes evidence \(t_m\) above \(T\) is wasteful. Therefore the manager prefers either \(t_m = t\) or \(t_m = T\). If the initial evidence exceeds the threshold \((t \geq T)\), the manager does not engage in evidence management and simply presents \(t_m = t\). Otherwise, the manager compares the expected payoffs from two choices, \(t_m = t\) and \(t_m = T\). If the manager chooses \(t_m = t\), or \(m = 0\), he receives a payoff of 0. If he chooses \(t_m = T\) or \(m = T - t > 0\), he incurs a cost of \(C(T - t)\) in return for an expected benefit

\[
\Delta(t) = \Pr(\omega = G|t)\delta + \Pr(\omega = B|t)(1 - \tau)(\delta - \phi).
\]

The expected benefit \(\Delta(t)\) depends on the realization of the economic substance \(\omega\) at \(t = 2\). If \(\omega\) turns out good, which occurs with probability \(\Pr(\omega = G|t)\), the manager receives the favorable treatment and enjoy a benefit of \(\delta\). But if \(\omega\) is bad and regulatory enforcement is ineffective, which occurs with probability \(\Pr(\omega = B|t)(1 - \tau)\), the manager receives the favorable treatment after making a side payment \(D^*(B, 0) = \phi\) to the auditor and therefore enjoys a net benefit of \(\delta - \phi\). In all other cases, the manager receives the unfavorable treatment and a net benefit of 0. This yields the expression
for $\Delta(t)$.

For a given standard with threshold $T$ and initial evidence $t < T$, the benefit of evidence management $\Delta(t)$ is strictly increasing in $t$. To see this, note that $\Delta_t = \frac{\partial \Pr(\omega = G|t)}{\partial t} (\delta - (1 - \tau)(\delta - \phi)) > 0$ where $\frac{\partial \Pr(\omega = G|t)}{\partial t} > 0$ is due to the MLRP property of the evidence distribution. The manager with more favorable initial evidence is more confident that his firm will turn out to be good at $t = 2$ and thus the benefit of meeting the threshold $T$ at $t = 1$ is larger. On the other hand, the cost of evidence management $C(T - t)$ is strictly decreasing in $t$. It takes the manager with a higher $t$ less manipulation to satisfy the threshold. Combining these two observations, the manager’s net incentive to engage in evidence management is increasing monotonically in initial evidence $t$. Therefore, the optimal evidence management strategy is characterized by a unique evidence threshold $\hat{T}$ at which the manager is indifferent between manipulation or not. That is, evidence management threshold $\hat{T}$ as a function of the threshold $T$ is defined implicitly by the equation

$$\Delta(\hat{T}(T)) - C(T - \hat{T}(T)) = 0. \quad (12)$$

Having determined the marginal type of the manager who engages in evidence management, we can characterize the manager’s optimal evidence management strategy at $t = 1$:

$$m^*(t; T) = \begin{cases} T - t & \text{if } t \in (\hat{T}(T), T) \\ 0 & \text{otherwise}. \end{cases} \quad (13)$$

Both functions $\hat{T}(T)$ and $m^*(t; T)$ capture some aspect of evidence management. For any given $T$, $\hat{T}(T)$ determines the marginal type of manager who manipulates evidence and $m^*(t; T)$ measures the magnitude of evidence management by each manager with initial evidence $t$.

The determination of the manager’s evidence management strategy and the side
payment strategy allows us to specify the false alarm error and the undue optimism error as follows:

\[
\Pr(r = b|\omega = G) = \int_{-\infty}^{\hat{T}(T)} f^G(x)dx
\]

\[
\Pr(r = g|\omega = b) = q_B L_B (1 - \tau) \int_{\hat{T}(T)}^{\infty} f^B(x)dx.
\]

Finally, at date 0, rationally anticipating the manager’s evidence management strategy at \(t = 1\) and the side payment strategy at \(t = 2\), the standard setter chooses the threshold \(T\) to minimize the investors’ loss function.

\[
\min_T L = q_G L_G \int_{-\infty}^{\hat{T}(T)} f^G(x)dx + q_B L_B (1 - \tau) \int_{\hat{T}(T)}^{\infty} f^B(x)dx
\]

subject to equation 12. (14)

The optimal threshold \(T^*\) is determined by the first-order condition

\[
L_{T|T=T^*} = (q_G L_G f^G(\hat{T}(T)) - q_B L_B (1 - \tau) f^B(\hat{T}(T))) \frac{\partial \hat{T}(T)}{\partial T} = 0. \quad (15)
\]

The intuition for determining the optimal threshold is clear. Threshold \(T\) affects the investors’ loss function through two channels. First, \(T\) affects the manager’s incentive to engage in evidence management. A standard with threshold \(T\) induces all managers with initial evidence exceeding \(\hat{T}(T)\) but lower than \(T\) to engage in evidence management. The relationship between \(T\) and \(\hat{T}(T)\) is implicitly defined by equation 12. Differentiating it with respect to \(T\), we obtain

\[
\frac{\partial \hat{T}(T)}{\partial T} = \frac{C_m}{C_m + \Delta_T} > 0.
\]

