Attention Allocation Over the Business Cycle

Marcin Kacperczyk∗ Stijn Van Nieuwerburgh† Laura Veldkamp‡

First version: May 2009, This version: February 2010§

Abstract

The invisibility of information precludes a direct test of attention allocation theories. To surmount this obstacle, we develop a model that uses an observable variable – the state of the business cycle – to predict attention allocation. Attention allocation, in turn, predicts aggregate investment patterns. Because the theory begins and ends with observable variables, it becomes testable. We apply our theory to a large information-based industry, actively managed equity mutual funds, and study its investment choices and returns. Consistent with the theory, which predicts cyclical changes in attention allocation, we find that in recessions, funds’ portfolios (1) covary more with aggregate payoff-relevant information, (2) exhibit more cross-sectional dispersion, and (3) generate higher returns. The results suggest that some, but not all, fund managers process information in a value-maximizing way for their clients and that these skilled managers outperform others.

∗Department of Finance Stern School of Business and NBER, New York University, 44 W. 4th Street, New York, NY 10012; mjakperc@stern.nyu.edu; http://www.stern.nyu.edu/~mkacperc.
†Department of Finance Stern School of Business, NBER, and CEPR, New York University, 44 W. 4th Street, New York, NY 10012; sviniewe@stern.nyu.edu; http://www.stern.nyu.edu/~svinuewe.
‡Department of Economics Stern School of Business, NBER, and CEPR, New York University, 44 W. 4th Street, New York, NY 10012; lveldkam@stern.nyu.edu; http://www.stern.nyu.edu/~lveldkam.
§We thank Joseph Chen, Xavier Gabaix, Vincent Glode, Ralph Koijen, Matthijs van Dijk, and seminar participants at NYU Stern, University of Vienna, Australian National University, University of Melbourne, University of New South Wales, University of Sydney, University of Technology Sydney, Erasmus University, University of Mannheim, Duke University, Stanford University, the University of California at Berkeley, the University of Alberta, the University of Toulouse, the Amsterdam Asset Pricing Retreat, the Society for Economic Dynamics meetings in Istanbul, the CEPR Financial Markets conference in Gerzensee, the UBC Summer Finance conference, and the Econometric Society meeting in Atlanta for useful comments and suggestions. Finally, we thank the Q-group for their generous financial support.
“What information consumes is rather obvious: It consumes the attention of its recipients. Hence a wealth of information creates a poverty of attention, and a need to allocate that attention efficiently among the overabundance of information sources that might consume it.” Simon (1971)

Most decision makers are faced with an abundance of available information and must choose how to allocate their limited attention. Recent work has shown that introducing attention constraints into decision problems can help explain observed price-setting, consumption, and financial investment patterns. Unfortunately, the invisibility of information precludes direct testing of whether agents actually allocate their attention in a value-maximizing way. To surmount this obstacle, we develop a model of investment that uses an observable variable – the state of the business cycle – to predict attention allocation. Attention, in turn, predicts aggregate investment patterns. Because the theory begins and ends with observable variables, it becomes testable. To carry out these tests, we use data on actively managed equity mutual funds. A wealth of detailed data on portfolio holdings and returns makes this industry an ideal setting in which to test the rationality of attention allocation.

A better understanding of attention allocation sheds new light on a central question in the financial intermediation literature: Do investment managers add value for their clients? What makes this an important question is that a large and growing fraction of individual investors delegate their portfolio management to professional investment managers. This intermediation occurs despite a significant body of evidence that finds that actively managed portfolios tend to underperform passive investment strategies, on average, net of fees, and after controlling for differences in systematic risk exposure. This evidence of negative average “alpha” has led many to conclude that investment managers have no skill. By devel-

---


2In 1980, 48% of U.S. equity was directly held by individuals – as opposed to being held through intermediaries; by 2007 that fraction has been down to 21.5%. See French (2008), Table 1. At the end of 2008, $9.6 trillion was invested with such intermediaries in the U.S. Of all investment in domestic equity mutual funds, about 85% is actively managed (2009 Investment Company Factbook). A related theoretical literature studies delegated portfolio management; e.g., Basak, Pavlova, and Shapiro (2007), Cuoco and Kaniel (2007), Vayanos and Woolley (2008), and Chien, Cole, and Lustig (2009).

oping a theory of managers’ information and investment choices and finding evidence for its predictions in the mutual fund industry data, we conclude that the data are consistent with a world in which a small fraction of investment managers have skill. However, the model is also consistent with the empirical literature’s finding that skill is hard to detect, on average. The model identifies recessions as times when information choices lead to investment choices that are more revealing of skill.

We argue that recessions and expansions imply different optimal attention allocation strategies for skilled investment managers. Different learning strategies, in turn, prompt different investment strategies, causing the differential performance in recessions and expansions. Specifically, we build a general equilibrium model in which a fraction of investment managers have skill, meaning that they can acquire and process informative signals about the future values of risky assets. These skilled managers can observe a fixed number of signals and choose what fraction of those signals will contain aggregate versus stock-specific information. We think of aggregate signals as macroeconomic data that affect future cash flows of all firms, and of stock-specific signals as firm-level data that forecast the part of firms’ future cash flows that is independent of the aggregate shocks. Based on their signals, skilled managers form portfolios, choosing larger portfolio weights for assets that are more likely to have high returns.

The model’s predictions fall into three categories. The first one relates to attention allocation. As in most learning problems, risks that are large in scale and high in volatility are more valuable to learn about. In our model, aggregate shocks are large in scale, because many asset returns are affected by them, but they have low volatility. Stock-specific shocks are smaller in scale but have higher volatility. As in the data, aggregate shocks are more volatile in recessions, relative to stock-specific shocks. The increased volatility of aggregate shocks makes it optimal to devote relatively more attention to aggregate shocks in recessions and stock-specific shocks in expansions.

The second category of predictions pertains to portfolio dispersion and helps to distinguish our theory from a non-informational one. In recessions, when aggregate shocks to asset payoffs are larger in magnitude, asset payoffs exhibit more comovement. Thus, any portfolio

---

4 The finding that some managers have skill is consistent with a number of recent papers in the empirical mutual fund literature, e.g., Cohen, Coval, and Pástor (2005), Kacperczyk, Sialm, and Zheng (2005, 2008), Kacperczyk and Seru (2007), Kojen (2008), Baker, Litov, Wachter, and Wurgler (2009), Huang, Sialm, and Zhang (2009).

5 We show below that the idiosyncratic risk in stock returns, averaged across stocks, does not vary significantly over the business cycle. In contrast, the aggregate risk averaged across stocks is almost twenty-five percent higher in recessions in our sample.
strategies that put (exogenously) fixed weights on assets would have returns that also co-move more in recessions. In contrast, when investment managers learn about asset payoffs and manage their portfolios according to what they learn, recessionary fund returns comove less. The reason is that when aggregate shocks become more volatile, and less predictable, managers who learn about aggregate shocks end up having more heterogeneous beliefs. They put less weight on their common prior beliefs, which have less predictive power, and more weight on their heterogeneous signals. More heterogeneous beliefs in recessions generates more heterogeneous investment strategies and fund returns as well.

Third, the model predicts time variation in fund performance. The average fund can only outperform the market if there are other, non-fund investors who underperform. Therefore, the model also includes unskilled non-fund investors. Due to their lack of skill, they reside mostly in the left tail of the return distribution. When return dispersion rises, in recessions, left-tail investors underperform by more and the average fund’s outperformance rises.

We test the model’s three main predictions on the universe of actively managed U.S. mutual funds. To detect evidence of cyclical changes in attention, we estimate the covariance of each fund’s portfolio holdings with the aggregate payoff shock, proxied by innovations in industrial production growth. We call this covariance reliance on aggregate information (RAI). RAI indicates a manager’s ability to time the market by increasing (decreasing) her portfolio positions in anticipation of good (bad) macroeconomic news. We find that the average RAI across funds is higher in recessions. We also calculate the covariance of a fund’s portfolio holdings with asset-specific shocks, proxied by innovations in earnings. We call this variable reliance on stock-specific information (RSI). RSI measures managers’ ability to pick stocks that subsequently experience unexpectedly high earnings. We find that RSI is higher in expansions.

Second, we test for cyclical changes in portfolio dispersion. In recessions, we find a higher portfolio concentration, measured as the sum of squared deviations of portfolio weights from those of the market portfolio. When funds hold portfolios that differ more from the market, which is the average portfolio, they are also holding portfolios that differ more from one another. Also consistent with the concentration hypothesis, we find higher idiosyncratic risk in fund returns in recessions. The increased dispersion additionally appears in fund returns, alphas, and betas. All these are predictions of our theory. Figure 1 shows a 30% increase of the cross-sectional standard deviation of fund alphas in recessions for our mutual fund data.

Third, we document fund outperformance in recessions. Risk-adjusted excess fund re-

---

Kosowski (2006), Lynch and Wachter (2007), and Glode (2008) also document such evidence, but their
returns (alphas) are around 1.8 to 2.4% per year higher in recessions, depending on the specification. Gross alphas (before fees) are not statistically different from zero in expansions, but they are positive in recessions. Net alphas (after fees) are negative in expansions and positive in recessions. These cyclical differences are statistically and economically significant. Indeed, Figure 2 shows that, over the period 1980-2005, actively managed mutual funds have earned 2.1% risk-adjusted excess returns (alphas) per year in recessions but only 0.3% in expansions. What remains for investors (net of fees) is 1.0% in recessions and -0.9% in expansions; the difference of 1.9% per year is both economically and statistically significant.

In our model, recessions are periods of high aggregate payoff and return volatility. The same is true in the data. Identifying high-volatility periods as recessions allows us to make contact with the existing macroeconomics literature on rational inattention, e.g., Mackowiak and Wiederholt (2009a, 2009b). Arguably, periods of low and high economic activity are common knowledge whereas measuring earnings volatility requires paying close attention to aggregate earnings data, which our theory predicts not all managers choose to do. Nevertheless, our results are robust to using a measure of aggregate payoff volatility instead of the recession indicator as explanatory variable. In a horse race with both recession and aggregate volatility, we find that both contribute about equally to our three main results. An extended model, in which not only the quantity of risk but also the price of risk changes with the business cycle, provides a rationale for the effect of recessions that is not captured by volatility.

Because our theory tells us how skilled managers should invest, it suggests how to construct metrics that could help us identify skilled managers. To show that skilled managers exist, we select the top 25 percent of funds in terms of their stock-picking ability in expansions and show that the same group has significant market-timing ability in recessions; the other funds show no such market-timing ability. Furthermore, these funds have higher unconditional returns. They tend to manage smaller, more active funds. By matching fund-level to manager-level data, we find that these skilled managers are more likely to attract new money flows and are more likely to depart later in their careers to hedge funds. Presumably, both are market-based reflections of their ability. Finally, we construct a skill index based on observables and show that it is persistent and that it predicts future performance.

---

This is quite different from the typical approach in the literature, which has studied stock picking and market timing in isolation, and unconditional on the state of the economy. The consensus view from that literature is that there is some evidence for stock-picking ability (on average over time and across managers), but no evidence for market timing (e.g., Graham and Harvey (1996), Daniel, Grinblatt, Titman, and Wermers (1997), Wermers (2000), Kacperczyk and Seru (2007), and Breon-Drish and Sagi (2008)).
The rest of the paper is organized as follows. Section 1 lays out our model. After describing the setup, we characterize the optimal information and investment choices of skilled and unskilled investors. We show how equilibrium asset prices are formed. We derive theoretical predictions for funds’ attention allocation, portfolio dispersion, and performance. Section 2 contains the empirical analysis for actively managed mutual funds and tests the model’s predictions. Section 3 discusses the results that use volatility as a conditioning variable instead of or in addition to the recession indicator. Section 4 uses the model’s insights to identify a group of skilled mutual funds in the data. Section 5 discusses alternative explanations.

1 Model

We develop a stylized model whose purpose is to understand the optimal attention allocation of investment managers and its implications for asset holdings and equilibrium asset prices.

1.1 Setup

We consider a three-period static model. At time 1, skilled investment managers choose how to allocate their attention across aggregate and idiosyncratic shocks. At time 2, all investors choose their portfolios of risky and riskless assets. At time 3, asset payoffs and utility are realized. Since this is a static model, the investment world is either in the recession (R) or in the expansion state (E).

Our main model holds each manager’s total attention fixed and studies its allocation in recessions and expansions. In Section 1.7, we allow a manager to choose how much capacity for attention to acquire.

Assets The model features three assets. Assets 1 and 2 have random payoffs $f$ with respective loadings $b_1, b_2$ on an aggregate shock $a$, and face an idiosyncratic shock $s_1, s_2$. The third asset, $c$, is a composite asset. Its payoff has no idiosyncratic shock and a loading of one on the aggregate shock. We use this composite asset as a stand-in for all other assets to avoid the curse of dimensionality in the optimal attention allocation problem. Formally,

---

We do not consider transitions between recessions and expansions, although such an extension would be trivial in our setting because assets are short lived and their payoffs are realized and known to all investors at the end of each period. Thus, a dynamic model simply amounts to a succession of static models that are either in the expansion or in the recession state.
\[ f_i = \mu_i + b_i a + s_i, \; i \in \{1, 2\} \]
\[ f_c = \mu_c + a \]

where the shocks \( a \sim N(0, \sigma_a) \) and \( s_i \sim N(0, \sigma_i) \), for \( i \in \{1, 2\} \). At time 1, the distribution of payoffs is common knowledge; all investors have common priors about payoffs \( f \sim N(\mu, \Sigma) \). Let \( E_1, V_1 \) denote expectations and variances conditioned on this information. Specifically, \( E_1[f_i] = \mu_i \). The prior covariance matrix of the payoffs, \( \Sigma \), has the following entries: \( \Sigma_{ii} = b_i^2 \sigma_a + \sigma_i \) and \( \Sigma_{ij} = b_i b_j \sigma_a \). In matrix notation:

\[
\Sigma = bb' \sigma_a + \begin{bmatrix}
\sigma_1 & 0 & 0 \\
0 & \sigma_2 & 0 \\
0 & 0 & 0
\end{bmatrix}
\]

where the vector \( b \) is defined as \( b = [b_1 \; b_2 \; 1]' \). In addition to the three risky assets, there exists a risk-free asset that pays a gross return, \( r \).

We model recessions as periods with higher aggregate risk, that is, the prior variance of the aggregate shock in recessions is higher than the one in expansions: \( \sigma_a(R) > \sigma_a(E) \). Section 2.2 justifies this assumption by showing that aggregate risk of stocks increases substantially in recessions while idiosyncratic risk does not.

**Investors** We consider a continuum of atomless investors. In the model, the only ex-ante difference between investors is that a fraction \( \chi \) of them have *skill*, meaning that they can choose to observe a set of informative signals about the payoff shocks \( a \) or \( s_i \). We describe this signal choice problem below. The remaining unskilled investors observe no information other than their prior beliefs.

Some of the unskilled investors are investment managers. As in reality, there are also non-fund investors, all of whom we assume are unskilled.\(^9\) The reason for modeling non-fund investors is that without them, the sum of all funds’ holdings would have to equal the market (market clearing) and therefore, the average fund return would have to equal the market return. There could be no excess return in expansions or recessions.

\(^9\)For our results, it is sufficient that the fraction of non-fund investors that are unskilled is higher than that for the investment managers (funds).
Bayesian Updating  At time 2, each skilled investment manager observes signal realizations. Signals are random draws from a distribution that is centered around the true payoff shock, with a variance equal to the inverse of the signal precision that was chosen at time 1. Thus, skilled manager $j$’s signals are $\eta_{aj} = a + e_{aj}$, $\eta_{1j} = s_1 + e_{1j}$, and $\eta_{2j} = s_2 + e_{2j}$, where $e_{aj} \sim N(0, \tau_{aj})$, $e_{1j} \sim N(0, \tau_{1j})$, and $e_{2j} \sim N(0, \tau_{2j})$ are independent of each other and across fund managers. Managers combine signal realizations with priors to update their beliefs, using Bayes’ law. Of course, asset prices contain payoff-relevant information as well. We can allow managers to infer this information and subtract the amount of attention required to infer this information from their total attention endowment. However, Lemma S.2 in the Supplementary Appendix\textsuperscript{10} establishes that managers would always prefer not to use their attention to process the information in prices, when they could instead use the same amount of capacity to learn about private signals. Therefore, we model managers as if they observed prices, but did not exert the mental effort required to infer the payoff-relevant signals.

Since the resulting posterior beliefs (conditional on time-2 information) are such that payoffs are normally distributed, they can be fully described by posterior means, $(\hat{a}_j, \hat{s}_{ij})$, and variances, $(\hat{\sigma}_{aj}, \hat{\sigma}_{ij})$. More precisely, posterior precisions are the sum of prior and signal precisions: $\hat{\sigma}_{aj}^{-1} = \sigma_{aj}^{-1} + \tau_{aj}^{-1}$ and $\hat{\sigma}_{ij}^{-1} = \sigma_{i}^{-1} + \tau_{ij}^{-1}$. The posterior means of the idiosyncratic shocks, $\hat{s}_{ij}$, are a precision-weighted linear combination of the prior belief that $s_i = 0$ and the signal $\eta_i$: $\hat{s}_{ij} = \tau_{ij}^{-1} \eta_{ij}/(\tau_{ij}^{-1} + \sigma_{i}^{-1})$. Simplifying yields $\hat{s}_{ij} = (1 - \hat{\sigma}_{ij} \sigma_{i}^{-1}) \eta_{ij}$ and $\hat{a}_j = (1 - \hat{\sigma}_{aj} \sigma_{aj}^{-1}) \eta_{aj}$. Next, we convert posterior beliefs about the underlying shocks into posterior beliefs about the asset payoffs. Let $\hat{\Sigma}_j$ be the posterior variance-covariance matrix of payoffs $f$:

$$
\hat{\Sigma}_j = bb^t \hat{\sigma}_{aj} + \begin{bmatrix} \hat{\sigma}_{1j} & 0 & 0 \\ 0 & \hat{\sigma}_{2j} & 0 \\ 0 & 0 & 0 \end{bmatrix}
$$

Likewise, let $\hat{\mu}_j$ be the vector of posterior expected payoffs:

$$
\hat{\mu}_j = [\mu_1 + b_1 \hat{a}_j + \hat{s}_{1j}, \mu_2 + b_2 \hat{a}_j + \hat{s}_{2j}, \mu_c + \hat{a}_j]^t
$$

For any unskilled manager or investor: $\hat{\mu}_j = \mu$ and $\hat{\Sigma}_j = \Sigma$.

Portfolio Choice Problem  We solve this model by backward induction. We first solve for the optimal portfolio at time 2 and substitute in that solution into the time-1 optimal

\textsuperscript{10}References denoted S are in the paper’s separate appendix, available from the authors’ websites or at \url{http://pages.stern.nyu.edu/~lveldkam/pdfs/mfund_KVNV_appdx.pdf}
attention allocation problem.

Investors are each endowed with initial wealth, $W_0$. They have mean-variance preferences over time-3 wealth, with a risk aversion coefficient $\rho$. Let $E_2$ and $V_2$ denote expectations and variances conditioned on all information known at time 2. Thus, investor $j$ chooses $q_j$ to maximize time-2 expected utility, $U_{2j}$:

$$U_{2j} = \rho E_2[W_j] - \frac{\rho^2}{2} V_2[W_j]$$

subject to the budget constraint:

$$W_j = rW_0 + q_j'(f - pr)$$

After having received the signals and having observed the prices of the risky assets, $p$, the investment manager chooses risky asset holdings, $q_j$, where $p$ and $q_j$ are 3-by-1 vectors.

**Asset Prices**  Equilibrium asset prices are determined by market clearing:

$$\int q_j d_j = \bar{x} + x,$$

where the left-hand side of the equation is the vector of aggregate demand and the right-hand side is the vector of aggregate supply. As in the standard noisy rational expectations equilibrium model, the asset supply is random to prevent the price from fully revealing the information of informed investors. We denote the $3 \times 1$ noisy asset supply vector by $\bar{x} + x$, with a random component $x \sim N(0, \sigma_x I)$.

**Attention Allocation Problem**  At time 1, a skilled investment manager $j$ chooses the precisions of signals about the payoff-relevant shocks $a$, $s_1$, or $s_2$ that she will receive at time 2. We denote these signal precisions by $\tau_{aj}^{-1}$, $\tau_{1j}^{-1}$, and $\tau_{2j}^{-1}$, respectively. These choices maximize time-1 expected utility, $U_{1j}$, over the fund’s terminal wealth:

$$U_{1j} = E_1 \left[ \rho E_2[W_j] - \frac{\rho^2}{2} V_2[W_j] \right],$$

subject to two constraints.

The first constraint is the *information capacity constraint*. It states that the sum of the
signal precisions must not exceed the information capacity:

\[ \tau_{ij}^{-1} + \tau_{kj}^{-1} + \tau_{aj}^{-1} \leq K. \]  

(6)

Unskilled investors have no information capacity, \( K = 0 \). In Bayesian updating with normal variables, observing one signal with precision \( \tau^{-1} \) or two signals, each with precision \( \tau^{-1}/2 \), is equivalent. Therefore, one interpretation of the capacity constraint is that it allows the manager to observe \( N \) signal draws, each with precision \( K/N \), for large \( N \). The investment manager then chooses how many of those \( N \) signals will be about each shock.[11]

The second constraint is the no-forgetting constraint, which ensures that the chosen precisions are non-negative:

\[ \tau_{ij}^{-1} \geq 0 \quad \tau_{kj}^{-1} \geq 0 \quad \tau_{aj}^{-1} \geq 0. \]  

(7)

It prevents the manager from erasing any prior information, to make room to gather new information about another shock.

