The Factor Structure in Equity Options*

Peter Christoffersen, University of Toronto, CBS and CREATES
Mathieu Fournier, University of Toronto
Kris Jacobs, University of Houston and Tilburg University

March 18, 2013

Abstract

Principal component analysis of equity options on Dow-Jones firms reveals a strong factor structure. The first principal component explains 77% of the variation in the equity volatility level, 77% of the variation in the equity option skew, and 60% of the implied volatility term structure across equities. Furthermore, the first principal component has a 92% correlation with S&P500 index option volatility, a 64% correlation with the index option skew, and a 80% correlation with the index option term structure. We develop an equity option valuation model that captures this factor structure. The model assumes a Heston (1993) style stochastic volatility model for the market return but additionally allows for stochastic idiosyncratic volatility for each firm. The model predicts that firms with higher betas have higher implied volatilities, consistent with the empirical findings in Duan and Wei (2009). It also predicts that firms with higher betas have steeper moneyness and term structure slopes. We provide a tractable approach for estimating the model on index and equity option data. The model provides a good fit to a large panel of options on stocks in the Dow-Jones index, and the data support the cross-sectional implications of the model.

JEL Classification: G10; G12; G13.

Keywords: Factor models; equity options; implied volatility; option-implied beta.

*We thank Yakov Amihud, Menachem Brenner, Redouane Elkamhi, Rob Engle, Bruno Feunou, Jean-Sebastien Fontaine, Jose Fajardo, Joel Hasbrouck, Matthew Richardson, Stijn Van Nieuwerburgh, Jason Wei, Alan White, Robert Whitelaw, and participants at NYU, the Mathematical Finance Days (IFM2), NFA, IFSID, OptionMetrics, Laval University, Ryerson University, University of Houston, and the SoFiE conference in Rio de Janeiro for helpful comments, and SSHRC for financial support. Correspondence to: Peter Christoffersen, Rotman School of Management, 105 St. George Street Toronto, Ontario, Canada M5S 3E6. E-mail: peter.christoffersen@rotman.utoronto.ca.

1 Introduction

While factor models are standard for understanding equity prices, classic equity option valuation models make no attempt at modeling a factor structure in the underlying equity prices. Typically, a stochastic process is assumed for each underlying equity price and the option is priced on this stochastic process, ignoring any links the underlying equity price may have with other equity prices through common factors. Seminal papers in this vein include Black and Scholes (1973), Wiggins (1987), Hull and White (1987), and Heston (1993).

When considering a single stock option, ignoring an underlying factor structure may be relatively harmless. However, in portfolio applications it is crucial to understand links between the underlying stocks. Risk managers need to understand the total exposure to the underlying risk factors in a portfolio of stocks and stock options. Equity portfolio managers who use equity options to hedge large downside moves in individual stocks need to know their overall market exposure. Dispersion traders who sell (expensive) index options and buy (cheaper) equity options to hedge need to understand the market exposure of individual equity options. See for example Driessen, Maenhout, and Vilkov (2009) for evidence on the market exposure of equity options.

Our empirical analysis of approximately four hundred thousand index quotes and more than two million equity option quotes reveals a very strong factor structure. We study three characteristics of option prices: short-term implied volatility (IV) levels, the slope of IV curves across option moneyness, and the slope of IV curves across option maturity.

First, we construct daily time series of short-term at-the-money implied volatility (IV) on the stocks in the Dow Jones Industrials Average and extract their principal components. The first common component explains 77% of the cross-sectional variation in IV levels and the common component has an 92% correlation with the short-term implied volatility constructed from S&P 500 index options. Short-term equity option IV appears to be characterized by a common factor.

Second, a principal component analysis of equity option IV moneyness, known as the option skew, reveals a significant common component as well. 77% of the variation in the skew across equities is captured by the first principal component. Furthermore, this common component has a correlation of 64% with the skew of market index options. Third, 60% of the variation in the term structure of equity IV is explained by the first principal component. This component has a correlation of 80% with the IV term slope from S&P 500 index options.

We use the findings from the principal component analysis as guidance to develop a structural model of equity option prices that incorporates a market factor structure. In line with well-known empirical facts in the literature on index options (see for example Bakshi, Cao and Chen, 1997; Heston and Nandi, 2000; Bates, 2000; and Jones, 2003), the model allows for mean-reverting stochastic volatility and correlated shocks to returns and volatility. Motivated by the principal component analysis, we allow for idiosyncratic shocks to equity returns which also have mean-reverting stochastic volatility and a separate leverage effect.

Individual equity returns are linked to the market index using a standard linear factor model with a constant factor loading. The model belongs to the affine class, which yields closed-form option pricing formulas. It can be extended to allow for market-wide and idiosyncratic jumps.[1] The model has three important cross-sectional implications. First, it predicts that firms with higher betas have higher implied volatilities, consistent with the empirical findings in Duan and Wei (2009). Second, it predicts that firms with higher betas have steeper moneyness slopes. Third, higher beta firms are expected to have a greater positive (negative) slope when the market variance term-structure is upward (downward) sloping.

We develop a convenient approach to estimating the model from option data. When estimating the model on the firms in the Dow-Jones index, we find that it provides a good fit to observed equity option prices, and the cross-sectional implications of the model are supported by the data. While it is not the main focus of this paper, our model provides option-implied estimates of market betas, which is a topic of recent interest, studied by for example Chang, Christoffersen, Jacobs, and Vainberg (2012), and Buss and Vilkov (2012). Multiple applications in asset pricing and corporate finance require estimates of beta, such as cost of capital estimation, performance evaluation, portfolio selection, and abnormal return measurement.

Our paper is also related to the recent empirical literature on equity options. Dennis and Mayhew (2002) investigate the relationship between firm characteristics and risk-neutral skewness. Bakshi and Kapadia (2003) investigate the volatility risk premium for equity options. Bakshi, Kapadia, and Madan (2003) derive a skew law for individual stocks, decomposing individual return skewness into a systematic and idiosyncratic component. They find that individual firms display much less (negative) option-implied skewness than the market index. Bakshi, Cao, and Zhong (2012) investigate the performance of jump models for equity option valuation. Engle and Figlewski (2012) develop time series models of implied volatilities and study their correlation dynamics. Perhaps most relevant for our work, Duan and Wei (2009) demonstrate empirically that systematic risk matters for the observed prices of equity options on the firm's stock.[2]

Our paper is also related to recent theoretical advances. Mo and Wu (2007) develop an international CAPM model which has features similar to our model. Elkamhi and Ornthanalai (2010) develop a bivariate discrete-time GARCH model to extract the market jump risk premia implicit in

[1] Pan (2002), Broadie, Chernov, and Johannes (2007), and Bates (2008) among others have documented the importance of modeling jumps in index options.

[2] See also Goyal and Saretto (2009), Vasquez (2011), and Jones and Wang (2012) for recent empirical work on equity option returns.

individual equity option prices. Finally, Serban, Lehoczky, and Seppi (2008) develop a non-affine model to investigate the relative pricing of index and equity options.

The reminder of the paper is organized as follows. In Section 2 we describe the data set and present the principal components analysis. In Section 3 we develop the theoretical model. Section 4 highlights a number of cross-sectional implications of the model. In Section 5 we estimate the model and investigate its fit to observed index and equity option prices. In Section 6 we explore the model's implications for expected option return and for portfolio risk management. Section 7 concludes.

2 Common Factors in Equity Option Prices

In this section we first introduce the data set used in our study. We then look for evidence of commonality in three crucial features of the cross-section of equity options: Implied volatility levels, moneyness slopes (or skews), and volatility term structures. We rely on a principal component analysis (PCA) of the firm-specific levels of short-term at-the-money implied volatility (IV), the slope of IV with respect to option moneyness, and the slope of IV with respect to option maturity. The results from this model-free investigation help identify desirable features of a structural model of equity option prices.

2.1 Data

We rely on end-of-day option data from OptionMetrics starting on January 2, 1996 and ending on October 29, 2010, which was the time span available at the time of writing. We use the S&P 500 index to proxy for the market factor. For our sample of individual equities, we choose the firms in the Dow Jones Industrial Average index at the end of the sample. Of the 30 firms in the index we excluded Kraft Foods for which data are not available throughout the sample. We filter out bid-ask option pairs with missing quotes or zero bids, and options that violate standard arbitrage restrictions. For each option maturity, interest rates are estimated by linear interpolation using zero coupon Treasury yields. Dividends are obtained from OptionMetrics and are assumed to be known during the life of each option.

Our study focuses on medium-term options, i.e. options with more than 20 days and less than 365 days to maturity (DTM). Following Bakshi, Cao and Chen (1997), we use mid-quotes (average bid-ask spread) in all computations, and eliminate options with moneyness (S/K) less than 0.9 and greater than 1.1. We also filter out quotes smaller than \$3/8, with implied volatility greater than 150%, and for which the present value of dividends is larger than 4% of the stock price.

The S&P 500 index options are European, but the individual equity options are American style, and their prices are influenced by early exercise premiums. To avoid biases due to the presence of early exercise premia and dividends, we do not include in the sample in-the-money (ITM) options for which the early exercise premium matters most.[3]

Table 1 presents the number of option contracts, the number of calls and puts, the average days-to-maturity, and the average implied volatility. The S&P 500 index has by far the greatest number of option contracts. We have a total of 393,429 index option quotes and 2,370,951 equity option quotes across the 29 firms. The average implied volatility for the market is 20.51% during the sample period. Cisco has the highest average implied volatility (40.78%) while Johnson & Johnson has the lowest average implied volatility (22.90%). Table 1 also shows that the data set is balanced with respect to the number of OTM calls and puts.

Table 2 reports the average, minimum, and maximum implied volatility, as well as the average option vega. Note that apart from Cisco the average implied volatilities of OTM puts are always higher than the average implied volatilities of OTM calls.

Figure 1 plots the daily average short-term (20 < DTM < 60) at-the-money ($0.95 < S/K < 1.05$) implied volatility (IV) for six firms (black lines) as well as for the S&P 500 index (grey lines). Figure 1 shows that the variation in the short-term at-the-money (ATM) equity volatility for each firm is highly related to S&P 500 volatility.

2.2 Methodology

We want to assess the extent to which the time-varying volatilities of equities share one or more common components. In order to gauge the degree of commonality in risk-neutral volatilities, we need daily estimates of the level and slope of the implied volatility curve, and slope of the term structure of implied volatility for all firms and the index. For each day t we run the following regression for firm j

$$IV_{j,l,t} = a_{j,t} + b_{j,t} \cdot \left(S_t^j / K_{j,l}\right) + c_{j,t} \cdot \left(DTM_{j,l}\right) + \epsilon_{j,l,t} \tag{2.1}$$

where l denotes the contracts available for firm j on day t. The regressors are standardized each day by subtracting the mean and dividing by the standard deviation. We run the same regression on the index option IVs. We interpret $a_{j,t}$ as a measure of the level of implied volatilities of firm j on day t. Similarly, $b_{j,t}$ captures the slope of implied volatility curve while $c_{j,t}$ proxies for the slope of the term structure of implied volatility.

[3]Table 2 in Bakshi, Kapadia, and Madan (2003) shows that for OTM calls and puts, the early excercise premia are negligible. Elkamhi and Ornthanalai (2010) get a similar result. See also Duan and Wei (2009).

Once the regression coefficients have been estimated on each day and for each firm, we run a PCA analysis on each of the matrices $\{a_{j,t}\}$, $\{b_{j,t}\}$, and $\{c_{j,t}\}$. Tables 3-5 contain the results from the PCA analysis and Figures 2-4 plot the first principal component as well as the corresponding index option coefficients, $a_{I,t}$, $b_{I,t}$, and $c_{I,t}$.

2.2.1 Common Factors in the Level of Implied Equity Volatility

Table 3 contains the results for implied volatility levels. We report the loading of each equity IV on the first three components. At the bottom of the table we show the average, minimum, and maximum loading across firms for each component. We also report the total variation captured as well as the correlation of each component with S&P 500 IV. The results in Table 3 are quite striking. The first component captures 77% of the total cross-sectional variation in short-term IV and it has a 92% correlation with the S&P 500 index IV. This suggests that the equity IVs have a very strong common component highly correlated with index option IVs. Note that the loadings on the first component are positive for all 29 firms, illustrating the pervasive nature of the common factor.

The top panel of Figure 2 shows the time series of short-term IV for index options. The bottom panel plots the time series of the first PCA component of equity IV. The strong relationship between the two series is readily apparent.

The second PCA component in Table 3 explains 13% of the total variation and the third component explains 3%. The average loadings on these two components are close to zero and the loadings take on a wide range of positive and negative values. The sizeable second PCA component and the wide range of the loadings suggest the need for a second, firm-specific, source of variation in equity volatility.

2.2.2 Common Factors in the Moneyness Slope

Table 4 contains the results for IV moneyness slopes. The moneyness slopes contain a significant degree of co-movement. The first principal component explains 77% of cross-sectional variation in the moneyness slope. The second and third components explain 6% and 4% respectively. The first component has positive loadings on all 29 firms where as the second and third components have positive and negative loadings across firms, and average loadings very close to zero.

Table 4 also shows that the first principal component has a 64% correlation with the moneyness slope of S&P 500 implied volatility. Equity moneyness slope dynamics clearly seem driven to a non-trivial extent by the market moneyness slope.

Figure 3 plots the S&P 500 index IV moneyness slope in the top panel and the first principal component from the equity moneyness slopes in the bottom panel. The relationship between the first principal component and the market moneyness slope is readily apparent, but not as strong as for the volatility level in Figure 2.

2.2.3 Common Factors in the Term Structure Slope

Table 5 contains the results for IV term structure slopes. The variation in the term structure slope captured by the first principal component is 60%, which is lower than for spot volatility (Table 3) and the moneyness slope (Table 4). The loadings on the first component are positive for all 29 firms. The correlation between the first component and the term slope of S&P 500 index option IV is 80%, which is higher than for the moneyness slope in Table 4 but lower than for the variance level in Table 3. The second and third components capture 14% and 5% of the variation respectively and the wide range of loadings on this factor suggest a scope for firm-specific variation in the IV term structure for equity options.

Figure 4 plots the S&P 500 index IV term structure slope in the top panel and the first principal component from the equity term slopes in the bottom panel. Most of the spikes in the S&P 500 term structure slope are clearly evident in the first principal component as well. Comparing Figures 2 and 4, we see that the term structure slope is close to zero when volatility is low and strongly negative when volatility is high.

We conclude that the market volatility term structure captures a substantial share of the variation in equity volatility term structures.

2.3 Other Stylized Facts in the Cross-Section of Equity Option Prices

The fast-growing literature on equity options has documented a number of important cross-sectional stylized facts. Bakshi, Kapadia, and Madan (2003) derive a skew law for individual stocks, decomposing individual return skewness into a systematic and idiosyncratic component. They theoretically investigate and empirically document the relationship between risk-neutral market and equity skewness, which affects the relationship between the moneyness slope for equity and index options. They find that the volatility smile for the market index is on average more negatively sloped than volatility smiles for individual firms. They also show that the more negatively skewed the risk-neutral distribution, the steeper the volatility smile. Finally, they find that the risk-neutral equity distributions are on average less skewed to the left than index distributions.

Other studies document cross-sectional relationships between betas, estimated using historical data, and characteristics of the equity IVs. Dennis and Mayhew (2002) find that option-implied skewness tends to be more negative for stocks with larger betas. Duan and Wei (2009) find that the level of implied equity volatility is related to the systematic risk of the firm and that the slope of the implied volatility curve is related to systematic risk as well. Finally, Driessen, Maenhout and Vilkov (2009) find a large negative index variance risk premium, but find no evidence of a negative risk premium on individual variance risk.

These findings are at first blush not directly related to the findings of the PCA analysis above, which merely documents a strong factor structure of various aspects of implied equity volatilities. We next outline a structural equity option modeling approach with a factor structure that captures the results from the PCA analysis outlined above, but is also able to match the cross-sectional relationships between betas and implied volatilities documented by these studies.

3 Equity Option Valuation Using a Single-Factor Structure

We model an equity market consisting of n firms driven by a single market factor, I_t. The individual stock prices are denoted by S_t^j, for $j = 1, 2, ..., n$. Investors also have access to a risk-free bond which pays a return of r.

The market factor evolves according to the process

$$\frac{dI_t}{I_t} = (r + \mu_I)dt + \sigma_{I,t} dW_t^{(I,1)} \tag{3.1}$$

where μ_I is the instantaneous market risk premium and where volatility is stochastic and follows the standard square root process

$$d\sigma_{I,t}^2 = \kappa_I(\theta_I - \sigma_{I,t}^2)dt + \delta_I \sigma_{I,t} dW_t^{(I,2)} \tag{3.2}$$

As in Heston (1993), θ_I denotes the long-run variance, κ_I captures the speed of mean reversion of $\sigma_{I,t}^2$ to θ_I, and δ_I measures volatility of volatility. The innovations to the market factor return and volatility are correlated with coefficient ρ_I. Conventional estimates of ρ_I are negative and large capturing the so-called leverage effect in aggregate market returns.

Individual equity prices are driven by the market factor as well as an idiosyncratic term which also has stochastic volatility

$$\begin{aligned} \frac{dS_t^j}{S_t^j} - rdt &= \alpha_j dt + \beta_j \left(\frac{dI_t}{I_t} - rdt \right) + \sigma_{j,t} dW_t^{(j,1)} && (3.3) \\ d\sigma_{j,t}^2 &= \kappa_j(\theta_j - \sigma_{j,t}^2)dt + \delta_j \sigma_{j,t} dW_t^{(j,2)} && (3.4) \end{aligned}$$

where α_j denotes the excess return and β_j is the market beta of firm j.

The innovations to idiosyncratic returns and volatility are correlated with coefficient ρ_j. As suggested by the skew laws derived in Bakshi, Kapadia, and Madan (2003), asymmetry of the idiosyncratic return component is required to explain the differences in the price structure of individual equity and index options.

Note that this model of the equity market has a total of $2(n+1)$ innovations.

3.1 The Risk Neutral Distribution

In order to use our model of the equity market to value derivatives we need to postulate a change of measure from the physical (P-measure) distribution developed above to the risk-neutral (Q-measure) distribution. Following the literature, we assume a change-of-measure of the exponential form

$$\frac{dQ}{dP}(t) = \exp\left(-\int_0^t \gamma_u dW_u - \frac{1}{2}\int_0^t \gamma_u' d\left\langle W, W'\right\rangle_u \gamma_u\right) \tag{3.5}$$

where $W_u \equiv \left[W_u^{(1,1)}, W_u^{(1,2)}, .., W_u^{(I,1)}, W_u^{(I,2)}\right]'$ is a $2(n+1)\times 1$ vector containing the innovations, $\gamma_u \equiv \left[\gamma_u^{(1,1)}, \gamma_u^{(1,2)}, .., \gamma_u^{(I,1)}, \gamma_u^{(I,2)}\right]'$ is the vector of market prices of risk, and $d\langle .,.\rangle$ is the covariance operator in our framework.

In the spirit of Cox, Ingersoll, and Ross (1985) and Heston (1993) among others, we assume a price of market variance risk of the form $\lambda_I \sigma_{I,t}$. We also assume that idiosyncratic variance risk is not priced. These assumptions yield the following result.