Thus, a higher threshold \(T\) makes it more difficult for managers to manipulate the evidence to satisfy the standard and thus moves upward the marginal type of manager who engages in evidence management.
Second, the induced evidence management affects the investors’ decision making. An increase in $\hat{T}$ increases the false alarm error at the rate of $q_G f^G(\hat{T})$ and simultaneously reduces the undue optimism error at the rate of $q_B (1 - \tau) f^B(\hat{T})$. Each error costs the investors $L_G$ and $L_B$, respectively. Thus, the marginal effect of raising $\hat{T}$ on the investors’ decision making is

$$H(\hat{T}) \equiv q_G L_G f^G(\hat{T}) - q_B L_B (1 - \tau) f^B(\hat{T}).$$  

(16)

Therefore, the optimal standard $T^*$ is chosen so as to induce a $\hat{T}^*$ that balance these two errors weighted by their respective costs to the investors, that is, $H(\hat{T}(T^*)) = 0$. This explains the first-order condition of equation 15.

Given the general distributions of evidence, the second order condition is not globally positive. Since we focus on the properties of the optimal threshold, we only look at the cases in which the second order condition is satisfied. In other words, for any optimal threshold $T^*$, we have $\frac{\partial H(\hat{T}(T))}{\partial \hat{T}}|_{T=T^*} > 0$. At the marginal point of evidence management threshold $\hat{T}(T^*)$ the false alarm error cost is increasing while the undue optimism error cost is diminishing.

In sum, we have solved the model and the next result characterizes the equilibrium.

**Proposition 2** The standard setter’s choice of threshold $T^*$ is characterized by equation (15). The manager’s evidence management strategy $\hat{T}(T^*)$ and $m(t; T^*)$ are characterized by equations (12) and (13). The manager’s side payment decision $D^*(\omega, \mu)$ and the auditor’s reporting decision $r^*(t_m, \mu, \omega)$ are summarized in equation (11). The equilibrium is fully characterized by $T^*$, $m(t; T^*)$, $\hat{T}(T^*)$, $D^*(\omega, \mu)$, and $r^*(t_m, \mu, \omega)$.
3.3 Properties of the optimal standard

Having characterized the optimal standard, we now characterize the properties of the optimal threshold $T^*$.

**Proposition 3** The optimal standard relies more on the auditor’s professional judgement if

1. the regulatory enforcement is more effective (i.e., $\frac{dT^*}{d\tau} < 0$ and $\frac{dT^*}{d\phi} < 0$); or

2. it is more costly for the manager to engage in evidence manager cost (i.e., $\frac{dT^*}{dc} < 0$); or

3. the manager’s incentive for the favorable treatment is lower (i.e., $\frac{dT^*}{d\delta} > 0$); or

4. the undue optimism error (false alarm error) is less (more) costly to investors (i.e., $\frac{dT^*}{dL_B} > 0$ and $\frac{dT^*}{dL_G} < 0$).

To understand the preceding comparative statics, note that a standard with threshold $T$ induces the marginal evidence management level $\hat{T}$ that, in turn, balances the two types of measurement errors. Therefore, a change in any parameter in our environment potentially affects the optimal standard in two ways: it affects the expected cost of measurement errors directly and/or does so indirectly by changing the manager’s incentives to engage evidence management.

We first illustrate the effect of the effectiveness of regulatory enforcement $\tau$ on the optimal standard. Because an effective enforcement regime ensures that the auditor appropriately uses her information in her attestation decision, the parameter $\tau$ can be viewed as a proxy for auditor’s independence. Making it more difficult (by increasing $\tau$) for the manager to influence the auditor has two effects. First, it directly reduces the probability of the undue optimism error by allowing the auditor to utilize the circumstantial information independently. This allows the standard setter to lower the
threshold to reduce the false alarm errors. Second, because the favorable treatment requires both favorable evidence and the auditor’s endorsement, an effective enforcement regime also has an indirect effect of reducing evidence management. As the manager anticipates that the favorable evidence obtained through evidence management is less likely to yield the favorable treatment, his incentives to engage in evidence management diminishes, resulting in a higher $\hat{T}$. The increase in the evidence management threshold $\hat{T}$ upsets the balance of the measurement errors and requires that the standard setter lowers the threshold to push down $\hat{T}$ to restore the balance. In sum, a more effective enforcement regime increases the auditor’s independence and allows the standard setter to incorporate more professional judgement in the optimal standard.

Consider another proxy for the auditor’s independence, $\phi$, the size of the side payment necessary to compromise the auditor’s judgement. A larger $\phi$ implies that it is costlier for the manager to get the auditor to do what the manager prefers. While a higher $\phi$ does not directly affect the trade-off between the two types of measurement errors, it does attenuate the threat of evidence management induced by the standard. As $\phi$ increases, the manager anticipates that he is less likely to influence the auditor so that the benefit of manipulating evidence up to the threshold becomes smaller. The reduction in evidence management (a higher $\hat{T}$) disturbs the balance of the measurement errors. As a result, the standard setter lowers the threshold to reduces the evidence management threshold $\hat{T}$.