### 1.2 Model Solution

Substituting the budget constraint (3) into the objective function (2) and taking the first-order condition with respect to \( q_j \) reveals that optimal holdings are increasing in the investor’s risk tolerance, precision of beliefs, and expected return on the assets:

\[ q_j = \frac{1}{\rho} \hat{\Sigma}_j^{-1}(\hat{\mu}_j - pr). \]  

(8)

Since uninformed managers and non-fund investors have identical beliefs, \( \hat{\mu}_j = \mu \) and \( \hat{\Sigma}_j = \Sigma \), they hold identical portfolios \( \rho^{-1}\Sigma^{-1}(\mu - pr) \).

Appendix 7 utilizes the market-clearing condition (4) to prove that equilibrium asset prices are linear in payoffs and supply shocks, and to derive expressions for the coefficients \( A, B, \) and \( C \) in the following lemma:

**Lemma 1.** \( p = \frac{1}{r} (A + Bf + Cx) \)

Substituting optimal risky asset holdings from equation (8) into the first-period objective

[11] The results are not sensitive to the additive nature of the information capacity constraint. They also hold, for example, for a product constraint on precisions. The entropy constraints often used in information theory take this multiplicative form.
function (5) yields: 

\[ U_{1j} = \frac{1}{2} E_1 \left[ (\hat{\mu}_j - pr) \hat{\Sigma}_j^{-1} (\hat{\mu}_j - pr) \right] \].

Because asset prices are linear functions of normally distributed payoffs and asset supplies, expected excess returns, \( \hat{\mu}_j - pr \), are normally distributed as well. Therefore, \( (\hat{\mu}_j - pr) \hat{\Sigma}_j^{-1} (\hat{\mu}_j - pr) \) is a non-central \( \chi^2 \)-distributed variable, with mean \[ 12 U_{1j} = \frac{1}{2} \text{trace}(\hat{\Sigma}_j^{-1} V_1 [\hat{\mu}_j - pr]) + \frac{1}{2} E_1 [\hat{\mu}_j - pr]' \hat{\Sigma}_j^{-1} E_1 [\hat{\mu}_j - pr]. \] (9)

1.3 Bridging The Gap Between Model and Data

The following three sections explain the model’s three key predictions: attention allocation, dispersion in investors’ portfolios, and average performance. For each prediction, we state a hypothesis and explain how to test it. But the payoffs and quantities that have analytical expressions in a CARA-normal model do not correspond neatly to the returns and portfolio weights that are commonly measured in the data. To bridge this gap, we introduce empirical measures of attention, dispersion, and performance. These standard definitions of returns and portfolio weights have no known moment-generating functions in our model. For example, the asset return is a ratio of normally distributed variables. Therefore, Appendix S.1 uses a numerical example to demonstrate that the empirical and theoretical measures have the same comparative statics.

Specifically, our empirical measures use conventional definitions of asset returns, portfolio returns, and portfolio weights. Risky asset returns are defined as \( R^i \equiv \frac{f_i - p_{i-1}}{p_i} \), for \( i \in \{1, 2, c\} \), while the risk-free asset return is \( R^0 \equiv \frac{1 + r}{1 + r} - 1 = r \). We define the market return as the value-weighted average of the individual asset returns: \( R^m \equiv \sum_{i=1}^{3} w^m_i R^i \), where \( w^m_i \equiv \frac{p_i q_j^i}{\sum_{i=1}^{3} p_i q_j^i} \). Likewise, a fund \( j \)'s return is \( R^j \equiv \sum_{i=0}^{3} w^j_i R^i \), where \( w^j_i \equiv \frac{p_i q_j^i}{\sum_{i=0}^{3} p_i q_j^i} \). It follows that end-of-period wealth (assets under management) equals beginning-of-period wealth times the fund return: \( W^j = W^j_0 (1 + R^j) \).

1.4 Hypothesis 1: Attention Allocation

Each skilled manager \((K > 0)\) solves for the choice of signal precisions \( \tau_{aj}^{-1} \geq 0 \) and \( \tau_{ij}^{-1} \geq 0 \) that maximize her time-1 expected utility (5). The choice of signal precision \( \tau_{aj}^{-1} \geq 0 \) is implied by the capacity constraint (6). A robust prediction of our model is that it becomes

\[ 12 \text{If } z \sim N(E[z], Var[z]), \text{ then } E[z'z] = \text{trace}(Var[z]) + E[z]'E[z], \text{ where trace is the matrix trace (the sum of its diagonal elements). Setting } z = \hat{\Sigma}_j^{-1/2} (\hat{\mu}_j - pr) \text{ delivers the result. Appendix 7.2 contains the expressions for } E_1 [\hat{\mu}_j - pr] \text{ and } V_1 [\hat{\mu}_j - pr]. \]
relatively more valuable to learn about the aggregate shock, $a$, when the prior aggregate variance increases, that is, in recessions.

**Proposition 1.** If aggregate variance is not too high ($\sigma_a \leq 1$), then the marginal value of a given investor $j$ reallocating an increment of capacity from stock-specific shock $i \in \{1, 2\}$ to the aggregate shock is increasing in the aggregate shock variance: If $K_{aj} = \bar{K}$ and $K_{ij} = K - \bar{K}$, then $\partial^2 U / \partial \bar{K} \partial \sigma_a > 0$.

The proofs of this and all further propositions are in Appendix 7. Intuitively, in most learning problems, investors prefer to learn about large shocks that are an important component of the overall asset supply, and volatile shocks that have high prior payoff variance. Aggregate shocks are larger in scale, but are less volatile than stock-specific shocks. Recessions are times when aggregate volatility increases, which makes aggregate shocks more valuable to learn about. As a result, in recessions, skilled investment managers allocate a relatively larger fraction of their attention to learning about the aggregate shock. The converse is true in expansions. The parameter restriction $\sigma_a < 1$, is a sufficient, but not necessary condition.\(^{13}\)

Appendix S.1 presents a detailed numerical example in which parameters are chosen to match the observed volatilities of the aggregate and individual stock returns in expansions and recessions. For our benchmark parameter values, all skilled managers exclusively allocate attention to idiosyncratic shocks in expansions. In contrast, the bulk of skilled managers learn about the aggregate shock in recessions (87%, with the remaining 13% equally split between shocks 1 and 2). Thus, managers may want to reallocate their attention over the business cycle.

We have verified that similarly large swings in attention allocation occur for a wide range of parameters. The result breaks down when assets become very asymmetric so that one learning decision is dominant in recessions and expansions. For example, if the average supply of the composite asset, $\bar{x}_c$, is too large relative to the supply of the individual asset supplies, $\bar{x}_1$ and $\bar{x}_2$, the aggregate shock will be so valuable to learn about that all skilled managers will want to learn about it all the time. Similarly, if the aggregate volatility, $\sigma_a$, is too low, then nobody ever wants to learn about the aggregate shock.

Investors’ optimal attention allocation decisions are reflected in their portfolio holdings. In recessions, skilled investors predominantly allocate attention to the aggregate payoff shock,\(^{13}\)

\(^{13}\)Of the seven terms in expected utility, six can be signed without parameter restrictions and one requires this restriction for the derivative to be positive. This constraint does not seem tight in the sense that a value for $\sigma_a$ of 0.13 in expansions and 0.25 in recessions are the parameter choices that replicate the observed volatility of aggregate stock market returns in our simulation.
a. They use the information they observe to form a portfolio that covaries with \( a \). In times when they learn that \( a \) will be high, they hold more risky assets whose returns are increasing in \( a \). This positive covariance can be seen from equation (8) in which \( q \) is increasing in \( \hat{\mu}_j \) and from equation (11) in which \( \hat{\mu}_j \) is increasing in \( \hat{a}_j \), which is further increasing in \( a \). The positive covariances between the aggregate shock and funds’ portfolio holdings in recessions, on the one hand, and between idiosyncratic shocks and the portfolio holdings in expansions, on the other hand, directly follow from optimal attention allocation decisions switching over the business cycle. As such, these covariances are the key moments that enable us to test the attention allocation predictions of the model.

We define a fund’s reliance on aggregate information, \( RAI \), as the covariance between its portfolio weights in deviation from the market portfolio weights, \( w^j_i - w^m_i \), and the aggregate payoff shock, \( a \):

\[
RAI^j_t = \frac{1}{N} \sum_{i=1}^{N} (w^j_{it} - w^m_{it})(a_{t+1}),
\]

(10)

where \( N \) is the number of individual assets. The subscript \( t \) on the portfolio weights and the subscript \( t + 1 \) on the aggregate shock signify that the aggregate shock is unknown at the time of portfolio formation. In our static model, time \( t \) is period 2 and time \( t + 1 \) is period 3. Relative to the market, a fund with a high \( RAI \) overweights assets that have high (low) sensitivity to the aggregate shock in anticipation of a positive (negative) aggregate shock realization and underweights assets with a low (high) sensitivity.

\( RAI \) is closely related to measures of market-timing ability. \( Timing \) measures how a fund’s holdings of each asset, relative the market, covary with the systematic component of the stock return:

\[
Timing^j_t = \frac{1}{N} \sum_{i=1}^{N} (w^j_{it} - w^m_{it})(\beta_{it} R^m_{t+1}),
\]

(11)

where \( \beta_i \) measures the covariance of asset \( i \)’s return, \( R^i \), with the market return, \( R^m \), divided by the variance of the market return. The object \( \beta_i R^m \) measures the systematic component of returns of asset \( i \). The time subscripts indicate that the systematic component of the return is unknown at the time of portfolio formation. Before the market return rises, a fund with a high \( Timing \) ability overweights assets that have high beta. Likewise, it underweights assets with a high beta in anticipation of a market decline.

To confirm that \( RAI \) and \( Timing \) accurately represent the model’s prediction that skilled investors allocate more attention to the aggregate state in recessions, we resort to a numerical simulation. Appendix S.1 details the procedure and the construction of the empirical
measures. For brevity, we only discuss the comparative statics in the main text. The simulation results show that $RAI$ and $Timing$ are higher for skilled investors in recessions than they are in expansions. Because of market clearing, not all investors can time the market. Unskilled investors have negative timing ability in recessions. When the aggregate state $a$ is low, most skilled investors sell, pushing down asset prices, $p$, and making prior expected returns, $(\mu - pr)$, high. Equation 8 shows that uninformed investors’ asset holdings increase in $(\mu - pr)$. Thus, their holdings covary negatively with aggregate payoffs, making their $RAI$ and $Timing$ measures negative. Since no investors learn about the aggregate shock in expansions, $RAI$ and $Timing$ are close to zero for both skilled and unskilled. When averaged over all funds (including both skilled and unskilled funds but excluding non-fund investors), we find that $RAI$ and $Timing$ are higher in recessions than in expansions.

When skilled investment managers allocate attention to stock-specific payoff shocks, $s_i$, information about $s_i$ allows them to choose portfolios that covary with $s_i$. We define reliance on stock-specific information, $RSI$, which measures the covariance of a fund’s portfolio weights of each stock, relative to the market, with the stock-specific shock, $s_i$:

$$RSI_i^j = \frac{1}{N} \sum_{i=1}^{N} (w_{it}^j - w_{it}^m)(s_{it+1})$$

(12)

How well the manager can choose portfolio weights in anticipation of future asset-specific payoff shocks is closely linked to her stock-picking ability. $Picking_i^j$ measures how a fund’s holdings of each stock, relative to the market, covary with the idiosyncratic component of the stock return:

$$Picking_i^j = \frac{1}{N} \sum_{i=1}^{N} (w_{it}^j - w_{it}^m)(R_{it+1}^i - \beta_iR_{it+1}^m)$$

(13)

A fund with a high $Picking$ ability overweights assets that have subsequently high idiosyncratic returns and underweights assets with low subsequent idiosyncratic returns. In our simulation, we find that skilled funds have positive $RSI$ and $Picking$ ability in expansions, when they allocate their attention to stock-specific information. Unskilled investors have negative $Picking$ in expansions for the same reason that they have negative $Timing$ in recessions: Price fluctuations induce them to buy when returns are low and sell when returns are high. Across all funds, the model predicts lower $RSI$ and $Picking$ in recessions.
1.5 Hypothesis 2: Dispersion

A second, more fundamental question is whether investment managers are processing information at all. One prediction that speaks directly to that question is portfolio dispersion. In recessions, as aggregate shocks become more volatile, the idiosyncratic shocks to assets’ payoffs account for less of the variation, and the comovement in stock payoffs rises. Since asset payoffs comove more, the payoffs to all investment strategies that put fixed weights on assets should also comove more.

When investment managers are processing information, this prediction is reversed. To see why, consider the Bayesian updating formula for the posterior mean of asset payoffs. It is a weighted average of the prior mean \( \mu \) and the fund \( j \)’s signal \( \eta_j | f \sim N(f, \Sigma_\eta) \), where each is weighted by their relative precision:

\[
E[f|\eta_j] = (\Sigma^{-1} + \Sigma_\eta^{-1})^{-1} (\Sigma^{-1} \mu + \Sigma_\eta^{-1} \eta_j)
\]  

(14)

In recessions, when the variance of the aggregate shock \( \sigma_a \) rises, the prior beliefs about asset payoffs are more uncertain, \( \Sigma \) rises, and \( \Sigma^{-1} \) falls. This makes the weight on prior beliefs \( \mu \) decrease and the weight on the signal \( \eta_j \) increase. The prior \( \mu \) is common across agents, while the signal \( \eta_j \) is heterogeneous. When informed managers weigh their heterogeneous signals more, their resulting posterior beliefs become more different from each other and more different from the beliefs of uninformed managers. More disagreement about what asset payoffs will result in more heterogeneous portfolios and portfolio returns.

Thus, the model’s second prediction is that in recessions, the cross-sectional dispersion in funds’ investment strategies and returns rises. The following proposition shows that funds’ portfolio returns, \( q_j'(f - pr) \), display higher cross-sectional dispersion when aggregate risk is higher, in recessions.

**Proposition 2.** If the average manager has sufficiently low capacity, \( \chi K < \sigma_a^{-1} \), then an increase in aggregate risk, \( \sigma_a \), increases the dispersion of funds’ portfolios \( E[(q_j - \bar{q})(q_j - \bar{q})'] \), and their portfolio returns \( E[((q_j - \bar{q})'(f - pr))^2] \), where \( \bar{q} \equiv \int q_jdj \).

As before, the parameter restriction is sufficient, but not necessary.

To connect our model to the data, we use several measures of portfolio dispersion, commonly used in the empirical literature. The first one, proposed by Kacperczyk, Sialm, and Zheng (2005), is the sum of squared deviations of fund \( j \)’s portfolio weight in asset \( i \) at time
from the average fund’s portfolio weight in asset \( i \) at time \( t \), summed over all assets:

\[
Concentration_{jt} = \sum_{i=1}^{N} (w_{jt}^i - w_{it}^m)^2
\]  

We label this measure \( Concentration \) because, as any Herfindahl index, it is a measure of portfolio concentration. Cross-sectional dispersion and concentration are two sides of the same coin. Because markets must clear, funds cannot all hold concentrated portfolios without dispersion across their portfolios. Our numerical example shows that \( Concentration \) is higher for all funds in recessions than it is in expansions. In recessions, the portfolios of the informed managers differ more from each other and more from the uninformed investors. Some of this difference comes from a change in the composition of the risky asset portfolio and some comes from differences in the fraction of assets held in riskless securities. Fund \( j \)’s portfolio weight \( w_{jt}^i \) is a fraction of the fund’s assets, including both risky and riskless, held in asset \( i \). Thus, when one informed fund gets a bearish signal about the market, its \( w_{jt}^i \) for all risky assets \( i \) falls. Concentration can rise when funds take different positions in risky assets, even if the fractional allocation among the risky assets remains identical.

Because more concentrated portfolios are less diversified, the model predicts that a skilled fund’s returns contain higher idiosyncratic risk in recessions. We define idiosyncratic portfolio risk as the residual standard deviation, \( \sigma_{jt}^i \), from a CAPM regression for fund \( j \):

\[
R_{jt} = \alpha_j + \beta_j R_{mt} + \sigma_{jt}^i \varepsilon_{jt}
\]  

In the simulation, skilled funds take on more idiosyncratic risk than the unskilled ones, and more so in recessions than in expansions. As a result, idiosyncratic risk, our second measure of portfolio dispersion, is higher in recessions than it is in expansions for all funds.

The higher dispersion across funds’ portfolio strategies translates into a higher cross-sectional dispersion in fund returns. We look at dispersion in the funds’ abnormal returns, \( R_j - R^m \), CAPM alphas, \( \alpha_j \) from equation (16), and CAPM betas, \( \beta_j \). To facilitate comparison with the data, we define the dispersion of variable \( X \) as the average over funds of \( |X_j - \bar{X}| \). The notation \( \bar{X} \) denotes the equally weighted cross-sectional average across all investment managers (excluding non-fund investors). Our numerical results show a higher

\[14\] The terminology idiosyncratic risk is slightly misleading in our context. In fact, the portfolio is not riskier as skilled managers obtain information which reduces risk. They optimally trade off the benefits from information against the costs of a reduction in diversification. The standard CAPM equation does not capture this tradeoff because it does not condition on what the manager knows.
dispersion of fund abnormal returns, alphas, and betas.

1.6 Hypothesis 3: Performance

The third prediction of the model is that the average performance of investment managers is higher in recessions than it is in expansions. The following proposition shows that skilled funds’ abnormal portfolio returns, defined as their portfolio return, $q_j'(f - pr)$, minus the market return, $\bar{q}'(f - pr)$, are higher when aggregate risk is higher, that is, in recessions.

**Proposition 3.** If some managers are uninformed, $\chi < 1$, but all informed managers learn about aggregate risk, and the average manager has sufficiently low capacity, $\chi K < \sigma_a^{-1}$, then an increase in aggregate risk, $\sigma_a$, increases the expected profit of an informed fund, $E[(q_j - \bar{q})'(f - pr)]$, where $\bar{q} \equiv \int q_j dj$.

Because asset payoffs are more uncertain, recessions are times when information is more valuable. Therefore, the advantage of the skilled over the unskilled increases in recessions. This informational advantage generates higher returns for informed managers. In equilibrium, market clearing dictates that alphas average to zero across all investors. However, because our data only include mutual funds, our model calculations similarly exclude non-fund investors. Since investment managers are skilled or unskilled, while other investors are only unskilled, an increase in the skill premium implies that the average manager’s alpha rises in recessions. The same argument holds for the abnormal return.

Our numerical simulations confirm that abnormal returns and alphas, defined as in the empirical literature, and averaged over all funds, are higher in recessions than in expansions. Skilled investment managers have positive excess returns, while the uninformed ones have negative excess returns. Aggregating across skilled and unskilled funds results in higher average alphas in recessions, the third main prediction of the model.

1.7 Endogenous Capacity Choice

So far, we have assumed that skilled investment managers choose how to allocate a fixed information-processing capacity, $K$. We now extend the model to allow for skilled managers to add capacity at a cost $C(K)$.

---

We model this cost as a utility penalty, akin to the disutility from labor in business cycle models. Since there are no wealth effects in our setting, it would be equivalent to modeling a cost of capacity through the budget constraint. For a richer treatment of information production modeling, see Veldkamp (2006).
Propositions 1-3 hold for any chosen level of capacity $K$, below an upper bound, no matter the functional form of $C$. Endogenous capacity only has quantitative, not qualitative implications. Second, because the marginal utility of learning about the aggregate shock is increasing in its prior variance (Proposition 1), skilled managers choose to acquire higher capacity in recessions. This extensive-margin effect amplifies our benchmark intensive-margin results. Third, the degree of amplification depends on the convexity of the cost function, $C(K)$. The convexity determines how elastic equilibrium capacity choice is to the cyclical changes in the marginal benefit of learning. Appendix S.1.4 discusses numerical simulation results from the endogenous-$K$ model; they are similar to our benchmark results.

2 Evidence from Equity Mutual Funds

Our model studies attention allocation over the business cycle, and its consequences for investors’ strategies. We now turn to a specific set of investment managers, active mutual fund managers, to test the predictions of the model. The richness of the data makes the mutual fund industry a great laboratory for this test. In principle, similar tests could be conducted for hedge funds, other professional investment managers, or even individual investors.

2.1 Data

Our sample builds upon several data sets. We begin with the Center for Research on Security Prices (CRSP) survivorship bias-free mutual fund database. The CRSP database provides comprehensive information about fund returns and a host of other fund characteristics, such as size (total net assets), age, expense ratio, turnover, and load. Given the nature of our tests and data availability, we focus on actively managed open-end U.S. equity mutual funds. We further merge the CRSP data with fund holdings data from Thomson Financial. The total number of funds in our merged sample is 3,477.