Proposition 1 *Given the change-of-measure in (3.5) the process governing the market factor under the Q-measure is given by*

$$\begin{aligned} \frac{dI_t}{I_t} &= rdt + \sigma_{I,t} d\tilde{W}_t^{(I,1)} && (3.6) \\ d\sigma_{I,t}^2 &= \tilde{\kappa}_I\left(\tilde{\theta}_I - \sigma_{I,t}^2\right)dt + \delta_I \sigma_{I,t} d\tilde{W}_t^{(I,2)} && (3.7) \\ \textit{with } \tilde{\kappa}_I &= \kappa_I + \delta_I \lambda_I, \textit{ and } \tilde{\theta}_I = \frac{\kappa_I \theta_I}{\tilde{\kappa}_I} && (3.8) \end{aligned}$$

and the processes governing the individual equities under the Q-measure are given by

$$\begin{aligned} \frac{dS_t^j}{S_t^j} &= rdt + \beta_j\left(\frac{dI_t}{I_t} - rdt\right) + \sigma_{j,t} d\tilde{W}_t^{(j,1)} && (3.9) \\ d\sigma_{j,t}^2 &= \kappa_j\left(\theta_j - \sigma_{j,t}^2\right)dt + \delta_j \sigma_{j,t} d\tilde{W}_t^{(j,2)} && (3.10) \end{aligned}$$

where $d\tilde{W}_t$ denotes the risk-neutral counterpart of dW_t for which

$$d\tilde{W}_t = dW_t + d\left\langle W, W' \right\rangle_t \gamma_t \tag{3.11}$$

where

$$\gamma_t^{(I,1)} = \frac{\mu_I - \rho_I \lambda_I \sigma_{I,t}^2}{\sigma_{I,t}(1-\rho_I^2)} \text{ and } \gamma_t^{(I,2)} = \frac{\lambda_I \sigma_{I,t}^2 - \rho_I \mu_I}{\sigma_{I,t}(1-\rho_I^2)}$$

$$\gamma_t^{(j,1)} = \frac{\alpha_j}{\sigma_{j,t}(1-\rho_j^2)} \text{ and } \gamma_t^{(j,2)} = -\frac{\rho_j \alpha_j}{\sigma_{j,t}(1-\rho_j^2)}$$

This proposition provides several insights. Note that the market factor structure is preserved under Q. Consequently, the market beta is the same under the risk-neutral and physical distributions. This is consistent with Serban, Lehoczky, and Seppi (2008), who document that the risk-neutral and objective betas are economically and statistically close for most stocks. Note that this result makes betas estimated from option data appropriate for applications of the CAPM such as capital budgeting.

It is also important to note that in our modeling framework, higher moments and their premiums, as defined by the difference between the moment under Q and P, are affected by the drift adjustment in the variance processes. We will discuss this further below.

3.2 Closed-Form Option Valuation

The model has been cast in an affine framework, which implies that the characteristic function for the logarithm of the index level and the logarithm of the equity price can both be derived analytically. The characteristic function for the index is identical to that in Heston (1993). Consider now individual equity options. We need the following proposition:

Proposition 2 *The risk-neutral conditional characteristic function $\tilde{\phi}^j(\tau, u)$ for the equity price, S_T^j, is given by*

$$\begin{aligned} \tilde{\phi}^j(\tau, u) &\equiv E_t^Q\left[\exp\left(iu\ln\left(S_T^j\right)\right)\right] \\ &= \left(S_t^j\right)^{iu}\exp\left(iur\tau - \left(A(\tau,u) + B(\tau,u)\right) - C(\tau,u)\sigma_{I,t}^2 - D(\tau,u)\sigma_{j,t}^2\right) \end{aligned} \tag{3.12}$$

where $\tau = T - t$ and the expressions for $A(\tau, u)$, $B(\tau, u)$, $C(\tau, u)$, and $D(\tau, u)$ are provided in Appendix A.

Proof. See Appendix A. ■

Given the characteristic function for the log spot price under Q, the price of a European equity call option with strike price K and maturity $\tau = T - t$ is

$$C_t^j(S_t^j, K, \tau) = S_t^j \Pi_1^j - K e^{-r\tau} \Pi_2^j \tag{3.13}$$

where the risk-neutral probabilities Π_1^j and Π_2^j are defined by

$$\Pi_1^j = \frac{1}{2} + \frac{e^{-r\tau}}{\pi S_t^j} \int_0^\infty \mathrm{Re}\left[\frac{e^{-iu \ln K} \tilde{\phi}^j(\tau, u - i)}{iu}\right] du \tag{3.14}$$

$$\Pi_2^j = \frac{1}{2} + \frac{1}{\pi} \int_0^\infty \mathrm{Re}\left[\frac{e^{-iu \ln K} \tilde{\phi}^j(\tau, u)}{iu}\right] du \tag{3.15}$$

While these integrals must be evaluated numerically, they are well-behaved and can be estimated at relatively low computational cost.

4 Model Predictions

In this section we derive a number of important cross-sectional implications from the model and assess if it captures the stylized facts documented in Section 2.[4]

4.1 The Level of Equity Option Volatility

Duan and Wei (2009) show empirically that firms with higher systematic risk have a higher level of risk-neutral variance. We now investigate if our model is consistent with this empirical finding.

First, define total spot variance for firm j at time t

$$V_{j,t} \equiv \beta_j^2 \sigma_{I,t}^2 + \sigma_{j,t}^2$$

and define the expectations under P and Q of the corresponding integrated variance by

$$E_t^P[V_{j,t:T}] \equiv E_t^P\left[\int_t^T V_{j,s} ds\right] \quad \text{and} \quad E_t^Q[V_{j,t:T}] \equiv E_t^Q\left[\int_t^T V_{j,s} ds\right]$$

By splitting up the P-expectation between the integrated market variance and the idiosyncratic

[4]For convenience we assume that beta is positive for all firms in the cross-section. This is not required by the model but it simplifies the interpretation of certain expressions.

one, we have

$$E_t^P[V_{j,t:T}] = \beta_j^2 E_t^P[\sigma_{I,t:T}^2] + E_t^P[\sigma_{j,t:T}^2]$$

where $\sigma_{I,t:T}^2$, and $\sigma_{j,t:T}^2$ correspond to the integrated variances from t to T.

Given our model, the expectation of the integrated total variance for equity j under Q is

$$E_t^Q[V_{j,t:T}] = \beta_j^2 E_t^Q[\sigma_{I,t:T}^2] + E_t^Q[\sigma_{j,t:T}^2] = \beta_j^2 E_t^Q[\sigma_{I,t:T}^2] + E_t^P[\sigma_{j,t:T}^2]$$

Note that the second equation holds as long as idiosyncratic risk is not priced (i.e. $E_t^P[\sigma_{j,t:T}^2] = E_t^Q[\sigma_{j,t:T}^2]$).

For any two firms having the same level of expected total variance under the P-measure ($E_t^P[V_{1,t:T}] = E_t^P[V_{2,t:T}]$) we have

$$E_t^P[\sigma_{1,t:T}^2] - E_t^P[\sigma_{2,t:T}^2] = -(\beta_1^2 - \beta_2^2) E_t^P[\sigma_{I,t:T}^2]$$

Therefore

$$\begin{aligned} E_t^Q[V_{1,t:T}] - E_t^Q[V_{2,t:T}] &= (\beta_1^2 - \beta_2^2) E_t^Q[\sigma_{I,t:T}^2] + \left(E_t^Q[\sigma_{1,t:T}^2] - E_t^Q[\sigma_{2,t:T}^2]\right) \\ &= (\beta_1^2 - \beta_2^2) E_t^Q[\sigma_{I,t:T}^2] + \left(E_t^P[\sigma_{1,t:T}^2] - E_t^P[\sigma_{2,t:T}^2]\right) \\ &= (\beta_1^2 - \beta_2^2)\left(E_t^Q[\sigma_{I,t:T}^2] - E_t^P[\sigma_{I,t:T}^2]\right) \end{aligned}$$

When the market variance premium is negative, we have $\tilde{\theta}_I > \theta_I$ which implies that $E_t^Q[\sigma_{I,t:T}^2] > E_t^P[\sigma_{I,t:T}^2]$. We therefore have that

$$\beta_1 > \beta_2 \Leftrightarrow E_t^Q[V_{1,t:T}] > E_t^Q[V_{2,t:T}]$$

We conclude that our model is consistent with the finding in Duan and Wei (2009) that firms with high betas tend to have a high level of risk-neutral variance.

4.2 Equity Option Skews

To understand the slope of equity option implied volatility moneyness curves, we need to understand how beta influences the skewness of the risk-neutral equity return distribution. The next proposition is key to comprehend how beta, systematic risk, and index skewness impact equity skewness.

Proposition 3 *The conditional total skewness of the integrated returns of firm j under P, denoted* *by TSk_j^P, is given by*

$$TSk^P_{j,t:T} \equiv Sk^P\left(\int_t^T \frac{dS^j_u}{S^j_u}\right) = Sk^P_I \cdot \left(A^P_{j,t:T}\right)^{3/2} + Sk^P_j \cdot \left(1 - A^P_{j,t:T}\right)^{3/2} \tag{4.1}$$

The conditional total skewness of the integrated returns of firm j under Q, denoted by TSk_j^Q, is given by

$$TSk^Q_{j,t:T} \equiv Sk^Q\left(\int_t^T \frac{dS^j_u}{S^j_u}\right) = Sk^Q_I \cdot \left(A^Q_{j,t:T}\right)^{3/2} + Sk^Q_j \cdot \left(1 - A^Q_{j,t:T}\right)^{3/2} \tag{4.2}$$

where

$$A^P_{j,t:T} \equiv \frac{E^P_t[\beta_j^2 \sigma^2_{I,t:T}]}{E^P_t[V_{j,t:T}]} \quad \text{and} \quad A^Q_{j,t:T} \equiv \frac{E^Q_t[\beta_j^2 \sigma^2_{I,t:T}]}{E^Q_t[V_{j,t:T}]}$$

are the proportion of systematic risk of firm j under P and Q, and $Sk_I = Sk\left(\int_t^T \frac{dI_s}{I_s}\right)$ and $Sk_j = Sk\left(\int_t^T \sigma_{j,s} dW_s^{(j,1)}\right)$ are the market and idiosyncratic skewness.

Proof. See Appendix B. ■

This result shows that β_j matters to determine firm j's conditional total skewness. Equation (4.2) indicates that under the risk neutral measure, β_j affects the slope of the equity implied volatility curve through $TSk^Q_{j,t:T}$ by influencing the level of systematic risk proportion $A^Q_{j,t:T}$. A higher $A^Q_{j,t:T}$ mechanically implies a higher loading on the market risk-neutral skewness Sk^Q_I. Consider two firms with the same expected total variance under Q and $\beta_1 > \beta_2$, which implies $A^Q_{1,t:T} > A^Q_{2,t:T}$. As a result, firm 1 has a greater loading on index risk-neutral skewness than firm 2. In an economy where the index Q-distribution is more negatively skewed than the idiosyncratic equity distribution,[5] we have the following cross-sectional prediction: higher-beta firms will have more negatively skewed Q-distributions. Note that this prediction is in line with the cross-sectional empirical findings of Duan and Wei (2009) and Dennis and Mayhew (2002).

Figure 5 plots the implied Black-Scholes volatility from model option prices. Each line has a different beta but the same amount of unconditional total equity variance $V_j = \beta_j^2 \theta_I + \theta_j = 0.1$. We set the current spot variance to $\sigma^2_{I,t} = 0.01$ and $V_{j,t} = 0.05$, and define the idiosyncratic variance as the residual $\sigma^2_{j,t} = V_{j,t} - \beta_j^2 \sigma^2_{I,t}$. The market index parameters are $\kappa_I = 5,\ \theta_I = 0.04,\ \delta_I = 0.5,\ \rho_I = -0.8$, and the individual equity parameters are $\kappa_j = 1,\ \delta_j = 0.4$, and $\rho_j = 0$. The risk-free rate is 4% per year and option maturity is 3 months. Figure 5 shows that beta has a substantial impact on the moneyness slope of equity IV even when keeping the total variance constant: The higher the beta, the larger the moneyness slope.

[5] This statement is in line with the empirical findings of Bakshi, Kapadia, and Madan (2003).

The model also has cross-sectional implications for the skewness premium. It implies that the skewness premium of the individual equity S_t^j takes the following form when $Sk_j^P = Sk_j^Q = 0$

$$TSk_{j,t:T}^{Q} - TSk_{j,t:T}^{P} = \left(A_{j,t:T}^{P}\right)^{3/2}\left[Sk_I^Q \cdot \left(\frac{A_{j,t:T}^{Q}}{A_{j,t:T}^{P}}\right)^{3/2} - Sk_I^P\right] \tag{4.3}$$

Recall that when the market variance risk premium is negative, we have $A_{j,t:T}^{Q} > A_{j,t:T}^{P}$. Combining this with a negative market skewness premium (i.e. $Sk_I^Q < Sk_I^P$) implies that the expression in square brackets is negative. Given that beta increases the proportion of systematic risk, $A_{j,t:T}$, controlling for total physical variance, high-beta firms should have lower skewness premiums.

Finally, consider the general implications of the model for the pricing of options under the risk-neutral measure. The model implies

$$A_{j,t:T}^{Q} > A_{j,t:T}^{P} \Leftrightarrow E_t^Q[\sigma_{I,t:T}^2] > E_t^P[\sigma_{I,t:T}^2] \tag{4.4}$$

Systematic risk is thus of relatively greater importance under the Q measure than under the P measure. This suggests that systematic risk and factor structure are likely to be of even greater importance under the risk-neutral measure to price options than under the physical measure to explain historical returns. Systematic risk may therefore be helpful in explaining the co-movements in the implied volatility moneyness slopes for equity options documented in Section 2.

4.3 The Term Structure of Equity Volatility

We next investigate the model's implication for the term structure of equity volatility. The role of the market beta turns out to be crucial once again.

Our model implies the following two-component term-structure of equity variance

$$E_t^Q[V_{j,t:T}] = \left(\beta_j^2\tilde{\theta}_I + \theta_j\right) + \beta_j^2\left(\sigma_{I,t}^2 - \tilde{\theta}_I\right)e^{-\tilde{\kappa}_I(T-t)} + \left(\sigma_{j,t}^2 - \theta_j\right)e^{-\kappa_j(T-t)} \tag{4.5}$$

This expression shows how the term structure of market variance affects the term structure of variance for firm j. Given different systematic and idiosyncratic mean reverting speeds ($\tilde{\kappa}_I \neq \kappa_j$), β_j has important implications for the term-structure of volatilities. In the empirical work below, we find that the idiosyncratic variance process is more persistent than the market variance. When the idiosyncratic variance process is more persistent ($\tilde{\kappa}_I > \kappa_j$), higher values of beta imply a faster reversion toward the unconditional total variance ($\tilde{V}_j = \beta_j^2\tilde{\theta}_I + \theta_j$). As a result, when the market variance process is less persistent than the idiosyncratic variance, in the cross-section, firms with higher betas are likely to have steeper volatility term-structures. In other words, higher beta firms are expected to have a greater positive (negative) slope when the market variance term-structure is upward (downward) sloping.

Figure 6 plots the implied Black-Scholes volatility from model prices against option maturity. Each line has a different beta but the same amount of unconditional total equity variance $V_j = \beta_j^2\theta_I + \theta_j = 0.1$. We set the current spot variance to $\sigma_{I,t}^2 = 0.01$ and $V_{j,t} = 0.05$, and define the idiosyncratic variance as the residual $\sigma_{j,t}^2 = V_{j,t} - \beta_j^2\sigma_{I,t}^2$. The parameter values are as in Figure 5. Figure 6 shows that beta has a non-trivial effect on the IV term structure: The higher the beta, the steeper the term structure when the term structure is upward sloping.

In summary, our model suggests that–ceteris paribus–firms with higher betas should have higher levels of volatility, steeper moneyness slopes, and higher absolute maturity slopes. We now estimate the model in order to assess if these patterns are indeed observed in the option data.

5 Estimation and Fit

In this section, we first describe our estimation methodology. Subsequently we report on parameter estimates and model fit. Finally we relate the estimated betas to patterns in observed equity option IVs.

5.1 Estimation Methodology

Several approaches have been proposed in the literature for estimating stochastic volatility models. Jacquier, Polson, and Rossi (1994) use Markov Chain Monte Carlo to estimate a discrete-time stochastic volatility model. Pan (2002) uses GMM to estimate the objective and risk neutral parameters from returns and option prices. Serban, Lehoczky, and Seppi's (2008) estimation strategy is based on simulated maximum likelihood using the EM algorithm and the particle filter.

Another approach treats the latent volatility states as parameters to be estimated and thus avoids filtering the latent volatility factor. This strategy has been adopted by Bates (2000) and Santa-Clara and Yan (2010) among others. We follow this strand of literature.

Recall that we need to estimate two vectors of latent variables $\{\sigma_{I,t}^2,\ \sigma_{j,t}^2\}$ and two sets of structural parameters $\{\Theta_I,\ \Theta_j\}$, where $\Theta_I \equiv \{\tilde{\kappa}_I, \tilde{\theta}_I, \delta_I, \rho_I\}$ and $\Theta_j \equiv \{\kappa_j, \theta_j, \delta_j, \rho_j, \beta_j\}$. Our methodology involves two main steps.

In the first step, we estimate the market index dynamic $\left\{\Theta_I, \sigma_{I,t}^2\right\}$ based on S&P 500 option prices alone. In the second step, we use equity options for firm j only, we take the market index dynamic as given, and we estimate the firm-specific dynamics $\left\{\Theta_j, \sigma_{j,t}^2\right\}$ for each firm conditional on estimates of $\left\{\Theta_I, \sigma_{I,t}^2\right\}$. This step-wise estimation procedure is not fully econometrically efficient but it enables us to estimate our model for 29 equities while ensuring that the same dynamic is imposed for the market-wide index for each of the 29 firms. We have confirmed that this estimating technique has good finite sample properties in a Monte Carlo study which is available from the authors upon request.

Each of the two steps contains an iterative procedure which we now describe in detail.

Step 1: Parameter Estimation for the Index

Given a set of starting values, Θ_I^0, for the structural parameters characterizing the index, we first estimate the spot market variance each day by solving

$$\hat{\sigma}_{I,t}^2 = \arg\min_{\sigma_{I,t}^2} \sum_{m=1}^{N_{I,t}} (C_{I,t,m} - C_m(\Theta_I^0, \sigma_{I,t}^2))^2 / Vega_{I,t,m}^2, \text{ for } t = 1, 2, ...T \tag{5.1}$$

where $C_{I,t,m}$ is the market price of index option contract m quoted at t, $C_m(\Theta_I, \sigma_{I,t}^2)$ is the model index option price, $N_{I,t}$ is the number of index contracts available on day t, and $Vega_{I,t,m}$ is the Black-Scholes sensitivity of the option price with respect to volatility evaluated at the implied volatility. These vega-weighted dollar price errors are a good approximation to implied volatility errors and the computational cost involved is much lower.[6]

Once the set of T market spot variances is obtained, we solve for the set of parameters characterizing the index dynamic as follows

$$\hat{\Theta}_I = \arg\min_{\Theta_I} \sum_{m,t}^{N_I} (C_{I,t,m} - C_m(\Theta_I, \hat{\sigma}_{I,t}^2))^2 / Vega_{I,t,m}^2, \tag{5.2}$$

where $N_I \equiv \sum_t^T N_{I,t}$ represents the total number of index option contracts available.

We iterate between (5.1) and (5.2) until the improvement in fit is negligible, which typically requires 5-10 iterations.

[6] This approximation has been used in Carr and Wu (2007) and Trolle and Schwartz (2009) among others.

Step 2: Parameter Estimation for Firm Equity

Given an initial value Θ_j^0 and the estimated $\hat{\sigma}_{I,t}^2$ and $\hat{\Theta}_I$ we can estimate the spot equity variance each day by solving

$$\hat{\sigma}_{j,t}^2 = \arg\min_{\sigma_{j,t}^2} \sum_{m=1}^{N_{j,t}} (C_{j,t,m} - C_m(\Theta_j^0, \hat{\Theta}_I, \hat{\sigma}_{I,t}^2, \sigma_{j,t}^2))^2 / Vega_{j,t,m}^2, \text{ for } t = 1, 2, ...T \tag{5.3}$$

where $C_{j,t,m}$ is the price of equity option m for firm j quoted at t, $C_m(\Theta_j, \Theta_I, \sigma_{I,t}^2, \sigma_{j,t}^2)$ is the model equity option price, $N_{j,t}$ is the number of equity contracts available on day t, and $Vega_{j,t,m}$ is the Black-Scholes Vega of the equity option.