We now turn to the cost of evidence management $c$. As $c$ increases, it does not directly affect the trade-off between the two types of measurement errors. However, it increases the manager’s evidence management threshold. Like the effect of $\phi$, the higher evidence management threshold $\hat{T}$ induced by a higher cost of evidence management makes the false alarm error more costly so that the standard setter lowers the optimal threshold in order to reduce the false alarm error. Thus, as the cost of evidence management increases, the optimal threshold is lower and the optimal standard relies
more on the auditor’s professional judgement.

Similarly, while the manager’s preference for a favorable treatment captured by the parameter $\delta$ doesn’t affect the trade-off between the two types of measurement errors, a higher $\delta$ increases the incentive for evidence management, which constrains the standard setter from lowering the threshold.

Finally, while the decision-making cost of measurement errors does not affect the incentive for evidence management, it affects the trade-off between the types of measurement errors. As the cost of false alarm error increases relative to the cost of undue optimism, (i.e., as $L_G$ increases relative to $L_G$), the standard setter lowers the threshold to reduce the cost of false alarm errors so that the standard relies more on principles-based elements.

4 Discussion/Conclusion

In a second-best environment plagued with potential evidence management and audit failure, we have shown that the optimal standard relies on rules-based and principles-based elements. The optimal standard takes a simple form: for sufficiently favorable evidence, it relies on the auditor’s professional judgement; otherwise, a strict rule is applied.

In deriving the optimal standard, we made an important and simplifying assumption: whenever the auditor is called upon to make a professional judgement, both the manager and auditor have perfect information about the economic substance of the transaction. By endowing the auditor with perfect information, we have assumed away auditor competence issues and focused exclusively on auditor’s independence issues. Auditor professional judgement relies on both auditor independence and competence. An interesting extension would be to relax the assumption that the auditor is always competent. This extension would allow us to examine the interaction between
auditor competence and independence issues and their impact on the optimal standard.
References


5 Appendix

Proof of Proposition 3. We consider a generic parameter \( x \in \{ \tau, c, \delta, \phi, L_G, L_B \} \).

By applying the implicit function theorem to the standard setter’s first order condition (equation 15), we have
\[
\frac{dT^*}{dx} = -\frac{1}{SOC} \left( \frac{\partial H(T^*;x)}{\partial T^*} \right) \left( \frac{\partial T^*}{\partial x} \right) + \left( \frac{\partial H(T^*;x)}{\partial x} \right).
\]
Parameter \( x \) affects the optimal threshold through two channels. First, \( x \) affects the relationship between the optimal threshold \( T^* \) and the manipulation threshold \( \hat{T} = \frac{\partial H(T^*;x)}{\partial T^*} \). Second, \( x \) also affects the marginal effect of the manipulation threshold \( \hat{T} = \frac{\partial H(T^*;x)}{\partial x} \).

The denominator \( SOC \) is the second order condition of the standard setter’s minimization problem and thus \( SOC > 0 \). Moreover, \( \frac{\partial H(T^*;x)}{\partial T} > 0 \) because \( \frac{\partial H(T^*;x)}{\partial T} = \frac{SOC}{\partial T} \) and \( \frac{\partial H(T^*;x)}{\partial x} > 0 \). Thus, the sign of \( \frac{dT^*}{dx} \) is determined by that of \( \frac{\partial H(T^*;x)}{\partial x} \).

For any given \( T^* \), we differentiate equation 12 with respect to the parameters and obtain
\[
\begin{align*}
\frac{\partial \hat{T}(T^*; \tau)}{\partial \tau} &= -\frac{\Delta_t}{\Delta_t + C_m} > 0 \\
\frac{\partial \hat{T}(T^*; \phi)}{\partial \phi} &= -\frac{\Delta_\phi}{\Delta_t + C_m} > 0 \\
\frac{\partial \hat{T}(T^*; \delta)}{\partial \delta} &= -\frac{\Delta_\delta}{\Delta_t + C_m} < 0 \\
\frac{\partial \hat{T}(T^*; c)}{\partial c} &= \frac{C_c}{\Delta_t + C_m} > 0 \\
\frac{\partial \hat{T}(T^*; L_G)}{\partial L_G} &= \frac{\partial \hat{T}(T^*; L_B)}{\partial L_B} = 0
\end{align*}
\]
Similarly, for any given \( \hat{T} \), we differentiate equation 16 with respect to the parameters and obtain

\[
\frac{\partial H(\hat{T}; \tau)}{\partial \tau} = q_B L_B f^B(\hat{T}) > 0 \\
\frac{\partial H(\hat{T}; \phi)}{\partial \phi} = \frac{\partial H(\hat{T}; \delta)}{\partial \delta} = \frac{\partial H(\hat{T}; c)}{\partial c} = 0 \\
\frac{\partial H(\hat{T}; L_G)}{\partial L_G} = q_G f^G(\hat{T}) > 0 \\
\frac{\partial H(\hat{T}; L_B)}{\partial L_B} = -q_B (1 - \tau) f^B(\hat{T}) < 0.
\]