The Systematic Component of Monetary Policy in SVARs: An Agnostic Identification Procedure

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Abstract

In this paper, we identify monetary policy shocks in structural vector autoregressions (SVARs) by imposing sign and zero restrictions on the systematic component of monetary policy while leaving the remaining equations in the system unrestricted. As in Uhlig (2005), no restrictions are imposed on the response of output to a monetary policy shock. We find that an exogenous increase in the federal funds rate leads to a persistent decline in output and prices. Our results show that the contractionary effects of monetary policy shocks do not hinge on questionable exclusion restrictions, but are instead consistent with agnostic identification schemes. The analysis is robust to various specifications of the systematic component of monetary policy widely used in the literature.

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1 Introduction

Following [Sims (1972, 1980, 1986)], researchers have analyzed the effects of monetary policy on output using structural vector autoregressions (SVARs). Most of them have concluded that an increase in the federal funds rate or a decrease in the money supply are contractionary—i.e., they have a significant negative effect on output. The set of studies supporting this view includes [Bernanke and Blinder (1992); Christiano et al. (1996); Leeper et al. (1996); and Bernanke and Mihov (1998)]. This intuitive result has become the cornerstone rationale behind New Keynesian dynamic stochastic general equilibrium (DSGE) models. Researchers also estimate New Keynesian models by matching the dynamic responses to a monetary policy shock implied by the model with those implied by a SVAR—see [Rotemberg and Woodford (1997) and Christiano et al. (2005)].

The consensus about the contractionary effects of monetary policy shocks on output has been challenged by [Uhlig (2005)], who found no evidence to support such a view using an agnostic identification strategy. Uhlig’s (2005) critique is that traditional SVARs require the researcher to identify all shocks in the system and impose a tremendous number of possibly spurious restrictions. He therefore proposes to identify monetary policy shocks by imposing sign restrictions on just the impulse response functions of prices and nonborrowed reserves to the shock. These restrictions eliminate the well-known price and liquidity puzzles while remaining agnostic about the responses of other variables, particularly output, to the monetary policy shock. Furthermore, this approach does not restrict the response of any variable to the remaining structural shocks. This means that Uhlig (2005) does not identify a single model but rather a set of models that are coherent with his sign restrictions. In other words, he does not identify the structural parameters themselves but instead set-identifies them.

In this paper, we endorse the agnostic approach, but instead of imposing restrictions on impulse response functions to a monetary policy shock, we impose them on the monetary policy equation. In particular, we use an agnostic identification scheme to restrict the systematic component of monetary policy. Our approach is inspired by the line of work of [Leeper et al. (1996); Leeper et al. (1996)].
and Zha (2003); and Sims and Zha (2006a), which emphasizes the need to specify and estimate behavioral relationships for monetary policy. Policy choices in general, and monetary policy choices in particular, do not evolve independently of economic conditions: *even the harshest critics of monetary authorities would not maintain that policy decisions are unrelated to the economy* (Leeper et al., 1996). Thus, to isolate exogenous changes in policy, one needs to model how policy reacts to the economy.

We identify monetary policy shocks by imposing sign and zero restrictions on the systematic component of monetary policy. We propose three alternative sets of restrictions, inspired by three specifications of the systematic component that are widely used in the literature. The first specification derives from standard SVARs, such as the one prominently used by Christiano et al. (1996), and implies that the federal funds rate responds positively to output and prices. The second specification originates from Taylor-type rules widely used in DSGE models and implies that the federal funds rate responds to inflation and a measure of economic activity. The third specification considers the class of money rules described in Leeper et al. (1996); Leeper and Zha (2003); and Sims and Zha (2006a,b). In contrast to these papers, we set-identify the SVAR because we only impose sign and zero restrictions on the monetary policy equation and we leave the non-policy equations unrestricted. Hence, our approach shares two features with Uhlig (2005). First, we do not impose any restriction on the response of output to monetary shocks. Second, we do not identify a single model but rather a set of models that are coherent with our sign and zero restrictions.

We highlight two results. First, we find that an exogenous increase in the federal funds rate has persistent contractionary effects on output. The decline in real activity, together with the decline in prices, causes a medium-term loosening of the monetary policy stance. Hence, our agnostic identification scheme recovers the consensus regarding the effects of monetary policy shocks while addressing Uhlig (2005)’s critique. Second, we show that the identification scheme in Uhlig (2005) violates our restrictions on the systematic component of monetary policy. Following Leeper et al. (1996); Leeper and Zha (2003); and Sims and Zha (2006a), a corollary to our findings is that the shocks identified in Uhlig (2005) are not monetary policy shocks because the systematic component of monetary policy is counterfactual and does not control for the endogenous response of monetary
policy to economic activity.

To further understand the relationship between the identification schemes, we combine the sign restrictions on impulse response functions in Uhlig (2005) with our restrictions on the systematic component. We find that our restrictions substantially shrink the set of models originally identified by Uhlig (2005), and that excluding models with counterfactual monetary policy equations suffices to generate a negative response of output and thereby recover the consensus. The restrictions in Uhlig (2005) also refine the set of admissible models obtained using our approach, as they exclude models that generate the price puzzle. But this refinement has modest impact on the results, as the subset of excluded models is small.