Once the set of T market spot variances is obtained, we solve for the set of parameters characterizing the equity dynamic as follows

$$\hat{\Theta}_j = \arg\min_{\Theta_j} \sum_{m,t}^{N_j} (C_{j,t,m} - C_m(\Theta_j, \hat{\Theta}_I, \hat{\sigma}_{I,t}^2, \hat{\sigma}_{j,t}^2)) / Vega_{j,t,m}^2 \tag{5.4}$$

where $N_j \equiv \sum_t^T N_{j,t}$ is the total number of contracts available for security j.

We again iterate between (5.3) and (5.4) until the improvement in fit is negligible.

5.2 Parameter Estimates

This section presents estimation results for the market index and the 29 firms for the 1996-2010 period. For the equity options we use contracts on each trading day. For index options we estimate the structural parameters in (5.2) on Wednesday data only because the computational burden is exorbitantly large if all trading days are used.

Table 6 reports estimates of the structural parameters that characterize the dynamics of the systematic variance and the idiosyncratic variance, as well as estimates of the betas. The top row shows estimates for the S&P 500 index.

The unconditional risk-neutral market index variance $\tilde{\theta}_I = 0.0542$ corresponds to 23% volatility per year. Based on the average index spot variance path for the sample, $\frac{1}{T}\sum_{t=1}^T \sigma_{I,t}^2$, we obtain a volatility of 21.74%. The difference between these two numbers provides a rough estimate of the volatility risk premium. The idiosyncratic θ_j estimates range from 0.0093 for General Electric to 0.0887 for Cisco.

The estimate of the mean-reversion parameter for the market index variance $\tilde{\kappa}_I$ is equal to 1.24, which corresponds to a daily variance persistence of $1 - 1.24/365 = 0.9966$. The idiosyncratic κ_j range from 0.53 for Chevron to 1.53 for 3M, indicating that idiosyncratic volatility is highly persistent as well. Only three firms in the sample (3M, Hewlett-Packard and IBM) have an idiosyncratic variance process that is less persistent than the market variance.

The estimate of ρ_I is strongly negative (-0.860), capturing the so-called leverage effect in the index. The idiosyncratic ρ_j are also strongly negative, ranging from -0.978 for Microsoft to -0.482 for Disney. The equity option data clearly require the idiosyncratic volatility component to provide additional skewness.

The estimates of beta are reasonable and vary from 0.70 for Johnson & Johnson to 1.30 for JP Morgan. The average beta across the 29 firms is 0.99.

The average total spot volatility (ATSV) for firm j is computed as

$$\text{ATSV} = \sqrt{\frac{1}{T}\sum_{t=1}^{T} V_{j,t}} = \sqrt{\frac{1}{T}\sum_{t=1}^{T} \left(\beta_j^2 \sigma_{I,t}^2 + \sigma_{j,t}^2\right)}$$

Comparing the beta column with the ATSV column in Table 6 shows that ATSV is generally high when beta is high.

The final column of Table 6 reports the systematic risk ratio (SSR) for each firm. It is computed from the spot variances as follows

$$\text{SSR} = \frac{\sum_{t=1}^{T} \beta_j^2 \sigma_{I,t}^2}{\sum_{t=1}^{T} \left(\beta_j^2 \sigma_{I,t}^2 + \sigma_{j,t}^2\right)}$$

Table 6 shows that the systematic risk ratio varies from close to 30% for Cisco and Hewlett-Packard to above 66% for Chevron. The systematic risk ratio is 45% on average, indicating that the estimated factor structure is strongly present in the equity option data. Comparison of the beta column with the SSR column in Table 6 shows that firms with similar betas can have radically different SSR and, vice versa, firms with very different betas can have roughly similar SSRs. This finding of course suggests a key role for the idiosyncratic variance dynamic in the model.

5.3 Model Fit

We measure model fit using the root mean squared error (RMSE) based on the vegas, which is consistent with the criterion function used in estimation

$$\text{Vega RMSE} \equiv \sqrt{\frac{1}{N}\sum\nolimits_{m,t}^{N} (C_{m,t} - C_{m,t}(\Theta))^2 / Vega_{m,t}^2}$$

We also report the implied volatility RMSE defined as

$$\text{IVRMSE} \equiv \sqrt{\frac{1}{N}\sum\nolimits_{m,t}^{N}(IV_{m,t} - IV(C_{m,t}(\Theta)))^2}$$

where $IV_{m,t}$ denotes market IV for option m on day t and $IV(C_{m,t}(\Theta))$ denotes model IV. We use Black-Scholes to compute IV for both model and market prices.

Table 7 reports model fit for the market index and for each of the 29 firms. We report results for all contracts, as well as separate results for out-of-the-money (OTM) calls and puts, and for short and long term at-the-money (ATM) contracts. We also report the IVRMSE divided by the average market IV in order to assess relative IV fit. Several interesting findings emerge from Table 7.

- First, the Vega RMSE approximates the IVMRSE closely for the index and for all firms. This suggests that using Vega RMSE in estimation does not bias the IVRMSE results.

- Second, the average IVRMSE across firms is 1.78% and the relative IV (IVRMSE / Average IV) is 6% on average. The fit does not vary much across firms. Overall the fit of the model is thus quite good across firms. The best pricing performance for equity options is obtained for Coca Cola with an IVRMSE of 1.41%. The worst fit is for Bank of America with an IVRMSE of 2.40%. Based on the relative IVRMSE, the best fit is for Intel at 4.14% and the worst is for AT&T at 8.15%.

- Third, the average IVRMSE fit across firms for OTM calls is 1.74% and for OTM puts it is 1.82%. Using this metric the model fits OTM calls and puts roughly equally well.

- Fourth, the average IVRMSE fit across firms for short-term ATM options is 1.67% and for long-term ATM options it is 1.59%. The model thus fits short-term and long-term ATM options equally well on average.

Figure 7 reports the average fit over time for different moneyness categories for each firm. Moneyness is on the horizontal axis, measured by S/K, so that OTM calls (and ITM puts) are shown on the left side and ITM calls (and OTM puts) are shown on the right side.

Figure 7.A reports on the first fifteen firms and Figure 7.B reports on the last 14 firms as well as the index. Note that in order to properly see the different patterns across firms, the vertical axis scale differs in each subplot, but the range of implied volatility values is kept fixed at 10% across firms to facilitate comparisons.

Figure 7 shows that the smiles computed using market prices (solid black lines) vary considerably across firms, both in terms of level and shape. Consider for example Cisco in Figure 7.A (third row, second column) which has a relatively steep smirk and high levels of IV, versus Bank of America (top row, third column) which has a more symmetric smile and lower levels of IV. The model (dashed grey lines) fits the different IV moneyness shapes remarkably well. The IV errors by moneyness are small in general and no dramatic outliers are apparent. If anything, the model tends to underprice the extreme OTM calls (ITM puts) on the left side of the graphs. The bias is small, however, relative to the overall IV level. This bias could be caused by an incomplete adjustment for the early exercise premium, which affects mostly ITM puts. The bottom right panel in Figure 7.B confirms the finding in Bakshi, Kapadia and Madan (2003) that the market index is much more (negatively) skewed than individual firms. The bottom right panel also shows that the model requires additional negative skewness to fit the relatively expensive OTM puts trading on the market index. This can be achieved by including jumps in returns (Bates, 2000). Interestingly, while the Heston (1993) model is unable to adequately capture OTM index put option IV levels, our model is able to fit OTM equity put options quite well. Nevertheless, expanding the index model we use by modeling jumps is a worthy topic for future study.

Figure 8 reports for each firm the average (over time) implied volatility as a function of time to maturity (in years). We split the data set into two groups: Days with upward-sloping IV term structure (grey lines) and days with downward sloping IV term structure (black lines). We report the average market IVs (solid lines) as well as the average model IVs (dashed lines). The downward sloping black lines use the left-hand axis and the upward sloping grey lines use the right-hand axis. In order to facilitate comparison between model and market IVs the level of IVs differ between the left and right axis and they differ across firms. In order to facilitate comparison between term structures the difference between the minimum and maximum on each axis is fixed at 10% across all firms.

Figure 8 shows that the term structure of IV differs considerably across firms. Some firms such as Alcoa and American Express have rather flat downward sloping term structures, whereas other firms such as General Electric and Hewlett-Packard have much steeper term structures. Generally, across firms, the downward sloping black lines appear to be steeper than the upward sloping grey lines. This pattern is matched well by the model. Figure 8 does not reveal any systematic model biases in the term structure of IVs.

We conclude from Table 7 and Figures 7 and 8 that the model fits the observed equity option data quite well. Encouraged by this finding, we next analyze in some detail how our estimated betas are related to observed patterns in equity option IVs.

5.4 Equity Betas and Equity Option IVs

The three main cross-sectional predictions of our model, as discussed in Section 4, are as follows:

1. Firms with higher betas have higher risk-neutral variance.

2. Firms with higher betas have steeper moneyness slopes. This is equivalent to stating that firms with higher betas are characterized by more negative skewness.

3. Firms with higher betas have steeper positive volatility term structures when the term structure is upward sloping, and steeper negative volatility term structures when the term structure is downward sloping.

We now document if these theoretical model implications are supported by the estimates for the 29 Dow-Jones firms. Consider first the level of option-implied volatility. In the top panel of Figure 9, we scatter plot the time-averaged intercepts from the implied volatility regression in (2.1), $\frac{1}{T}\sum_{t=1}^{T} a_{j,t}$ against the beta estimate from Table 6 for each firm j. We then run a regression on the 29 points in the scatter and assess the significance and fit. The slope has a t-statistic of 4.29 and the regression fit (R^2) is quite high at 50%. The regression line shows the positive relationship between the estimated betas and the average implied volatility observed in the market prices of equity options.

In the middle panel of Figure 9, we scatter plot the moneyness slope coefficients from the IV regression in (2.1), $\frac{1}{T}\sum_{t=1}^{T} b_{j,t}$ against the beta estimate from Table 6 for each firm j. In the moneyness slope regression, the sensitivity to beta has a t-statistic of 4.12 and an R^2 of 25%. Clearly the moneyness scatter is noisy and has several outliers, including Alcoa with a beta higher than one but also the lowest moneyness slope in the sample. Nevertheless, Panel B shows that higher beta estimates are associated with steeper slopes of the IV moneyness smile. The two firms with the highest betas (JP Morgan and American Express) have very high moneyness slopes, suggesting that the relationship may be nonlinear.

Finally, in the bottom panel of Figure 9 we scatter plot the absolute value of the term structure slope coefficients from (2.1), $\frac{1}{T}\sum_{t=1}^{T} c_{j,t}$ against the beta estimate from Table 6 for each firm. In the term slope regression, the sensitivity to beta has a t-statistic of 4.05 and the R^2 is quite high at 40%. Panel C shows that higher betas are associated with higher absolute slopes of the term structure in equity IVs: Firms with high betas will tend to have a term structure of implied volatility curve that decays more quickly to the unconditional level of volatility compared with firms with low betas.

We conclude that the estimates of beta are related to the model-free measures of IV level, slope, and term structure in a way that is consistent with the three main model predictions from Section 4.

6 Discussion

In this section we further explore the empirical results, and we also discuss some additional implications of the model. First we compare the option-implied betas with historical betas. Then we discuss the cross-sectional structure of the idiosyncratic volatilities, and we relate our findings to the existing literature. We also the model's implications for equity option risk management by computing the most important option price sensitivity measures, and we derive the expected return on options implied by the model.

6.1 Option-Implied and Historical Betas

As discussed in section 5.2, the estimated betas seem reasonable. They vary from 0.70 for Johnson & Johnson to 1.30 for JP Morgan, and the average beta across the 29 firms is 0.99. To provide additional perspective we also compute historical betas for the same 29 firms. To be consistent with the option-based estimate, we estimate a constant beta using daily data for the entire sample. The historical beta is 0.97 on average across firms.

Figure 10 provides a scatter plot of the option-implied betas versus the historical betas. It also shows the results of a regression of the historical on the option-implied betas. A number of important conclusions obtain. First, the option-implied betas are positively correlated with the historical betas. In fact, the relation between the two beta estimates is very strong, which is evidenced by the high R-square of the regression (78%) and the fact that Figure 10 contains very few outliers. Second, option-implied betas have a smaller dispersion than historical betas. This is interesting in light of the well-known statistical biases in estimating historical betas, and the common practice of shrinking the betas toward one to account for this bias. Note that this larger dispersion of the historical betas also yields a regression slope larger than one and a negative regression intercept.

We conclude that overall the relationship between historical and option-implied beta is surprisingly strong. It may prove interesting to see if this relationship also holds for betas computed over shorter windows, but this exercise is computationally demanding for option-implied betas.

6.2 The Cross-Section of Idiosyncratic Risk

A number of recent studies investigate co-movements between firm-level volatilities. Engle and Figlewski (2012) model the dynamics of correlations between implied volatilities, and investigate the role of VIX as a factor in explaining firm-level implied volatilities. Kelly, Lustig, and Van Nieuwerburgh (2012) show that there is a strong factor structure in firm-level historical volatility, distinct from the common variation in returns. Surprisingly, they find that idiosyncratic volatility contains a factor structure that is similar to total volatility.

Motivated by these findings, Table 8 presents the correlation matrix between the idiosyncratic variances for the 29 firms estimated from the model. Clearly Table 8 confirms the results of Kelly, Lustig, and Van Nieuwerburgh (2012), which are obtained using historical returns data. While Table 8 may be interpreted as suggesting the need for a richer factor model, note that the results of Kelly, Lustig, and Van Nieuwerburgh (2012) are robust to the inclusion of additional factors.

6.3 Equity Option Risk Management

In classic equity option valuation models, partial derivatives are used to assess the sensitivity of the option price to the underlying stock price (delta) and equity volatility (vega). In our model the equity option price additionally is exposed to changes in the market level and market variance. Portfolio managers with diversified equity option holdings need to know the sensitivity of the equity option price to these market level variables in order to properly manage risk. The following proposition provides the model's implications for the sensitivity to the market level and market volatility.

Proposition 4 *For a derivative contract f^j written on the stock price, S_t^j, the sensitivity of f^j with respect to the index level, I_t (the market delta) is given by:*

$$\frac{\partial f^j}{\partial I_t} = \frac{\partial f^j}{\partial S_t^j} \frac{S_t^j}{I_t} \beta_j$$

The sensitivity of f^j with respect to the market variance (the market vega) is given by:

$$\frac{\partial f^j}{\partial \sigma_{I,t}^2} = \frac{\partial f^j}{\partial V_{j,t}} \beta_j^2$$

Proof. *See Appendix C.* ■

This proposition shows that the beta of the firm in a straightforward way provides the link between the usual stock price delta $\frac{\partial f^j}{\partial S_t}$ and the market delta, $\frac{\partial f^j}{\partial I_t}$, as well as the link between the usual equity vega, $\frac{\partial f^j}{\partial V_{j,t}}$, and the market vega $\frac{\partial f^j}{\partial \sigma_{I,t}^2}$.

This result allows market participants with portfolios of equity options on different firms to measure and manage their total exposure to the index level and to the market volatility. It also allows investors engaged in dispersion trading, who sell index options and buy equity options, to measure and manage their overall exposure to market risk and market volatility risk.

6.4 Expected Returns on Equity Options

So far we have focused on option prices. In applications such as the management of option portfolios, option returns are of interest as well. The following proposition provides an expression for the expected (P-measure) equity option return as a function of the expected market return.[7]

Proposition 5 *For a derivative f^j written on the stock price, S_t^j, the expected excess return on the derivative contract is given by:*

$$\frac{1}{dt}E_t^P\left[\frac{df^j}{f^j}-rdt\right]=\frac{\partial f^j}{\partial S_t^j}\frac{S_t^j}{f^j}\left(\alpha_j+\beta_j\mu_I\right)=\frac{\partial f^j}{\partial S_t^j}\frac{S_t^j}{f^j}\alpha_j+\frac{\partial f^j}{\partial I_t}\frac{I_t}{f^j}\mu_I$$

where $\frac{\partial f^j}{\partial I_t}$ is given by Proposition 4.

Proof. *See Appendix D.* ■

This proposition indicates that the beta of the stock provides a simple link between the expected return on the index and the expected return on the equity option via the delta of the option. The model thus decomposes the excess return on the option in two parts: The delta of the equity option and the beta of the stock. Put differently, equity options provide investors which two sources of leverage: First, the beta with respect to the market, and second, the elasticity of the option price with respect to changes in the stock price.

7 Summary and Conclusions

Principal Component Analysis reveals a strong factor structure in equity options. The first common component explains 77% of the cross-sectional variation in IV and the common component has a 92% correlation with the short-term implied volatility constructed from S&P 500 index options. Furthermore, 77% of the variation in the equity skew is captured by the first principal component. This common component has a correlation of 64% with the skew of market index options. Also, 60% of the variation in the term structure of equity IV is explained by the first principal component. This component has a correlation of 80% with the term slope of the option IV from S&P500 index options.

Motivated by the findings from the principal component analysis, we develop a structural model of equity option prices that incorporates a market factor. The model allows for mean-reverting

[7] Recent empirical work on equity and index option returns includes Broadie, Chernov, and Johannes (2009), Goyal and Saretto (2009), Constantinides, Czerwonko, Jackwerth, and Perrakis (2011), Vasquez (2011), and Jones and Wang (2012).

stochastic volatility and correlated shocks to returns and volatility. Motivated by the principal components analysis, we allow for idiosyncratic shocks to equity prices which also have mean-reverting stochastic volatility and a separate leverage effect. Individual equity returns are linked to the market index using a standard linear factor model with a constant beta factor loading. We derive closed-form option pricing formulas as well as results for option hedging and option expected returns.

We develop a convenient estimation method for estimation and filtering based on option prices. When estimating the model on the firms in the Dow-Jones index, we find that it provides a good fit to observed equity option prices. Moreover, we show that the estimates strongly confirm the three main cross-sectional model implications.

Several issues are left for future research. First, it would be interesting to empirically study the implications of our models for option returns. Second, it may be useful to extend the model, for instance by allowing for two stochastic volatility factors in the market price process, as in Bates (2000), or by allowing for jumps in the market price (Bollerslev and Todorov, 2011). Third, combining option information with high-frequency returns (Patton and Verardo, 2012; Hansen, Lunde, and Voev, 2012) may lead to better estimates of betas. Fourth, characterizing the time-variation in option-implied betas would be of significant interest. Finally, using the option-implied betas to study cross-sectional stock returns (Conrad, Dittmar and Ghysels, 2013) is a promising avenue for research.

Appendix

This appendix collects proofs of the propositions.

