Title: From Statistical Learning to Game-Theoretic Learning

This is joint work with Karthik Sridharan (TTI-Chicago) and Ambuj Tewari (UT Austin)

Abstract:
Statistical Learning Theory studies the problem of estimating (learning) an unknown function given a class of hypotheses and an i.i.d. sample of data. Classical results show that combinatorial parameters (such as Vapnik-Chervonenkis and scale-sensitive dimensions) and complexity measures (such as covering numbers, Rademacher averages) govern learnability and rates of convergence.
Further, it is known that learnability is closely related to the uniform Law of Large Numbers for function classes.

In contrast to the i.i.d. case, in the online learning framework the learner is faced with a sequence of data appearing at discrete time intervals, where the data is chosen by the adversary. Unlike statistical learning, where the focus has been on complexity measures, the online learning research has been predominantly algorithm-based.
That is, an algorithm with a non-trivial guarantee provides a certificate of learnability.

We develop tools for analyzing learnability in the game-theoretic setting of online learning without necessarily providing a computationally feasible algorithm. We define complexity measures which capture the difficulty of learning in a sequential manner. Among these measures are analogues of Rademacher complexity, covering numbers and fat shattering dimension from statistical learning theory. These can be seen as temporal generalizations of classical results.
A further generalization beyond external regret covers an array of known frameworks, such as internal and Phi-regret, Blackwell's Approachability, global non-additive notions of cumulative loss, and more. In particular, we apply our tools to the problem of calibration of forecasters.