Our work is related to several studies in the literature. A similar identification strategy to the one used in this paper is employed in Caldara and Kamps (2012), who identify tax and government spending shocks by putting discipline on the systematic component of fiscal policy. They combine zero restrictions with empirically plausible bounds on the output elasticities of fiscal variables. Baumeister and Hamilton (2014) study how informative are the data relative to the prior distributions in the estimation of SVARs. In one of their examples they impose prior distributions on the systematic component of monetary policy of a very simple three equations model. Arias et al. (2014) develop the theoretical foundation to identify SVARs by jointly imposing sign and zero restrictions. They apply their methodology to revisit the identification of optimism shocks in Beaudry et al. (2011) and the identification of fiscal shocks in Mountford and Uhlig (2009). Both applications impose restrictions on impulse response functions, while we apply their methodology to impose restrictions directly on the SVAR equations. We also study identification schemes that combine restrictions on the SVAR equations with restrictions on impulse response functions. Some recent applications of SVAR identification based on sign and zero restrictions on impulse response functions include Baumeister and Benati (2010), who identify the effects of unconventional monetary policy; Binning (2013), who identifies anticipated government spending shocks; and Peersman and Wagner (2014), who identify shocks to bank lending.

The structure of the paper is as follows. In Section 2 we describe the SVAR methodology and describe our baseline identification scheme. In Section 3 we describe the results and compare
them with [Uhlig (2005)](#). In Section 4, we consider alternative specifications of the monetary policy equation. In Section 5, we conclude.

## 2 Methodology

Let us consider the following SVAR

\[
y_t' A_0 = \sum_{\ell=1}^{p} y_{t-\ell}' A_{\ell} + c + \varepsilon_t' \quad \text{for} \quad 1 \leq t \leq T, \tag{1}
\]

where \(y_t\) is an \(n \times 1\) vector of endogenous variables, \(\varepsilon_t\) is an \(n \times 1\) vector of structural shocks, \(A_\ell\) is an \(n \times n\) matrix of structural parameters for \(0 \leq \ell \leq p\) with \(A_0\) invertible, \(c\) is a \(1 \times n\) vector of parameters, \(p\) is the lag length, and \(T\) is the sample size. The vector \(\varepsilon_t\), conditional on past information and the initial conditions \(y_0, \ldots, y_{1-p}\), is Gaussian with mean zero and covariance matrix \(I_n\) (the \(n \times n\) identity matrix). The model described in equation (1) can be written as

\[
y_t' A_0 = x_t' A_+ + \varepsilon_t' \quad \text{for} \quad 1 \leq t \leq T, \tag{2}
\]

where \(A_+ = \begin{bmatrix} A_1' & \cdots & A_p' & c' \end{bmatrix}\) and \(x_t' = \begin{bmatrix} y_{t-1}' & \cdots & y_{t-p}' & 1 \end{bmatrix}\) for \(1 \leq t \leq T\). The dimension of \(A_+\) is \(m \times n\), where \(m = np+1\). We call \(A_0\) and \(A_+\) the structural parameters. The reduced-form representation implied by equation (2) is

\[
y_t' = x_t' B + u_t' \quad \text{for} \quad 1 \leq t \leq T,
\]

where \(B = A_+ A_0^{-1}\), \(u_t' = \varepsilon_t' A_0^{-1}\), and \(E[u_t' u_t'] = \Sigma = (A_0 A_0')^{-1}\). The matrices \(B\) and \(\Sigma\) are the reduced-form parameters. Finally, the impulse response functions (IRFs) are as follows.

**Definition 1.** Let \((A_0, A_+)\) be any value of structural parameters, the IRF of the \(i\)-th variable to the \(j\)-th structural shock at finite horizon \(h\) corresponds to the element in row \(i\) and column \(j\) of
the matrix

\[ L_h(A_0, A_+) = \left( A_0^{-1} J F^h J \right)' , \text{ where } F = \begin{bmatrix}
A_1 A_0^{-1} & I_n & \cdots & 0 \\
\vdots & \vdots & \ddots & \vdots \\
A_{p-1} A_0^{-1} & 0 & \cdots & I_n \\
A_p A_0^{-1} & 0 & \cdots & 0
\end{bmatrix}, \text{ and } J = \begin{bmatrix} I_n \\ 0 \\ \vdots \end{bmatrix} . \]

Papers in the literature involving set identification of structural parameters typically impose sign and/or zero restrictions on either \( A_0 \) or the IRFs. The identification approach that we propose in this paper combines sign and zero restrictions on \( A_0 \) or IRFs or both. We use restrictions on \( A_0 \) to discipline the systematic component of monetary policy and restrictions on the IRFs to restrict the dynamics of the structural shocks.

Our methodology is based on Rubio-Ramírez et al. (2010) and Arias et al. (2014). For details, we refer the reader to the mentioned papers, but we can summarize the characterization of the restrictions as follows. Let us assume that we want to impose restrictions on some elements of \( A_0 \) and on some IRFs at different horizons. It is convenient to stack \( A_0 \) and the IRFs for all the relevant horizons into a single matrix of dimension \( k \times n \), which we denote by \( f(A_0, A_+) \). For example, if we impose restrictions at horizons zero and one, then

\[
f(A_0, A_+) = \begin{bmatrix} A_0 \\ L_0(A_0, A_+) \\ L_1(A_0, A_+) \end{bmatrix}, \text{ where } k = 3n \text{ in this case.}
\]

We represent the sign restrictions on \( f(A_0, A_+) \) used to identify structural shock \( j \) by a matrix \( S_j \), where the number of columns in \( S_j \) is equal to \( k \) and \( 1 \leq j \leq n \). \( S_j \) is a selection matrix and thus has one non-zero entry in each row. If the rank of \( S_j \) is \( s_j \), then \( s_j \) is the number of sign restrictions imposed to identify the \( j - \text{th} \) structural shock. Similarly, we represent the zero restrictions on \( f(A_0, A_+) \) used to identify structural shock \( j \) by selection matrices \( Z_j \), where the number of columns in \( Z_j \) is also equal to \( k \) and each row has one non-zero entry. If the rank of \( Z_j \) is \( z_j \), then \( z_j \) is the number of zero restrictions imposed to identify the \( j - \text{th} \) structural shock.
shock. When we only impose sign restrictions, we draw from the posterior distribution of the structural parameters using algorithms in [Rubio-Ramírez et al. (2010)]. When we impose sign and zero restrictions, we draw using algorithms in [Arias et al. (2014)].