A. Proof of Proposition 2

For ease of notation, we define $\tilde{W}^1_{\sigma_{k,t:T}} \equiv \int_t^T \sigma_{k,u} d\tilde{W}^{(k,1)}_u$ for $k \in \{I, j\}$. Given the Q -processes, one can apply Ito's lemma to $dln(S^j_t)$ and obtain after integration the following expression for individual equity log-returns

$$\ln\left(\frac{S^j_T}{S^j_t}\right) = r\tau - \frac{1}{2}\left(\sigma^2_{j,t:T} + \beta^2_j \sigma^2_{I,t:T}\right) + \tilde{W}^1_{\sigma_{j,t:T}} + \beta_j \tilde{W}^1_{\sigma_{I,t:T}} \tag{7.1}$$

where $\tau = T - t$. Therefore, the conditional characteristic function of the risk-neutral log-returns takes the form

$$\tilde{\phi}^{LR}(\tau, u) = E_t^Q \left[\exp\left(iu \left(r\tau - \frac{1}{2}\left(\sigma^2_{j,t:T} + \beta_j^2 \sigma^2_{I,t:T}\right) + \tilde{W}^1_{\sigma_{j,t:T}} + \beta_j \tilde{W}^1_{\sigma_{I,t:T}} \right)\right)\right] \tag{7.2}$$

Making use of the definition of the stochastic exponential $\xi(\cdot)$

$$\xi\left(\eta \tilde{W}^1_{\sigma_{k,t:T}}\right) = \exp\left(\eta \tilde{W}^1_{\sigma_{k,t:T}} - \frac{(\eta)^2}{2}\left\langle \tilde{W}^1_{\sigma_k}, \tilde{W}^1_{\sigma_k}\right\rangle_{t,T}\right) = \exp\left(\eta \tilde{W}^1_{\sigma_{k,t:T}} - \frac{1}{2}\eta^2 \sigma^2_{k,t:T}\right) \tag{7.3}$$

where η can be real or complex (see Protter (1990) p85), we can write (7.2) as

$$\tilde{\phi}^{LR}(\tau, u) = \exp(iur\tau) E_t^Q \left[\xi\left(iu\beta_i \tilde{W}^1_{\sigma_{I,t:T}}\right) \xi\left(iu \tilde{W}^1_{\sigma_{j,t:T}}\right) \exp\left(-\left(g_1 \sigma^2_{I,t:T} + g_2 \sigma^2_{j,t:T}\right)\right)\right] \tag{7.4}$$

where $g_1 = \frac{iu}{2}\beta_j^2(1 - iu)$ and $g_2 = \frac{iu}{2}(1 - iu)$. Following Carr and Wu (2004) or Detemple and Rindisbacher (2010), we define the following change of measure

$$\frac{dC}{dQ}(t) \equiv \xi\left(iu\beta_j \tilde{W}^1_{\sigma_{I,0:t}}\right) \xi\left(iu \tilde{W}^1_{\sigma_{j,0:t}}\right) \tag{7.5}$$

Combining (7.4) with the change of measure (7.5) allows us to write

$$\tilde{\phi}^{LR}(\tau, u) = \exp(iur\tau) E_t^Q \left[\frac{\frac{dC}{dQ}(T)}{\frac{dC}{dQ}(t)} \exp\left(-\left(g_1 \sigma^2_{I,t:T} + g_2 \sigma^2_{j,t:T}\right)\right)\right]$$

$$\Rightarrow \tilde{\phi}^{LR}(\tau, u) = \exp(iur\tau) E_t^C \left[\exp(-g_1 \sigma^2_{I,t:T})\right] E_t^C \left[\exp\left(-g_2 \sigma^2_{j,t:T}\right)\right] \tag{7.6}$$

Given an extension of the Girsanov theorem to the complex plane, under the C-measure we have

$$dW_t^{C,(I,2)} = d\tilde{W}_t^{(I,2)} - (iu\rho_I \beta_j \sigma_{I,t})dt$$

$$dW_t^{C,(j,2)} = d\tilde{W}_t^{(j,2)} - (iu\rho_j \sigma_{j,t})dt$$

As a result,

$$d\sigma^2_{k,t} = \kappa_k^C(\theta_k^C - \sigma^2_{k,t})dt + \delta_k \sigma_{k,t} dW_t^{C,(k,2)} \tag{7.7}$$

where

$$\kappa_I^C = \tilde{\kappa}_I - iu\rho_I \beta_j \delta_I, \quad \theta_I^C = \frac{\tilde{\kappa}_I \tilde{\theta}_I}{\kappa_I^C}, \; \kappa_j^C = \kappa_j - iu\rho_j \delta_j, \text{ and } \theta_j^C = \frac{\kappa_j \theta_j}{\kappa_j^C}$$

We make use of the closed-form for the moment generating function of $E_t^C[\exp(-g\sigma_{t:T}^2)]$ to obtain an expression for $\tilde{\phi}^{LR}(\cdot)$:

$$\tilde{\phi}^{LR}(\tau,u) = \exp\left(iur\tau - (A(\tau,u)+B(\tau,u)) - C(\tau,u)\sigma_{I,t}^2 - D(\tau,u)\sigma_{j,t}^2\right) \tag{7.8}$$

with

$$A(\tau,u) = \frac{\tilde{\kappa}_I\tilde{\theta}_I}{\delta_I^2}\left\{2\ln(1-\frac{(\Psi_1-\kappa_I^C)}{2\Psi_1}\left(1-e^{-\Psi_1\tau}\right) + \left(\Psi_1-\kappa_I^C\right)\tau\right\} \tag{7.9}$$

$$B(\tau,u) = \frac{\kappa_j\theta_j}{\delta_j^2}\left\{2\ln\left(1-\frac{(\Psi_2-\kappa_j^C)}{2\Psi_2}\left(1-e^{-\Psi_2\tau}\right)\right) + \left(\Psi_2-\kappa_j^C\right)\tau\right\} \tag{7.10}$$

$$C(\tau,u) = \frac{2g_1(1-e^{-\Psi_1\tau})}{2\Psi_1-(\Psi_1-\kappa_I^C)(1-e^{-\Psi_1\tau})} \tag{7.11}$$

$$D(\tau,u) = \frac{2g_2(1-e^{-\Psi_2\tau})}{2\Psi_2-\left(\Psi_2-\kappa_j^C\right)(1-e^{-\Psi_2\tau})} \tag{7.12}$$

and where

$$\Psi_1 = \sqrt{(\kappa_I^C)^2+2\delta_I^2 g_1} \text{ and } \Psi_2 = \sqrt{(\kappa_j^C)^2+2\delta_j^2 g_2}$$

with

$$g_1 = \frac{iu}{2}\beta_j^2(1-iu) \text{ and } g_2 = \frac{iu}{2}(1-iu)$$

and

$$\kappa_I^C = \tilde{\kappa}_I - iu\rho_I\beta_j\delta_I \text{ and } \kappa_j^C = \kappa_j - iu\rho_j\delta_j$$

Using the fact that $\tilde{\phi}^j(\tau,u) = e^{iu\ln(S_t^j)}\tilde{\phi}^{LR}(\tau,u)$, the previous equations can be used to compute the price of a call written on S^j.

B. Proof of Proposition 3

The following argument is derived under the P measure; however, a similar argument can be developed under the risk-neutral measure. Given the definition of skewness, the total (conditional) skewness of the integrated returns of firm "j" is

$$Sk^P\left(\int_t^T\frac{dS_u^j}{S_u^j}\right) = E_t^P\left[\left(\int_t^T\frac{dS_u^j}{S_u^j} - E_t^P\left[\int_t^T\frac{dS_u^j}{S_u^j}\right]\right)^3\right] \Big/ \left(E_t^P\left[\left(\int_t^T\frac{dS_u^j}{S_u^j} - E_t^P\left[\int_t^T\frac{dS_u^j}{S_u^j}\right]\right)^2\right]\right)^{3/2} \tag{7.13}$$

Given that

$$\int_t^T\frac{dS_u^j}{S_u^j} - E_t^P\left[\int_t^T\frac{dS_u^j}{S_u^j}\right] = \beta_j W_{\sigma_{I,t:T}}^1 + W_{\sigma_{j,t:T}}^1$$

equation (7.13) can be simplified to

$$Sk^P\left(\int_t^T \frac{dS_u^j}{S_u^j}\right) = \frac{E_t^P\left[\left(\beta_j W^1_{\sigma_{I,t:T}} + W^1_{\sigma_{j,t:T}}\right)^3\right]}{\left(E_t^P\left[\left(\beta_j W^1_{\sigma_{I,t:T}} + W^1_{\sigma_{j,t:T}}\right)^2\right]\right)^{3/2}}$$

By the properties of the Ito's integrals and the independence of $W^{(I,1)}$ and $W^{(j,1)}$, we have

$$E_t^P\left[\left(\beta_j W^1_{\sigma_{I,t:T}} + W^1_{\sigma_{j,t:T}}\right)^2\right] = E_t^P\left[\beta_j^2\sigma_{I,t:T}^2\right] + E_t^P\left[\sigma_{j,t:T}^2\right] = E_t^P[V_{j,t:T}]$$

and

$$E_t^P\left[\left(\beta_j W^1_{\sigma_{I,t:T}} + W^1_{\sigma_{j,t:T}}\right)^3\right] = E_t^P\left[\left(\beta_j W^1_{\sigma_{I,t:T}}\right)^3\right] + E_t^P\left[\left(W^1_{\sigma_{j,t:T}}\right)^3\right]$$

Consequently, the total (conditional) skewness of the integrated returns of firm "j" takes the form

$$\begin{aligned}
Sk^P\left(\int_t^T \frac{dS_u^j}{S_u^j}\right) &= \frac{E_t^P\left[\left(\beta_j W^1_{\sigma_{I,t:T}}\right)^3\right]}{\left(E_t^P[V_{j,t:T}]\right)^{3/2}} + \frac{E_t^P\left[\left(W^1_{\sigma_{j,t:T}}\right)^3\right]}{\left(E_t^P[V_{j,t:T}]\right)^{3/2}} \\
&= \frac{E_t^P\left[\left(W^1_{\sigma_{I,t:T}}\right)^3\right]}{\left(E_t^P[\sigma_{I,t:T}^2]\right)^{3/2}} \cdot \left(\frac{E_t^P[\beta_j^2\sigma_{I,t:T}^2]}{E_t^P[V_{j,t:T}]}\right)^{3/2} \cdot sign(\beta_j) \\
&\quad + \frac{E_t^P\left[\left(W^1_{\sigma_{j,t:T}}\right)^3\right]}{\left(E_t^P\left[\sigma_{j,t:T}^2\right]\right)^{3/2}} \cdot \left(\frac{E_t^P\left[\sigma_{j,t:T}^2\right]}{E_t^P[V_{j,t:T}]}\right)^{3/2}
\end{aligned}$$

Defining $A_{j,t:T}^P \equiv E_t^P[\beta_j^2\sigma_{I,t:T}^2]/E_t^P[V_{j,t:T}]$ and given the definition of skewness, we obtain for positive beta firms

$$Sk^P\left(\int_t^T \frac{dS_u^j}{S_u^j}\right) = Sk_I^P \cdot \left(A_{j,t:T}^P\right)^{3/2} + Sk_j^P \cdot \left(1 - A_{j,t:T}^P\right)^{3/2} \tag{7.14}$$

where

$$Sk_I^P = Sk^P\left(\int_t^T \frac{dI_s}{I_s}\right) \quad \text{and} \quad Sk_j^P = Sk^P\left(\int_t^T \sigma_{j,s} dW_s^{(j,1)}\right)$$

are the market and idiosyncratic skewness.

C. Proof of Proposition 4

Within our model, the index price (I_t) takes the form under the risk-neutral probability

$$I_t = I_0 \exp(rt - \frac{\sigma^2_{I,0:t}}{2} + \tilde{W}^1_{\sigma_{I,0:t}})$$

We know from Malliavin calculus that I_t is a differentiable function with respect to $\tilde{W}^1_{\sigma_{I,0:t}}$ (see Di Nunno, Øksendal, Proske, 2008). Taking the derivative of the index price I_t with respect to $\beta_j \tilde{W}^1_{\sigma_{I,0:t}}$ gives

$$\frac{\partial I_t}{\partial \beta_j \tilde{W}^1_{\sigma_{I,0:t}}} = \frac{\partial I_t}{\partial \tilde{W}^1_{\sigma_{I,0:t}}} \frac{\partial \tilde{W}^1_{\sigma_{I,0:t}}}{\partial \beta_j \tilde{W}^1_{\sigma_{I,0:t}}} = \frac{\partial I_t}{\partial \tilde{W}^1_{\sigma_{I,0:t}}} \left(\frac{\partial \beta_j \tilde{W}^1_{\sigma_{I,0:t}}}{\partial \tilde{W}^1_{\sigma_{I,0:t}}} \right)^{-1} = \frac{I_t}{\beta_j}$$

where the second equality makes use of the inverse function theorem which holds as long as $\beta_j \neq 0$. Moreover, as long as $I_t \neq 0$ the inverse function theorem also implies that

$$\frac{\partial \beta_j \tilde{W}^1_{\sigma_{I,0:t}}}{\partial I_t} = \left(\frac{\partial I_t}{\partial \beta_j \tilde{W}^1_{\sigma_{I,0:t}}} \right)^{-1} = \frac{\beta_j}{I_t} \tag{7.15}$$

Furthermore, within our model the equity price is given by

$$S^j_t = S^j_0 \exp(rt - \frac{1}{2} \left(\sigma^2_{j,0:t} + \beta^2_j \sigma^2_{I,0:t} \right) + \tilde{W}^1_{\sigma_{j,0:t}} + \beta_j \tilde{W}^1_{\sigma_{I,0:t}})$$

Therefore,

$$\frac{\partial S^j_t}{\partial \beta_j \tilde{W}^1_{\sigma_{I,0:t}}} = S^j_t \tag{7.16}$$

Combining (7.15) and (7.16) implies

$$\frac{\partial S^j_t}{\partial I_t} = \frac{\partial S^j_t}{\partial \beta_j \tilde{W}^1_{\sigma_{I,0:t}}} \frac{\partial \beta_j \tilde{W}^1_{\sigma_{I,0:t}}}{\partial I_t} = \frac{S^j_t}{I_t} \beta_j$$

Therefore, for any derivative f^j written on S^j the sensitivity of f^j with respect to market value, I_t (market delta) is

$$\frac{\partial f^j}{\partial I_t} = \frac{\partial f^j}{\partial S^j_t} \frac{\partial S^j_t}{\partial I_t} = \frac{\partial f^j}{\partial S^j_t} \frac{S^j_t}{I_t} \beta_j$$

For the sensitivity of f^j with respect to market variance (market vega), we have

$$\frac{\partial f^j}{\partial \sigma_{I,t}^2} = \frac{\partial f^j}{\partial V_{j,t}} \frac{\partial V_{j,t}}{\partial \sigma_{I,t}^2} = \frac{\partial f^j}{\partial V_{j,t}} \frac{\partial(\beta_j^2 \sigma_{I,t}^2 + \sigma_{j,t}^2)}{\partial \sigma_{I,t}^2} = \frac{\partial f^j}{\partial V_{j,t}} \beta_j^2$$

D. Proof of Proposition 5

The proof of this proposition is adapted from Broadie, Chernov, and Johannes (2009) to our set-up. By application of Ito's lemma to f^j, a derivative written on S^j, combined with the pricing PDE allows us to write the dynamic of df^j under P as

$$\begin{aligned} df^j &= \left\{ rf^j - f_S^j r S_t^j - f_{V_j}^j \beta_j^2 \tilde{\kappa}_I (\tilde{\theta}_I - \sigma_{I,t}^2) + \tilde{\kappa}_j (\tilde{\theta}_j - \sigma_{j,t}^2) \right\} dt \\ &\quad + f_S^j dS_t^j + f_{V_j}^j dV_{j,t} \end{aligned}$$

$$\begin{aligned} \Leftrightarrow \quad df^j = &\left\{ rf^j - f_S^j r S_t^j - f_{V_j}^j \beta_j^2 \tilde{\kappa}_I (\tilde{\theta}_I - \sigma_{I,t}^2) + \kappa_j (\theta_j - \sigma_{j,t}^2) \right\} dt \\ &+ f_S^j dS_t^j + f_{V_j}^j dV_{j,t} \end{aligned} \tag{7.17}$$

where f_x^j stands for the partial derivative of f^j with respect to x. Note that in the previous equation, we have assumed that the idiosyncratic risk is not priced consistent with Proposition 1 (i.e. $\tilde{\theta}_j = \theta_j$ and $\tilde{\kappa}_j = \kappa_j$). Moreover,

$$\begin{aligned} \tfrac{E_t^P[dS_t^j]}{dt} &= (r + \alpha_j + \beta_j \mu_I) S_t^j \\ \tfrac{E_t^P[dV_{j,t}]}{dt} &= \beta_j^2 \kappa_I \left(\theta_I - \sigma_{I,t}^2 \right) + \kappa_j (\theta_j - \sigma_{j,t}^2) \end{aligned} \tag{7.18}$$

Consequently, (7.17) and (7.18) leads to

$$\begin{aligned} \frac{1}{dt} E_t^P \left[\frac{df^j}{f^j} - rdt \right] &= \frac{f_S^j}{f^j} E_t^P \left[dS_t^j - rS_t^j dt \right] + \frac{f_{V_j}^j}{f^j} \beta_j^2 E_t^P \left[d\sigma_{I,t}^2 - \tilde{\kappa}_I (\tilde{\theta}_I - \sigma_{I,t}^2) dt \right] \\ &\quad + \frac{f_{V_j}^j}{f^j} E_t^P \left[d\sigma_{j,t}^2 - \kappa_j (\theta_j - \sigma_{j,t}^2) dt \right] \end{aligned}$$

which simplifies to

$$\frac{1}{dt} E_t^P \left[\frac{df^j}{f^j} - rdt \right] = f_S^j \frac{S_t^j}{f^j} \left(\alpha_j + \beta_j \mu_I \right) + f_{V_j}^j \frac{\beta_j^2}{f^j} (\tilde{\kappa}_I \tilde{\theta}_I - \kappa_I \theta_I) \tag{7.19}$$

In the previous expression, Q is the risk-neutral distribution defined by 3.5 for which $\tilde{\kappa}_I\tilde{\theta}_I = \kappa_I\theta_I$ hold. Consequently, we obtain

$$\frac{1}{dt}E_t^P\left[\frac{df^j}{f^j} - rdt\right] = f_S^j\frac{S_t^j}{f^j}\left(\alpha_j + \beta_j\mu_I\right) = \frac{\partial f^j}{\partial S_t^j}\frac{S_t^j}{f^j}\alpha_j + \frac{\partial f^j}{\partial I_t}\frac{I_t}{f^j}\mu_I$$

where the second equation uses of the result in Proposition 4.

References

[1] Bakshi, G., Cao, C., Z. Chen, Z. (1997) Empirical performance of alternative option pricing models, Journal of Finance, 52, 2003–2050.

[2] Bakshi, G., C. Cao, and Zhong, Z. (2012) Assessing models of individual equity option prices, Working Paper, University of Maryland

[3] Bakshi, G., Kapadia, N. (2003) Delta-hedged gains and the negative market volatility risk premium, Review of Financial Studies, 16, 527–566.

[4] Bakshi, G., Kapadia, N., Madan, D. (2003) Stock return characteristics, skew laws, and differential pricing of individual equity options, Review of Financial Studies, 10, 101–143.

[5] Bates, D. (2000) Post-'87 crash fears in the S&P500 futures option market, Journal of Econometrics, 94, 181-238.

[6] Bates, D. (2008) The market for crash risk, Journal of Economic Dynamics and Control, 32, 2291-2321.

[7] Black, F., Scholes, M. (1973) Valuation of options and corporate liabilities, Journal of Political Economy, 81, 637–654.

[8] Bollerslev, T., Todorov, B. (2011), Tails, fears, and risk premia, Journal of Finance, 66, 2165-2211.

[9] Broadie, M., Chernov, M., Johannes, M. (2007) Model specification and risk premiums: evidence from futures options, Journal of Finance, 62, 1453-1490.

[10] Broadie, M. Chernov, M., Johannes, M. (2009) Understanding index option returns, Review of Financial Studies, 22, 4493-4529.

[11] Buss, A., Vilkov, G., (2012) Measuring equity risk with option-implied correlations, Manuscript, Goethe University, Frankfurt.

[12] Carr, P., Wu, L. (2004) Time-changed Lévy processes and option pricing, Journal of Financial Economics, 71, 113–141.

[13] Carr, P., Wu, L. (2007) Stochastic skew in currency options, Journal of Financial Economics, 86, 213–247.

[14] Carr, P., Wu, L. (2009) Variance risk premiums, Review of Financial Studies, 22, 1311–1341.