In addition, for some of our exercises, we map funds to the names of their managers using information from CRSP, Morningstar, Nelson’ Directory of Investment Managers, Zoominfo, and Zabasearch. This mapping results in a sample with 4,267 managers. We also use the CRSP/Compustat stock-level database, which is a source of information on individual stocks’ return, market capitalization, book-to-market ratio, momentum, liquidity, and standardized unexpected earnings (SUE). We use changes in monthly industrial production as a proxy for aggregate shocks. Industrial production is seasonally adjusted; the data are from the Federal Reserve Statistical Release.
Finally, we measure recessions using the definition of the National Bureau of Economic Research (NBER) business cycle dating committee. The start of the recession is the peak of economic activity and its end is the trough. Our aggregate sample spans 312 months of data from January 1980 until December 2005, among which 38 are NBER recession months (12%). We consider several alternative recession indicators and find our results to be robust.

2.2 Recessions Are Periods of Higher Aggregate Risk

Before testing our main hypotheses, we present empirical evidence for the main assumption in our model: Recessions are periods in which individual stocks contain more aggregate risk. Table II shows that an average stock’s aggregate risk increases substantially in recessions whereas the change in idiosyncratic risk is not statistically different from zero. The table uses monthly returns for all stocks in the CRSP universe. For each stock and each month, we estimate a CAPM equation based on a twelve-month rolling-window regression, delivering the stock’s beta, $\beta_i t$, and its residual standard deviation, $\sigma_{i t}^\epsilon$. We define the aggregate risk of stock $i$ in month $t$ as $|\beta_i t \sigma_{i t}^m|$ and its idiosyncratic risk as $\sigma_{i t}^\epsilon$, where $\sigma_{i t}^m$ is formed monthly as the realized volatility from daily return observations. Panel A reports the results from a time-series regression of the aggregate risk averaged across stocks (Columns 1 and 2) and of the idiosyncratic risk averaged across stocks (Columns 3 and 4) on the NBER recession indicator variable.

The aggregate risk is twenty percent higher in recessions than it is in expansions (6.69% versus 8.04% per month), an economically and statistically significant difference. In contrast, the stock’s idiosyncratic risk is essentially identical in expansions and in recessions. The results are similar whether one controls for other aggregate risk factors (Columns 2 and 4) or not (Columns 1 and 3). Panel B reports estimates from panel regressions of a stock’s aggregate risk (Columns 1 and 2) or idiosyncratic risk (Columns 3 and 4) on the recession indicator variable, $Recession$, and additional stock-specific control variables including size, book-to-market ratio, and leverage. The panel results confirm the time-series findings.

We have confirmed our results using a dummy for negative real consumption growth, the Chicago Fed National Activity Index (CFNAI), and a dummy for the 25% lowest stock market returns as alternative recession indicators. While its salience makes the NBER indicator a natural benchmark, the other measures may be available in a timelier manner. Also, the CFNAI has the advantage that it is a continuous variable, measuring the strength of economic activity. As an example, Table S.12 in the separate appendix shows that the results on performance are, if anything, stronger with the CFNAI measure than with the NBER indicator. The other results are omitted for brevity but available from the authors upon request.

The reported results are for equally weighted averages. Unreported results confirm that value-weighted averaging across stocks delivers the same conclusion.
A large literature in economics and finance presents evidence supporting the results in Table 1. First, Ang and Chen (2002), Ribeiro and Veronesi (2002), and Forbes and Rigobon (2002) document that stocks exhibit more comovement in recessions, consistent with stocks carrying higher systematic risk in recessions. Second, Schwert (1989), Hamilton and Lin (1996), Campbell, Lettau, Malkiel, and Xu (2001), and Engle and Rangel (2008) show that aggregate stock market return volatility is much higher during periods of low economic activity. Diebold and Yilmaz (2008) find a robust cross-country link between volatile stock markets and volatile fundamentals. Third, Bloom, Floetotto, and Jaimovich (2009) find that the volatilities of GDP and industrial production growth, obtained by GARCH estimation, and the volatility implied by stock options are much higher during recessions. The same is true for the uncertainty in several establishment-, firm- and industry-level payoff measures they consider.

### 2.3 Testing Hypothesis 1: Attention Allocation

We begin by testing the first and most direct prediction of our model, that skilled investment managers reallocate their attention over the business cycle. Learning about the aggregate payoff shock in recessions makes managers choose portfolio holdings that covary more with the aggregate shock. Conversely, in expansions their holdings covary more with stock-specific information. To this end, we estimate the following regression model:

\[
\text{Attention}_j^t = a_0 + a_1 \text{Recession}_t + a_2 \text{X}_j^t + \epsilon^t, \tag{17}
\]

where \( \text{Attention}_j^t \) denotes a generic attention variable, observed at month \( t \) for fund \( j \). \( \text{Recession}_t \) is an indicator variable equal to one if the economy in month \( t \) is in recession, as defined by the NBER, and zero otherwise. \( \text{X} \) is a vector of fund-specific control variables, including the fund age (natural logarithm of age in years since inception, \( \log(\text{Age}) \)), the fund size (natural logarithm of total net assets under management in millions of dollars, \( \log(\text{TNA}) \)), the average fund expense ratio (in percent per year, \( \text{Expenses} \)), the turnover rate (in percent per year, \( \text{Turnover} \)), the percentage flow of new funds (defined as the ratio of \( \text{TNA}_t - \text{TNA}_{t-1}(1 + R_t^j) \) to \( \text{TNA}_{t-1} \), \( \text{Flow} \)), and the fund load (the sum of front-end and back-end loads, additional fees charged to the customers to cover marketing and other expenses, \( \text{Load} \)). Also included are the fund style characteristics along the size, value, and momentum dimensions.\(^{18}\) To mitigate the impact of outliers on our estimates, we winorize

\(^{18}\)The size style of a fund is the value-weighted score of its stock holdings’ percentile scores calculated with respect to their market capitalizations (1 denotes the smallest size percentile; 100 denotes the largest
Flow and Turnover at the 1% level.

We estimate this and most of our subsequent regression specifications using pooled (panel) regression and calculating standard errors by clustering at the fund and time dimensions. This approach addresses the concern that the errors, conditional on independent variables, might be correlated within fund and time dimensions (e.g., Moulton (1986) and Thompson (2009)). Addressing this concern is especially important in our context since our variable of interest, Recession, is constant across all fund observations in a given time period. Also, we demean all control variables so that the constant \( a_0 \) can be interpreted as the level of the attention variable in expansions, and \( a_1 \) indicates how much the variable increases in recessions.

The first attention variable we examine is reliance on aggregate information, \( RAI \), as in equation (10). We proxy for the aggregate payoff shock with the innovation in log industrial production growth. A time series for \( RAI_j^t \) is obtained by computing the covariance of the innovations and each fund \( j \)’s portfolio weights using twelve-month rolling windows. Our hypothesis is that \( RAI \) should be higher in recessions, which means that the coefficient on Recession, \( a_1 \), should be positive.

Our estimates of the parameters appear in Table 2. Column 1 shows the results for a univariate regression. In expansions, \( RAI \) is not different from zero, implying that funds’ portfolios do not comove with future macroeconomic information in those periods. In recessions, \( RAI \) increases. Both findings are consistent with the model. The increase amounts to ten percent of a standard deviation of \( RAI \). It is measured precisely, with a t-statistic of 3. To remedy the possibility of a bias in the coefficient due to omitted fund characteristics correlated with recession times, we turn to a multivariate regression. Our findings, presented in Column 2, remain largely unaffected by the inclusion of the control variables.

Next, we repeat our analysis using funds’ reliance on stock-specific information (RSI) as a dependent variable. Using equation (12), the RSI metric is computed in each month \( t \) as a cross-sectional covariance across the assets between the fund’s portfolio weights and size percentile). The value style is the value-weighted score of its stock holdings’ percentile scores calculated with respect to their book-to-market ratios (1 denotes the smallest B/M percentile; 100 denotes the largest B/M percentile). The momentum style is the value-weighted score of a fund’s stock holdings’ percentile scores calculated with respect to their past twelve-month returns (1 denotes the smallest return percentile; 100 denotes the largest return percentile). These style measures are similar in spirit to those defined in Kacperczyk, Sialm, and Zheng (2005) and Huang, Sialm, and Zhang (2009).

\(^{19}\)We regress log industrial production growth at \( t+1 \) on log industrial production growth in month \( t \), and use the residual from this regression. Because industrial production growth is nearly i.i.d, the same results obtain if we simply use the log change in industrial production between \( t \) and \( t+1 \).
firm-specific earnings shocks. In the model, the fund’s portfolio holdings and its returns covary more with subsequent firm-specific shocks in expansions. Therefore, our hypothesis is that \( RSI \) should fall in recessions, meaning that \( a_1 \) should be negative.

Columns 3 and 4 of Table 2 show that the average \( RSI \) across funds is positive in expansions and substantially lower in recessions. The effect is statistically significant at the 1% level. It is also economically significant: \( RSI \) decreases by approximately ten percent of one standard deviation. Overall, the data support the model’s prediction that portfolio holdings are more sensitive to aggregate shocks in recessions and more sensitive to firm-specific shocks in expansions.

Next, we examine market-timing, \( Timing_j^t \), and stock-picking ability, \( Picking_j^t \), defined in equations (11) and (13). The benefit of using these variables is that they have an exact analog in the model. In contrast, for \( RAI \) and \( RSI \), we need to take a stance on the empirical proxy for the aggregate and idiosyncratic shocks. The stock betas, \( \beta_i \), in \( Timing \) and \( Picking \) are computed using the twelve-month rolling-window regressions of stock excess returns on market excess returns.

Columns 5 and 6 of Table 2 show that the average market-timing ability across funds increases significantly in recessions. In turn, we find no evidence of market timing in expansions. Since expansion months make up the bulk of our sample, this result is consistent with the literature which fails to find evidence for market timing, on average. However, we find that market timing is positive and statistically different from zero in recessions. The increase is 25 percent of a standard deviation of the \( Timing \) measure, which is economically meaningful. Likewise, Columns 7 and 8 show that stock-picking ability deteriorates substantially in recessions, again consistent with the theory. The reduction in recessions is about 20 percent of a standard deviation of the \( Picking \) measure.

Table S.5 performs several robustness checks. First, we compute an alternative \( RAI \) measure, in which the aggregate shock is proxied by surprises in non-farm employment growth, another salient macroeconomic variable, instead of industrial production growth. Second, we compute an alternative \( RSI \) measure in which earnings surprises are defined as the residual from a regression of earnings per share in a given year on earnings per share in that same quarter one year earlier (instead of one quarter earlier), as in Bernard and

---

\(^{20}\)We regress earnings per share in a given quarter on earnings per share in the previous quarter (earnings are reported quarterly), and use the residual from this regression. Suppose month \( t \) and \( t + 3 \) are end-of-quarter months. Then \( RSI \) in months \( t, t + 1, \) and \( t + 2 \) are computed using portfolio weights from month \( t \) and earnings surprises from month \( t + 3 \).

\(^{21}\)We have checked that the firm-specific earnings shocks are uncorrelated with the aggregate earnings shocks. The median correlation across stocks is below 0.01, with a cross-sectional standard deviation of 0.28.
Thomas (1989). Third, to check the market-timing results, we also study the $R^2$ from a CAPM regression at the fund level, as in equation (16). It measures how the funds’ excess returns (as opposed to their portfolio weights) covary with the aggregate state, as measured by the market’s excess return. All the results are similar to our benchmark result, and in the case of employment growth, are estimated even more precisely.

To further understand how funds improve their market timing in recessions, we conduct several exercises. We find they increase their cash holdings, reduce their holdings of high-beta stocks, and tilt their portfolios towards more defensive sectors. Tables S.6, S.7, and S.8 present the results; a more detailed discussion is in Appendix S.2.1.

### 2.4 Testing Hypothesis 2: Dispersion

The second prediction of the model is that heterogeneity in fund investment strategies and portfolio returns rises in recessions. To test this hypothesis, we estimate the following regression specification, using various return and investment heterogeneity measures, denoted as $Dispersion_j^t$, the dispersion of fund $j$ at month $t$.

$$Dispersion^t_j = b_0 + b_1Recession^t + b_2X^t_j + \epsilon^t_j, \quad (18)$$

The definitions of $Recession$ and other control variables mirror those in regression (17). Our coefficient of interest is $b_1$.

We begin by examining dispersion in investment strategies. The results are in Table 3. Our first measure is a fund’s portfolio $Concentration$, defined in equation (15). Funds whose holdings deviate more from the S&P 500 portfolio, and therefore from other investors, have higher levels of portfolio concentration; they pursue more active investment strategies. In contrast, when all funds hold the market portfolio, average concentration and portfolio dispersion are zero. The results, in Columns 1 and 2, indicate an increase in average $Concentration$ across funds in recessions. The increase is statistically significant at the 1% level. It is also economically significant: The value of stock concentration in recessions goes up by about 15% of a standard deviation.

An alternative way to assess a fund’s concentration level is to look at its degree of idiosyncratic risk. A more concentrated portfolio carries more idiosyncratic risk, $\sigma^j_\epsilon$, according to the CAPM regression (16). Columns 3 and 4 show that the idiosyncratic volatility increases in recessions. The increase is highly significant, statistically and economically. One concern with the CAPM-based measure of idiosyncratic risk is that it might not capture the possi-
bility that some fund returns load on passive factors besides the market return. Therefore, we recompute idiosyncratic volatility, controlling for a fund’s exposure to size (SMB), value (HML), and momentum (UMD) factors. The resulting Recession coefficient in a univariate regression is 0.347 and the intercept is 1.189. Controlling for fund characteristics changes the coefficients by 1% or less.

Since dispersion in fund strategies should generate dispersion in fund returns, we next look for evidence of higher return dispersion in recessions. To measure dispersion in return variable $X$, we use the absolute deviation between fund $j$’s value and the equally weighted cross-sectional average, $|X_j^t - \bar{X}_t|$, as the dependent variable in (18). Columns 5 and 6 of Table 3 present the results for the dispersion in the funds’ CAPM alphas, which are obtained from twelve-month rolling-window regressions of fund excess returns on market excess returns. Comparing the slope $b_1$ to the intercept $b_0$, we find a 50% dispersion increase in recessions. The effect is measured precisely. Columns 7 through 8 show that using four-factor alphas in place of CAPM alphas does not change the result. Finally, Columns 9 and 10 show that the CAPM-beta dispersion also increases by about 30% in recessions, as investment managers take different directional bets in their investment strategies. The increased dispersions in abnormal returns, alphas, and betas are all consistent with the predictions of our model.

Table S.3 (in the Appendix) considers additional measures of portfolio and return dispersion. For example, we show that managers shift their investment styles more in recessions, consistent with more active portfolio management. Their funds also exhibit greater industry concentration in recessions. Next, we show that the dispersion of fund returns minus the market return nearly doubles in recessions. In unreported results, we obtain similar results for the dispersion of CAPM alpha and betas that are calculated by estimating their dependence on the aggregate dividend-price ratio, the term spread, the short-term interest rate, and the default spread, in one full-sample regression (Avramov and Wermers 2006). Finally, we study the dispersion in the information ratio, defined as the ratio of the CAPM alpha to the CAPM residual volatility. These results further strengthen the evidence of the increased dispersion in recessions.

### 2.5 Testing Hypothesis 3: Performance

The third prediction of our model is that recessions are times when information allows funds to earn higher average risk-adjusted returns, on average. We evaluate this hypothesis using
the following regression specification:

$$\text{Performance}_t^j = c_0 + c_1 \text{Recession}_t + c_2 X_t^j + \epsilon_t^j$$

(19)

where $\text{Performance}_t^j$ denotes the fund $j$’s performance in month $t$ using previously introduced measures of abnormal fund returns, CAPM, three-factor, and four-factor alphas. $\text{Recession}$ and the control variables, $X$, are defined as before. All returns are expressed net of management fees. Our coefficient of interest is $c_1$.

Table 4, Column 1, shows that the average fund’s net return is 3bp per month less than the market return in expansions, but it is 34bp per month higher in recessions. This difference is highly statistically significant and becomes even larger (42bp), after we control for fund characteristics (Column 2). Similar results (Columns 3 and 4) obtain when we use the CAPM alpha as a measure of fund performance, except that the alpha in expansions becomes negative. When we use alphas based on the three-factor and four-factor models, the recession return premium diminishes (Columns 5 through 8). But in recessions, the four-factor alpha still represents a non-trivial 1% per year risk-adjusted excess return, 1.6% higher than the -0.6% recorded in expansions (significant at the 1% level).

The cross-sectional regression model allows us to include a host of fund-specific control variables, making use of rich panel data. But because performance is measured using past twelve-month rolling-window regressions, a given observation for the dependent variable can be classified as a recession when some or even all of the remaining eleven months of the window are expansions. To verify the robustness of our results, we also employ a time-series approach. In each month, we form the equally weighted portfolio of funds and calculate its net return, in excess of the risk-free rate. We then regress this time series of fund portfolio returns on $\text{Recession}$ and common risk factors. We adjust standard errors for heteroscedasticity and autocorrelation (Newey and West 1987). Table S.10 shows that our previous results remain largely unchanged.

Our results are robust to alternative performance measures. Table S.11 uses gross fund returns and alphas. In unreported results, we also use the information ratio (the ratio of the CAPM alpha to the CAPM residual volatility) as a performance measure. It increases sharply in recessions. Finally, we find similar results when we lead alpha on the left-hand side by one month instead of using a contemporaneous alpha. All results point in the same direction: Outperformance clusters in periods of recessions.

While our model is silent about the distinction between funds and fund managers, in practice, skill could be embodied in the manager or be produced by the organizational setup.
the fund provides that manager. We rerun our main results using the cross-section of data at the manager level, tracking the fund(s) a manager works for over time. Table S.14 shows that the recession effects on RAI/RSI, dispersion, and performance are essentially unchanged, suggesting that the distinction is not important for our results.

3 Recession versus Volatility

In our model recessions are times of higher aggregate payoff volatility, and hence higher stock return volatility. In Section 2.2, we show that the same is true in the data. This motivates our use of recessions in the empirical work. The link between recessions and aggregate volatility, however, suggests an additional way of testing the model, namely to replace the recession indicator with an indicator for high aggregate payoff volatility. The high-volatility indicator variable we construct equals one in months with the highest volatility of aggregate earnings growth, where aggregate volatility is estimated from Shiller’s S&P 500 earnings growth data.\footnote{We chose the volatility cutoff such that 12% of months are selected, the same fraction as NBER recession months.} As predicted by our theory, we find that RAI, dispersion, and performance all rise in high volatility months, while RSI falls. For illustration, Table S.13 describes the performance results; the other results are omitted for brevity.

Given that both recessions and aggregate volatility qualitatively deliver the same predictions, it is natural to ask which of the two effects is stronger. To that end, we include both NBER recession and high aggregate payoff volatility indicators as explanatory variables in an empirical horse race. For brevity, Table 5 combines the headline results for RAI/RSI, portfolio dispersion, and performance in one table. It shows that both recession and volatility contribute to a lower RSI in expansions and a higher RAI in recessions, to a higher portfolio dispersion in recessions (concentration, alpha dispersion, and beta dispersion), and to a higher performance in recessions (four-factor alpha).\footnote{The results without controls are similar, as are the results for other dispersion and performance measures.} For some of the results the recession effect is slightly stronger, while for others the volatility effect is slightly stronger. Clearly, there is an effect of recessions beyond the one coming through volatility.

To understand why recessions have an incremental effect over volatility in explaining attention allocation, we consider an augmented model in which both the quantity and the price of risk rise in recessions. The idea that the price of risk rises in recessions is supported by a large asset pricing literature (e.g., the external habit model of Campbell and Cochrane (1999) or the variable rare disasters model of Gabaix (2009)). The parameter that governs
the price of risk in our model is risk aversion. The following result shows that an increase in
the price of risk (risk aversion) in recessions is an independent force driving the reallocation
of attention from stock-specific to aggregate shocks. Because of this additional channel,
recessions should generate more attention reallocation than a rise in aggregate volatility
alone, just as we see in the data. The proof is in Appendix 7.7.

**Proposition 4.** If the size of the composite asset $\bar{x}_c$ is sufficiently large, then an increase
in risk aversion increases the marginal utility of reallocating a unit of capacity from the
aggregate shock to the idiosyncratic shock: $\partial/\partial \rho (\partial U / \partial (\hat{\sigma}_{aj}^{-1} - \hat{\sigma}_{1j}^{-1})) > 0$.

The intuition for this result is that aggregate shocks affect a large fraction of the value
of one’s portfolio. Therefore, a marginal reduction in the uncertainty about an aggregate
shock reduces total portfolio risk by more than the same-sized reduction in the uncertainty
about a stock-specific shock. In other words, learning about aggregate shocks is the most
efficient way to reduce portfolio risk. The more risk-averse an agent is, the more attractive
aggregate attention allocation becomes.

4 Using Theory and Data to Identify Skilled Managers

Our analysis so far shows that the data are consistent with the three main predictions
of the model. This suggests we can use it to identify skilled investment managers. In
particular, we exploit the model’s prediction that skilled managers display market-timing
ability in recessions and stock-picking ability in expansions. We define market-timing and
stock-picking ability as in equations (11) and (13). Since the funds’ portfolio holdings in
each stock are observed at most quarterly, we assume that funds use buy-and-hold strategies
in non-disclosure periods. In these periods, the portfolio weights, $w_{jt}$, would only vary to
the extent that market prices vary.