To highlight the implications of our identification scheme, we choose a widely used specification of the reduced-form VAR model. In particular, we make our results comparable to [Uhlig (2005)], and we use Bayesian methods to estimate the same reduced-form model as [Uhlig (2005)] on his dataset, which spans U.S. monthly data from 1965:I to 2003:XII, using his priors. Given that the priors, the reduced-form model, and the data have been extensively discussed by [Uhlig (2005)], for our purposes it suffices to mention that the VAR specification includes output (real GDP), $y_t$; the GDP deflator, $p_t$; an index of commodity prices; $p_{c,t}$; total reserves, $tr_t$; nonborrowed reserves, $nbr_t$; and the federal funds rate, $r_t$. We take the natural logarithm of all variables except for the federal funds rate, and without loss of generality, we assume in all our identification schemes that variables follow the order of listing above. This vector of endogenous variables is standard in the literature and has been used, among others, by [Christiano et al. (1996)] and [Bernanke and Mihov (1998)]. The VAR specification includes 12 lags ($p = 12$) and does not include any deterministic term.

### 2.1 Sign Restrictions on IRFs

Agnostic identification schemes are commonly associated with the imposition of sign restrictions on IRFs. A seminal paper in this literature is [Uhlig (2005)]. This paper examines the effects of monetary policy shocks on output. In order to identify monetary policy shocks, he imposes the following restrictions.

**Restriction 1.** A monetary policy shock leads to a negative response of the GDP deflator, commodity prices, and nonborrowed reserves, and to a positive response of the federal funds rate, all at horizons $t = 0, \ldots, 5$.

Restriction 1 rules out the price puzzle—a positive response of the price level following a monetary contraction—and the liquidity puzzle—a positive response of monetary aggregates. [Uhlig (2005)]

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3We repeat the analysis using an updated version of the dataset running until 2007, and a version with quarterly data. Results reported in the following sections are robust to the use of these datasets and are available upon request.
motivates these restrictions as a way to rule out implausible price and reserve behaviors, so that the set of admissible SVARs does not include models that we would find not interesting from a theoretical perspective. Restriction 1 implies non-linear restrictions on \((A_0, A_+). But the crucial features of the identification described by Restriction 1 are that (i) it remains agnostic about the response of output after an increase in the federal funds rate and (ii) it only identifies monetary policy shocks. This implies that Restriction 1 does not identify the structural parameters but only set-identifies them, allowing a set of models, rather than a single model, to be compatible with the restrictions.

Without loss of generality, if we let the monetary policy shock be the first structural shock, we characterize Restriction 1 with the matrices described below:

\[
f(A_0, A_+) = \begin{bmatrix}
L_0(A_0, A_+) \\
\vdots \\
L_5(A_0, A_+)
\end{bmatrix}, \quad S_1 = \begin{bmatrix}
S_{10} & 0_{m,n} & \cdots & 0_{m,n} \\
0_{m,n} & \ddots & \ddots & \vdots \\
\vdots & \ddots & \ddots & 0_{m,n} \\
0_{m,n} & \cdots & 0_{m,n} & S_{15}
\end{bmatrix}
\]

\[
S_{1t} = \begin{bmatrix}
0 & -1 & 0 & 0 & 0 & 0 \\
0 & 0 & -1 & 0 & 0 & 0 \\
0 & 0 & 0 & -1 & 0 & 0 \\
0 & 0 & 0 & 0 & -1 & 0 \\
0 & 0 & 0 & 0 & 0 & 1
\end{bmatrix}
\] for \(t = 0, \ldots, 5\), where \(m = 6\) and \(n = 4\).

In Figure 1, we plot the IRFs to an exogenous tightening of monetary policy identified by imposing Restriction 1. Throughout the paper, we normalize the size of the shock to be equal to one standard deviation. All results are based on 10,000 draws from the posterior distribution of the structural parameters. The shadowed area shows the 68% confidence bands and the solid lines show the median IRFs. This figure replicates Figure 6 in Uhlig (2005). Panel (A) shows that the median response of output is positive. In addition, there is evidence that in the short run the 68% confidence bands include zero, indicating that the estimated impulse responses are not statistically significant.

\footnote{In the paper, it is always the case that the monetary policy shock is the first structural shock.}
confidence bands do not contain zero. Panels (B) and (C) show the response of the GDP deflator and the commodity price index, respectively, which are restricted to be negative for six months to exclude the price puzzle. Panels (D) and (E) show the response of total reserves and nonborrowed reserves, both of which are negative in the short run. The reduction in nonborrowed reserves is more significant because the response of this variable is restricted to be negative for six months to exclude the liquidity puzzle. Finally, Panel (F) shows the response of the federal funds rate, which is restricted to be positive for the first six months and it becomes negative 18 months after the shock.

![Figure 1: IRFs to a Monetary Policy Shock Identified Using Restrictions](image)

Figure 1: IRFs to a Monetary Policy Shock Identified Using Restrictions

Hence, consistent with Uhlig (2005), the main result shown in Figure 1 is the lack of support for the contractionary effects on output of an exogenous increase in the federal funds rate. This result presents a challenge to the consensus view that output decreases in response to a tightening of monetary policy. Replicating Uhlig (2005) is important because in the next section we show that, despite its appeal, Restriction implies a counterfactual systematic component of monetary
policy and therefore does not identify monetary policy shocks.

2.2 Systematic Component of Monetary Policy

The identification of monetary policy shocks either requires or implies the specification of how policy usually reacts to economic conditions. Leeper et al. (1996); Leeper and Zha (2003); and Sims and Zha (2006a) emphasize the need to specify and estimate the behavior of the systematic component of monetary policy. Uhlig (2005) deviates from this paradigm, as discussed in the previous section, but we argue that the implied systematic component provides a useful way to check whether the set of identified models is a sensible one.