[15] Chang, B.Y, Christoffersen, P., Jacobs, K., Vainberg, G. (2012) Option-implied measures of equity risk, Review of Finance, 16, 385–428.

[16] Christensen, B., Prabhala, N. (1998) The relation between implied and realized volatility, Journal of Financial Economics, 50, p 125–150.

[17] Constantinides, G., Czerwonko, M., Jackwerth, J., Perrakis, S. (2011), Are options on index futures profitable for risk averse investors? Empirical evidence, Journal of Finance, 66, 1407–1437.

[18] Conrad, J., Dittmar, R., Ghysels, E. (2013), Ex-ante skewness and expected stock returns, Journal of Finance, 68, 85-124.

[19] Cox, J.C., Ingersoll, J.E., Ross, S.R. (1985) A theory of the term structure of interest rates, Econometrica, 53, 385–408.

[20] Dennis, P., Mayhew, S. (2002) Risk-neutral skewness: evidence from stock options, Journal of Financial and Quantitative Analysis, 37, 471–93.

[21] Detemple, J. B., Rindisbacher, M. (2010) Dynamic asset allocation: portfolio decomposition formula and applications, Review of Financial Studies, 23, p 25–100.

[22] Di Nunno, G., Øksendal, B., Proske, F. (2008) Malliavin calculus for Lévy processes with applications to Finance, Springer-Verlag.

[23] Driessen, J., Maenhout, P., Vilkov, G. (2009) The price of correlation risk: evidence from equity options, Journal of Finance, 64, 1377-1406.

[24] Duan, J-C., Wei, J. (2009) Systematic risk and the price structure of individual equity options, Review of Financial Studies, 22, 1981–2006.

[25] Duffie, D., Kan, R. (1996) A yield-factor model of interest rates, Mathematical Finance, 6, 379–406.

[26] Elkamhi, R., Ornthanalai, C. (2010) Market jump risk and the price structure of individual equity options, Working Paper, Rotman School, University of Toronto.

[27] Engle, R. and Figlewski, S. (2012) Modeling the dynamics of correlations among implied volatilities, Working Paper, Stern School, New York University.

[28] Goyal A., Saretto A. (2009) Cross-section of option returns and volatility. Journal of Financial Economics, 94, 310-326.

[29] Hansen, P., Lunde, A., Voev, V. (2012), Realized beta GARCH: a multivariate GARCH model with realized measures of volatility and co-volatility, Working Paper, European University Institute.

[30] Heston, S. (1993) Closed-form solution for options with stochastic volatility, with application to bond and currency options, Review of Financial Studies, 6, 327–343.

[31] Heston, S., Nandi, S. (2000) A closed-form GARCH option pricing model, Review of Financial Studies, 13, 585–626.

[32] Hull, J., White, A. (1987) The pricing of options on assets with stochastic volatilities, Journal of Finance, 42, 281–300.

[33] Jacquier, E., Polson, N.G., Rossi, P.E. (1994) Bayesian analysis of stochastic volatility models (with discussion), Journal of Business and Economic Statistics, 12, 371-417.

[34] Jones, C.S. (2003) The dynamics of stochastic volatility: evidence from underlying and options markets, Journal of Econometrics, 116, 181–224.

[35] Jones, C.S., Wang, T. (2012), The term structure of equity option implied volatility, Working Paper, USC Marshall.

[36] Kelly, B., Lustig, H., Van Nieuwerburgh, S. (2012), The volatility factor structure, Working Paper, NYU Stern.

[37] Mo, H., Wu, L. (2007) International capital asset pricing: evidence from options, Journal of Empirical Finance, 14, 465-498.

[38] Pan, J. (2002) The jump risk premia implicit in options: Evidence from an integrated time Series Study, Journal of Financial Economics, 63, p. 3-50.

[39] Patton, A., Verardo, M. (2012), Does beta move with news? Firm-specific information flows and learning about profitability, Review of Financial Studies, 25, 2789–2839.

[40] Protter, P. (1990) Stochastic integration and differential equations, Springer-Verlag.

[41] Santa-Clara, P., Yan, S. (2010) Crashes, volatility, and the equity premium: lessons from S&P500 options, Review of Economics and Statistics, 92, 435-451.

[42] Serban, M., Lehoczky, J., Seppi, D. (2008) Cross-sectional stock option pricing and factor models of returns, Working Paper, CMU.

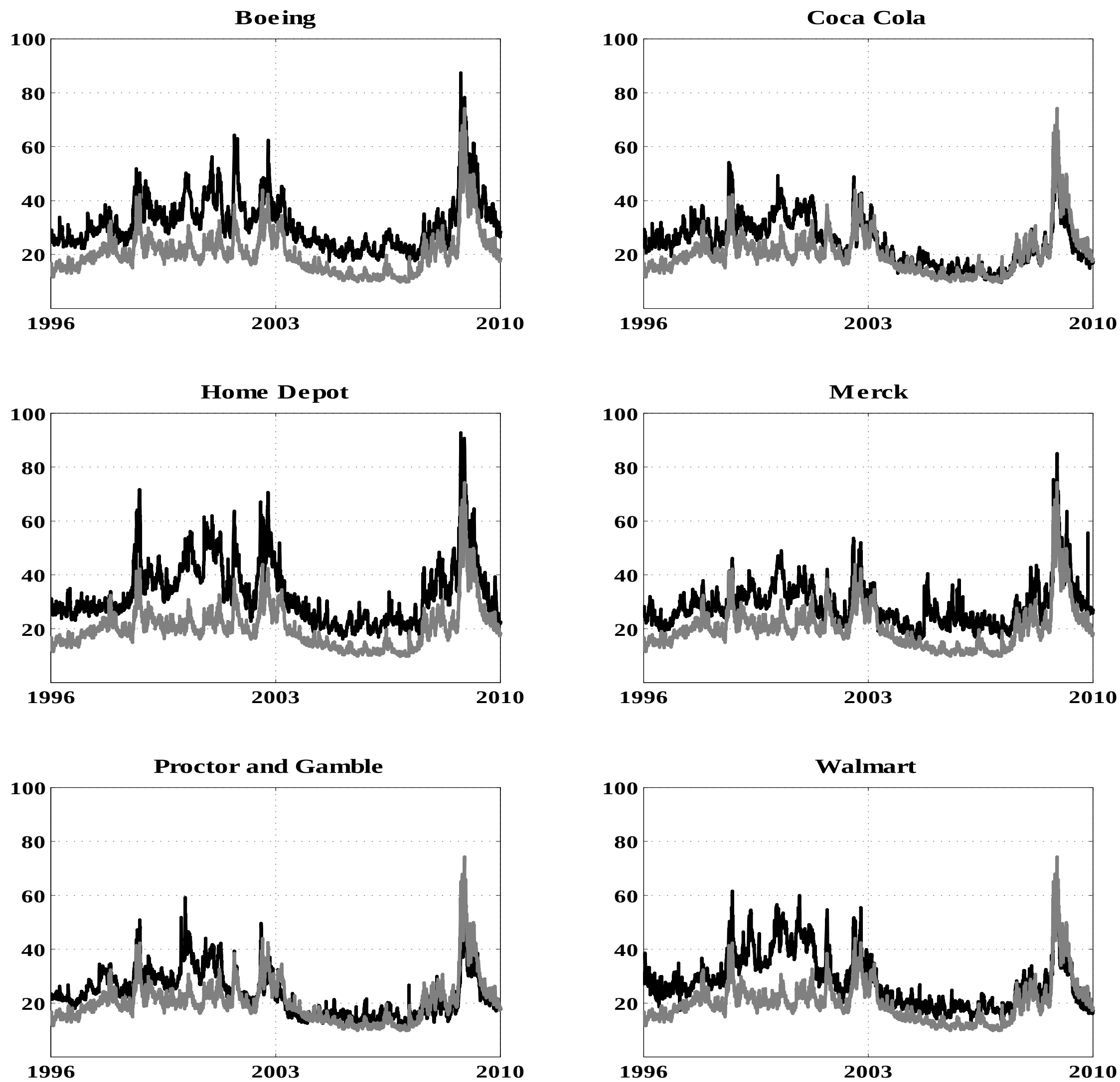
[43] Trolle, A., Schwartz, E. (2009) Unspanned stochastic volatility and the pricing of commodity derivatives, Review of Financial Studies, 22, 4423-4461.

[44] Vasquez, A. (2011), Volatility term structure and the cross-section of option returns, Working Paper, ITAM.

[45] Wiggins, J. (1987) Option values under stochastic volatility: theory and empirical evidence, Journal of Financial Economics, 19, 351-372.

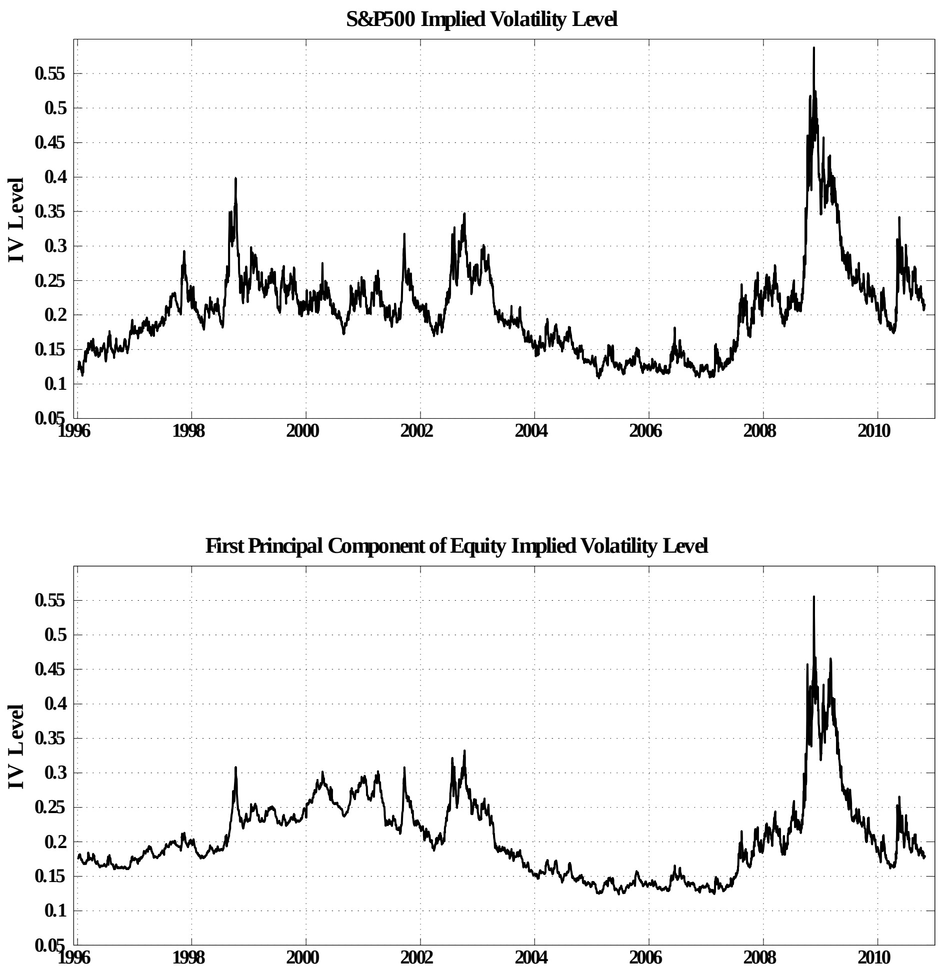
[46] Wu, L. (2008) Modeling financial security returns using Lévy processes, Handbooks in Operations Research and Management Science: Financial Engineering, J. Birge and V. Linetsky (eds.), Elsevier.

Figure 1: Short-Term At-the-Money Implied Volatility. Six Firms and the S&P 500 Index



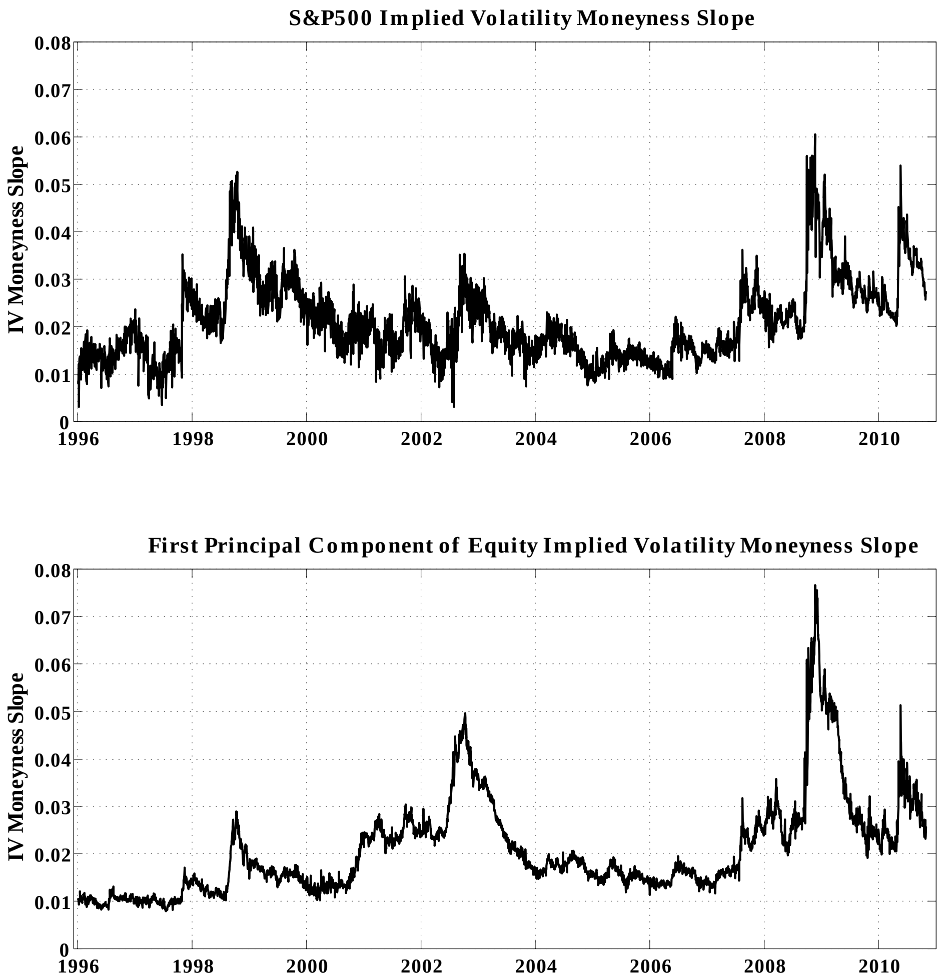
Notes to Figure: We plot the time series of implied volatility for six firms (black) and the S&P 500 index (grey). On each day we use contracts with between 20 and 60 days to maturity and a moneyness (S/K) between 0.95 and 1.05. For every trading day and every security, we average the available implied volatilities to obtain an estimate of short-term at-the-money implied volatility.

Figure 2: Implied Volatility Level:
S&P 500 Index and the First Principal Component from 29 Firms



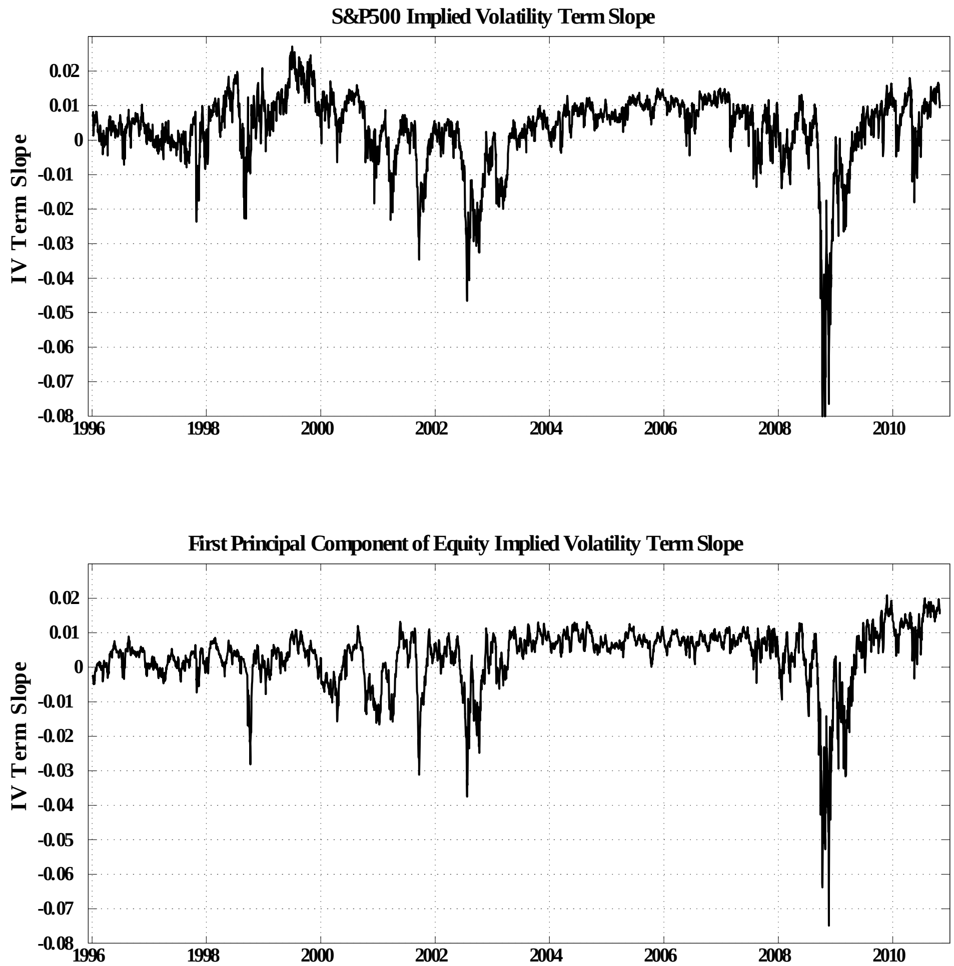
Notes to Figure: The top panel plots implied volatility from short-term at-the-money (ATM) S&P 500 index options. The bottom panel plots the first principal component of implied short-term ATM implied volatility from options on 29 equities in the Dow-Jones index.