4.1 The Same Managers Do Switch Strategies

We first test the prediction that the same investment managers with stock-picking ability in
expansions display market-timing ability in recessions. To this end, we first identify funds
with superior stock-picking ability in expansions: For all expansion months, we select all
fund-month observations that are in the highest 25% of the $Picking_{jt}$ distribution. We form
an indicator variable $Skill Picking (SP_j \in \{0, 1\})$ that is equal to 1 for the 25% of funds (884
funds) with the highest fraction of observations in the top, relative to the total number of
observations (in expansions) for that fund. Then, we estimate the following pooled regression model, separately for expansions and recessions:

$$ Ability^d_t = d_0 + d_1 SP^d_t + d_2 X^d_t + e^d_t, \quad (20) $$

where $Ability$ denotes either $Timing$ or $Picking$. $X$ is a vector of previously defined control variables. Our coefficient of interest is $d_1$.

Table 6, Column 3, confirms that $SP$ funds are significantly better at picking stocks in expansions, after controlling for fund characteristics. This is true by construction. The main point, however, is that these same $SP$ funds are also good at market timing in recessions. This result is evident from the recession-based market-timing regression in Column 2, in which the coefficient on $SP$ is statistically significant at the 5% level. Finally, we note that the funds in $SP$ do not exhibit superior market-timing ability in expansions (Column 1) nor superior stock-picking ability in recessions (Column 4), which confirms that $SP$ funds switch strategies.

Having identified a subset of skilled funds based on their time-varying investment strategies, the model predicts that this group should outperform the unskilled funds not only in recessions but also in expansions. Table 7 compares the unconditional performance of the $SP$ portfolio to that composed of all other funds. After controlling for various fund characteristics, the CAPM, three-factor, and four-factor alphas are 70-90 basis points per year higher for the $SP$ portfolio, a difference that is statistically and economically significant.

In Panel A of Table 8, we further compare the characteristics of the funds in the $Skill-Picking$ portfolio to those not included in the portfolio. We note several salient differences. First, funds in $SP$ are on average younger (by five years). Second, they have less wealth under management (by $400$ million), suggestive of decreasing returns to scale at the fund level, as in Berk and Green (2004) and Chen, Hong, Huang, and Kubik (2004). Third, they tend to charge higher expenses (by 0.26% per year), suggesting rent extraction from customers for the skill they provide. Fourth, they exhibit much higher turnover rates (about 130% per year, versus 80% per year for other funds), consistent with their more active-management styles. Fifth, they receive higher inflows of new assets to manage, consistent with their superior performance, and presumably a market-based reflection of their skill. Sixth, the $SP$ funds tend to hold more concentrated portfolios, with fewer stocks and higher stock-level and industry-level Herfindahl concentration. Seventh, their betas deviate more from their peers suggesting a strategy with different systematic risk exposure. Finally, they rely significantly more on aggregate information. Taken together, these findings begin to
paint a picture of what a typical skilled fund looks like.

To what extent can observable characteristics predict skill (SP)? Table S.15 reports the estimates from a linear-probability regression model of the SP indicator on fund characteristics, such as age, TNA, expenses, and turnover. The regression $R^2$ equals 14%. Including attributes that our theory links to skilled managers, such as stock and industry concentration, beta deviation, and RAI, increases the $R^2$ to 19%. Table S.15 Panel B, examines manager characteristics. SP fund managers are 2.6% more likely to have an MBA, are one year younger, and have 1.7 fewer years of experience. Interestingly, they are much more likely to depart for hedge funds later in their careers, suggesting that the market judges them to have superior skills.

The existence of skilled mutual funds with cyclical learning and investment strategies is not a fragile result. First, the results continue to hold if we change the cutoff levels for the inclusion in the SP portfolio. Second, we show that the top 25% RSI funds in expansions have higher RAIs in recessions and higher unconditional alphas (Tables S.16 and S.17). Third, we verify our results using Daniel, Grinblatt, Titman, and Wermers (1997)’s definitions of market timing (CT) and stock picking (CS). Finally, we reverse the sort, to show that funds in the top 25% of market-timing ability in recessions, have statistically higher stock-picking ability in expansions and higher unconditional alphas (Tables S.18 and S.19).

4.2 Creating a Skill Index

If one is going to use the model to identify skilled investment managers, it is important that she can identify these managers in real time, without looking at the full sample of the data. To this end, we construct a Skill Index that is informed by the main predictions of our model that attention allocation and investment strategies change over the business cycle. We define the Skill Index as a weighted average of Timing and Picking measures, in which the weights we place on each measure depend on the state of the business cycle:

$$\text{Skill Index}_t^j(z) = w(z_t)\text{Timing}_t^j + (1 - w(z_t))\text{Picking}_t^j, \text{ with } z_t \in \{E, R\}.$$  

We demean Timing and Picking, divide each by its standard deviation, and set $w(R) = 0.8 > w(E) = 0.2$ (the exact number is not crucial).

Subsequently, we examine whether the time-$t$ Index can predict future fund performance, measured by the CAPM, three-factor, and four-factor alphas one month (and one year) later. Table 9 shows that funds with a higher Skill Index have on average higher alphas.
For example, when Skill Index is zero (its mean), the alpha is -4bp per month. However, when the Skill Index is one standard deviation (0.83%) above its mean, the alpha is 1.1% (four-factor) or 2.4% (CAPM) higher per year. The three most right columns show similar predictive power of the Skill Index for one-year ahead alphas. As a robustness check, we construct a second skill index based on RAI and RSI instead of Timing and Picking. A one-standard-deviation increase in this skill index increases one-month-ahead alphas by 0.3-0.5% per year, a statistically significant effect (Table S.20).

A large literature investigates whether measures of skill persist through time (e.g., Carhart (1997), Brown and Goetzmann (1995)). To investigate whether the skill we identify exhibits persistence, we sort funds into quintiles based on their values of the Skill Index. We then track the Skill Index of the funds in each quintile over the next twelve months. Figure 3 shows a substantial amount of skill persistence, which is slowly declining over time. One interesting observation is that the best funds display the most persistence, which is in contrast with most of the literature, which usually finds persistence among the worst but not the best funds.

5 Alternative Explanations

We briefly explore other candidate explanations. The first alternative is that no skill exists. In that case, all the recession effects in fund returns would have to arise mechanically from the properties of asset returns. To rule this out, we calculate means, volatilities, alphas, betas, and idiosyncratic volatilities of individual stock returns, in the same way as we do it for mutual fund returns. None of these moments differ between expansions and recessions (except for higher volatility of asset returns in recessions, our driving force). Using a simulation, we verify that a mechanical mutual fund investment policy that randomly selects 50, 75, or 100 stocks cannot produce the observed counter-cyclical fund returns.

Second, we consider a potential sample selection issue. Suppose that managers have heterogeneous skill, but they do not display the cyclical variation in attention allocation we envision. Furthermore, suppose that the best managers leave the sample in good times, maybe because they go to a hedge fund. Then the composition effect would deliver lower alphas and less dispersion in expansions. If for some reason skill is associated with high RAI and low RSI, it could also explain the attention allocation results. There are at least three ways to refute this story. First, we redo our results with managers (instead of funds) as the unit of observation and include manager fixed effects. Table S.21 shows that our results
go through unchanged. Such fixed effects regressions are the standard response to sample selection concerns. Similarly, fund fixed effects do not change our fund-level results. Second, in Section 4 we showed that it is the same managers who have high RSI in expansions and high RAI in recessions, also going against the composition effect explanation. Third, Table S.22 shows not only a higher chance of being promoted or picked off by a hedge fund in expansions, it also shows a higher likelihood of being fired or demoted in recessions. The latter effect works in the opposite direction of the first.

Third, we consider career concerns as a potential explanation. Chevalier and Ellison (1999) show that young managers with career concerns may have an incentive to herd. Now imagine that in expansions the incentive to herd is strong, while in recessions young managers have to deviate from the pack to safeguard their jobs. We would then see higher dispersion in strategies and performance in recessions. In order to investigate this hypothesis, we first run our dispersion regressions at the manager-level adding the manager’s log age and log age interacted with the recession indicator as independent variables. The career concern hypothesis predicts a negative sign on the interaction term: younger managers should deviate more from the pack in recessions. Instead, we find a significantly positive interaction effect for our Concentration measure. The effect on idiosyncratic risk and beta dispersion is not statistically different from zero, while the effect on alpha dispersion is negative but only significant at the 10% level. It is worth noting that the sign on manager age itself is positive and significant, in line with the findings of Chevalier and Ellison (1999).²⁴ In sum, the results do not provide much evidence for the career hypothesis. Second, it is not clear how the hypothesis would account for the RAI/RSI and performance results. Third, we find no systematic differences between the age, educational background or experience of our managers in recessions versus expansions (Table S.23). While labor market considerations may be important to understand many aspects of the behavior of mutual fund managers, the above argument suggests that they cannot account for the patterns we document.

Fourth, Glode (2008) argues that funds outperform in recessions because their investors’ marginal utility is highest in such periods. While complementary to our explanation, his work remains silent on what strategies investment managers pursue to achieve this differential performance, and hence on our first and second hypothesis. In sum, while various explanations can account for some of the facts, we conclude that they are unlikely to account for all facts jointly.

²⁴Detailed results are omitted for brevity, but available upon request.
6 Conclusion

Do investment managers add value for their clients? The answer to this question matters for problems ranging from the discussion of market efficiency to a practical portfolio advice for households. The large amount of randomness in financial asset returns makes it a difficult question to answer. The multi-billion investment management business is first and foremost an information-processing business. We model investment managers not only as agents making optimal portfolio decisions, but also as ones who optimally allocate a limited amount of attention or information-processing capacity. Since the optimal attention allocation varies with the state of the economy, so do investment strategies and fund returns. As long as a subset of investment managers can process information about future asset payoffs, the model predicts a higher covariance of portfolio holdings with aggregate information, more dispersion in returns across funds, and a higher average outperformance, in recessions. We observe these patterns in investments and returns of actively managed U.S. mutual funds. Hence, the data are consistent with a world in which some investment managers have skill, but that skill is often hard to detect. Recessions are times when differences in performance are magnified and skill is easier to detect.

Beyond the mutual fund industry, a sizeable fraction of GDP now comes from industries that produce and process information. Increasing access to information through the internet has made the problem of how to best allocate a limited amount of information-processing capacity even more relevant. While information choices have consequences for real outcomes, they are often poorly understood because they are difficult to measure. By predicting how information choices are linked to observable variables (such as the state of the economy) and by tying information choices to real outcomes (such as portfolio investment), we show how models of information choices can be brought to the data. This information-choice-based approach could be useful in examining other information-processing sectors of the economy.
References


7 Proofs of Propositions

7.1 Mathematical Preliminaries

Expressing matrices in terms of fundamental variances To determine the effect of changes in aggregate shock variance on dispersion and profits, we need to express some of the matrices in terms of \( \sigma_a^{-1} \). If we can decompose the matrices into components that depend on \( \sigma_a \) and those that do not, we can differentiate the expressions more easily.

First, we decompose the payoff precision matrices. To do this decomposition, we need to invert \( \Sigma \). Doing it by hand yields

\[
\Sigma^{-1} = \begin{bmatrix}
\sigma_1^{-1} & 0 & -b_1\sigma_1^{-1} \\
0 & \sigma_2^{-1} & -b_2\sigma_2^{-1} \\
-b_1\sigma_1^{-1} & -b_2\sigma_2^{-1} & \sigma_a^{-1} + b_1^2\sigma_1^{-1} + b_2^2\sigma_2^{-1}
\end{bmatrix}
\] (21)

Similarly, the posterior precision matrix for investor \( j \) is

\[
\hat{\Sigma}_j^{-1} = \begin{bmatrix}
\hat{\sigma}_1^{-1} & 0 & -b_1\hat{\sigma}_1^{-1} \\
0 & \hat{\sigma}_2^{-1} & -b_2\hat{\sigma}_2^{-1} \\
-b_1\hat{\sigma}_1^{-1} & -b_2\hat{\sigma}_2^{-1} & \hat{\sigma}_a^{-1} + b_1^2\hat{\sigma}_1^{-1} + b_2^2\hat{\sigma}_2^{-1}
\end{bmatrix}
\] (22)

It is useful to separate out the terms that depend on \( \sigma_a \) from those that do not. Define

\[
S \equiv \begin{bmatrix}
\sigma_1^{-1} & 0 & -b_1\sigma_1^{-1} \\
0 & \sigma_2^{-1} & -b_2\sigma_2^{-1} \\
-b_1\sigma_1^{-1} & -b_2\sigma_2^{-1} & \sigma_a^{-1} + b_1^2\sigma_1^{-1} + b_2^2\sigma_2^{-1}
\end{bmatrix}
\] (23)

and let \( \hat{S} \) be the posterior \( S \), meaning that each \( \sigma_1 \) is replaced with the posterior variance \( \hat{\sigma}_1 \) and each \( \sigma_2 \) is replaced with the posterior variance \( \hat{\sigma}_2 \).

\[
\Upsilon_a \equiv \begin{bmatrix}
0 & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & 1
\end{bmatrix} \quad \text{and} \quad \Upsilon_1 \equiv \begin{bmatrix}
1 & 0 & -b_1 \\
0 & 0 & 0 \\
-b_1 & 0 & b_1^2
\end{bmatrix}
\] (24)

so that \( \partial \hat{\Sigma}_j^{-1}/\partial \sigma_a^{-1} = \Upsilon_a \) and \( \partial \hat{\Sigma}_j^{-1}/\partial \hat{\sigma}_a^{-1} = \Upsilon_1 \).

Then,

\[
\Sigma^{-1} = S + \sigma_a^{-1}\Upsilon_a
\] (25)

\[
\hat{\Sigma}_j^{-1} = \hat{S} + \hat{\sigma}_a^{-1}\Upsilon_a
\] (26)

Define the average posterior precision matrix when a fraction \( \chi \) of investment managers have capacity to be \( (\bar{\Sigma})^{-1} = \int_j \hat{\Sigma}_j^{-1} dj \). Similarly, let \( S^a = \int_j \hat{S} dj \) and let \( \bar{K}_a \) be the average amount of capacity that an agent devotes to processing aggregate information. For example, if a fraction \( \chi \) of investors are skilled, and all skilled investors devote all their capacity \( K \) to processing aggregate information, \( \bar{K}_a = \chi K \). Then,

\[
(\bar{\Sigma})^{-1} = S^a + (\sigma_a^{-1} + \chi K^a)\Upsilon_a
\] (27)
An expression that recurs frequently below is the difference between the precision of an informed manager’s posterior beliefs and the precision of the average manager’s posterior beliefs. This difference becomes

$$\Sigma^{-1} - (\bar{\Sigma})^{-1} = S - S^a + (1 - \chi)K \Upsilon_a$$

(28)

Second, we decompose the variance matrices. In particular, we need to know average variance, which requires inverting $(\bar{\Sigma})^{-1}$. Replacing $\sigma_a$ with $(\sigma_a^{-1} + \chi K)^{-1}$, and following the same inversion steps backwards, we get

$$\bar{\Sigma} = (\sigma_a^{-1} + \bar{K}_a)^{-1} bb' + \Phi,$$

(29)

where $b$ is the $3 \times 1$ vector of loadings of each asset on aggregate risk, and if $\bar{K}_1$ and $\bar{K}_2$ represent the average amount of capacity devoted to processing information about assets 1 and 2,

$$\Phi = \begin{bmatrix} (\sigma_1^{-1} + \bar{K}_1)^{-1} & 0 & 0 \\ 0 & (\sigma_2^{-1} + \bar{K}_2)^{-1} & 0 \\ 0 & 0 & 0 \end{bmatrix}.$$

Finally, let $\tilde{\sigma}_a \equiv (\sigma_a^{-1} + \bar{K}_a)^{-1}$.

**Portfolio holdings** The optimal portfolio for investor $j$ is

$$q_j = \frac{1}{\rho} \bar{\Sigma}_j^{-1}(\bar{\mu}_j - pr)$$

(30)

This comes from the first order condition and is a standard expression in any portfolio problem with CARA or mean-variance utility.

Next, compute the portfolio of the average investor. Let the average of all investors’ signal precision be $(\bar{\Sigma})^{-1} \equiv \int \hat{\Sigma}_j^{-1} dj$. Use the fact that $\bar{\mu}_j = \Sigma_j \Sigma_j^{-1} \mu + (I - \Sigma_j \Sigma_j^{-1}) \eta_j$ and the fact that the signal noise is mean-zero to get that $\int \hat{\Sigma}_j^{-1} \bar{\mu}_j dj = \Sigma^{-1} \mu + ((\bar{\Sigma})^{-1} - \Sigma^{-1}) f$. This is true because the mean of all investors’ signals are the true payoffs $f$ and because the signal errors are uncorrelated with (but of course, not independent of) signal precision.

$$\bar{q} \equiv \int q_j dj = \frac{1}{\rho} (\Sigma^{-1} \mu + ((\bar{\Sigma})^{-1} - \Sigma^{-1}) f - (\bar{\Sigma})^{-1} pr)$$

(31)

Using Bayes’ rule for the posterior variance of normal variables, we can rewrite this as

$$\bar{q} \equiv \int q_j dj = \frac{1}{\rho} (\Sigma^{-1} \mu + (\Sigma_a^a)^{-1} f - (\bar{\Sigma})^{-1} pr)$$

(32)

### 7.2 Proof of Lemma

**Proof.** Following Admati (1985), we conjecture that the price vector $p$ is linear in the payoff vector $f$ and the supply vector $x$: $pr = A + B f + C x$. We now verify that conjecture by imposing market clearing

$$\int q_j dj = \bar{x} + x$$

(33)
Using (31) to substitute out the left hand side, and rearranging,

\[ \text{pr} = -\rho \bar{\Sigma}(\bar{x} + x) + f + \bar{\Sigma}\Sigma^{-1}(\mu - f) \]

Thus, the coefficients \( A \), \( B \), and \( C \) are given by

\[ A = -\rho \bar{\Sigma}\bar{x} + \bar{\Sigma}\Sigma^{-1}\mu \] (34)
\[ B = I - \bar{\Sigma}\Sigma^{-1} \] (35)
\[ C = -\rho \bar{\Sigma} \] (36)

which verifies our conjecture. \( \square \)

7.3 Lemma 2: Investors Prefer Not To Learn Price Information

The idea behind this result is that an investor who learns from price information, will infer that the asset is valuable when its price is high and infer that the asset is less valuable when its price is low. Buying high and selling low is generally not a way to earn high profits. This effect shows up as a positive correlation between \( \hat{\mu} \) and \( \text{pr} \), which reduces the variance \( V_1[\hat{\mu}_j - \text{pr}] \).

Mathematical Preliminaries: Note that \( B^{-1}(\text{pr} - A) = f + B^{-1}Cx \). Since \( x \) is a mean-zero shock, this is an unbiased signal about the true asset payoff \( f \). The precision of this signal is \( \Sigma_p^{-1} \equiv \sigma_x^{-1}B'(CC')^{-1}B \).

Lemma 2. A manager who could choose either learning from prices and observing a signal \( \tilde{\eta} | f \sim N(f, \tilde{\Sigma}_\eta) \) or not learning from prices and instead getting a higher-precision signal \( \eta | f \sim N(f, \Sigma_\eta) \), where the signals are conditionally independent across agents, and where \( \Sigma_\eta^{-1} = \Sigma_p^{-1} + \tilde{\Sigma}_\eta^{-1} \), would prefer not to learn from prices.

Proof. From (\text{9}) in the main text, we know that expected utility is

\[ \text{U}_{1j} = \frac{1}{2} \text{trace}(\hat{\Sigma}_j^{-1}V_1[\hat{\mu}_j - \text{pr}]) + \frac{1}{2}E_1[\hat{\mu}_j - \text{pr}]^\prime \hat{\Sigma}_j^{-1}E_1[\hat{\mu}_j - \text{pr}] \]

Since the two options yield equally informative signals, by Bayes’ rule, they yield equally informative posterior beliefs: \( \hat{\Sigma}_j^{-1} = \Sigma^{-1} + \Sigma_\eta^{-1} \), which is also equal to \( \Sigma^{-1} + \Sigma_p^{-1} + \tilde{\Sigma}_\eta^{-1} \). Likewise, since both possibilities give the manager unbiased signals, beliefs are a martingale, meaning that \( E_1[\hat{\mu}_j - \text{pr}] \), is identical under the two options.