In order to characterize the systematic component of monetary policy, it is important to note that labeling a structural shock in the SVAR as the monetary policy shock is equivalent to specifying the same equation as the monetary policy equation. Thus, the first equation of the SVAR,

\[ y_t' a_{0,1} = \sum_{\ell=1}^{p} y_{t-\ell} a_{\ell,1} + \varepsilon_{1t} \quad \text{for } 1 \leq t \leq T, \tag{3} \]

is the monetary policy equation, where \( \varepsilon_{1t} \) denotes the first entry of \( \varepsilon_t \), \( a_{\ell,1} \) denotes the first column of \( A_\ell \) for \( 0 \leq \ell \leq p \), and \( a_{\ell,ij} \) denotes the \((i,j)\) entry of \( A_\ell \). Consequently, \( \sum_{\ell=1}^{p} y'_{t-\ell} a_{\ell,1} \) describes the systematic component of monetary policy.

When analyzing the systematic component of monetary policy, we borrow from the literature three specifications of the monetary policy equation. The benchmark specification, discussed in this section, is motivated by Christiano et al. (1996). The second and third specifications, discussed in Section 4, are motivated, respectively, by Taylor (1993, 1999); and Leeper et al. (1996); Leeper and Zha (2003); Sims and Zha (2006a); and Sims and Zha (2006b). Even though each of these approaches characterizes the systematic component of monetary policy in a particular way, they deliver similar results.

The monetary policy equation implied by Christiano et al. (1996) makes two important identification assumptions about the systematic component of monetary policy. They are summarized as follows.
Restriction 2. The federal funds rate is the monetary policy instrument and it only reacts contemporaneously to output and prices.

Restriction 2 comprises two parts. First, the fact that the federal funds rate is the policy instrument is supported by empirical and anecdotal evidence. Except for a short period between October 1979 and October 1982 when the Federal Reserve explicitly targeted nonborrowed reserves, monetary policy in the U.S. since 1965 can be characterized by a direct or indirect interest rate targeting regime. Sims and Zha (2006) also provide support for this view in their finding that the federal funds rate was the policy instrument for most of their sample, which runs from 1959 to 2003. Even so, they also suggest that one should be careful when applying the Taylor formalism to interpret specific historical periods; for example, as in Bernanke and Blinder (1992), they find that policy behavior was better characterized by nonborrowed reserves targeting in the first three years of Paul Volcker’s tenure as Chairman of the Federal Reserve from October 1979 to October 1982, as well as in the first years of Arthur Burns’ tenure as Chairman of the Fed in the early 1970s. With these exceptions in mind, one could conclude that the Fed has used the federal funds rate as its monetary policy instrument almost continuously since 1965, although the federal funds rate has only formally been the Federal Reserve’s policy instrument since 1997.

Second, the federal funds rate does not react to changes in reserve aggregates. Bernanke and Blinder (1992) and Christiano et al. (1996) also include reserve aggregates because in the mid-1990s they were viewed as alternative instruments for characterizing the conduct of monetary policy. Nevertheless, when the federal funds rate is the monetary instrument in these papers, reserve aggregates do not enter the monetary equation.

Next, we impose qualitative restrictions on the response of the federal funds rate to economic conditions, which we summarize as follows.

Restriction 3. The contemporaneous reaction of the federal funds rate to output and prices is nonnegative.

5See Bernanke and Blinder (1992) and Chappell Jr et al. (2005).
6Christiano et al. (1996) study also a monetary rule where nonborrowed reserves is the policy instrument. We do not explore this specification because the analysis in Christiano et al. (1996) is not robust to extending the sample beyond 1995. This is consistent with the view that nonborrowed reserves were used as an explicit policy instrument only in the early 1980s.
Restriction 3 is implicit in the Federal Reserve Act, according to which the objectives of monetary policy are maximum employment, stable prices, and moderate long-term interest rates. From a more general perspective, it is a reflection of the modern conduct of monetary policy, which is less mechanical than it was at the beginning of 20th century, and is based instead on achieving certain economic goals, such as full employment and price stability, as mentioned above (see Woodford (2003)).

We see the set of behavioral policy equations that are consistent with Restriction 2 and 3 as the largest set describing the historical conduct of U.S. monetary policy toward fulfilling these objectives. Importantly, we stress that Restrictions 2 and 3 are sign and zero restrictions on the coefficients of the monetary policy equation and they do not impose restrictions on the response of variables to the monetary policy shocks, nor restrict the sign of such responses. For this reason, we remain agnostic about the response of output to an increase in the federal funds rate. It is also the case that, contrary to Christiano et al. (1996), we leave the remaining equations unrestricted and therefore only identify monetary policy shocks. Thus, as in Uhlig (2005), Restrictions 2 and 3 do not identify the structural parameters but only set identify them, allowing a set of models to be compatible with the restrictions rather than a single one.7

If we only concentrate on the contemporaneous coefficients, we can rewrite equation (3) as

\[ r_t = \psi_y y_t + \psi_p p_t + \psi_{pc} p_{c,t} + \psi_{tr} r_{t-1} + \psi_{nbr} nbr_t + a^{-1}_{0,61} \varepsilon_{1,t} \]  

where \( \psi_y = a^{-1}_{0,61} a_{0,11} \), \( \psi_p = a^{-1}_{0,61} a_{0,21} \), \( \psi_{pc} = a^{-1}_{0,61} a_{0,31} \), \( \psi_{tr} = a^{-1}_{0,61} a_{0,41} \), and \( \psi_{nbr} = a^{-1}_{0,61} a_{0,51} \). Equipped with this representation of the monetary policy equation, we describe Restrictions 2 and 3 as follows.