Figure 3: Implied Volatility Moneyness Slopes:
S&P 500 Index and the First Principal Component from 29 Firms



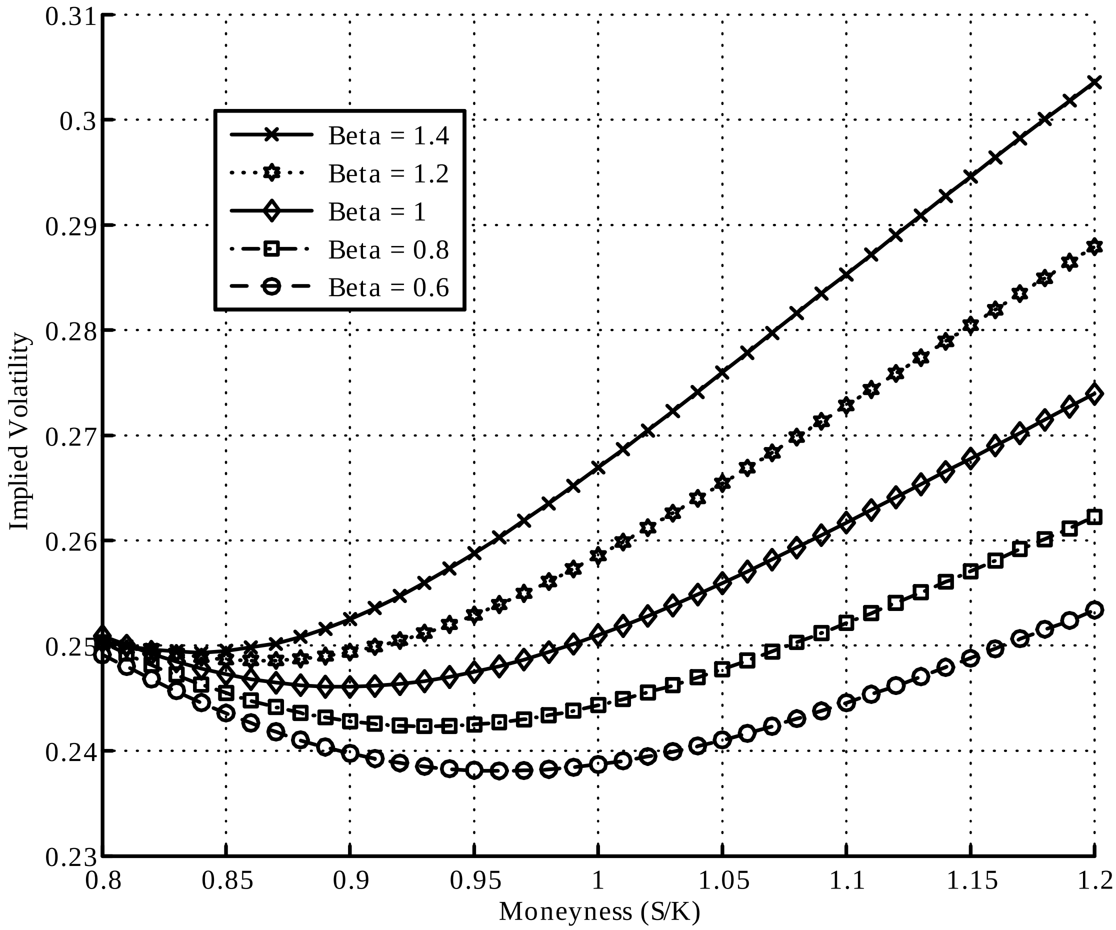
Notes to Figure: The top panel plots the time series of the slope of implied volatility with respect to moneyness from short-term S&P 500 index options. The bottom panel plots the first principal component of the implied volatility moneyness slopes from options on 29 equities in the Dow-Jones index.

Figure 4: Implied Volatility Term Structure Slopes:
S&P 500 Index and the First Principal Component from 29 Firms



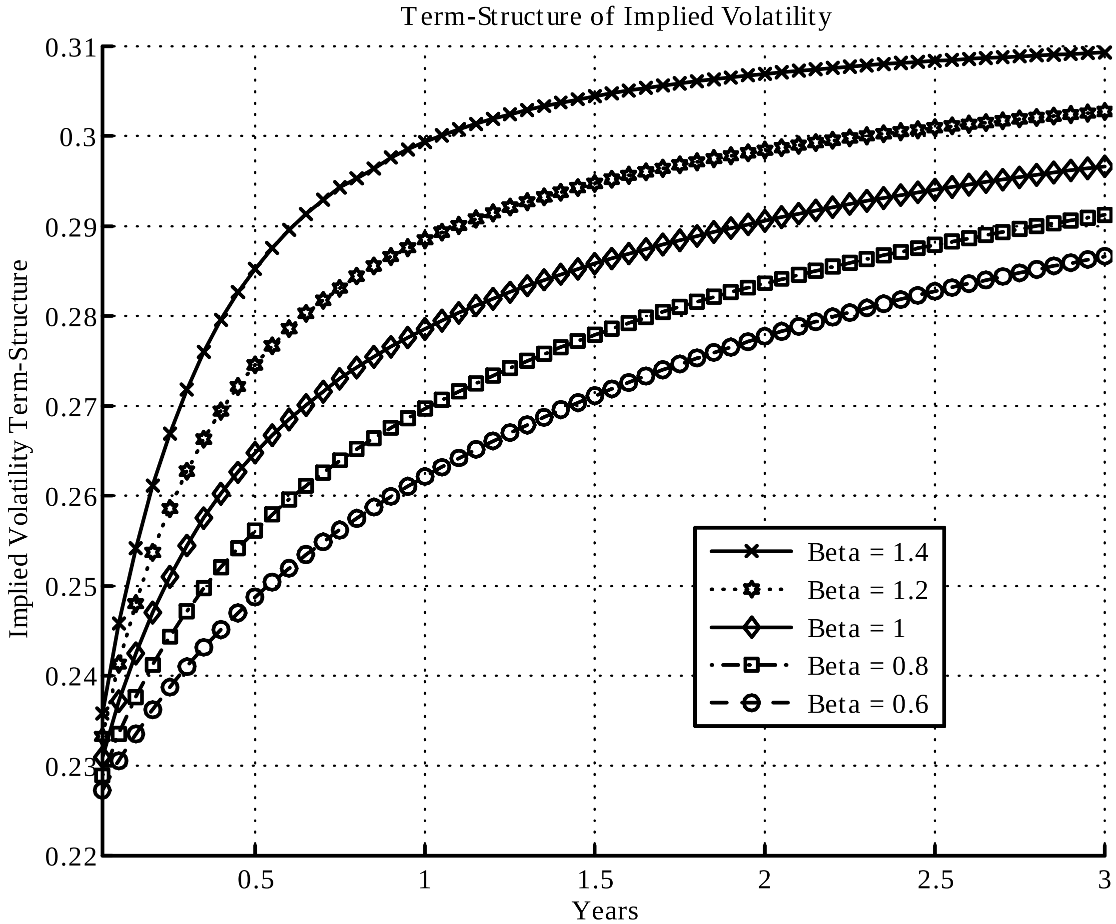
Notes to Figure: The top panel plots the slope of the implied volatility term structure from S&P 500 index options. The bottom panel plots the first principal component of the implied volatility term structure from options on 29 equities in the Dow-Jones index.

Figure 5: Beta and Implied Volatility Across Moneyness



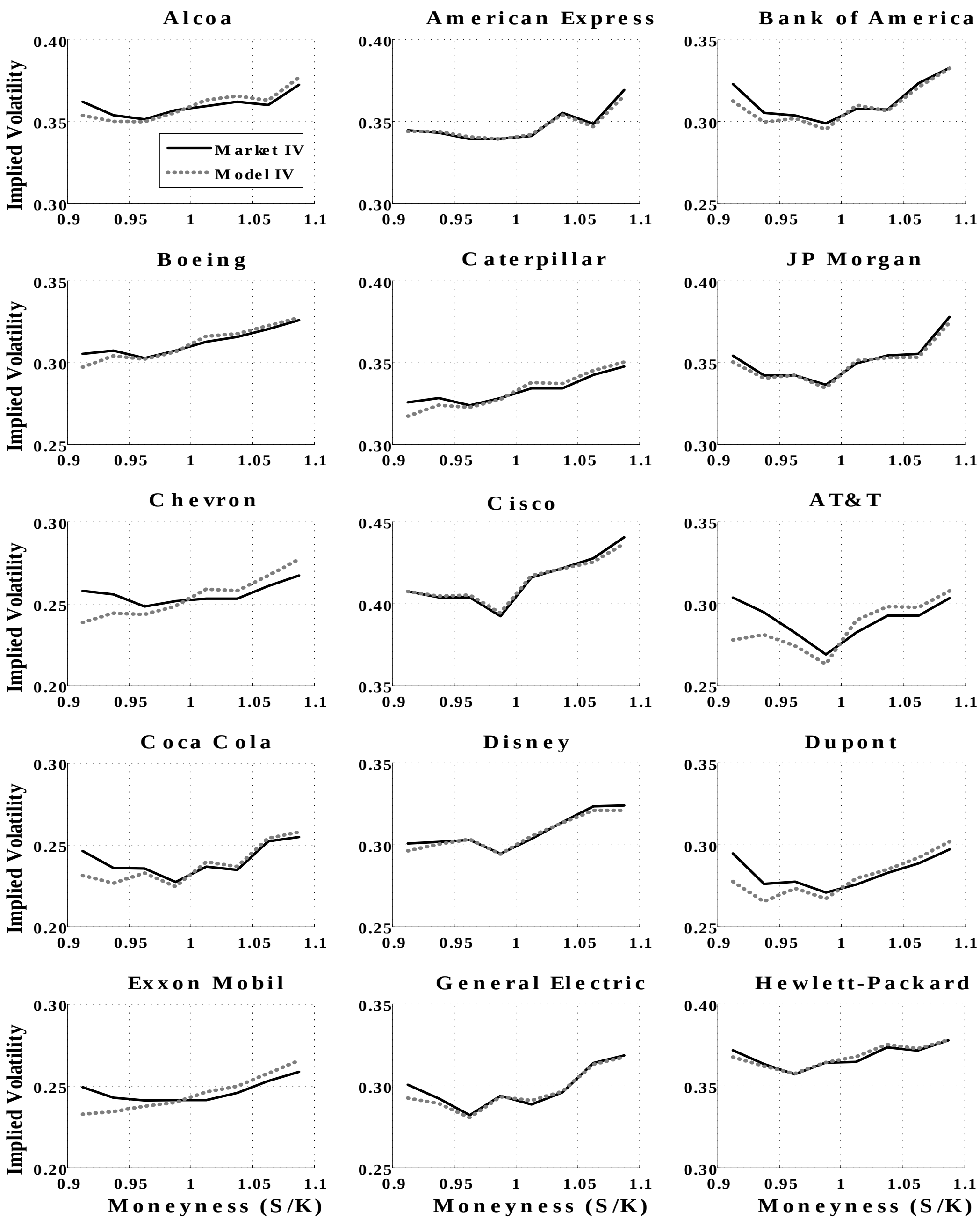
Notes to Figure: We plot implied Black-Scholes volatility from model prices. Each line has a different beta but the same amount of unconditional total equity variance $V_j = \beta_j^2 \theta_I + \theta_j = 0.1$. We set the current spot variance to $\sigma_{I,t}^2 = 0.01$ and $V_{j,t} = 0.05$, and define the idiosyncratic variance as the residual $\sigma_{j,t}^2 = V_{j,t} - \beta_j^2 \sigma_{I,t}^2$. The market index parameters are $\kappa_I = 5,\ \theta_I = 0.04,\ \delta_I = 0.5,\ \rho_I = -0.8$, and the individual equity parameters are $\kappa_j = 1,\ \delta_j = 0.4$, and $\rho_j = 0$. The risk-free rate is 4% per year and option maturity is 3 months.

Figure 6: Beta and the Implied Volatility Term Structure. At-the-Money Options



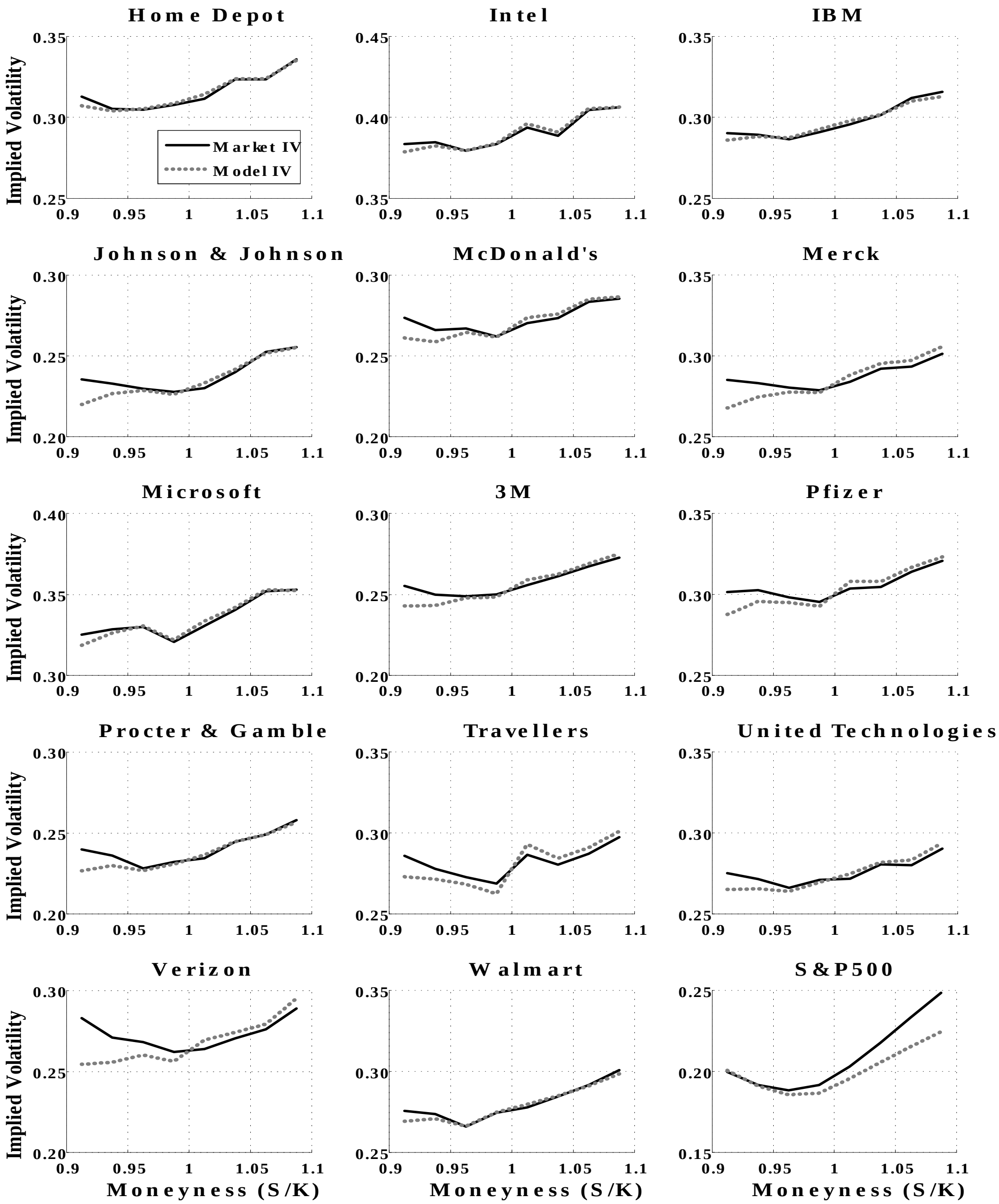
Notes to Figure: We plot implied Black-Scholes volatility from model prices. Each line has a different beta but the same amount of unconditional total equity variance $V_j = \beta_j^2\theta_I + \theta_j = 0.1$. We set the current spot variance to $\sigma_{I,t}^2 = 0.01$ and $V_{j,t} = 0.05$, and define the idiosyncratic variance as the residual $\sigma_{j,t}^2 = V_{j,t} - \beta_j^2\sigma_{I,t}^2$. The market index parameters are $\kappa_I = 5,\ \theta_I = 0.04,\ \delta_I = 0.5,\ \rho_I = -0.8$, and the individual equity parameters are $\kappa_j = 1,\ \delta_j = 0.4$, and $\rho_j = 0$. The risk-free rate is 4% per year.

Figure 7.A: Average Market- and Model-Implied Volatility Smile



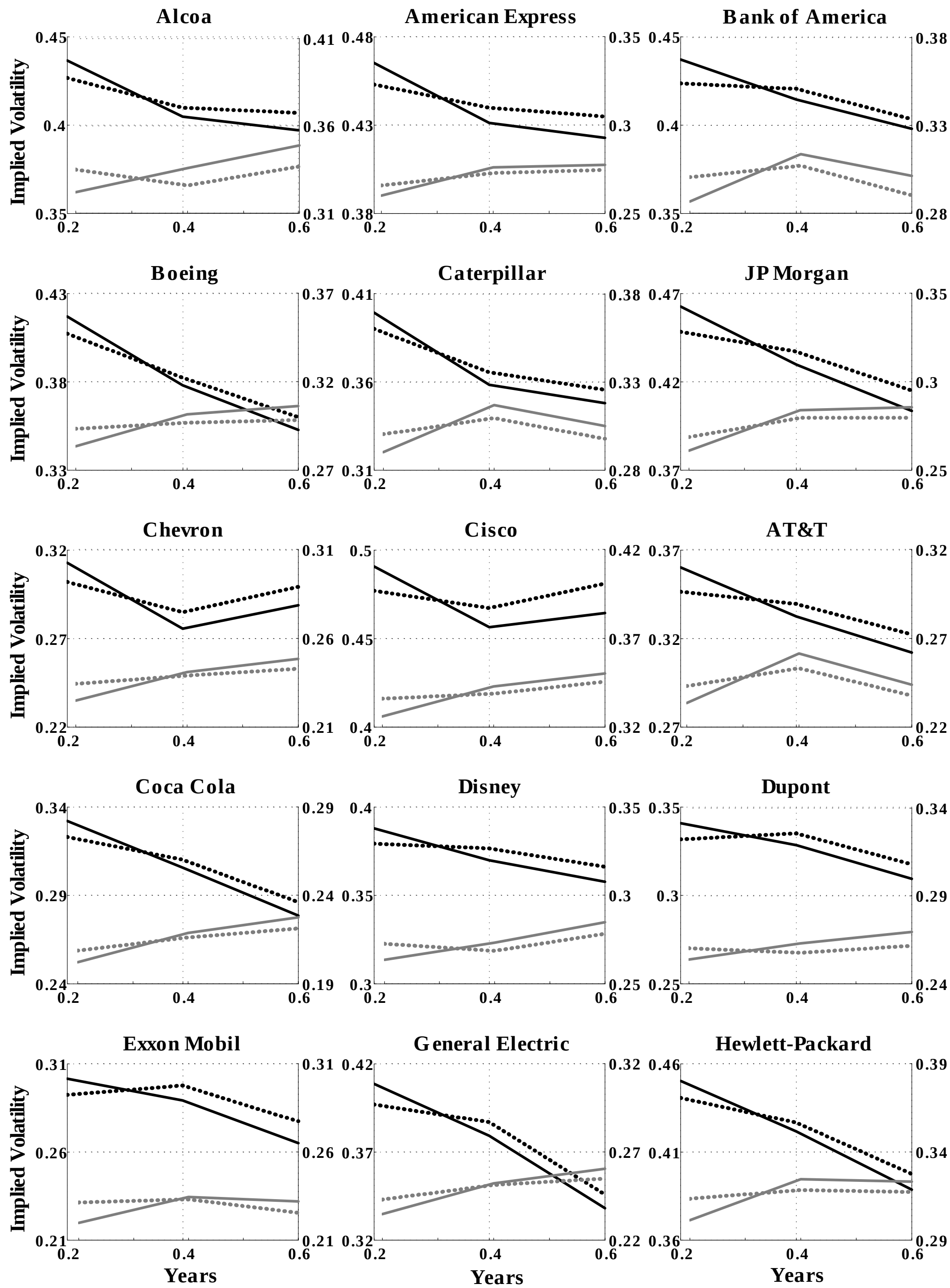
Notes to Figure: We plot implied volatility, averaged over time, against moneyness for 15 firms. The solid black line denotes market IVs and the dashed grey denotes model IVs.