Thus, the only term in expected utility that is affected by the decision to learn information from prices is \( V_1[\hat{\mu}_j - \text{pr}] \). Let \( \hat{\mu}_j = E[f | \eta] \) be the posterior expected value of payoffs for the manager who learns from the conditionally independent signal. By Bayes’ Law,

\[ \hat{\mu}_j = \hat{\Sigma}_j(\Sigma^{-1}\mu + \Sigma_\eta^{-1}\eta) \]

The signal \( \eta \) can be broken down into the true payoff, plus noise: \( \eta = f + \epsilon \), where \( \epsilon \sim N(0, \Sigma_\eta) \). Using the
expression for $\hat{\mu}_j$ and the pricing equation $pr = A + Bf + Cx$, we write

$$\hat{\mu}_j - pr = \hat{\Sigma}_j\Sigma^{-1}\mu - A + \hat{\Sigma}_j\Sigma^{-1}\epsilon + (\hat{\Sigma}_j\Sigma^{-1} - B)f - Cx.$$ 

Since $\mu$ and $A$ are constants, and $\epsilon$, $f$, and $x$ are mutually independent, the variance of this expression is

$$V_1[\hat{\mu}_j - pr] = \hat{\Sigma}_j\Sigma^{-1}\hat{\Sigma}_j + (\hat{\Sigma}_j\Sigma^{-1} - B)\Sigma(\hat{\Sigma}_j\Sigma^{-1} - B)' - \sigma_xCC'.$$

Next, consider the manager who chooses to learn information in prices. This person will have different posterior belief about $f$. Let $E[f | p, \tilde{\eta}] = \tilde{\mu}$. Using Bayes’ law, he will combine information from his prior, prices and the signal $\tilde{\eta}$ his posterior belief:

$$\tilde{\mu} = \hat{\Sigma}_j(\Sigma^{-1}\mu + \Sigma_p^{-1}B^{-1}(pr - A) + \Sigma_\eta^{-1}\tilde{\eta}).$$

Again, breaking up the signal into truth and noise ($\tilde{\eta} = f + \epsilon$), and using the price equation, we can write

$$\tilde{\mu}_j - pr = \hat{\Sigma}_j\Sigma^{-1}\mu + \hat{\Sigma}_j\hat{\Sigma}_\eta^{-1}(f + \epsilon) + (\hat{\Sigma}_j\Sigma_p^{-1}B^{-1} - I)(A + Bf + Cx) - \hat{\Sigma}_j\Sigma_p^{-1}B^{-1}A.$$

$$= \hat{\Sigma}_j\Sigma^{-1}\mu + (\hat{\Sigma}_j\Sigma_p^{-1}(I - B^{-1}) - I)A + (\hat{\Sigma}_j\hat{\Sigma}_\eta^{-1} + \hat{\Sigma}_j\Sigma_p^{-1}B - B)f + \hat{\Sigma}_j\hat{\Sigma}_\eta^{-1}\epsilon + (\hat{\Sigma}_j\Sigma_p^{-1} - I)Cx.$$

Since $\mu$ and $A$ are constants, and $\epsilon$, $f$, and $x$ are mutually independent, the variance of this expression is

$$V_1[\tilde{\mu} - pr] = (\hat{\Sigma}_j\hat{\Sigma}_\eta^{-1} + \hat{\Sigma}_j\Sigma_p^{-1}B - B)\Sigma(\hat{\Sigma}_j\hat{\Sigma}_\eta^{-1} + \hat{\Sigma}_j\Sigma_p^{-1}B - B)' + \hat{\Sigma}_j\hat{\Sigma}_\eta^{-1}\hat{\Sigma}_j + \sigma_x(\hat{\Sigma}_j\Sigma_p^{-1} - I)CC'(\hat{\Sigma}_j\Sigma_p^{-1} - I)'$$

We have assumed that $\hat{\Sigma}_\eta^{-1} + \Sigma_p^{-1} = \Sigma^{-1}$. Therefore, the first term $\hat{\Sigma}_j\hat{\Sigma}_\eta^{-1} + \hat{\Sigma}_j\Sigma_p^{-1}B - B = \hat{\Sigma}_j\Sigma_p^{-1}B - B$, which is the same quantity as in the first term of $V_1[\hat{\mu}_j - pr]$.

Thus, when we subtract one expression from the other,

$$V_1[\tilde{\mu}_j - pr] - [V_1[\tilde{\mu} - pr] = \hat{\Sigma}_j(\Sigma^{-1} - \Sigma_\eta^{-1})\hat{\Sigma}_j - \sigma_x(\hat{\Sigma}_j\Sigma_p^{-1}CC'\Sigma_p^{-1}\hat{\Sigma}_j - 2\hat{\Sigma}_j\Sigma_p^{-1}CC').$$

Since $\Sigma_\eta^{-1} = \hat{\Sigma}_\eta^{-1} + \Sigma_p^{-1}$ and $\Sigma_p^{-1}$ is positive semi-definite (an inverse variance matrix always is), $\hat{\Sigma}_j(\Sigma_\eta^{-1} - \hat{\Sigma}_\eta^{-1})\hat{\Sigma}_j$ is positive semi-definite. Thus, the difference is positive semi-definite if $2I - \Sigma_p^{-1}\hat{\Sigma}_j$ is. Since for the investor that learns about prices, Bayes’ rule tells us that $\hat{\Sigma}_j^{-1} = \Sigma^{-1} + \Sigma_p^{-1} + \Sigma_\eta^{-1}$. This means that $I - \Sigma_p^{-1}\hat{\Sigma}_j = (\Sigma_\eta^{-1} - \hat{\Sigma}_\eta^{-1})\hat{\Sigma}_j$, which is positive semi-definite. Therefore, $2I - \Sigma_p^{-1}\hat{\Sigma}_j$ is also positive semi-definite.

Thus, the difference in utility from learning conditionally independent information and learning price information is, $1/2trace(\hat{\Sigma}_j^{-1}(V_1[\hat{\mu}_j - pr] - [V_1[\tilde{\mu} - pr]]))$. Since the expression inside the trace is a product of positive semi-definite matrices, the trace and therefore the difference in expected utilities is positive. 

### 7.4 Proof of Proposition 1

If aggregate variance is not too high ($\sigma_a < 1$), then the marginal value of a given investor $j$ reallocating an increment of capacity from stock-specific shock $i \epsilon \{1, 2\}$ to the aggregate shock is increasing in the aggregate shock variance: If $K_{aj} = \tilde{K}$ and $K_{ij} = K - \tilde{K}$, then $\partial^2U/\partial\tilde{K}\partial\sigma_a > 0$. 

40
Proof. From (9) in the main text, we know that expected utility is

\[ U_{1j} = \frac{1}{2} \text{trace}(\hat{\Sigma}_{1j}^{-1}V_1[\hat{\mu}_j - pr]) + \frac{1}{2}E_1[\hat{\mu}_j - pr] \hat{\Sigma}_1^{-1}E_1[\hat{\mu}_j - pr] \]

The first step is to work out the variance \( V_1[\hat{\mu}_j - pr] \).

\[ \hat{\mu}_j - pr = \hat{\Sigma}_j(\Sigma^{-1} \mu + \Sigma_{nj}^{-1} \eta_j) - A - Bf - Cx \]

The signal \( \eta \) can be expressed as the true asset payoff, plus orthogonal signal noise \( \epsilon_j \).

\[ \hat{\mu}_j - pr = \hat{\Sigma}_j\Sigma^{-1} \mu - A + (\hat{\Sigma}_j \Sigma_{nj}^{-1} - B)f + \hat{\Sigma}_j \Sigma_{nj}^{-1} \epsilon_j - Cx \]

Since \( \mu \) and \( A \) are known constants and \( f, \epsilon_j, \) and \( x \) are independent, with variances \( \Sigma, \Sigma_{nj}, \) and \( \sigma_x \) respectively,

\[ V_1[\hat{\mu}_j - pr] = (\hat{\Sigma}_j \Sigma_{nj}^{-1} - B)\Sigma(\hat{\Sigma}_j \Sigma_{nj}^{-1} - B)' + \hat{\Sigma}_j \Sigma_{nj}^{-1} \hat{\Sigma}_j + C\Sigma' \sigma_x \]

Substituting in for the price coefficients using (34), (35), and (36) yields

\[ V_1[\hat{\mu}_j - pr] = (\Sigma - \hat{\Sigma}_j)\Sigma^{-1} (\Sigma - \hat{\Sigma}_j)' + \hat{\Sigma}_j \Sigma_{nj}^{-1} \hat{\Sigma}_j + \rho^2 \Sigma \Sigma \sigma_x \]

Next, work out the second term by using the expression above for \( \hat{\mu}_j - pr \) and taking the expectation: 

\[ E[\hat{\mu}_j - pr] = \hat{\Sigma}_j\Sigma^{-1} \mu - A + (\hat{\Sigma}_j \Sigma_{nj}^{-1} - B)\mu. \]

Substituting in the coefficients \( A \) and \( B \), and simplifying reveals that 

\[ E[\hat{\mu}_j - pr] = \rho \Sigma \bar{x}. \]

Thus, expected utility is

\[ U_{1j} = \frac{1}{2} \text{trace} \left( \Sigma^{-1} (\Sigma - \hat{\Sigma}_j) \Sigma^{-1} (\Sigma - \hat{\Sigma}_j)' + \Sigma_{nj}^{-1} \hat{\Sigma}_j + \rho^2 \Sigma \Sigma \sigma_x \right) + \frac{\rho^2}{2} \bar{x}' \Sigma \Sigma \sigma_x \bar{x} \]

In the next step, we want to take a cross-partial derivative of utility with respect to \( \sigma_a \) and \( \hat{K} \). To do this, we will substitute out the \( \hat{\Sigma}_{1j}^{-1} \) terms using (23), (24), and (27). Then, we will use the fact that, by the chain rule, \( \partial U/\partial \hat{K} = \partial U/\partial K_{aj} - \partial U/\partial K_{ij} \). Therefore, \( \partial^2 U/\partial \hat{K} \partial \sigma_a = \partial^2 U/\partial K_{aj} \partial \sigma_a - \partial^2 U/\partial K_{ij} \partial \sigma_a \). We consider each of these two cross-partial derivatives separately.

Part a: The marginal value of a given investor \( j \) having additional capacity \( K_{aj} \) devoted to learning about the aggregate shock \( a \) is increasing in the aggregate shock variance: \( \partial^2 U/\partial K_{aj} \partial \sigma_a > 0 \).

Sign last term: \( \frac{\rho^2}{2} \bar{x}' \hat{\Sigma} \Sigma \bar{x} \).

Note that \( K_{aj} \) appears only in \( \hat{\Sigma}_{1j}^{-1} \). Recall that \( \hat{\Sigma}_{1j}^{-1} / \partial \sigma_a^{-1} = \Upsilon_a \). Since \( \sigma_a^{-1} = \sigma_a^{-1} + K_{aj} \), the chain rule implies that \( \hat{\Sigma}_{1j}^{-1} / \partial K_{aj} = \Upsilon_a \). Thus, the last term has \( K_{aj} \) derivative \( (\rho^2/2) \bar{x}' \Sigma \Upsilon_a \Sigma \bar{x} \). The only term in this expression that varies in \( \sigma_a \) is \( \Upsilon_a \). Since \( \Sigma \) has every entry increasing in \( \sigma_a \) (equation (24)), and \( \Sigma \) and \( \Upsilon_a \) are positive semi-definite matrices, this term has a positive cross-partial derivative \( \partial^2 / \partial K_{aj} \partial \sigma_a > 0 \).

Thus, a sufficient condition for \( \partial^2 U/\partial K_{j} \partial \sigma_a > 0 \) is for the trace term to have a positive cross partial
derivative. Since the trace of a sum is the sum of the traces, we can break this term up into 3 major parts.

Sign term 3: \( \text{Tr}(\rho^2 \Sigma_{ij}^{-1} \Sigma \sigma_x) \).
This takes the same form as the term outside the trace. \( K_{a_j} \) appears only in \( \Sigma_j^{-1} \). Its derivative is \( \partial \Sigma_{ij}^{-1}/\partial K_{a_j} = \Upsilon_a \). Thus, the term has derivative \( \rho^2 \Upsilon_a \Sigma \Sigma \sigma_x \). The only term in this expression that varies in \( \sigma_a \) is \( \Sigma \). Since \( \Sigma \) and \( \Upsilon_a \) are positive semi-definite matrices, and \( \rho^2 \) and \( \sigma_x \) are positive constants, the trace must be positive. Since \( \Sigma \) has every entry increasing in \( \sigma_a \) (equation [29]), the \( \partial \text{Tr}(\cdot)/\partial K_{a_j} \) is increasing in \( \sigma_a \). In other words, \( \partial^2 /\partial K_{a_j} \sigma_a > 0 \).

Sign term 2: \( \text{Tr}(\Sigma_{nj}^{-1} \Sigma \sigma_x) \).
By Bayes’ Law, the signal precision is the difference between the posterior and prior precisions, \( \Sigma_{nj}^{-1} = \Sigma_j^{-1} - \Sigma^{-1} \). Substituting this into the trace term yields \( \text{Tr}(I - \Sigma_j^{-1}) = 3 - \text{Tr}(\Sigma_1^{-1} \Sigma_j) \), where the 3 comes from the fact that the variance matrices are all \( (3 \times 3) \). The \( -\text{Tr}(\Sigma_j^{-1} \Sigma_j) \) will cancel out with the first term.

Sign term 1: \( \text{Tr}(\Sigma_j^{-1} (\Sigma - \Sigma_j \Upsilon a) - \Sigma a a^{-1} (\Sigma - \Sigma_j)') \).
We can further break this term up into a sum of three parts, which I will call 1a, 1b and 1c.

Term 1a is \( \text{Tr}(\Sigma_j^{-1} \Sigma_j^{-1} (\Sigma - \Sigma_j \Upsilon a) \Sigma a a^{-1} (\Sigma - \Sigma_j)') \), which is equal to \( \text{Tr}(\Sigma_j^{-1} \Sigma_j) \). This term cancels out the \( -\text{Tr}(\Sigma_j^{-1} \Sigma_j) \) from term 2.

Term 1b is \( -2 \text{Tr}(\Sigma_j^{-1} \Sigma_j^{-1} \Sigma_j') \), which is equal to \( -2 \text{Tr}(\Sigma_j^{-1} \Sigma_j') \). This term only depends on prior variance and average posterior variance, not on investor \( j \)'s information choice. Since it has no \( K_j \) in it, its derivative with respect to \( K_j \) is 0.

Term 1c is \( \text{Tr}(\Sigma_j^{-1} \Sigma_j^{-1} \Sigma_j') \). Investor \( j \)'s information choice \( K_{a_j} \) shows up only in \( \Sigma_j^{-1} \), while \( \partial \Sigma_j^{-1} /\partial K_{a_j} = \Upsilon_a \). Thus \( \partial \text{Tr}(\cdot)/\partial K_{a_j} = \text{Tr}(\Sigma_j^{-1} \Sigma_j') \). Next, replace \( \Sigma \) with (29) and \( \Sigma^{-1} \) with (25) and then take the derivative with respect to \( \sigma_a^{-1} \). That delivers

\[
\frac{\partial^2 \text{Tr}(\cdot)}{\partial K_{a_j} \partial \sigma_a^{-1}} = \text{Tr} \left[ -2 \Upsilon_a \Sigma a (\sigma_a^{-1} + \bar{K}_a)^{-2} b b'((\sigma_a^{-1} + \bar{K}_a)^{-1} b b' + \bar{\Phi}) (S + \sigma_a^{-1} \Upsilon_a) + \Upsilon_a ((\sigma_a^{-1} + \bar{K}_a)^{-1} b b' + \bar{\Phi}) (S + \sigma_a^{-1} \Upsilon_a) \right]
\]

If we want the cross-partial derivative with \( \sigma_a \) to be positive, then we are looking for a negative sign here.

Since \( \bar{\Phi} \) only has non-zero entries in the (1,1) and (2,2) spots and \( \Upsilon_a \) has only a 1 in the (3,3) entry, \( \Upsilon_a \bar{\Phi} = 0 \). Thus,

\[
\frac{\partial^2 \text{Tr}(\cdot)}{\partial K_{a_j} \partial \sigma_a^{-1}} = (\sigma_a^{-1} + \bar{K}_a)^{-2} \text{Tr} \left[ -2(\sigma_a^{-1} + \bar{K}_a)^{-1} \Upsilon_a b b' b' (S + \sigma_a^{-1} \Upsilon_a) + \Upsilon_a b b' b' \Upsilon_a \right]
\]

Since \( \Upsilon_a \), \( b b' \) and \( S \) are all positive semi-definite matrices, their product must have a positive trace and
\( -2 \text{Tr}((\sigma_a^{-1} + \bar{K}_a)^{-1} \Upsilon_a b b' b') < 0 \). That leaves \( \text{Tr}(\Upsilon_a b b' b' \Upsilon_a) \), which is positive since \( \Upsilon_a \) and \( b b' \) are positive semi-definite, times a constant \( 1 - 2\sigma_a^{-1}/(\sigma_a^{-1} + \bar{K}_a) \). This is negative or zero as long as \( \bar{K}_a \leq \sigma_a^{-1} \).

We can get an alternative sufficient condition for this term to be positive by using the fact that \( \Sigma^{-1} = (S + \sigma_a^{-1} \Upsilon_a) \) and therefore \( \Upsilon_a = (\Sigma^{-1} - S) \sigma_a \). Thus, we can rewrite

\[
\frac{\partial^2 \text{Tr}(\cdot)}{\partial K_{a_j} \partial \sigma_a^{-1}} = (\sigma_a^{-1} + \bar{K}_a)^{-2} \text{Tr} \left[ \Upsilon_a b b' b' ((\Sigma^{-1} - S) \sigma_a - 2 \frac{1}{\sigma_a^{-1} + K_a} \Sigma^{-1}) \right]
\]

The term \( \text{Tr}[-\Upsilon_a b b' b' S \sigma_a] \) is negative, as before. The remaining term is \( \text{Tr}[\Upsilon_a b b' b' \Sigma^{-1}] \), which is positive,
times \((\sigma_a - 2/\sigma_a^2 + K_a)\). This is negative or zero if \(K_a \geq \sigma_a\).

If \(\sigma_a \leq 1\), then one of the sufficient conditions for the last term to have \(\partial^2/\partial K_{aj}\partial \sigma_a > 0\) is always satisfied. In sum, when \(\sigma_a \leq 1\), all the terms have non-negative cross-partial derivatives and therefore \(\partial^2 U/\partial K_{ij}\partial \sigma_a > 0\).

Part b: The marginal value of a given investor \(j\) having additional capacity \(K_{ij}\) devoted to learning about stock-specific shock \(i\) is constant in the aggregate shock variance: \(\partial^2 U/\partial K_{ij}\partial \sigma_a = 0\).

Without loss of generality, we consider reallocating capacity from the asset 1 shock to the aggregate shock \((i = 1)\). The same proof follows if it were asset 2 instead.

Sign last term: \(\rho^2 \Sigma \Sigma^{-1} x \Sigma \Sigma^{-1} x\).

Note that \(K_{1j}\) appears only in \(\Sigma_j^{-1}\). Recall that \(\partial \Sigma_j^{-1}/\partial \sigma_1 = \Upsilon_1\). Since \(\sigma_1^{-1} = \sigma_1^{-1} + K_{1j}\), using the chain rule, we get \(\partial \Sigma_j^{-1}/\partial K_{1j} = \Upsilon_1\). Therefore, \(\partial/\partial K_{1j}(\Sigma \Sigma^{-1} \Sigma \Sigma^{-1} x) = \Sigma \Upsilon_1 \Sigma \Sigma^{-1} x\).

Because of the structure of the \(\Upsilon_1\) matrix, it turns out that using (24) and (29) to multiply out the three matrices \(\Sigma \Upsilon_1 \Sigma\) delivers

\[
\Sigma \Upsilon_1 \Sigma = \begin{bmatrix}
\sigma_1^{-1} + K_1 \\
0 & 0 & 0 \\
0 & 0 & 0
\end{bmatrix}.
\]

Since this has no \(\sigma_a\) term in it and both \(\bar{x}\) and \(\rho\) are exogenous, the cross-partial derivative \(\partial^2/\partial K_{ij}\partial \sigma_a\) of the last terms is zero.

Sign term 3: \(\text{Tr}(\rho^2 \Sigma^{-1} \Sigma \Sigma \sigma_x)\).

This takes the same form as the term outside the trace. The term inside the trace has derivative \(\partial/\partial K_{1j} = \rho^2 \Sigma \Upsilon_1 \Sigma \sigma_x\). Since \(\rho\) and \(\sigma_x\) are exogenous and \(\Sigma \Upsilon_1 \Sigma\) does not depend on \(\sigma_a\), the derivative of the trace is invariant in \(\sigma_a\). Its cross-partial derivative \(\partial^2/\partial K_{1j}\partial \sigma_a = 0\).

Sign term 2: \(\text{Tr}(\Sigma^{-1} \Sigma)\).

As in part a, this term cancels out Term 1a.

Sign term 1: \(\text{Tr}(\Sigma_j^{-1}(\Sigma - \Sigma_j) \Sigma^{-1} (\Sigma - \Sigma_j)')\)

We can further break this term up into a sum of three parts, which I will call 1a, 1b and 1c.

Term 1a, \(\text{Tr}(\Sigma_j^{-1} \Sigma \Sigma^{-1} \Sigma')\), cancels out term 2.

Term 1b is \(-2\text{Tr}(\Sigma_j^{-1} \Sigma \Sigma^{-1} \Sigma')\), which is equal to \(-2\text{Tr}(\Sigma^{-1} \Sigma')\). This term only depends on prior variance and average posterior variance, not on investor \(j\)’s information choice. Since it has no \(K\) in it, its derivative with respect to \(K_{ij}\) is 0.