**Remark 1.** Restriction 2 implies that \( \psi_{tr} = \psi_{nbr} = 0 \), while Restriction 3 implies that \( \psi_y, \psi_p, \psi_{pc} \geq 0 \).

Let \( s_{10} \), the number of sign restrictions at horizon 0, be equal to 5, \( s_{1+} \), the number of sign restrictions at horizon greater than 1, be equal to 1, and \( z_{10} \), the number of zero restrictions at horizon 0, be equal to 5. Let \( z_{1+} \), the number of zero restrictions at horizon greater than 1, be equal to 1.

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7See Arias et al. (2014) for details on set identification in SVARs identified imposing sign and zero restrictions.
horizon zero, be equal to 2. If we let the monetary policy shock be the first structural shock, then Restrictions 2 and 3 and the normalization on the federal funds rate impose restrictions on \((A_0, A_+)\) that are characterized using the following matrices.

\[
f(A_0, A_+) = \begin{bmatrix}
A_0 \\
L_0 (A_0, A_+) \\
\vdots \\
L_5 (A_0, A_+)
\end{bmatrix}
\]

\[
S_1 = \begin{bmatrix}
S_{10} & 0_{s10,n} & \cdots & 0_{s10,n} \\
0_{s1+2n} & S_{11} & 0_{s1+n} & \cdots \\
\vdots & \vdots & 0_{m,n} & \ddots & \vdots \\
0_{s1+2n} & \vdots & \cdots & \cdots & S_{15}
\end{bmatrix},
\]

\[
Z_1 = \begin{bmatrix}
Z_{10} \\
0_{s10,5n}
\end{bmatrix},
S_{10} = \begin{bmatrix}
-1 & 0 & 0 & 0 & 0 & 0 \\
0 & -1 & 0 & 0 & 0 & 0 \\
0 & 0 & -1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 \\
0_{s_1} & \cdots & \cdots & \cdots & 0_{s_1} & S
\end{bmatrix},
S_1^t = S \text{ for } t = 1, \ldots, 5, \text{ and } Z_{10} = \begin{bmatrix}
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0
\end{bmatrix}.
\]

We impose two normalizations. First, when we impose Restrictions 2 and 3 only, we normalize the sign of the shock by assuming that the federal funds rate response remains positive for six months.\(^8\) Second, we restrict \(a_{0,61} > 0\) in order to satisfy the regularity conditions for \(f(A_0, A_+)\) specified in Arias et al. (2014).

In Section 3, we present results for the identification of monetary policy shocks that jointly imposes Restrictions 1, 2, and 3 on \((A_0, A_+)\). To characterize this identification scheme, it suffices

\(^8\)We choose six months as our baseline because there is ample evidence of short-run smoothing of policy rates (Rudebusch, 2006). Results are robust to imposing this normalization for both one and three months. We also apply this normalization to the policy rules considered in Section 4.
to modify the above set of matrices by setting $s_{10} = 8$, $s_{1+} = 3$, and matrix $S$ to

$$
S = \begin{bmatrix}
0 & -1 & 0 & 0 & 0 & 0 \\
0 & 0 & -1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & -1 & 0 \\
0 & 0 & 0 & 0 & 0 & 1
\end{bmatrix}.
$$

3 Results

In this section, we characterize the systematic component of monetary policy implied by Uhlig’s (2005) identification scheme and highlight the fact that it is counterfactual. We then present results for our agnostic identification scheme based on Restrictions 2 and 3.

3.1 Systematic Component of Monetary Policy and Uhlig (2005)

We now describe the systematic component of monetary policy that is consistent with the monetary policy shocks identified in Uhlig (2005). By construction, the set of models that satisfy Restriction 1 implies $\psi_{tr} \neq 0$ and $\psi_{nbr} \neq 0$, thus violating Restrictions 2. As explained in Arias et al. (2014), unless we condition on the zero restrictions to draw the structural parameters, the set of models that satisfy such zero restrictions has measure zero. More importantly, we show in Figure 2 that Restriction 1 also implies coefficients on output ($\psi_y$) and prices ($\psi_p$ and $\psi_{pc}$) that violate Restriction 3. Panels (A), (B), and (C) show the cumulative density functions (CDFs) of the coefficients $\psi_y$, $\psi_p$, and $\psi_{pc}$. The y-axes indicate the value of the CDFs and the x-axes indicate the support of these distributions. Restriction 1 allocates a significant probability mass to negative values of these coefficients and, as a byproduct, to events in which there is a monetary tightening in response to a decrease in either prices or output. Over 60% of the draws violate the sign restriction on $\psi_y$, about 15% of the draws violate the sign restriction on $\psi_p$, and 10% of the draws violate the sign restriction on $\psi_{pc}$. 80% of the draws violate at least one sign restriction and, as explained in the previous paragraph, all draws violate the sign and zero restrictions.

This exercise shows that Uhlig’s (2005) identification scheme implies a counterfactual systematic...
component of monetary policy that violates both Restrictions 2 and 3. Following Leeper et al. (1996); Leeper and Zha (2003); and Sims and Zha (2006a), a corollary to our findings is that the shocks identified by Restriction 1 are not monetary policy shocks because the systematic component of monetary policy is counterfactual and hence does not control for the endogenous response of monetary policy to economic activity as characterized by Restrictions 2 and 3.

3.2 Restricting the Systematic Component of Monetary Policy

We now present results derived by imposing Restrictions 2 and 3 on the monetary policy equation. We first combine those restrictions with the sign restrictions in Uhlig (2005) before applying them in isolation.

In Figure 3, we plot the IRFs to a monetary policy shock identified by jointly imposing Restrictions 1, 2, and 3. We emphasize two results. First, the output response is negative and it builds up over time. Second, the contour of the federal funds rate is similar to Uhlig (2005): positive for one year, and negative thereafter. But contrary to Uhlig (2005), we can rationalize this path with the systematic component of monetary policy, as the drop in the federal funds rate is the endogenous response of policy to the decline in real activity and prices.