Figure 7.B: Average Market- and Model-Implied Volatility Smile



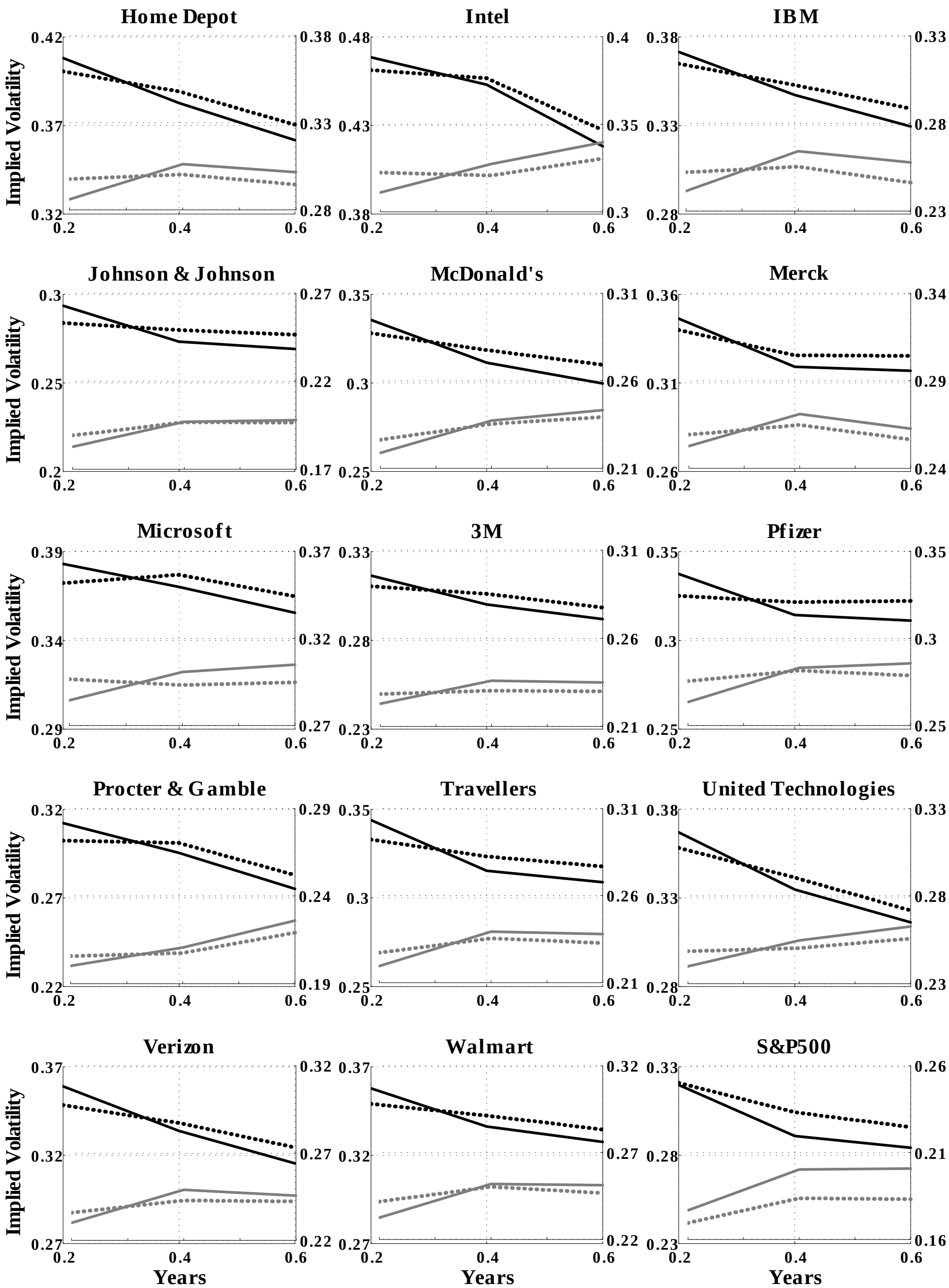
Notes to Figure: We plot implied volatility, averaged over time, against moneyness for 14 firms and the index. The solid black line denotes market IVs and the dashed grey denotes model IVs.

Figure 8.A: Market- and Model-Implied Term Structures for At-the-Money Implied Volatility



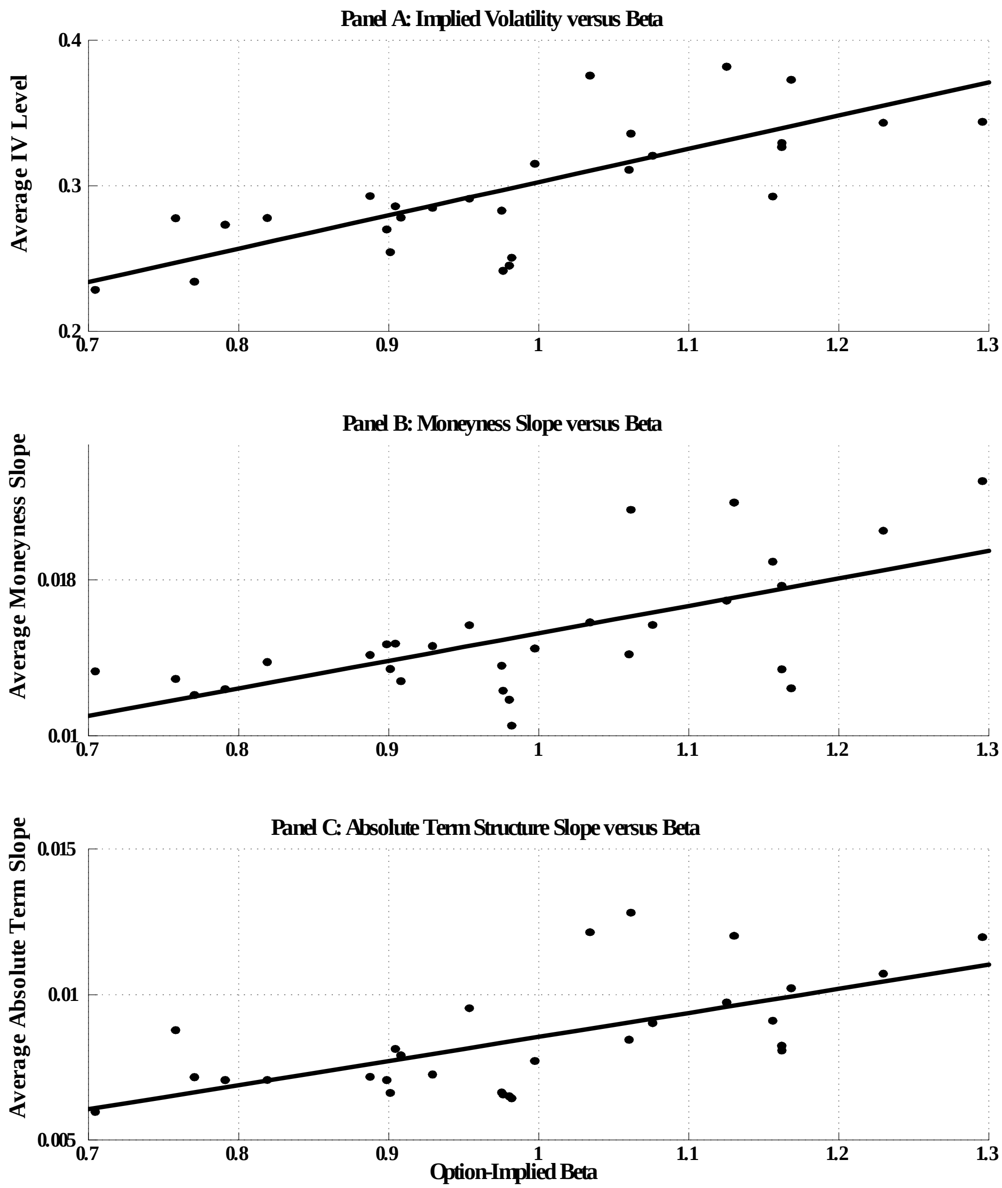
Notes to Figure: The solid black line (left axis) shows the average market IV for days with downward sloping term structures. The grey line is for days with upward sloping term structures (right axis). The dotted lines show the average model IVs. Moneyness (S/K) is between 0.95 and 1.05.

Figure 8.B: Market- and Model-Implied Term Structures for At-the-Money Implied Volatility



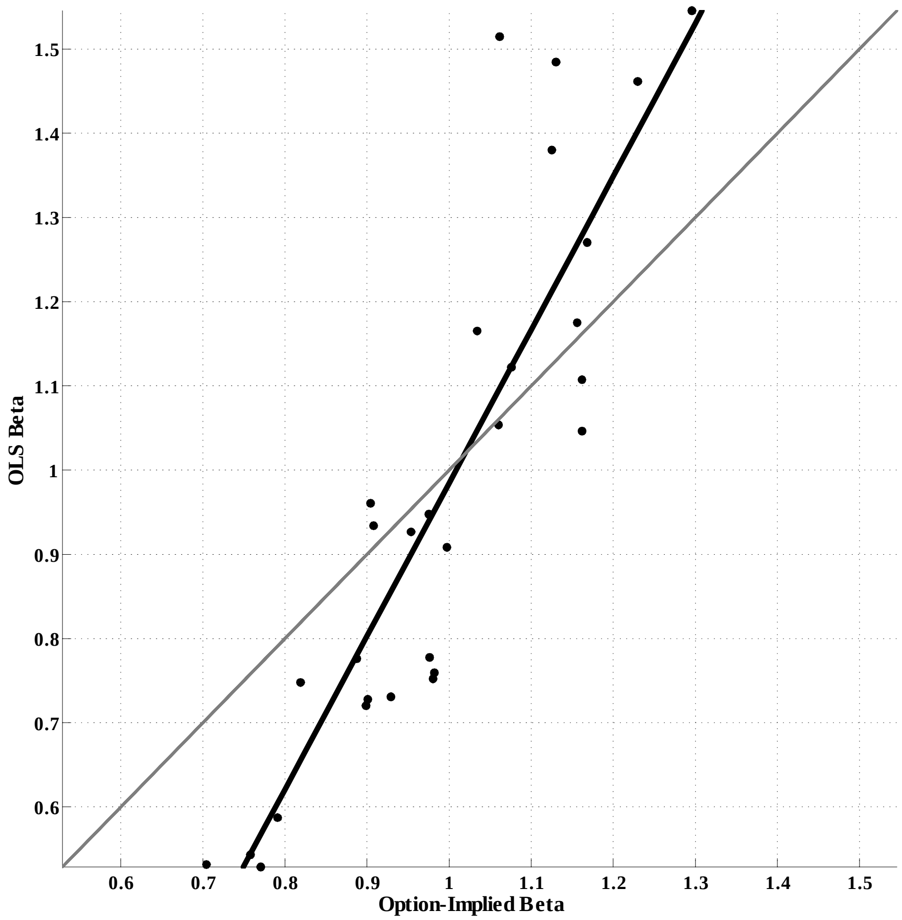
Notes to Figure: The solid black line (left axis) shows the average market IV for days with downward sloping term structures. The grey line is for days with upward sloping term structures (right axis). The dotted lines show the average model IVs. Moneyness (S/K) is between 0.95 and 1.05.

Figure 9: Implied Volatility Levels, Moneyness Slopes, and Term Structure Slopes Against Firm Beta



Notes to Figure: We plot cross-sectional regressions of the average implied volatility (IV) levels from Figure 2 (top panel), the average moneyness slopes from Figure 3 (middle panel), and the average absolute value of the term-structure slopes from Figure 4 (bottom panel) against the estimated betas from Table 6.

Figure 10: OLS Beta versus Option-Implied Beta



Notes to Figure: We plot the cross-sectional regression of OLS Betas against Option-Implied Betas. The OLS Betas are obtained from a CAPM regression using daily returns from the CRSP database over the period January 2, 1996 until October 29, 2010. The fitted line from the regression is displayed in dark while the grey line represents the 45° line.

Table 1: Companies, Tickers and Option Contracts

Company	Ticker	Total Number of Quotes			Average	Average
		All	Puts	Calls	DTM	IV
S&P500 Index	SPX	393,429	199,756	193,673	91	20.51%
Alcoa	AA	60,937	30,683	30,254	116	36.42%
American Express	AXP	95,246	47,937	47,309	120	33.65%
Bank of America	BAC	85,129	43,159	41,970	128	32.46%
Boeing	BA	91,446	45,964	45,482	127	31.28%
Caterpillar	CAT	94,211	47,238	46,973	123	33.03%
JP Morgan	JPM	100,235	50,627	49,608	125	34.07%
Chevron	CVX	91,143	46,069	45,074	130	25.23%
Cisco	CSCO	65,032	32,737	32,295	123	40.78%
AT&T	T	54,186	28,032	26,154	114	28.54%
Coca Cola	KO	84,738	43,114	41,624	130	24.19%
Disney	DIS	66,060	33,536	32,524	120	30.83%
Dupont	DD	81,191	41,216	39,975	121	28.26%
Exxon Mobil	XOM	82,362	41,681	40,681	125	24.54%
General Electric	GE	89,313	45,227	44,086	130	28.94%
Hewlett-Packard	HPQ	89,046	44,680	44,366	125	37.39%
Home Depot	HD	81,683	41,386	40,297	127	32.32%
Intel	INTC	75,533	38,081	37,452	123	37.76%
IBM	IBM	110,620	55,612	55,008	123	28.94%
Johnson & Johnson	JNJ	71,789	36,553	35,236	131	22.90%
McDonald's	MCD	80,828	40,899	39,929	126	27.46%
Merck	MRK	87,223	44,219	43,004	122	28.43%
Microsoft	MSFT	90,038	45,376	44,662	126	32.07%
3M	MMM	90,625	45,717	44,908	125	25.37%
Pfizer	PFE	79,480	40,450	39,030	128	29.37%
Procter & Gamble	PG	86,648	43,961	42,687	129	23.53%
Travellers	TRV	43,767	22,225	21,542	119	28.42%
United Technologies	UTX	86,063	43,213	42,850	126	27.70%
Verizon	VZ	67,948	34,995	32,953	118	27.43%
Walmart	WMT	88,431	44,624	43,807	130	27.87%
Average		81,757	41,352	40,405	124	29.97%

Note to Table: For each firm, we report the total number of options quotes, and the number of puts and calls quotes over the sample period 1996-2010. DTM refers to the average number of days-to-maturity in the option sample. Finally, IV denotes the average implied volatility in the sample.

Table 2: Summary Statistics on Implied Volatility 1996-2010

	Out-of-the-money Call Options				Out-of-the-money Put Options			
Ticker	Avg IV	max(IV)	min(IV)	Avg Vega	Avg IV	max(IV)	min(IV)	Avg Vega
SPX	19.6%	82.6%	5.4%	172.00	21.5%	83.5%	5.1%	181.58
AA	35.1%	143.4%	12.4%	8.12	36.7%	142.6%	17.6%	8.07
AXP	34.1%	149.7%	9.3%	12.40	35.3%	143.3%	11.8%	12.38
BAC	29.8%	149.9%	5.1%	11.31	32.5%	149.8%	9.9%	11.44
BA	30.4%	92.3%	11.2%	12.69	31.9%	92.9%	14.8%	12.67
CAT	32.1%	105.7%	14.8%	12.56	34.4%	113.6%	17.1%	12.61
JPM	34.0%	149.4%	6.7%	11.06	36.0%	148.9%	11.6%	11.03
CVX	23.9%	98.0%	7.3%	15.68	27.2%	100.1%	11.6%	15.85
CSCO	41.3%	109.1%	16.1%	9.34	41.2%	111.4%	15.4%	9.19
T	27.1%	100.4%	7.2%	7.05	30.5%	89.7%	9.6%	7.10
KO	22.9%	69.5%	5.2%	10.79	24.9%	70.5%	9.0%	10.85
DIS	30.3%	102.2%	6.7%	8.06	31.1%	105.1%	14.1%	7.94
DD	26.6%	92.2%	7.1%	9.82	29.7%	94.2%	12.6%	9.80
XOM	23.4%	89.1%	5.8%	13.26	25.8%	97.2%	8.2%	13.31
GE	28.8%	147.2%	6.1%	11.51	30.6%	145.1%	7.0%	11.46
HPQ	36.3%	112.6%	11.6%	11.28	37.1%	93.7%	13.9%	11.19
HD	30.9%	98.5%	8.7%	8.55	32.0%	106.4%	11.8%	8.45
INTC	38.6%	92.6%	10.4%	12.06	39.2%	90.6%	15.8%	11.91
IBM	29.2%	87.9%	7.5%	22.08	30.1%	87.5%	12.0%	22.06
JNJ	22.7%	71.3%	5.1%	13.93	24.6%	76.9%	8.7%	13.91
MCD	26.3%	90.5%	7.0%	9.44	28.0%	74.0%	11.0%	9.45
MRK	27.4%	84.7%	10.2%	12.48	29.8%	93.7%	11.7%	12.25
MSFT	33.1%	91.5%	8.3%	13.55	33.6%	93.8%	10.6%	13.32
MMM	24.6%	83.2%	7.4%	17.95	26.7%	84.0%	11.7%	18.04
PFE	29.5%	122.7%	7.5%	10.20	31.3%	74.9%	13.2%	10.01
PG	23.1%	72.2%	5.5%	15.44	24.8%	72.8%	9.2%	15.43
TRV	26.9%	144.5%	6.8%	9.42	29.3%	113.4%	13.1%	9.45
UTX	26.5%	90.3%	8.2%	15.59	28.5%	87.7%	12.3%	15.59
VZ	25.3%	90.9%	6.6%	8.94	29.0%	90.3%	10.6%	8.90
WMT	27.5%	70.6%	10.3%	10.39	28.5%	71.1%	10.5%	10.39
Average	29.2%	103.5%	8.4%	11.89	31.0%	100.5%	11.9%	11.86

Note to Table: For each firm, we report the average, max, and min of implied volatility. We use Black-Scholes to compute implied volatility (IV) for index and equity OTM calls, and we use binomial trees with 200 steps for OTM equity puts. Option vega is computed using Black-Scholes.

Table 3: Principal Component Analysis of Short-Term Implied Equity Volatility. Component Loadings and Properties

Company	1st Component	2nd Component	3rd Component
Alcoa	24.57%	30.87%	-20.02%
American Express	38.44%	50.96%	47.44%
Bank of America	15.63%	-0.71%	-5.85%
Boeing	13.05%	-13.72%	14.28%
Caterpillar	17.19%	-13.35%	-33.30%
JP Morgan	16.75%	1.33%	-1.39%
Chevron	10.96%	3.45%	-19.29%
Cisco	24.29%	11.76%	10.47%
AT&T	15.89%	-19.77%	12.98%
Coca Cola	10.24%	-14.91%	9.94%
Disney	9.78%	-12.05%	-18.07%
Dupont	18.24%	-18.97%	17.29%
Exxon Mobil	19.47%	2.09%	-36.25%
General Electric	15.34%	-7.85%	-9.45%
Hewlett-Packard	14.58%	-20.13%	3.14%
Home Depot	29.43%	18.73%	0.72%
Intel	17.45%	-7.70%	-15.78%
IBM	18.41%	-24.94%	1.40%
Johnson & Johnson	12.41%	2.69%	-8.32%
McDonald's	16.67%	9.16%	-3.35%
Merck	11.82%	7.82%	-19.85%
Microsoft	13.01%	-4.05%	-0.78%
3M	12.09%	-6.66%	7.07%
Pfizer	12.55%	-11.60%	13.40%
Procter & Gamble	28.44%	11.51%	-5.68%
Travellers	16.22%	-29.44%	8.50%
United Technologies	19.32%	-7.70%	-16.68%
Verizon	15.39%	-12.61%	-27.83%
Walmart	19.24%	-41.77%	33.13%
Average	17.48%	-4.05%	-2.14%
Min	9.78%	-41.77%	-36.25%
Max	38.44%	50.96%	47.44%
Variation Captured	77.18%	13.43%	2.47%
Correlation with S&P500 Short-Term Implied Volatility	91.94%	14.88%	-6.58%

Note to Table: This table present the loading of each individual company short term IV on the first three principal components. The estimates are obtained by regressing each individual equity short-term ATM implied volatility proxy on the components obtained from the PCA. We also report the Average, Min and Max of component loadings across firms. Finally we report the total cross sectional variation captured by each of the first three components as well as their correlation with the S&P500 short-term IV.