Term 1c is \(\text{Tr}(\Sigma_j^{-1} \Sigma \Sigma^{-1} \Sigma')\). Investor \(j\)’s information choice \(K_{ij}\) shows up only in \(\Sigma_j^{-1}\), where \(\partial \Sigma_j^{-1}/\partial K_{aj} = \Upsilon_1\). Therefore, \(\partial/\partial K_{1j}(\Sigma_j^{-1} \Sigma \Sigma^{-1} \Sigma') = \Upsilon_1 \Sigma \Sigma^{-1} \Sigma'\). As before, the sparse form of \(\Upsilon_1\) causes the matrix multiplication to turn out neatly. Using (24), (29) and (34) to multiply out the four matrices delivers

\[
\Upsilon_1 \Sigma \Sigma^{-1} \Sigma' = \begin{bmatrix}
\sigma_1^{-1} (\sigma_1^{-1} + K_1)^{-2} & 0 & 0 \\
0 & 0 & 0 \\
b_1 \sigma_1^{-1} (\sigma_1^{-1} + K_1)^{-2} & 0 & 0
\end{bmatrix}.
\]

The trace of this matrix is \(\sigma_1^{-1} (\sigma_1^{-1} + K_1)^{-2}\). Since this has no \(\sigma_a\) term in it, the cross-partial derivative \(\partial^2/\partial K_{ij}\partial \sigma_a\) is zero.

Since \(\partial^2 U/\partial K_{aj}\partial \sigma_a > 0\) and \(\partial^2 U/\partial K_{ij}\partial \sigma_a = 0\), the difference of the two terms is positive. Thus, the
marginal value of a given investor $j$ reallocating an increment of capacity from shock 1 to the aggregate shock is increasing in the aggregate shock variance: $\partial^2 U / \partial K \partial \sigma_a = \partial^2 U / \partial K_{ij} \partial \sigma_a - \partial^2 U / \partial K_{ij} \partial \sigma_a > 0$. ⊓⊔

7.5 Proof of Proposition 2

Part a: If the average manager has sufficiently low capacity $\chi K < \sigma_a^{-1}$, then an increase in aggregate risk $\sigma_a$ increases the dispersion of fund portfolios $E[(q_j - \bar{q})(q_j - \bar{q})]$, where $\bar{q} \equiv \int q_j dj$.

Proof. Using the optimal portfolio expressions, (30) and (32), and Bayes’ rule ($\hat{\Sigma}$ is the average manager’s signal precision).

Next, we need to take into account that signals and payoffs are correlated. To do this, replace the signal $f$ with the true payoff, plus signal noise: $\eta_j = f + e_j$,

$$ (q_j - \bar{q}) = \frac{1}{\rho} \left[ \Sigma^{-1} e_j + (\Sigma^{-1} - \Sigma^{-1})f + (\Sigma^{-1} - \Sigma^{-1})pr \right] $$

Bayes’ rule for variances of normal variables is $\hat{\Sigma}^{-1} = \Sigma^{-1} + \Sigma_a^{-1}$. Integrating the left and right sides of this expression over managers $j$ yields $\hat{\Sigma}^{-1} = \Sigma^{-1} + (\Sigma_a^{-1})^{-1}$. Subtracting one expression from the other yields $\hat{\Sigma}^{-1} - \Sigma^{-1} = \Sigma_a^{-1} - (\Sigma_a^{-1})^{-1}$. Define $\Delta = \hat{\Sigma}^{-1} - \Sigma^{-1}$. Substituting this in and combining terms yields

$$ (q_j - \bar{q}) = \frac{1}{\rho} \left[ \Sigma^{-1} e_j + \Delta(f - pr) \right]. \tag{39} $$

Now replace $pr$ with $A+ Bf + Cx$, where $A$, $B$, and $C$ are given by Appendix 7.2

$$ (q_j - \bar{q}) = \frac{1}{\rho} \left[ \Sigma^{-1} e_j + \Delta((I-B)f - A-Cx) \right] \tag{40} $$

Substituting in the coefficients in the pricing equation reveals that $(I - B)\mu - A = \rho \tilde{\Sigma} \tilde{x}$, that $I - B = \tilde{\Sigma} \Sigma^{-1}$, and that $C = -\rho \tilde{\Sigma}$.

To work out the expectation of this quantity squared, recognize that this is the square of a sum of one constant and three, independent, mean-zero, normal variables. Since $e_j$, $f - \mu$ and $x$ are independent, all the cross terms drop out, leaving

$$ E[(q_j - \bar{q})(q_j - \bar{q})] = Tr(\Sigma^{-1}) + Tr(\Sigma^{-1} \Delta \Delta \tilde{\Sigma}) + \rho^2 \tilde{x'} \tilde{\Sigma} \Delta \Delta \tilde{\Sigma} \tilde{x} + \rho^2 \sigma_x Tr(\Delta \Delta \tilde{\Sigma}) $$

The first term depends only on information choice variables. So, holding choices fixed, the partial derivative with respect to $\sigma_a$ is zero.

For the second term, it is easier to take the partial derivative with respect to $\sigma_a^{-1}$ and show that it is negative. This is equivalent to showing that the derivative with respect to $\sigma_a$ is positive. First, use (29) to show that $\partial \Sigma / \partial \sigma_a^{-1} = -(\sigma_a^{-1} + K_a)^{-2} bb'$. Next, use (25) to show that $\partial \Sigma^{-1} / \partial \sigma_a^{-1} = \gamma_a$. Recall that $\Delta$ depends only on information choices, which we hold fixed.

44
Then, using the product rule, the derivative is the sum of two terms:

$$\frac{\partial}{\partial \sigma_a} Tr(\Sigma^{-1} \Delta \Delta \Sigma) = -2 \left( \frac{1}{\sigma_a^{-1} + K_a} \right)^2 Tr(\Sigma \Delta \Delta bb') + Tr(\Sigma \Upsilon_a \Delta \Delta \Sigma)$$

Using (29), we can write $\Sigma \Upsilon_a = (\sigma_a^{-1} + K_a)^{-1} bb' \Upsilon_a + \Phi \Upsilon_a$. Recall that $\Phi$ has only non-zero (1,1) and (2,2) entries and the $\Upsilon_a$ has only a non-zero (3,3) entry. Therefore, $\Phi \Upsilon_a = 0$. Letting $\bar{\sigma}_a \equiv (\sigma_a^{-1} + K_a)^{-1}$, we can rewrite

$$\frac{\partial}{\partial \sigma_a} Tr(\Sigma^{-1} \Delta \Delta \Sigma) = \bar{\sigma}_a Tr(\Sigma (\Upsilon_a - 2 \bar{\sigma}_a \Sigma^{-1}) \Delta \Delta bb')$$

This is negative iff $(\Upsilon_a - 2 \bar{\sigma}_a \Sigma^{-1})$ is negative semi-definite. Using (23) to rewrite $\Sigma^{-1}$ as $S + \sigma_a^{-1} \Upsilon_a$ and substituting in reveals that $((1 - 2 \bar{\sigma}_a) \Upsilon_a - 2 \bar{\sigma}_a S)$ must be negative semi-definite. Since $S$ is a positive semi-definite matrix, a sufficient condition is $1 - 2 \bar{\sigma}_a \leq 0$. Substituting back in the definition of $\bar{\sigma}_a$ and rearranging yields $K_a \leq \sigma_a^{-1}$. Thus, if $K_a \leq \sigma_a^{-1}$, the second term is decreasing in $\sigma_a^{-1}$ and therefore increasing in $\sigma_a$.

**Term 3:** The product $\partial/\partial \sigma_a^{-1}(x^\prime \Sigma \Delta \Delta x)$ is non-positive if $\partial/\partial \sigma_a^{-1} (\Sigma \Delta \Delta \Sigma)$ is negative semi-definite. As before $\Delta$ is a choice variable, which we hold fixed. $\partial \bar{\Sigma}/\partial \sigma_a^{-1} = -(\sigma_a^{-1} + K_a)^{-2} bb'$. Therefore,

$$\frac{\partial}{\partial \sigma_a} \bar{\Sigma} \Delta \Delta \Sigma = \frac{-2}{(\sigma_a^{-1} + K_a)^2} \bar{\Sigma} \Delta \Delta bb'$$

Since $\bar{\Sigma}$, $\Delta$ and $bb'$ are positive semi-definite, this is negative semi-definite.

**Term 4:** As shown in the previous step, $\partial/\partial \sigma_a^{-1} (\Sigma \Delta \Delta \Sigma)$ is negative semi-definite. Therefore, the derivative of the trace, which is the trace of the derivative, is negative: $\partial/\partial \sigma_a^{-1} Tr(\Sigma \Delta \Delta \Sigma) < 0$.

Since all four terms in the expression for dispersion are decreasing in $\sigma_a^{-1}$, dispersion is increasing in $\sigma_a$.

**Part b:** If the average manager has sufficiently low capacity, $\chi K < \sigma_a^{-1}$, then an increase in aggregate risk, $\sigma_a$, increases the dispersion of funds’ portfolio returns $E[(q_j - \bar{q})'(f - pr)2]$.

**Proof.** From the previous part (equation 40), we know that we can use the optimal portfolio expressions, (30) and (32), and Bayes’ rule to express the portfolio difference as a function of three underlying random variables, $e_j$, $f - \mu$ and $x$. The $f - pr$ can likewise be expressed as a function of $f - \mu$ and $x$:

$$(q_j - \bar{q})'(f - pr) = \frac{1}{\rho} \left[ \Sigma^{-1} e_j + \Delta((I - B)f - A - Cx) \right]'((I - B)f - A - Cx)$$

(41)

Substituting in the price coefficients, this is

$$= \frac{1}{\rho} \left[ \Sigma^{-1} e_j + \Delta \Sigma(\rho x + \Sigma^{-1}(f - \mu)\rho x) \right]'(\rho x + \Sigma^{-1}(f - \mu)\rho x)$$

(42)

Since a linear combination of two normal variables is also a normal variable, we can write $\bar{\Sigma}(\Sigma^{-1}(f - \mu)\rho x) = V z$ where $z \sim N(0, I)$ and $V \equiv \Sigma \Sigma^{-1/2} + \rho x^{-1/2} \Sigma$. Likewise, we can use a shorthand for the constant term $w \equiv \rho \bar{\Sigma} x$. Then, dispersion in fund profits becomes

$$E[(q_j - \bar{q})'(f - pr)2] = E \left[ \left( \frac{1}{\rho} \Sigma^{-1} e_j + \Delta V z + \Delta w \right)'(w + V z) \right]^2$$

(43)
Only terms with even powers of the normal variables are non-zero. This leaves

\[
\frac{1}{\rho^2} \left[ \Sigma^{-1} + (w' \Delta w)^2 + w' w (V \Delta V) + 4 Tr (V \Delta V) w' \Delta w + Tr (V V) w' \Delta w + Tr (\Delta V V V \Delta) + Tr (\Delta V V)^2 \right]
\]

(44)

where the last two terms come from the expectation of a multivariate normal variable \( z \), raised to the fourth.

To sign the partial derivative of dispersion, with respect to \( \sigma_a \), we proceed term-by-term. The first term \( \Sigma^{-1} \) depends only on choice variables, which we hold fixed. Similarly, \( \Delta \) is also choice variables.

The constant \( w \) is \( \rho \bar{\Sigma} \bar{x} \), where \( \rho \) and \( \bar{x} \) are positive constants and every entry of \( \bar{\Sigma} \) is increasing in \( \sigma_a \). Therefore, all the \( w' w \) and \( \bar{x} \) are increasing in \( \sigma_a \). Furthermore, since wherever \( \Delta \) appears, it shows up twice, whether it is positive or negative makes no difference. If it is negative, the two negative signs cancel. Thus, it remains to be shown that \( \partial V / \partial \sigma \) is positive or negative makes no difference. If it is negative, the two negative signs cancel. Thus, the formula for the expectation of a chi-square variable,

\[
\text{If some managers are uninformed } \chi < 1, \text{ but all informed managers learn about aggregate risk, and the average manager has sufficiently low capacity } \chi K < \sigma_a^{-1}, \text{ then an increase in aggregate risk } \sigma_a \text{ increases the expected profit of an informed fund, } E((q_j - \bar{q})'(f - pr)), \text{ where } \bar{q} \equiv \int q_j dj.
\]

Proof. Assume that all informed investors use their capacity \( K \) to learn about the aggregate risk. We show that when \( \sigma_a^{-1} \) falls (in recessions), that expected excess returns of the informed traders rise.

Begin by taking the expectation of (41) to get expected profits. Since the supply shocks and the signal noise are mean-zero and independent of all other shocks, we can take their expectations separately. Using the formula for the expectation of a chi-square variable,

\[
E((q_j - \bar{q})'(f - pr)) = \frac{-\sigma_x}{\rho} Tr[C'(1 - \chi) K \Sigma_a C] + \frac{(1 - \chi)K}{\rho} E \left\{ (I - B)' \Sigma_a ((I - B)f - A) \right\}
\]

(45)

Since \((I - B) f - A\) is normally distributed, the remaining expectation is also the mean of a chi square

\[
E((q_j - x - \bar{x})'(f - pr)) = \frac{\sigma_x(1 - \chi) K}{\rho} Tr[C' \Sigma_a C] + \frac{(1 - \chi)K}{\rho} ((I - B) \mu - A)' \Sigma_a ((I - B) \mu - A) + \frac{(1 - \chi)K}{\rho} Tr((I - B)' \Sigma_a (I - B))
\]

Finally, substitute in for \( A, B, \) and \( C \) from (31), (33), and (30).

\[
E((q_j - x - \bar{x})'(f - pr)) = (1 - \chi) K \left\{ \sigma_x \rho Tr[\Sigma \Sigma_a \Sigma] + \rho \bar{x}' \Sigma_a \Sigma \bar{x} + \frac{1}{\rho} Tr[\Sigma \Sigma^{-1} \Sigma_a \Sigma] \right\}
\]

(46)
\( \Sigma \) is equal to \( 1/(\sigma_a^{-1} + \chi K)bb' \) in its 3rd column and 3rd row entries, which are the only entries that the \( \Upsilon_a \) matrix does not zero out. Therefore, \( \Sigma \Upsilon_a \Sigma = (1/(\sigma_a^{-1} + \chi K))^2bb' \Upsilon_a bb' \), and

\[
E[(q_j-x-\bar{x})(f-pr)] = (1-\chi)K \left\{ \frac{\sigma_a}{(\sigma_a^{-1} + \chi K)^2} Tr[bb' \Upsilon_a bb'] + \frac{\rho}{(\sigma_a^{-1} + \chi K)^2} \bar{x}'bb' \Upsilon_a \bar{x} + \frac{1}{\rho} Tr[\Sigma \Upsilon_a \Sigma] \right\}
\]

Since \( bb' \Upsilon_a bb' \) does not depend on \( \sigma_a \), but is positive semi-definite, and \((1/(\sigma_a^{-1} + \chi K))^2 \) is increasing in \( \sigma_a \), the first two terms are increasing in \( \sigma_a \).

The last term can be rewritten using the relationship that \( \Sigma \) has non-negative eigenvalues. Even \( \Upsilon_a \) has non-negative eigenvalues. Since the trace is the sum of the eigenvalues and sums and products of non-negative eigenvalues are non-negative, the first term is positive. Furthermore, \( \Sigma \) has every entry increasing in \( \sigma_a \). Therefore, the first trace term is increasing in \( \sigma_a \).

In the second trace term, the matrix \( \Upsilon_a \Upsilon_a = \Upsilon_a \). Using the value derived for \( Tr[\Sigma \Upsilon_a \Sigma] \) above, we can rewrite the remaining term as

\[
\sigma_a^{-1} Tr[\Sigma \Upsilon_a \Sigma] = \frac{\sigma_a^{-1}}{(\sigma_a^{-1} + \chi K)^2} Tr[bb' \Upsilon_a bb']
\]

This is increasing in \( \sigma_a \) if \( \partial/\partial \sigma_a^{-1}(\sigma_a^{-1}/(\sigma_a^{-1} + \chi K)^2) < 0 \), which is true if

\[
\frac{\partial}{\partial \sigma_a^{-1}} \frac{\sigma_a^{-1}}{(\sigma_a^{-1} + \chi K)^2} = \frac{\chi K - \sigma_a^{-1}}{(\sigma_a^{-1} + \chi K)^3} < 0
\]

Thus,

\[
\chi K < \sigma_a^{-1}
\]

is a sufficient, but not a necessary, condition for profits to be increasing in \( \sigma_a \).

\[\square\]

### 7.7 Proof of Proposition 4

If the size of the composite asset \( \bar{x}_3 \) is sufficiently large, then an increase in risk aversion increases the marginal utility of reallocating a unit of capacity from the aggregate shock to the idiosyncratic shock: \( \partial/\partial \rho(\partial U/\partial (\hat{\sigma}_{a_j}^{-1} - \hat{\sigma}_{ij}^{-1})) > 0 \).

**Proof.** We can rewrite \( \partial/\partial \rho(\partial U/\partial (\hat{\sigma}_{a_j}^{-1} - \hat{\sigma}_{ij}^{-1})) \) as \( \partial^2 U/\partial \rho (\partial \hat{\sigma}_{a_j}^{-1} - \partial^2 U/\partial \rho \hat{\sigma}_{ij}^{-1}) \) > 0.

We will work out each of these two terms separately. But first, both depend on the partial derivative of utility with respect to risk aversion. Risk aversion shows up in utility only through the asset prices. Recall from (34)–(36) that price can be expressed as \( pr = -\rho \Sigma (\bar{x} + x) + f + \Sigma \Sigma (\mu - f) \).

Since \( E[x] = 0 \), the expected price has partial derivative \( \partial E[pr]/\partial \rho = \Sigma \bar{x} \). Note that price variance is \( V[pr] = \rho^2 e' \Sigma \Sigma + (I - \Sigma \Sigma) \Sigma (I - \Sigma \Sigma) \). The partial derivative of this is \( \partial V[pr]/\partial \rho = 2 \rho e' \Sigma \Sigma \). Taking the
partial derivative of utility and substituting these two expression in yields

\[ \frac{\partial U}{\partial \rho} = \rho \sigma_x T \Sigma_j^{-1} \Sigma \Sigma_j^{-1} \Sigma \bar{x}. \] (46)

The next step is to differentiate (46) with respect to \( \hat{\sigma}_{a_j}^{-1} \). Since \( \hat{\sigma}_{a_j}^{-1} \) is the precision of agent \( j \)'s information, it does not affect aggregate variables such as \( \Sigma \). Recalling that \( \partial \hat{\Sigma}_j^{-1} / \partial \hat{\sigma}_{a_j}^{-1} = Y_a \),

\[ \frac{\partial^2 U}{\partial \rho \partial \hat{\sigma}_{a_j}^{-1}} = \rho \sigma_x T \Sigma_y [\Sigma Y_a + \frac{1}{2} \bar{x}' \Sigma Y_a \Sigma \bar{x}]. \] (47)

Next, we follow the same steps to differentiate (46) with respect to \( \hat{\sigma}_{1_j}^{-1} \), which also affects only \( \hat{\Sigma}_j \). Recalling that \( \partial \hat{\Sigma}_j^{-1} / \partial \hat{\sigma}_{1_j}^{-1} = Y_1 \), and using the fact that the trace is invariant to matrix ordering,

\[ \frac{\partial^2 U}{\partial \rho \partial \hat{\sigma}_{1_j}^{-1}} = \rho \sigma_x T \Sigma Y_1 [\Sigma Y_a + \frac{1}{2} \bar{x}' \Sigma Y_a \Sigma \bar{x}]. \] (48)

The extent to which risk aversion affects the utility of reallocating precision from risk 1 to risk \( a \) is the difference of (47) and (48):

\[ \partial \rho (\partial U / \partial (\hat{\sigma}_{a_j}^{-1} - \hat{\sigma}_{1_j}^{-1})) = \rho \sigma_x T \Sigma (Y_a - Y_1) \Sigma + \frac{1}{2} \bar{x}' \Sigma (Y_a - Y_1) \Sigma \bar{x}. \] (49)

Multiplying out term-by-term \( \Sigma Y_a \Sigma \) reveals that it equals \( \hat{\sigma}^2_{a} bb' \). Multiplying out \( \Sigma Y_1 \Sigma \) reveals that it equals

\[ \Sigma Y_1 \Sigma = \begin{bmatrix} \hat{\sigma}^2_{a} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \]

Thus,

\[ T \Sigma (Y_a - Y_1) \Sigma \bar{x} = \hat{\sigma}^2_{a} (b_1^2 + b_2^2 + 1) - \hat{\sigma}^2_{1} \]

and

\[ \bar{x}' \Sigma (Y_a - Y_1) \Sigma \bar{x} = \hat{\sigma}^2_{a} (b_1 \bar{x}_1 + b_2 \bar{x}_2 + \bar{x}_3)^2 - \hat{\sigma}^2_{1} \]

If these two terms are positive, then \( \partial \rho (\partial U / \partial (\hat{\sigma}_{a_j}^{-1} - \hat{\sigma}_{1_j}^{-1})) > 0 \). Note that the whole second partial derivative is increasing in \( x_3 \), the supply of the composite asset:

\[ \partial \rho (\partial U / \partial (\hat{\sigma}_{a_j}^{-1} - \hat{\sigma}_{1_j}^{-1})) = \rho \sigma_x (\hat{\sigma}^2_{a} (b_1^2 + b_2^2 + 1) - \hat{\sigma}^2_{1}) + \frac{1}{2} \hat{\sigma}^2_{a} (b_1 \bar{x}_1 + b_2 \bar{x}_2 + \bar{x}_3)^2 - \frac{1}{2} \bar{x}_3^2 \hat{\sigma}^2_{1}. \] (50)

Thus, as long as the composite asset, meant to represent the entire market capitalization, aside from the two assets 1 and 2, is large enough relative to assets 1 and 2, the cross-partial derivative will be positive. □
Figure 1: Cross-Sectional Distribution of Outperformance

This figure shows the cross-sectional distribution in recessions (red) and in expansions (blue) of the four-factor alpha for the mutual funds in our sample. The data are from CRSP and are available monthly from January 1980 until December 2005.