As explained in Section 2.2, we normalize the response of the federal funds rate response to be positive when we apply Restrictions 2 and 3 in isolation, as this sign normalization is implicit in Restrictions 1. This is also the case in Section 4.

As mentioned in Section 2.2, this identification scheme remains silent about the effects of output to a monetary policy shock.
Finally, in Figure 4 we plot the IRFs to a monetary shock identified by imposing only Restrictions 2 and 3. Dropping Restriction 1 has little effect on our main finding: output drops following monetary tightening and, together with a drop in prices, leads to a long-run loosening of the policy stance. But Panel (B) shows that dropping Restriction 1 leads to the emergence of the price puzzle, though one of quantitatively modest size. Hence, the set of models characterized by Restrictions 2 and 3 include a small subset of models with a counterfactual response of prices to monetary shocks. Comparing Figures 3 and 4, we see that Restriction 1 helps refining the set of models by excluding models that generate the price puzzle, although the effect of imposing Restriction 1 on the IRFs is quantitatively modest.

It is also important to highlight that our results in Figures 3 and 4 contradict Uhlig’s (2005) claim that you need to restrict the initial response of output to zero in order to recover the consensus. In both figures the initial response is different from zero and output drops after the negative

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\[11\] To eliminate the price puzzle we would only need to impose that a monetary policy shock leads to a negative response of the GDP deflator, without imposing the remaining sign restrictions described in Restriction 1.
monetary policy shock.

![Figure 4: IRFs to a Monetary Policy Shock Identified Using Restrictions 2 and 3](image)

All told, three messages emerge from this section. First, imposing some discipline in the systematic component of monetary policy is crucial to recover the conventional effects of monetary policy. Second, once the systematic behavior of monetary policy is restricted, imposing additional sign restrictions on IRFs as motivated by Uhlig (2005) helps refine the set of admissible models but is not crucial for the results. Third, it is not necessary to have a dogmatic zero restriction on the output response to a monetary policy shock in order to rescue the conventional effects of monetary policy.

4 Alternative Systematic Components of Monetary Policy

In this section, we consider two alternative specifications of the monetary policy equation. The first specification is a rule motivated by the use in DSGE models of interest rate rules reacting to inflation and some measure of economic activity. We refer to this class of rules as Taylor rules.
The second rule is a money rule motivated by the work of Leeper et al. (1996); Leeper and Zha (2003); Sims and Zha (2006a); and Sims and Zha (2006b). Each of these specifications has received wide attention in the empirical monetary literature and provides alternative descriptions of the systematic component.

4.1 Taylor-Type Rule

In the specification of the monetary policy equation studied in Section 2, the federal funds rate responds to output and price levels. But researchers, especially those working with DSGE models, often consider Taylor-type monetary policy equations in which the funds rate responds to inflation and a measure of economic activity instead. Inspired by the Taylor rules used in the literature, we model the systematic component of monetary policy using the following set of restrictions.

Restriction 4. The federal funds rate is the monetary policy instrument and it only reacts contemporaneously to output growth, GDP deflator inflation, and commodity prices inflation.

Restriction 5. The contemporaneous reaction of the federal funds rate to output growth and both measures of inflation is nonnegative.

We specify a rule in the growth rate of output and not in the output gap (or the growth rate of the output gap) as done in many DSGE models because our reduced-form specification does not include potential output. However, results are qualitatively similar for a specification that includes output instead of its growth rate.

As in Section 2, since Restrictions 4 and 5 just describe sign and zero restrictions on the coefficients of the monetary policy equation, we remain agnostic about the response of output to a monetary policy shock. These restrictions only identify the behavior of the monetary policy equation while leaving the remaining equations unrestricted.

\[ \text{In this exercise, while we set-identify the structural parameters, we exactly identify the IRFs associated with the monetary policy shock. That is, for any draw of the reduced-form parameters, there exists at most one column of the rotation matrix that satisfies the sign and zero restrictions. This is the case because in the Taylor-type rule identification scheme we impose } n - 1 \text{ zero restrictions on the monetary policy equation. If we relax the sign and zero restrictions on commodity price inflation, the IRFs to a monetary shock are set-identified and are very similar to those reported in Figures 5 and 6. See Arias et al. (2014) for additional details.} \]
Figure 5: IRFs to a Monetary Policy Shock Identified Using Restrictions [1, 4, and 5]

Since the federal funds rate is the policy instrument, if we concentrate on the contemporaneous coefficients, we can rewrite equation (3) as

\[ r_t = \psi_y \Delta y_t + \psi_p \pi_t + \psi_{mc,t} \pi_{ct,t} + \psi_{tr} tr_t + \psi_{nbr} nbr_t + a_{0,61}^{-1} \varepsilon_{1,t}, \]  

(5)

where \( \Delta y_t \) is the monthly output growth, \( \pi_t \) is the monthly inflation rate of the GDP deflator, \( \pi_{ct,t} \) is the monthly inflation rate of the index of commodity prices, \( \psi_y = a_{0,61}^{-1} a_{0,11}, \psi_p = a_{0,61}^{-1} a_{0,21}, \)
\( \psi_{mc} = a_{0,61}^{-1} a_{0,31}, \psi_{tr} = a_{0,61}^{-1} a_{0,41}, \) and \( \psi_{nbr} = a_{0,61}^{-1} a_{0,51}. \) Equipped with this representation of the monetary policy equation, we describe Restrictions [4] and [5] as follows.