Table 4: Principal Component Analysis of Equity IV Moneyness Slope. Component Loadings and Properties

Company	1st Component	2nd Component	3rd Component
Alcoa	19.40%	-14.18%	-6.86%
American Express	26.14%	-9.89%	50.00%
Bank of America	15.46%	-6.14%	-4.64%
Boeing	11.71%	10.72%	-1.07%
Caterpillar	18.22%	1.25%	-17.08%
JP Morgan	17.32%	-3.05%	-10.04%
Chevron	15.12%	-5.56%	-4.72%
Cisco	25.40%	18.20%	41.16%
AT&T	17.34%	18.98%	-4.31%
Coca Cola	12.12%	18.16%	-19.15%
Disney	14.44%	-10.47%	-8.27%
Dupont	18.84%	32.62%	-5.51%
Exxon Mobil	19.99%	-35.30%	-7.49%
General Electric	19.45%	-11.75%	-21.89%
Hewlett-Packard	15.98%	5.41%	-14.49%
Home Depot	30.31%	-20.67%	33.50%
Intel	16.56%	-4.77%	-8.16%
IBM	16.62%	21.62%	-13.12%
Johnson & Johnson	13.12%	-1.95%	-9.65%
McDonald's	20.11%	-15.94%	-8.08%
Merck	16.24%	-9.38%	-5.24%
Microsoft	15.47%	1.71%	-12.90%
3M	13.86%	9.98%	-13.53%
Pfizer	15.89%	3.67%	-12.87%
Procter & Gamble	29.96%	-10.19%	18.81%
Travellers	15.40%	0.87%	-2.62%
United Technologies	20.60%	-6.86%	-4.11%
Verizon	17.73%	14.90%	-35.19%
Walmart	12.12%	64.38%	20.35%
Average	17.96%	1.94%	-3.00%
Min	11.71%	-35.30%	-35.19%
Max	30.31%	64.38%	50.00%
Variation Captured	76.67%	5.57%	3.72%
Correlation with S&P500 Moneyness Slope	63.71%	5.32%	31.42%

Note to Table: For the first three principal components of implied volatility (IV) moneyness slope we report the loadings of each firm as well as the average, min and max loading across firms. The time series of moneyness slopes are obtained by regressing equity IV on moneyness for each firm on each day. We also report the total cross sectional variation captured by each of the first three components as well as their correlation with the S&P500 moneyness slope.

Table 5: Principal Component Analysis of Equity IV Term Structure Slope. Component Loadings and Properties

Company	1st Component	2nd Component	3rd Component
Alcoa	18.53%	21.29%	-21.80%
American Express	35.89%	67.42%	30.07%
Bank of America	18.75%	-7.41%	-2.74%
Boeing	13.07%	-9.36%	-5.24%
Caterpillar	17.77%	-16.56%	-10.04%
JP Morgan	15.37%	-0.45%	-15.07%
Chevron	14.62%	-8.90%	-22.02%
Cisco	21.19%	15.42%	21.43%
AT&T	18.36%	-17.78%	4.29%
Coca Cola	12.05%	-8.02%	-10.52%
Disney	11.55%	-10.75%	-13.28%
Dupont	16.62%	-19.56%	10.45%
Exxon Mobil	17.31%	-1.34%	-21.94%
General Electric	17.79%	-4.65%	-15.76%
Hewlett-Packard	12.30%	-10.64%	-5.14%
Home Depot	27.37%	18.40%	-3.93%
Intel	19.57%	-8.35%	-6.20%
IBM	17.06%	-15.23%	18.99%
Johnson & Johnson	15.66%	-4.05%	-14.15%
McDonald's	18.25%	2.19%	-18.20%
Merck	15.02%	-5.77%	-26.27%
Microsoft	15.52%	-4.77%	-6.75%
3M	12.24%	-5.21%	-14.54%
Pfizer	13.01%	-9.51%	-6.59%
Procter & Gamble	28.31%	18.41%	5.07%
Travellers	18.97%	-29.79%	31.85%
United Technologies	18.24%	-10.81%	-1.90%
Verizon	15.82%	-9.27%	-11.05%
Walmart	21.00%	-31.43%	55.14%
Average	17.83%	-3.67%	-2.62%
Min	11.55%	-31.43%	-26.27%
Max	35.89%	67.42%	55.14%
Variation Captured	59.55%	13.57%	4.87%
Correlation with S&P500 Term Structure Slope	79.87%	9.03%	-8.59%

Note to Table: For the first three principal components of implied volatility (IV) term structure slope we report the loadings of each firm as well as the average, min and max loading across firms. The time series of tem structure slopes are obtained by regressing equity IV on maturity for each firm on each day. We also report the total cross sectional variation captured by each of the first three components as well as their correlation with the S&P500 term structure slope.

Table 6: Model Parameters and Properties. Index and Equity Options

Ticker	Kappa	Theta	Delta	Rho	Beta	Average Total Spot Volatility	Systematic Risk Ratio (R-Squared)
SPX	1.24	0.0542	0.366	-0.860		21.74%	
AA	0.98	0.0207	0.202	-0.659	1.17	41.06%	38.28%
AXP	0.73	0.0229	0.182	-0.801	1.23	39.16%	46.62%
BAC	0.76	0.0147	0.150	-0.775	1.06	40.76%	32.07%
BA	0.99	0.0508	0.317	-0.747	1.00	33.31%	42.37%
CAT	1.10	0.0329	0.269	-0.814	1.16	35.43%	50.84%
JPM	0.80	0.0184	0.172	-0.914	1.30	38.97%	52.27%
CVX	0.53	0.0357	0.195	-0.829	0.98	26.25%	66.13%
CSCO	0.97	0.0887	0.414	-0.943	1.13	44.95%	29.89%
T	0.75	0.0350	0.229	-0.909	0.98	30.36%	48.87%
KO	1.00	0.0340	0.260	-0.824	0.76	25.61%	41.43%
DIS	1.04	0.0319	0.257	-0.482	1.06	33.48%	47.41%
DD	1.06	0.0341	0.268	-0.907	0.98	30.20%	49.32%
XOM	1.18	0.0180	0.206	-0.800	0.98	25.79%	68.31%
GE	0.62	0.0093	0.107	-0.843	1.16	32.78%	58.78%
HPQ	1.44	0.0660	0.436	-0.697	1.03	41.11%	29.91%
HD	1.01	0.0264	0.231	-0.808	1.16	35.40%	50.93%
INTC	1.16	0.0533	0.350	-0.738	1.13	41.60%	34.58%
IBM	1.46	0.0235	0.262	-0.691	0.95	32.15%	41.60%
JNJ	1.05	0.0356	0.273	-0.822	0.70	23.81%	41.39%
MCD	1.31	0.0518	0.369	-0.774	0.79	28.54%	36.31%
MRK	1.71	0.0443	0.390	-0.846	0.93	29.95%	45.50%
MSFT	1.02	0.0240	0.217	-0.978	1.08	35.03%	44.60%
MMM	1.53	0.0248	0.275	-0.798	0.90	26.98%	52.71%
PFE	0.88	0.0601	0.326	-0.848	0.89	30.41%	40.28%
PG	0.95	0.0341	0.255	-0.787	0.77	24.55%	46.58%
TRV	0.84	0.0331	0.236	-0.823	0.90	31.38%	39.28%
UTX	1.22	0.0351	0.293	-0.764	0.91	29.74%	44.08%
VZ	1.04	0.0497	0.322	-0.876	0.90	29.00%	45.40%
WMT	0.59	0.0550	0.254	-0.727	0.82	29.38%	36.75%
Average	1.02	0.0367	0.27	-0.80	0.99	32.66%	44.91%

Note to Table: We use option data from 1996 to 2010 to estimate risk-neutral parameter values for the market index as well as the 29 individual equities. The individual equity parameters are estimated taking the market index parameter values as given. The last two columns report the average spot volatility through the sample and the proportion of total variance accounted for by the systematic market risk factor.

Table 7: Model Fit. Index and Equity Options

	All Contracts			Calls	Puts	Short Term	Long Term
Ticker	Vega RMSE	IVRMSE	IVRMSE / Average IV	OTM IVRMSE	OTM IVRMSE	ATM IVRMSE	ATM IVRMSE
SPX	1.90%	2.01%	9.79%	1.87%	2.14%	1.52%	1.91%
AA	1.82%	1.82%	5.01%	1.80%	1.85%	1.67%	1.70%
AXP	1.90%	1.91%	5.67%	1.86%	1.96%	1.59%	1.76%
BAC	2.39%	2.40%	7.40%	2.37%	2.43%	2.19%	1.95%
BA	1.65%	1.66%	5.30%	1.58%	1.73%	1.55%	1.47%
CAT	1.86%	1.87%	5.66%	1.82%	1.92%	1.78%	1.56%
JPM	2.36%	2.37%	6.95%	2.25%	2.49%	2.08%	2.01%
CVX	1.81%	1.81%	7.18%	1.82%	1.80%	1.85%	1.64%
CSCO	1.91%	1.91%	4.70%	1.93%	1.90%	1.75%	1.92%
T	2.31%	2.33%	8.15%	2.31%	2.35%	2.29%	2.09%
KO	1.41%	1.41%	5.83%	1.38%	1.44%	1.37%	1.27%
DIS	1.57%	1.57%	5.11%	1.46%	1.69%	1.43%	1.42%
DD	1.90%	1.91%	6.76%	1.80%	2.02%	1.84%	1.73%
XOM	1.55%	1.55%	6.33%	1.59%	1.51%	1.49%	1.54%
GE	2.02%	2.03%	7.01%	1.92%	2.14%	1.96%	1.51%
HPQ	1.66%	1.66%	4.45%	1.64%	1.69%	1.62%	1.47%
HD	1.69%	1.69%	5.24%	1.65%	1.74%	1.59%	1.53%
INTC	1.56%	1.56%	4.14%	1.56%	1.57%	1.47%	1.49%
IBM	1.63%	1.63%	5.63%	1.58%	1.69%	1.45%	1.57%
JNJ	1.48%	1.49%	6.52%	1.46%	1.53%	1.35%	1.21%
MCD	1.64%	1.65%	6.01%	1.62%	1.68%	1.64%	1.30%
MRK	1.70%	1.70%	5.99%	1.67%	1.74%	1.75%	1.54%
MSFT	1.73%	1.72%	5.38%	1.69%	1.76%	1.59%	1.75%
MMM	1.54%	1.54%	6.09%	1.49%	1.60%	1.41%	1.39%
PFE	1.60%	1.60%	5.46%	1.59%	1.62%	1.52%	1.36%
PG	1.60%	1.60%	6.82%	1.54%	1.67%	1.38%	1.39%
TRV	1.96%	1.96%	6.91%	1.96%	1.97%	1.76%	1.93%
UTX	1.57%	1.57%	5.69%	1.54%	1.62%	1.54%	1.51%
VZ	2.05%	2.07%	7.55%	2.03%	2.11%	2.05%	1.87%
WMT	1.45%	1.45%	5.22%	1.42%	1.49%	1.38%	1.16%
Average	1.77%	1.78%	6.00%	1.74%	1.82%	1.67%	1.59%

Note to Table: For the S&P500 index and for each firm we compute the implied volatility root mean squared error (IVRMSE) along with the vega-based approximation used in estimation and IVRMSE divided by the average market IV from Table 1. We also report IVRMSE for out-of-the-money (OTM) call and put options separately. Finally, we report IVRMSE for at-the-money (ATM) short term and long term options. At the money is defined by $0.975<S/K<1.025$ and short (and long) term are defined as less than (more than) six months to maturity.

Table 8: Idiosyncratic Variance Correlation Matrix

	AA	AXP	BAC	BA	CAT	JPM	CVX	CSCO	T	KO	DIS	DD	XOM	GE	HPQ	HD	INTC	IBM	JNJ	MCD	MRK	MSFT	MMM	PFE	PG	TRV	UTX	VZ	WMT
AA		0.87	0.83	0.48	0.69	0.71	0.50	-0.12	0.11	-0.01	0.25	0.51	0.16	0.76	-0.07	0.27	-0.03	-0.07	-0.19	-0.13	0.47	0.00	0.19	-0.02	-0.01	0.61	0.31	0.11	0.01
AXP	0.87		0.84	0.56	0.65	0.77	0.48	0.05	0.22	0.21	0.37	0.58	0.13	0.77	0.08	0.40	0.11	0.14	0.01	0.01	0.54	0.17	0.30	0.15	0.16	0.73	0.43	0.22	0.17
BAC	0.83	0.84		0.38	0.61	0.79	0.30	-0.10	0.02	0.06	0.20	0.46	-0.02	0.80	-0.07	0.19	-0.04	-0.03	-0.10	-0.14	0.38	0.03	0.15	0.08	0.05	0.51	0.23	0.02	0.02
BA	0.48	0.56	0.38		0.59	0.53	0.25	0.52	0.51	0.48	0.72	0.70	0.13	0.64	0.55	0.65	0.56	0.48	0.36	0.35	0.43	0.57	0.64	0.31	0.46	0.62	0.80	0.47	0.47
CAT	0.69	0.65	0.61	0.59		0.50	0.48	0.10	0.23	0.27	0.36	0.78	0.21	0.72	0.23	0.38	0.15	0.22	0.16	0.06	0.52	0.23	0.52	0.34	0.38	0.50	0.47	0.15	0.34
JPM	0.71	0.77	0.79	0.53	0.50		0.25	0.22	0.43	0.25	0.49	0.55	0.07	0.77	0.22	0.51	0.33	0.25	0.16	0.18	0.46	0.31	0.36	0.19	0.21	0.69	0.52	0.48	0.27
CVX	0.50	0.48	0.30	0.25	0.48	0.25		-0.18	0.19	0.12	0.08	0.39	0.79	0.22	-0.02	0.34	-0.09	0.04	0.00	0.15	0.52	-0.02	0.26	0.16	0.18	0.50	0.23	0.13	0.23
CSCO	-0.12	0.05	-0.10	0.52	0.10	0.22	-0.18		0.54	0.68	0.62	0.39	-0.03	0.23	0.86	0.64	0.87	0.81	0.65	0.50	0.19	0.83	0.67	0.40	0.64	0.25	0.66	0.55	0.68
T	0.11	0.22	0.02	0.51	0.23	0.43	0.19	0.54		0.53	0.70	0.55	0.31	0.29	0.67	0.70	0.69	0.63	0.62	0.69	0.43	0.62	0.59	0.36	0.54	0.60	0.73	0.86	0.64
KO	-0.01	0.21	0.06	0.48	0.27	0.25	0.12	0.68	0.53		0.54	0.63	0.14	0.24	0.73	0.65	0.65	0.84	0.84	0.60	0.46	0.75	0.70	0.71	0.90	0.35	0.63	0.49	0.86
DIS	0.25	0.37	0.20	0.72	0.36	0.49	0.08	0.62	0.70	0.54		0.61	0.04	0.45	0.67	0.67	0.70	0.60	0.54	0.57	0.37	0.66	0.61	0.43	0.49	0.63	0.82	0.69	0.60
DD	0.51	0.58	0.46	0.70	0.78	0.55	0.39	0.39	0.55	0.63	0.61		0.25	0.64	0.53	0.60	0.45	0.57	0.52	0.37	0.64	0.55	0.76	0.55	0.70	0.60	0.71	0.43	0.66
XOM	0.16	0.13	-0.02	0.13	0.21	0.07	0.79	-0.03	0.31	0.14	0.04	0.25		0.00	0.11	0.34	0.04	0.14	0.15	0.25	0.41	0.09	0.30	0.14	0.26	0.33	0.20	0.24	0.27
GE	0.76	0.77	0.80	0.64	0.72	0.77	0.22	0.23	0.29	0.24	0.45	0.64	0.00		0.25	0.40	0.30	0.25	0.14	0.03	0.45	0.32	0.41	0.22	0.26	0.58	0.51	0.29	0.22
HPQ	-0.07	0.08	-0.07	0.55	0.23	0.22	-0.02	0.86	0.67	0.73	0.67	0.53	0.11	0.25		0.72	0.87	0.84	0.73	0.63	0.30	0.82	0.73	0.48	0.74	0.34	0.73	0.63	0.78
HD	0.27	0.40	0.19	0.65	0.38	0.51	0.34	0.64	0.70	0.65	0.67	0.60	0.34	0.40	0.72		0.72	0.69	0.52	0.62	0.49	0.67	0.71	0.44	0.62	0.61	0.77	0.72	0.77
INTC	-0.03	0.11	-0.04	0.56	0.15	0.33	-0.09	0.87	0.69	0.65	0.70	0.45	0.04	0.30	0.87	0.72		0.83	0.67	0.56	0.30	0.85	0.66	0.43	0.61	0.38	0.73	0.73	0.69
IBM	-0.07	0.14	-0.03	0.48	0.22	0.25	0.04	0.81	0.63	0.84	0.60	0.57	0.14	0.25	0.84	0.69	0.83		0.82	0.59	0.40	0.89	0.73	0.63	0.80	0.34	0.69	0.61	0.82
JNJ	-0.19	0.01	-0.10	0.36	0.16	0.16	0.00	0.65	0.62	0.84	0.54	0.52	0.15	0.14	0.73	0.52	0.67	0.82		0.64	0.39	0.76	0.63	0.67	0.83	0.28	0.60	0.58	0.79
MCD	-0.13	0.01	-0.14	0.35	0.06	0.18	0.15	0.50	0.69	0.60	0.57	0.37	0.25	0.03	0.63	0.62	0.56	0.59	0.64		0.28	0.51	0.48	0.43	0.55	0.33	0.59	0.63	0.68
MRK	0.47	0.54	0.38	0.43	0.52	0.46	0.52	0.19	0.43	0.46	0.37	0.64	0.41	0.45	0.30	0.49	0.30	0.40	0.39	0.28		0.36	0.54	0.54	0.51	0.55	0.48	0.41	0.48
MSFT	0.00	0.17	0.03	0.57	0.23	0.31	-0.02	0.83	0.62	0.75	0.66	0.55	0.09	0.32	0.82	0.67	0.85	0.89	0.76	0.51	0.36		0.71	0.55	0.74	0.35	0.70	0.61	0.77
MMM	0.19	0.30	0.15	0.64	0.52	0.36	0.26	0.67	0.59	0.70	0.61	0.76	0.30	0.41	0.73	0.71	0.66	0.73	0.63	0.48	0.54	0.71		0.55	0.79	0.47	0.73	0.51	0.79
PFE	-0.02	0.15	0.08	0.31	0.34	0.19	0.16	0.40	0.36	0.71	0.43	0.55	0.14	0.22	0.48	0.44	0.43	0.63	0.67	0.43	0.54	0.55	0.55		0.70	0.23	0.46	0.37	0.70
PG	-0.01	0.16	0.05	0.46	0.38	0.21	0.18	0.64	0.54	0.90	0.49	0.70	0.26	0.26	0.74	0.62	0.61	0.80	0.83	0.55	0.51	0.74	0.79	0.70		0.32	0.60	0.45	0.88
TRV	0.61	0.73	0.51	0.62	0.50	0.69	0.50	0.25	0.60	0.35	0.63	0.60	0.33	0.58	0.34	0.61	0.38	0.34	0.28	0.33	0.55	0.35	0.47	0.23	0.32		0.67	0.60	0.39
UTX	0.31	0.43	0.23	0.80	0.47	0.52	0.23	0.66	0.73	0.63	0.82	0.71	0.20	0.51	0.73	0.77	0.73	0.69	0.60	0.59	0.48	0.70	0.73	0.46	0.60	0.67		0.71	0.69
VZ	0.11	0.22	0.02	0.47	0.15	0.48	0.13	0.55	0.86	0.49	0.69	0.43	0.24	0.29	0.63	0.72	0.73	0.61	0.58	0.63	0.41	0.61	0.51	0.37	0.45	0.60	0.71		0.58
WMT	0.01	0.17	0.02	0.47	0.34	0.27	0.23	0.68	0.64	0.86	0.60	0.66	0.27	0.22	0.78	0.77	0.69	0.82	0.79	0.68	0.48	0.77	0.79	0.70	0.88	0.39	0.69	0.58	

Note to Table: We use the time-series of idiosyncratic spot variances estimated to compute the correlation matrix for the 29 firms over the sample period 1996 to 2010.