Figure 2: Investment Performance in Recessions vs. Expansions

This figure shows four-factor alphas for all domestic equity mutual funds. They are obtained by, first, regressing fund returns in excess of the risk-free rate on the market return in excess of the risk-free rate, the return on a portfolio that is long in small firms and short in large firms (SMB), the return on a portfolio that is long in value firms and short in growth firms (HML), and the return on a portfolio that is long in winners and short in losers (UMD) in twelve-month rolling-window regressions. The fund alpha is the intercept of that regression. In a second step, we regress the fund alphas on a recession indicator variable in a panel regression, controlling for other fund characteristics. The intercept of that regression is the alpha in expansions, the sum of the coefficient on the dummy and the intercept is the alpha in recessions. We annualize monthly alphas by multiplying them by twelve. The data are from CRSP and available monthly from January 1980 until December 2005.
In a given month, all funds are ranked into quintiles based on their Skill Index, defined as
\[ \text{Skill Index}_j(t) = w(z_t) \text{Timing}_j(t) + (1 - w(z_t)) \text{Picking}_j(t), \]
with \( z_t \in \{ E, R \} \). Skill Index is standardized cross-sectionally to have mean zero and standard deviation one. We then compute and plot the Skill Index of the funds in each quintile in the subsequent twelve months. The data cover the period from January 1980 until December 2005.
Table 1: Individual Stocks Have More Aggregate Risk in Recessions

For each stock $i$ and each month $t$, we estimate a CAPM equation based on twelve months of data (a twelve-month rolling-window regression). This estimation delivers the stock’s beta, $\beta_{it}$, and its residual standard deviation, $\sigma_{it}^\epsilon$. We define stock $i$’s aggregate risk in month $t$ as $|\beta_{it}\sigma_{it}^m|$ and its idiosyncratic risk as $\sigma_{it}^\epsilon$, where $\sigma_{it}^m$ is formed as the realized volatility from daily return observations. Panel A reports the results from a time-series regression of the aggregate risk averaged across stocks, $\frac{1}{N} \sum_{i=1}^{N} |\beta_{it}\sigma_{it}^m|$, in Columns 1 and 2, and of the idiosyncratic risk averaged across stocks, $\frac{1}{N} \sum_{i=1}^{N} \sigma_{it}^\epsilon$, in Columns 3 and 4 on Recession. Recession is an indicator variable equal to one for every month the economy is in a recession according to the NBER, and zero otherwise. In Columns 2 and 4 we include several aggregate control variables: We regress the portfolio (net) return in excess of the risk-free rate on Recession and a set of four risk factors: the market excess return (MKTPREM), the return on the small-minus-big portfolio (SMB), the return on the high-minus-low book-to-market portfolio (HML), the return on the up-minus-down momentum portfolio (UMD). The data are monthly and cover the period 1980 to 2005 (309 months). Standard errors (in parentheses) are corrected for autocorrelation and heteroscedasticity. Panel B reports results of panel regressions of the aggregate risk of an individual stock, $|\beta_{it}\sigma_{it}^m|$, in Columns 1 and 2 and of its idiosyncratic risk, $\sigma_{it}^\epsilon$, in Columns 3 and 4 on Recession. In Columns 2 and 4 we include several firm-specific control variables: the log market capitalization of the stock, $\log(\text{Size})$, the ratio of book equity to market equity, $\frac{\text{B}}{\text{M}}$, the average return over the past year, Momentum, the stock’s leverage, Leverage, measured as the ratio of book debt to book debt plus book equity, and an indicator variable, NASDAQ, equal to one if the stock is traded on NASDAQ. All control variables are lagged one month. The data are monthly and cover all stocks in the CRSP universe for the period 1980 to 2005. Standard errors (in parentheses) are clustered at the stock and time dimensions.

<table>
<thead>
<tr>
<th></th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Aggregate Risk</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Recession</td>
<td>1.348</td>
<td>1.308</td>
<td>0.058</td>
<td>0.016</td>
</tr>
<tr>
<td></td>
<td>(0.693)</td>
<td>(0.678)</td>
<td>(1.018)</td>
<td>(1.016)</td>
</tr>
<tr>
<td>MKTPREM</td>
<td>-4.034</td>
<td>-1.865</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(3.055)</td>
<td>(3.043)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>SMB</td>
<td>8.110</td>
<td>12.045</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(3.780)</td>
<td>(4.293)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>HML</td>
<td>0.292</td>
<td>9.664</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(5.458)</td>
<td>(8.150)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>UMD</td>
<td>-1.279</td>
<td>-1.112</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(2.349)</td>
<td>(3.888)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Constant</td>
<td>6.694</td>
<td>6.748</td>
<td>13.229</td>
<td>13.196</td>
</tr>
<tr>
<td></td>
<td>(0.204)</td>
<td>(0.212)</td>
<td>(0.286)</td>
<td>(0.276)</td>
</tr>
<tr>
<td><strong>Idiosyncratic Risk</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Observations</td>
<td>309</td>
<td>309</td>
<td>309</td>
<td>309</td>
</tr>
</tbody>
</table>

**Panel A: Time-Series Regression**

<table>
<thead>
<tr>
<th></th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Aggregate Risk</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Recession</td>
<td>1.203</td>
<td>1.419</td>
<td>0.064</td>
<td>0.510</td>
</tr>
<tr>
<td></td>
<td>(0.242)</td>
<td>(0.238)</td>
<td>(0.493)</td>
<td>(0.580)</td>
</tr>
<tr>
<td>Log(\text{Size})</td>
<td>-0.145</td>
<td>-1.544</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.021)</td>
<td>(0.037)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>B-M Ratio</td>
<td>-0.934</td>
<td>-2.691</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.056)</td>
<td>(0.086)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Momentum</td>
<td>0.097</td>
<td>2.059</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.101)</td>
<td>(0.177)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Leverage</td>
<td>-0.690</td>
<td>-1.006</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.074)</td>
<td>(0.119)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>NASDAQ</td>
<td>0.600</td>
<td>1.937</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.075)</td>
<td>(0.105)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Constant</td>
<td>4.924</td>
<td>4.902</td>
<td>12.641</td>
<td>12.592</td>
</tr>
<tr>
<td></td>
<td>(0.092)</td>
<td>(0.095)</td>
<td>(0.122)</td>
<td>(0.144)</td>
</tr>
<tr>
<td>Observations</td>
<td>1,312,216</td>
<td>1,312,216</td>
<td>1,312,216</td>
<td>1,312,216</td>
</tr>
</tbody>
</table>
Table 2: Attention Allocation

The dependent variables are funds’ reliance on aggregate information (RAI), funds’ reliance on stock-specific information (RSI), funds’ market-timing ability (Timing), and funds’ stock-picking ability (Picking). A fund j’s $R_{AI,j}^t$ is defined as the (twelve-month rolling-window time-series) covariance between the funds’ holdings in deviation from the market $(w_{it}^j - w_{it}^m)$ in month $t$ and changes in industrial production growth between $t$ and $t+1$. A fund j’s $R_{SI,j}^t$ is defined as the (across stock) covariance between the funds’ holdings in deviation from the market $(w_{it}^j - w_{it}^m)$ in month $t$ and changes in earnings growth between $t$ and $t+1$. Timing is defined as follows: $\text{Timing}_j^t = \sum_{i=1}^{N} (w_{it}^j - w_{it}^m)(\beta_{it}R_{m,t+1}^t)$ and $\text{Picking}_j^t = \sum_{i=1}^{N} (w_{it}^j - w_{it}^m)(R_{i,t+1}^t - \beta_{it}R_{m,t+1}^t)$, where the stocks’ $\beta_{it}$ is measured over a twelve-month rolling window. $RAI, RSI, Timing$, and $Picking$ are all multiplied by 10,000 for ease of readability. $Recession$ is an indicator variable equal to one for every month the economy is in a recession according to the NBER, and zero otherwise. $Log(Age)$ is the natural logarithm of fund age. $Log(TNA)$ is the natural logarithm of a fund total net assets. $Expenses$ is the fund expense ratio. $Turnover$ is the fund turnover ratio. $Flow$ is the percentage growth in a fund’s new money. $Load$ is the total fund load. The last three control variables measure the style of a fund along the size, value, and momentum dimensions, calculated from the scores of the stocks in their portfolio in that month. They are omitted for brevity. All control variables are demeaned. Flow and Turnover are winsorized at the 1% level. The data are monthly and cover the period 1980 to 2005. Standard errors (in parentheses) are clustered at the fund and time dimensions.

<table>
<thead>
<tr>
<th></th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
<th>(5)</th>
<th>(6)</th>
<th>(7)</th>
<th>(8)</th>
</tr>
</thead>
<tbody>
<tr>
<td>RAI</td>
<td>0.011</td>
<td>0.011</td>
<td>-0.682</td>
<td>-0.696</td>
<td>0.140</td>
<td>0.139</td>
<td>-0.144</td>
<td>-0.146</td>
</tr>
<tr>
<td>RSI</td>
<td>(0.004)</td>
<td>(0.004)</td>
<td>(0.159)</td>
<td>(0.150)</td>
<td>(0.070)</td>
<td>(0.068)</td>
<td>(0.047)</td>
<td>(0.047)</td>
</tr>
<tr>
<td>Timing</td>
<td>-0.002</td>
<td>0.423</td>
<td>0.006</td>
<td>0.004</td>
<td>(0.01)</td>
<td>(0.060)</td>
<td>(0.006)</td>
<td>(0.004)</td>
</tr>
<tr>
<td>Picking</td>
<td>-0.001</td>
<td>-0.173</td>
<td>0.000</td>
<td>-0.003</td>
<td>(0.000)</td>
<td>(0.029)</td>
<td>(0.004)</td>
<td>(0.003)</td>
</tr>
<tr>
<td>Recession</td>
<td>-0.330</td>
<td>88.756</td>
<td>1.021</td>
<td>-0.815</td>
<td>(0.244)</td>
<td>(11.459)</td>
<td>(1.280)</td>
<td>(0.839)</td>
</tr>
<tr>
<td>Log(Age)</td>
<td>-0.002</td>
<td>0.423</td>
<td>0.006</td>
<td>0.004</td>
<td>(0.01)</td>
<td>(0.060)</td>
<td>(0.006)</td>
<td>(0.004)</td>
</tr>
<tr>
<td>Log(TNA)</td>
<td>-0.001</td>
<td>-0.173</td>
<td>0.000</td>
<td>-0.003</td>
<td>(0.000)</td>
<td>(0.029)</td>
<td>(0.004)</td>
<td>(0.003)</td>
</tr>
<tr>
<td>Expenses</td>
<td>-0.330</td>
<td>88.756</td>
<td>1.021</td>
<td>-0.815</td>
<td>(0.244)</td>
<td>(11.459)</td>
<td>(1.280)</td>
<td>(0.839)</td>
</tr>
<tr>
<td>Turnover</td>
<td>-0.008</td>
<td>1.692</td>
<td>-0.001</td>
<td>0.058</td>
<td>(0.010)</td>
<td>(0.639)</td>
<td>(0.078)</td>
<td>(0.088)</td>
</tr>
<tr>
<td>Flow</td>
<td>0.017</td>
<td>-9.644</td>
<td>0.033</td>
<td>0.156</td>
<td>(0.023)</td>
<td>(1.972)</td>
<td>(0.180)</td>
<td>(0.131)</td>
</tr>
<tr>
<td>Load</td>
<td>-0.001</td>
<td>3.084</td>
<td>0.007</td>
<td>-0.010</td>
<td>(0.001)</td>
<td>(0.069)</td>
<td>(0.070)</td>
<td>(0.024)</td>
</tr>
<tr>
<td>Constant</td>
<td>-0.001</td>
<td>3.084</td>
<td>0.007</td>
<td>-0.010</td>
<td>(0.001)</td>
<td>(0.069)</td>
<td>(0.070)</td>
<td>(0.024)</td>
</tr>
<tr>
<td>Observations</td>
<td>224,257</td>
<td>224,257</td>
<td>166,328</td>
<td>166,328</td>
<td>221,306</td>
<td>221,306</td>
<td>221,306</td>
<td>221,306</td>
</tr>
</tbody>
</table>
Table 3: Dispersion in Funds’ Portfolio Strategies and Returns

The dependent variables are Concentration, Idio Vol, and $|X_j^t - \bar{X}_t|$, where $X_j^t$ is the CAPM Alpha, 4 – Factor Alpha, or CAPM Beta, and $\bar{X}$ denotes the (equally weighted) cross-sectional average. Concentration for fund $j$ at time $t$ is calculated as the Herfindahl index of portfolio weights in stocks $i \in \{1, \cdots, N\}$ in deviation from the market portfolio weights $\sum_{i=1}^{N} (w_{it} - w_{it}^m)^2 \times 100$. Idio Vol is the idiosyncratic volatility from a twelve-month rolling-window CAPM regression at the fund level. The CAPM alpha (and four-factor alpha) and the CAPM beta are obtained from twelve-month rolling-window regressions of fund-level excess returns on excess market returns (and returns on SMB, HML, and MOM). Recession is an indicator variable equal to one for every month the economy is in a recession according to the NBER, and zero otherwise. Log(Age) is the natural logarithm of fund age. Log(TNA) is the logarithm of a fund total net assets. Expenses is the fund expense ratio. Flow is the percentage growth in a fund’s new money. Turnover is the fund turnover ratio. Load is the total fund load. The last three control variables measure the style of a fund along the size, value, and momentum dimensions, calculated from the scores of the stocks in their portfolio in that month. They are omitted for brevity. All control variables are demeaned. Flow and Turnover are winsorized at the 1% level. The data are monthly and cover the period 1980 to 2005. Standard errors (in parentheses) are clustered at the fund and time dimensions.

<table>
<thead>
<tr>
<th></th>
<th>(1) Concentration</th>
<th>(2) Idio Vol</th>
<th>(3) CAPM Alpha</th>
<th>(4) 4-Factor Alpha</th>
<th>(5) CAPM Beta</th>
<th>(6) Recession</th>
<th>(7) Log(Age)</th>
<th>(8) Log(TNA)</th>
<th>(9) Expenses</th>
<th>(10) Turnover</th>
<th>(11) Flow</th>
<th>(12) Load</th>
<th>(13) Constant</th>
<th>(14) Constant</th>
</tr>
</thead>
<tbody>
<tr>
<td>Recession</td>
<td>0.206 (0.027)</td>
<td>0.147 (0.026)</td>
<td>0.348 (0.127)</td>
<td>0.359 (0.104)</td>
<td>0.275 (0.054)</td>
<td>0.298 (0.050)</td>
<td>0.140 (0.028)</td>
<td>0.150 (0.025)</td>
<td>0.082 (0.015)</td>
<td>0.083 (0.014)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Log(Age)</td>
<td>-0.203 (0.028)</td>
<td>-0.181 (0.017)</td>
<td>-0.045 (0.004)</td>
<td>-0.111 (0.002)</td>
<td>-0.099 (0.002)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Log(TNA)</td>
<td>-0.179 (0.014)</td>
<td>0.039 (0.012)</td>
<td>0.017 (0.002)</td>
<td>-0.006 (0.001)</td>
<td>0.03 (0.001)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Expenses</td>
<td>28.835 (4.860)</td>
<td>54.365 (2.806)</td>
<td>9.468 (6.585)</td>
<td>8.58 (4.468)</td>
<td>5.460 (2.35)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Turnover</td>
<td>-0.092 (0.025)</td>
<td>0.358 (0.023)</td>
<td>0.050 (0.004)</td>
<td>0.059 (0.003)</td>
<td>0.02 (0.001)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Flow</td>
<td>0.122 (0.104)</td>
<td>0.196 (0.174)</td>
<td>0.315 (0.053)</td>
<td>0.242 (0.032)</td>
<td>0.02 (0.017)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Load</td>
<td>-1.631 (0.907)</td>
<td>-5.562 (0.490)</td>
<td>-1.123 (0.095)</td>
<td>-0.420 (0.070)</td>
<td>-0.444 (0.042)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Constant</td>
<td>1.525 (0.024)</td>
<td>1.524 (0.022)</td>
<td>2.103 (0.071)</td>
<td>2.104 (0.068)</td>
<td>0.586 (0.018)</td>
<td>0.585 (0.016)</td>
<td>0.497 (0.009)</td>
<td>0.497 (0.008)</td>
<td>0.229 (0.006)</td>
<td>0.229 (0.006)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Observations: 230,185
Table 4: Fund Performance: Cross-Section Approach

The dependent variables are funds’ Abnormal Return, CAPM Alpha, 3-Factor Alpha, and 4-Factor Alpha. All are obtained from twelve-month rolling-window regressions of fund-level excess returns on excess market returns for the CAPM alpha, additionally on the SMB and the HML factors for the three-factor alpha, and additionally on the UMD factor for the four-factor alpha. The abnormal return is the fund return minus the market return. Recession is an indicator variable equal to one for every month the economy is in a recession according to the NBER, and zero otherwise. Log(Age) is the natural logarithm of fund age. Log(TNA) is the natural logarithm of a fund total net assets. Expenses is the fund expense ratio. Flow is the percentage growth in a fund’s new money. Turnover is the fund turnover ratio. Load is the total fund load. The last three control variables measure the style of a fund along the size, value, and momentum dimension, calculated from the scores of the stocks in their portfolio in that month. They are omitted for brevity. All control variables are demeaned. Flow and Turnover are winsorized at the 1% level. The data are monthly and cover the period 1980 to 2005. Standard errors (in parentheses) are clustered at the fund and time dimensions.

<table>
<thead>
<tr>
<th></th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
<th>(5)</th>
<th>(6)</th>
<th>(7)</th>
<th>(8)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Abnormal Return</td>
<td>CAPM Alpha</td>
<td>3-Factor Alpha</td>
<td>4-Factor Alpha</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Recession</td>
<td>0.342 (0.056)</td>
<td>0.425 (0.058)</td>
<td>0.337 (0.048)</td>
<td>0.404 (0.047)</td>
<td>0.043 (0.034)</td>
<td>0.073 (0.028)</td>
<td>0.107 (0.041)</td>
<td>0.139 (0.032)</td>
</tr>
<tr>
<td>Log(Age)</td>
<td>-0.031 (0.009)</td>
<td>-0.036 (0.008)</td>
<td>-0.028 (0.006)</td>
<td>-0.039 (0.006)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Log(TNA)</td>
<td>0.046 (0.005)</td>
<td>0.033 (0.004)</td>
<td>0.009 (0.003)</td>
<td>0.012 (0.003)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Expenses</td>
<td>-1.811 (1.046)</td>
<td>-2.372 (0.945)</td>
<td>-7.729 (0.782)</td>
<td>-7.547 (0.745)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Turnover</td>
<td>-0.023 (0.016)</td>
<td>-0.044 (0.010)</td>
<td>-0.074 (0.010)</td>
<td>-0.065 (0.008)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Flow</td>
<td>2.978 (0.244)</td>
<td>2.429 (0.172)</td>
<td>1.691 (0.097)</td>
<td>1.536 (0.096)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Load</td>
<td>-0.809 (0.226)</td>
<td>-0.757 (0.178)</td>
<td>-0.099 (0.131)</td>
<td>-0.335 (0.141)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Constant</td>
<td>-0.027 (0.027)</td>
<td>-0.033 (0.026)</td>
<td>-0.059 (0.025)</td>
<td>-0.063 (0.024)</td>
<td>-0.059 (0.020)</td>
<td>-0.060 (0.018)</td>
<td>-0.050 (0.023)</td>
<td>-0.052 (0.021)</td>
</tr>
</tbody>
</table>
Table 5: Recession and Volatility

The dependent variables are funds’ reliance on aggregate information (RAI), funds’ reliance on stock-specific information (RSI), funds’ portfolio concentration (Concentration), the cross-sectional dispersion in fund CAPM alphas (CAPM Alpha Disp) and betas (CAPM Beta Disp), and the funds’ four-factor alpha (4-Factor Alpha). The definitions of the dependent variables are listed in the captions of Tables 2, 3, and 4. Recession is an indicator variable equal to one for every month the economy is in a recession according to the NBER, and zero otherwise. Volatility is a dummy variable indicating periods of high volatility in fundamentals. We calculate the twelve-month rolling-window standard deviation of aggregate earnings growth. Aggregate earnings growth is the year-to-year log change in the earnings of S&P 500 index constituents; the aggregate earnings data are from Robert Shiller for the period from 1926 until 2008. Volatility equals one if the standard deviation of aggregate earnings growth is in the highest 10% of months in the 1926-2008 sample. Twelve percent of months in our 1985-2005 sample are such high volatility months. Log(Age) is the natural logarithm of fund age. Log(TNA) is the natural logarithm of a fund total net assets. Expenses is the fund expense ratio. Turnover is the fund turnover ratio. Flow is the percentage growth in a fund’s new money. Load is the total fund load. The last three control variables measure the style of a fund along the size, value, and momentum dimensions, calculated from the scores of the stocks in their portfolio in that month. They are omitted for brevity. All control variables are demeaned. Flow and Turnover are winsorized at the 1% level. The data are monthly and cover the period 1980 to 2005. Standard errors (in parentheses) are clustered at the fund and time dimensions.