**Remark 2.** Restriction [4] implies that \( \psi_{tr} = \psi_{nbr} = -a_{0,11} + a_{1,11} = -a_{0,21} + a_{1,21} = -a_{0,31} + a_{1,31} = 0, \) while Restriction [5] implies that \( \psi_y, \psi_p, \psi_{mc} \geq 0. \)

Restrictions [4] and [5] map into restrictions on both \( A_0 \) and \( A_+ \) because we restrict growth rates for output and prices by placing constraints on the coefficients of lagged output and price levels. In fact, restrictions \(-a_{0,11} + a_{1,11} = -a_{0,21} + a_{1,21} = -a_{0,31} + a_{1,31} = 0\) equate the coefficients on
current and lagged output and current and lagged price levels in order to obtain growth rates.

Let $s_{10} = 5$, $s_{1+} = 1$, and $z_{10} = 5$. If we let the monetary policy shock be the first structural shock, then we summarize Restrictions 4 and 5 in the following matrices:

$$f(A_0, A_+) = \begin{bmatrix} A_0 \\ A_1 \\ L_0(A_0, A_+) \\ \vdots \\ L_5(A_0, A_+) \end{bmatrix}, \quad S_1 = \begin{bmatrix} S_{10} & 0_{s_{10},n} & \ldots & 0_{s_{10},n} \\ 0_{s_{1+},2n} & S_{11} & 0_{s_{1+},n} & \ldots \\ \vdots & \vdots & \ddots & \vdots \\ 0_{s_{1+},2n} & \vdots & \ldots & S_{15} \end{bmatrix}.$$  

$$Z_1 = \begin{bmatrix} Z_{10} \\ 0_{z_{10},6n} \end{bmatrix}', \quad S_{10} = \begin{bmatrix} -1 & 0 & 0 & 0 & 0 & 0_{1,2n} \\ 0 & -1 & 0 & 0 & 0 & \vdots \\ 0 & 0 & -1 & 0 & 0 & \vdots \\ 0 & 0 & 0 & 0 & 1 & 0_{1,2n} \\ 0_{s,1} & \ldots & \ldots & \ldots & 0_{s,1} & S \end{bmatrix}, \quad S = \begin{bmatrix} 0'_{s_{10},n} \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}.$$  

$S_{1t} = S$ for $t = 1, \ldots, 5$, and $Z_{10} = \begin{bmatrix} -1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}.$

As in Section 3, we also present results for the identification of monetary policy shocks that jointly impose Restrictions 1, 4, and 5. To characterize this identification scheme, we set $s_{10} = 7$, $s_{1+} = 4$, $z_{10} = 4$, and matrix S to
In Figure 5, we plot the IRFs to a monetary policy shock identified by imposing Restrictions 1, 4, and 5. Qualitatively, the results are similar to those obtained using Restrictions 1, 2, and 3: output declines after a negative monetary policy shock, and monetary policy loosens its stance in the long run. But all the IRFs, particularly the output response, are more precisely estimated which reinforces the message that a negative monetary policy shock is contractionary once the systematic component of monetary policy is taken into account.

In Figure 6, we plot the IRFs to a monetary policy shock identified by imposing only Restrictions 4 and 5, together with the sign normalization on the response of the federal funds rate. As in Section 3, dropping Restriction 1 leads to the emergence of the price puzzle. Nevertheless, the response of output to the monetary tightening is negative and persistent. The response of the other variables is also similar to Figure 5.

The analysis confirms the robustness of our findings. This alternative agnostic identification scheme that restricts the systematic behavior of monetary policy is consistent with the consensus regarding the effects of monetary policy on output. Imposing additional sign restrictions on IRFs as motivated by Uhlig (2005) helps to refine the set of admissible models that are consistent with the systematic component of monetary policy, but it is not crucial for the results.

4.2 Money Rule

Finally, the last specification of the monetary policy equation that we consider follows the money rules postulated in Leeper et al. (1996); Leeper and Zha (2003); and Sims and Zha (2006a,b). In these rules, only the federal funds rate and money enter the monetary policy equation. To model this rule, we follow Sims and Zha (2006b) and replace total reserves and nonborrowed reserves with

\[
S = \begin{bmatrix}
0_{1,n} & 0 & -1 & 0 & 0 & 0 & 0 \\
\vdots & 0 & 0 & -1 & 0 & 0 & 0 \\
\vdots & 0 & 0 & 0 & 0 & -1 & 0 \\
0_{1,n} & 0 & 0 & 0 & 0 & 0 & 1
\end{bmatrix}.
\]
money, as measured by M2.\textsuperscript{13} Except for this use of money instead of reserves, the reduced-form model is identical to the one we describe in Section 2.

We first replicate the main findings in Uhlig (2005) using the new reduced-form specification in order to show that his results are not a consequence of using reserves instead of money. To implement Uhlig's (2005) agnostic identification scheme, we replace the sign restrictions on nonborrowed reserves with sign restrictions on money. We thus characterize the agnostic identification scheme by the following Restriction.

**Restriction 6.** A monetary policy shock leads to a negative response of the GDP deflator, commodity prices, and money, and to a positive response of the federal funds rate, all at horizons $t = 0, \ldots, 5$.

As was the case with Restriction 1, Restriction 6 rules out the price and the liquidity puzzles and implies non-linear restrictions on $(A_0, A_+)$. But the crucial feature of the identification described

\textsuperscript{13}We use monthly data on M2 Money supply (M2SL) from the H.6 Money supply Measures of the Board of Governors of the Federal Reserve System downloaded from the Federal Reserve Bank of Saint Louis.
by Restriction 6 is that it still remains agnostic about the response of output after an increase in the federal funds rate and only identifies monetary policy shocks, allowing a set of models to be compatible with the restrictions.

We omit the description of the function $f(A_0, A_+)$ and the selection matrix $S_1$ which are necessary to implement Restrictions 6 because they follow trivially from the ones described in Section 2. We plot the resulting IRFs in Figure 7. As in Uhlig (2005)'s specification with reserves instead of money, an increase in the federal funds rate leads to an increase in output. The output response becomes negative after about six months, but zero is always included in the 68% credible set. Therefore, there is no evidence that negative monetary policy shocks are contractionary when Restrictions 6 is used to identify them: Uhlig (2005)'s results survive the swap of reserves for M2.

Next, we specify the identification assumptions that are consistent with the money rule as follows.

**Restriction 7.** The federal funds rate is the monetary policy instrument and it only reacts con-
temporarily to money.

**Restriction 8.** The contemporaneous reaction of the federal funds rate to money is nonnegative.

Here again, we only restrict the behavior of the monetary policy equation while leaving the remaining equations unrestricted. Therefore, we only identify monetary policy shocks and we remain agnostic about the response of output after an increase in the federal funds rate. As in the previous exercises, we do not identify the structural parameters but only set-identify them.

We rewrite the monetary policy equation, concentrating on the contemporaneous coefficients, as

$$ r_t = \psi_y y_t + \psi_p p_t + \psi_{pc} p_{c,t} + \psi_m m_t + a_{-1,0} \varepsilon_{1,t}, $$

where $\psi_y = a_{0,1} a_{0,11}$, $\psi_p = a_{0,21} a_{0,21}$, $\psi_{pc} = a_{0,31} a_{0,31}$, and $\psi_m = a_{0,41} a_{0,41}$. Equipped with this representation of the monetary policy equation, we summarize Restrictions 7 and 8 as follows.

**Remark 3.** Restriction 7 implies that $\psi_y = \psi_p = \psi_{pc} = 0$, while Restriction 8 implies that $\psi_m \geq 0$.

Note also that under Restriction 7 the monetary equation (6) becomes

$$ r_t = \psi_m m_t + a_{0,1} \varepsilon_{1,t}. $$

This equation has three possible interpretations. The first, which is consistent with how we specify equation (7), is that the federal funds rate responds to changes in the money supply. The second interpretation is that the money supply adjusts to changes in the federal funds rate. This interpretation is consistent with Sims and Zha's (2006b) view on how monetary policy was conducted between 1979 and 1982. A third interpretation is simply that both the federal funds rate and the money supply respond to Fed actions, and that both indicators are important in describing the effects of monetary policy on the economy (Belongia and Ireland, 2014). But inference is consistent with all three different interpretations, which only imply different normalizations in Restriction 8.

In its current form, Restriction 8 states that shocks that raise the money supply lead the Federal Reserve to increase the federal funds rate. An alternative interpretation is that a monetary policy
A shock leads to a simultaneous increase in the federal funds rate and a reduction in the money supply.

These restrictions are implemented by defining the function \( f(A_0, A_+) \) and the matrices \( S_1 \) and \( Z_1 \) as described below:

\[
f(A_0, A_+) = \begin{bmatrix} A_0 \\ L_0(A_0, A_+) \\ \vdots \\ L_5(A_0, A_+) \end{bmatrix}, \quad S_1 = \begin{bmatrix} S_{10} & 0_{s10,n} & \cdots & 0_{s10,n} \\ 0_{s11,n} & S_{11} & 0_{s11,n} & \cdots \\ \vdots & \vdots & \ddots & \vdots \\ 0_{s15,n} & \cdots & S_{15} \end{bmatrix}, \quad Z_1 = \begin{bmatrix} Z_{10} \\ 0_{s10,5n} \end{bmatrix}.
\]
In Figure 8, we plot the IRFs to a tightening in monetary policy that is consistent with Restrictions 6, 7, and 8. Qualitatively, impulse responses are similar to those plotted in Figure 3. The response of output is more hump-shaped than in Figure 3, with output returning to its pre-shock level within five years. The response of the federal funds rate rate is also hump-shaped, with the stance tightening but still accommodative after five years. As for the Taylor rule specification, the output response is more precisely estimated than in the baseline, which is in line with the evidence that M2 helps in forecasting output in VARs that include the federal funds rate (Belongia and Ireland, 2014).

In Figure 9, we plot the IRFs identified by imposing only Restrictions 7 and 8. The response of output remains negative and becomes more persistent, as it is still below its pre-shock level after five years. But the path for the federal funds rate is consistent with a tighter stance of monetary policy than in Figure 8: the initial increase is about 20 basis points higher and it remains positive for around 18 months, staying at zero thereafter. The response of the GDP deflator shows a more pronounced price puzzle than in Figure 4.

Overall, the evidence presented in this section confirms the results in Section 3: output declines after a contractionary monetary policy shock in SVARs identified by imposing some discipline on the systematic component of monetary policy.

\[ S_{10} = \begin{bmatrix} 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ o_{s,1} & \ldots & \ldots & \ldots & o_{s,1} \end{bmatrix}, \quad S = \begin{bmatrix} 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix}, \]

\[ S_{1t} = S \text{ for } t = 1, \ldots, 5, \quad Z_{10} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}. \]
5 Conclusion

The agnostic identification of monetary policy shocks by imposing sign restrictions on IRFs as proposed by Uhlig (2005) finds that increases in the federal funds rate are not contractionary. We re-examine this issue and show that the identification scheme in Uhlig (2005) implies a counterfactual characterization of the systematic component of monetary policy. We design an agnostic identification scheme that imposes sign and zero restrictions on the systematic component of monetary policy and find that an increase in the federal funds rate leads to a persistent decline in output and prices.

Overall, our results suggest that while set identification is appealing because it does not require inference to be based on very specific, and often questionable, exclusion restrictions, it is subject to the danger of including implausible models. Our suggestion is to impose restrictions on objects that can be easily evaluated, which in our application is the systematic component of monetary policy. The issue of how to specify agnostic restrictions in SVARs is not limited to the identifica-
tion of monetary policy, and the approach described in this paper can be applied to a variety of identification problems.
References