<table>
<thead>
<tr>
<th></th>
<th>(1) RAI</th>
<th>(2) RSI</th>
<th>(3) Concentration</th>
<th>(4) CAPM Alpha Disp</th>
<th>(5) CAPM Beta Disp</th>
<th>(6) 4-Factor Alpha</th>
</tr>
</thead>
<tbody>
<tr>
<td>Recession</td>
<td>0.011</td>
<td>-0.503</td>
<td>0.185</td>
<td>0.217</td>
<td>0.062</td>
<td>0.100</td>
</tr>
<tr>
<td></td>
<td>(0.004)</td>
<td>(0.166)</td>
<td>(0.027)</td>
<td>(0.050)</td>
<td>(0.013)</td>
<td>(0.035)</td>
</tr>
<tr>
<td>Volatility</td>
<td>0.000</td>
<td>-0.477</td>
<td>0.206</td>
<td>0.254</td>
<td>0.077</td>
<td>0.125</td>
</tr>
<tr>
<td></td>
<td>(0.003)</td>
<td>(0.106)</td>
<td>(0.030)</td>
<td>(0.035)</td>
<td>(0.018)</td>
<td>(0.064)</td>
</tr>
<tr>
<td>Log(Age)</td>
<td>-0.002</td>
<td>0.416</td>
<td>0.198</td>
<td>-0.040</td>
<td>-0.004</td>
<td>-0.036</td>
</tr>
<tr>
<td></td>
<td>(0.001)</td>
<td>(0.060)</td>
<td>(0.027)</td>
<td>(0.004)</td>
<td>(0.002)</td>
<td>(0.006)</td>
</tr>
<tr>
<td>Log(TNA)</td>
<td>-0.001</td>
<td>-0.167</td>
<td>-0.176</td>
<td>0.013</td>
<td>0.002</td>
<td>0.010</td>
</tr>
<tr>
<td></td>
<td>(0.000)</td>
<td>(0.029)</td>
<td>(0.014)</td>
<td>(0.003)</td>
<td>(0.001)</td>
<td>(0.003)</td>
</tr>
<tr>
<td>Expenses</td>
<td>-0.331</td>
<td>90.911</td>
<td>29.859</td>
<td>8.166</td>
<td>3.695</td>
<td>-8.187</td>
</tr>
<tr>
<td></td>
<td>(0.247)</td>
<td>(11.599)</td>
<td>(4.868)</td>
<td>(5.866)</td>
<td>(2.11)</td>
<td>(7.87)</td>
</tr>
<tr>
<td>Turnover</td>
<td>-0.004</td>
<td>-0.199</td>
<td>-0.089</td>
<td>0.044</td>
<td>0.012</td>
<td>-0.067</td>
</tr>
<tr>
<td></td>
<td>(0.001)</td>
<td>(0.063)</td>
<td>(0.025)</td>
<td>(0.004)</td>
<td>(0.001)</td>
<td>(0.008)</td>
</tr>
<tr>
<td>Flow</td>
<td>-0.008</td>
<td>1.668</td>
<td>0.108</td>
<td>0.316</td>
<td>0.008</td>
<td>1.536</td>
</tr>
<tr>
<td></td>
<td>(0.010)</td>
<td>(0.632)</td>
<td>(0.104)</td>
<td>(0.046)</td>
<td>(0.016)</td>
<td>(0.096)</td>
</tr>
<tr>
<td>Load</td>
<td>0.017</td>
<td>-10.009</td>
<td>-1.789</td>
<td>-0.895</td>
<td>-0.252</td>
<td>-0.223</td>
</tr>
<tr>
<td></td>
<td>(0.023)</td>
<td>(1.984)</td>
<td>(0.909)</td>
<td>(0.100)</td>
<td>(0.040)</td>
<td>(0.141)</td>
</tr>
<tr>
<td>Constant</td>
<td>-0.001</td>
<td>3.124</td>
<td>1.546</td>
<td>0.561</td>
<td>0.221</td>
<td>-0.065</td>
</tr>
<tr>
<td></td>
<td>(0.001)</td>
<td>(0.074)</td>
<td>(0.023)</td>
<td>(0.016)</td>
<td>(0.005)</td>
<td>(0.021)</td>
</tr>
<tr>
<td>Observations</td>
<td>224,257</td>
<td>224,257</td>
<td>226,745</td>
<td>226,745</td>
<td>226,745</td>
<td>226,745</td>
</tr>
</tbody>
</table>
Table 6: Same Funds with Stock-Picking Ability in Expansions Have Market-Timing Ability in Recessions

We divide all fund-month observations into Recession and Expansion subsamples. *Recession* is an indicator variable equal to one for every month the economy is in a recession according to the NBER, and zero otherwise; *Expansion* is equal to one every month the economy is not in recession. The dependent variables are our measure of a fund’s market timing, $\text{Timing}_t^j$, and our measure of the fund’s stock-picking ability, $\text{Picking}_t^j$. They are defined as follows: $\text{Timing}_t^j = \sum_{i=1}^{N} (w_{it}^j - w_{it}^m)(\beta_i R_{it}^m + 1)$ and $\text{Picking}_t^j = \sum_{i=1}^{N} (w_{it}^j - w_{it}^m)(R_{it} + 1 - \beta_i R_{it}^m + 1)$. *Skill Picking* is an indicator variable equal to one for all funds whose *Picking* measure in Expansion is in the highest 25th percentile of the distribution, and zero otherwise. $\log(Age)$ is the natural logarithm of fund age. $\log(TNA)$ is the natural logarithm of a fund total net assets. *Expenses* is the fund expense ratio. *Flow* is the percentage growth in a fund’s new money. *Turnover* is the fund turnover ratio. *Load* is the total fund load. The last three control variables measure the style of a fund along the size, value, and momentum dimensions, calculated from the scores of the stocks in their portfolio in that month. They are omitted for brevity. All control variables are demeaned. *Flow* and *Turnover* are winsorized at the 1% level. The data are monthly and cover the period 1980 to 2005. Standard errors (in parentheses) are clustered at the fund and time dimensions.

<table>
<thead>
<tr>
<th>(1) Market Timing</th>
<th>(2)</th>
<th>(3) Stock Picking</th>
<th>(4)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Skill Picking</strong></td>
<td>0.000</td>
<td>0.017</td>
<td>0.056</td>
</tr>
<tr>
<td></td>
<td>(0.004)</td>
<td>(0.009)</td>
<td>(0.004)</td>
</tr>
<tr>
<td><strong>Log(Age)</strong></td>
<td>0.009</td>
<td>-0.025</td>
<td>-0.001</td>
</tr>
<tr>
<td></td>
<td>(0.002)</td>
<td>(0.006)</td>
<td>(0.002)</td>
</tr>
<tr>
<td><strong>Log(TNA)</strong></td>
<td>-0.001</td>
<td>0.005</td>
<td>0.000</td>
</tr>
<tr>
<td></td>
<td>(0.001)</td>
<td>(0.003)</td>
<td>(0.001)</td>
</tr>
<tr>
<td><strong>Expenses</strong></td>
<td>0.868</td>
<td>1.374</td>
<td>-1.291</td>
</tr>
<tr>
<td></td>
<td>(0.321)</td>
<td>(1.032)</td>
<td>(0.376)</td>
</tr>
<tr>
<td><strong>Turnover</strong></td>
<td>0.009</td>
<td>-0.011</td>
<td>0.017</td>
</tr>
<tr>
<td></td>
<td>(0.003)</td>
<td>(0.007)</td>
<td>(0.004)</td>
</tr>
<tr>
<td><strong>Flow</strong></td>
<td>0.056</td>
<td>-0.876</td>
<td>0.138</td>
</tr>
<tr>
<td></td>
<td>(0.024)</td>
<td>(0.112)</td>
<td>(0.037)</td>
</tr>
<tr>
<td><strong>Load</strong></td>
<td>0.094</td>
<td>-0.076</td>
<td>0.131</td>
</tr>
<tr>
<td></td>
<td>(0.049)</td>
<td>(0.151)</td>
<td>(0.055)</td>
</tr>
<tr>
<td><strong>Constant</strong></td>
<td>0.016</td>
<td>0.059</td>
<td>-0.021</td>
</tr>
<tr>
<td></td>
<td>(0.001)</td>
<td>(0.004)</td>
<td>(0.001)</td>
</tr>
<tr>
<td><strong>Observations</strong></td>
<td>204,330</td>
<td>18,354</td>
<td>204,330</td>
</tr>
</tbody>
</table>

56
Table 7: Unconditional Performance of “Skill-Picking” Funds

We divide all fund-month observations into Recession and Expansion subsamples. Expansion equals one every month the economy is not in recession according to the NBER, and zero otherwise. We define the stock picking ability of a fund as \( Picking_j^t = \sum_{i=1}^{N} (w_{ij}^t - w_{im}^t) (R_i^{t+1} - \beta_i R_m^{t+1}) \). Skill Picking, \( SP \), is an indicator variable equal to one for all funds whose \( Picking \) measure in Expansion is in the highest 25th percentile of the distribution, and zero otherwise. The dependent variables are the CAPM alpha, three-factor alpha, or four-factor alpha of the mutual fund, obtained from a twelve-month rolling-window regression of a fund’s excess returns before expenses on a set of common risk factors. \( \text{Log}(Age) \) is the natural logarithm of fund age. \( \text{Log}(TNA) \) is the natural logarithm of a fund’s total net assets. \( \text{Expenses} \) is the fund expense ratio. \( \text{Flow} \) is the percentage growth in a fund’s new money. \( \text{Turnover} \) is the fund turnover ratio. \( \text{Load} \) is the total fund load. The last three control variables measure the style of a fund along the size, value, and momentum dimensions, calculated from the scores of the stocks in their portfolio in that month. They are omitted for brevity. All control variables are demeaned. \( \text{Flow} \) and \( \text{Turnover} \) are winsorized at the 1% level. The data are monthly and cover the period 1980 to 2005. Standard errors (in parentheses) are clustered at the fund and time dimensions.

<table>
<thead>
<tr>
<th></th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>CAPM Alpha</td>
<td>3-Factor Alpha</td>
<td>4-Factor Alpha</td>
</tr>
<tr>
<td>Skill Picking</td>
<td>0.076</td>
<td>0.056</td>
<td>0.064</td>
</tr>
<tr>
<td></td>
<td>(0.040)</td>
<td>(0.021)</td>
<td>(0.038)</td>
</tr>
<tr>
<td>Log(Age)</td>
<td>-0.039</td>
<td>-0.028</td>
<td>-0.038</td>
</tr>
<tr>
<td></td>
<td>(0.008)</td>
<td>(0.006)</td>
<td>(0.006)</td>
</tr>
<tr>
<td>Log(TNA)</td>
<td>0.032</td>
<td>0.013</td>
<td>0.014</td>
</tr>
<tr>
<td></td>
<td>(0.005)</td>
<td>(0.004)</td>
<td>(0.004)</td>
</tr>
<tr>
<td>Expenses</td>
<td>4.956</td>
<td>0.627</td>
<td>0.241</td>
</tr>
<tr>
<td></td>
<td>(1.066)</td>
<td>(0.793)</td>
<td>(0.739)</td>
</tr>
<tr>
<td>Turnover</td>
<td>-0.009</td>
<td>-0.047</td>
<td>-0.041</td>
</tr>
<tr>
<td></td>
<td>(0.014)</td>
<td>(0.012)</td>
<td>(0.009)</td>
</tr>
<tr>
<td>Flow</td>
<td>2.579</td>
<td>1.754</td>
<td>1.602</td>
</tr>
<tr>
<td></td>
<td>(0.173)</td>
<td>(0.102)</td>
<td>(0.101)</td>
</tr>
<tr>
<td>Load</td>
<td>-0.744</td>
<td>-0.090</td>
<td>-0.289</td>
</tr>
<tr>
<td></td>
<td>(0.214)</td>
<td>(0.136)</td>
<td>(0.145)</td>
</tr>
<tr>
<td>Constant</td>
<td>0.057</td>
<td>0.038</td>
<td>0.049</td>
</tr>
<tr>
<td></td>
<td>(0.017)</td>
<td>(0.015)</td>
<td>(0.018)</td>
</tr>
<tr>
<td>Observations</td>
<td>227,183</td>
<td>227,183</td>
<td>227,183</td>
</tr>
</tbody>
</table>
Table 8: Comparing “Skill-Picking” Funds to Other Funds

We divide all fund-month observations into Recession and Expansion subsamples. Expansion equals one every month the economy is not in recession according to the NBER, and zero otherwise. We define the stock picking ability of a fund as \( \text{Picking}_t^j = \sum_{i=1}^{N} (w^{m}_{it} - w^{m}_{it})(R^{i}_{t+1} - \beta_i R^{m}_{t+1}). \) Skill Picking is an indicator variable equal one for all funds whose Picking measure in Expansion is in the highest 25th percentile of the distribution, and zero otherwise. Panel A reports fund-level characteristics. Age is the fund age in years. TNA is the fund’s total net assets. Expenses is the fund expense ratio. Turnover is the fund turnover ratio. Flows is the fund’s net inflow of new assets to manage. Concentration is the concentration of the fund’s portfolio, measured as the Herfindahl index of portfolio weights in deviation from the market portfolio’s weights. Stock Number is the number of stocks in the fund’s portfolio. Industry is the industry concentration of the fund’s portfolio, measured as the Herfindahl index of portfolio weights in a given industry in deviation from the market portfolio’s weights. Beta Deviation is the absolute difference between the fund’s beta and the average beta in its style category. RAI is the manager reliance on aggregate information, defined as the R-squared from the regression of the fund’s portfolio returns on contemporaneous changes in industrial production. Panel B reports manager-level characteristics. MBA equals one if the manager obtained an MBA degree, and zero otherwise. Ivy equals one if the manager graduated from an Ivy League institution, and zero otherwise. Age is the fund manager age in years. Experience is the fund manager experience in years. Gender equals one if the manager is a male and zero if the manager is female. Hedge Fund equals one if the manager ever departed to a hedge fund, and zero otherwise. \( \text{SP}1 - \text{SP}0 \) is the difference between the mean values of the groups for which Skill Picking equals one and zero, respectively. The data are monthly and cover the period 1980 to 2005. \( p - \text{values} \) measure statistical significance of the difference.

<table>
<thead>
<tr>
<th>Skill Picking = 1</th>
<th>Skill Picking = 0</th>
<th>Difference</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>Stdev.</td>
<td>Median</td>
</tr>
<tr>
<td>Age</td>
<td>10.01</td>
<td>8.91</td>
</tr>
<tr>
<td>TNA</td>
<td>621.13</td>
<td>2027.04</td>
</tr>
<tr>
<td>Expenses</td>
<td>1.48</td>
<td>0.47</td>
</tr>
<tr>
<td>Turnover</td>
<td>130.41</td>
<td>166.44</td>
</tr>
<tr>
<td>Flows</td>
<td>0.22</td>
<td>7.39</td>
</tr>
<tr>
<td>Concentration</td>
<td>1.68</td>
<td>1.60</td>
</tr>
<tr>
<td>Stock Number</td>
<td>90.83</td>
<td>110.20</td>
</tr>
<tr>
<td>Industry</td>
<td>8.49</td>
<td>7.90</td>
</tr>
<tr>
<td>Beta Deviation</td>
<td>0.18</td>
<td>0.38</td>
</tr>
<tr>
<td>RAI</td>
<td>4.13</td>
<td>5.93</td>
</tr>
</tbody>
</table>

Panel B: Fund Manager Characteristics

| MBA               | 42.09            | 49.37      | 0                | 39.49            | 48.88       | 0    | 2.60   | 0.128  |
| Ivy               | 25.36            | 43.51      | 0                | 27.94            | 44.87       | 0    | -2.57  | 0.205  |
| Age               | 53.02            | 10.42      | 50               | 54.11            | 10.06       | 52   | -1.08  | 0.081  |
| Experience        | 26.45            | 10.01      | 24               | 28.14            | 10.00       | 26   | -1.69  | 0.003  |
| Gender            | 90.89            | 28.77      | 100              | 90.50            | 29.31       | 100  | 0.39   | 0.681  |
| Hedge Fund        | 10.43            | 30.57      | 0                | 6.12             | 23.96       | 0    | 4.31   | 0.000  |
Table 9: Skill Index Predicts Performance

The dependent variable is the fund’s cumulative CAPM, three-factor, or four-factor alpha, calculated from a twelve-month rolling regression of observations in month \( t + 2 \) in the three left columns and in month \( t + 13 \) in the three most right columns. For each fund, we form the following skill index in month \( t \).

\[
\text{Skill Index}_t = w(z_t) \cdot \text{Timing}_t + (1 - w(z_t)) \cdot \text{Picking}_t, \quad z_t \in \{\text{Expansion, Recession}\},
\]

\( w(\text{Recession}) = 0.8 > w(\text{Expansion}) = 0.2, \)

where

\[
\text{Timing}_t = \frac{1}{N} \sum_{i=1}^{N} (w_{it} - w_{it}^m)(\beta_i R_{it+1}^m)
\]

and

\[
\text{Picking}_t = \frac{1}{N} \sum_{i=1}^{N} (w_{it} - w_{it}^m)(R_{it+1} - \beta_i R_{it+1}^m).
\]

\( Picking \) and \( Timing \) are normalized so that they are mean zero and have a standard deviation of one over the full sample. \( \log(Age) \) is the natural logarithm of fund age. \( \log(TNA) \) is the natural logarithm of a fund total net assets. \( Expenses \) is the fund expense ratio. \( Flow \) is the percentage growth in a fund’s new money. \( Turnover \) is the fund turnover ratio. \( Load \) is the total fund load. The last three control variables measure the style of a fund along the size, value, and momentum dimensions, calculated from the scores of the stocks in their portfolio in that month. They are omitted for brevity. All control variables are demeaned. \( Flow \) and \( Turnover \) are winsorized at the 1% level. The data are monthly and cover the period 1980 to 2005. Standard errors (in parentheses) are clustered at the fund and time dimensions.

<table>
<thead>
<tr>
<th></th>
<th>One Month Ahead</th>
<th></th>
<th>One Year Ahead</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>CAPM Alpha</td>
<td>3-Factor Alpha</td>
<td>4-Factor Alpha</td>
<td>CAPM Alpha</td>
</tr>
<tr>
<td>Skill Index</td>
<td>0.239</td>
<td>0.118</td>
<td>0.107</td>
<td>0.224</td>
</tr>
<tr>
<td>(0.044)</td>
<td>(0.022)</td>
<td>(0.019)</td>
<td>(0.031)</td>
<td>(0.025)</td>
</tr>
<tr>
<td>Log(Age)</td>
<td>-0.034</td>
<td>-0.024</td>
<td>-0.036</td>
<td>-0.019</td>
</tr>
<tr>
<td>(0.009)</td>
<td>(0.006)</td>
<td>(0.007)</td>
<td>(0.008)</td>
<td>(0.005)</td>
</tr>
<tr>
<td>Log(TNA)</td>
<td>0.026</td>
<td>0.010</td>
<td>0.011</td>
<td>-0.016</td>
</tr>
<tr>
<td>(0.005)</td>
<td>(0.004)</td>
<td>(0.004)</td>
<td>(0.003)</td>
<td>(0.003)</td>
</tr>
<tr>
<td>(1.620)</td>
<td>(1.004)</td>
<td>(0.957)</td>
<td>(1.578)</td>
<td>(0.917)</td>
</tr>
<tr>
<td>Turnover</td>
<td>-0.010</td>
<td>-0.047</td>
<td>-0.039</td>
<td>-0.001</td>
</tr>
<tr>
<td>(0.016)</td>
<td>(0.014)</td>
<td>(0.010)</td>
<td>(0.016)</td>
<td>(0.014)</td>
</tr>
<tr>
<td>Flow</td>
<td>2.409</td>
<td>1.664</td>
<td>1.519</td>
<td>0.237</td>
</tr>
<tr>
<td>(1.511)</td>
<td>(0.097)</td>
<td>(0.095)</td>
<td>(0.119)</td>
<td>(0.086)</td>
</tr>
<tr>
<td>Load</td>
<td>-0.762</td>
<td>-0.093</td>
<td>-0.313</td>
<td>-0.683</td>
</tr>
<tr>
<td>(0.233)</td>
<td>(0.144)</td>
<td>(0.157)</td>
<td>(0.225)</td>
<td>(0.129)</td>
</tr>
<tr>
<td>Constant</td>
<td>-0.030</td>
<td>-0.055</td>
<td>-0.041</td>
<td>-0.043</td>
</tr>
<tr>
<td>(0.024)</td>
<td>(0.018)</td>
<td>(0.021)</td>
<td>(0.024)</td>
<td>(0.019)</td>
</tr>
</tbody>
</table>

59