Rollover Risk: Optimal but Inefficient

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Abstract

This paper presents a competitive-equilibrium model of financial institutions optimally choosing their debt maturity structure in the face of idiosyncratic as well as aggregate risk. Rollover risk arises endogenously from the interaction of creditors in a global-game framework. When only idiosyncratic risk is present short-term debt acts as an effective disciplining device but once aggregate risk is added a two-sided inefficiency arises. Good aggregate states lead to excessive risk-taking while bad aggregate states suffer from fire-sale liquidation – economic surplus is destroyed in both cases. In the competitive equilibrium with endogenous liquidation values, the two-sided inefficiency reinforces itself through a feedback effect. It increases the volatility of liquidation values and thereby amplifies the impact of aggregate risk.

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1 Introduction

Short-term debt and the rollover risk it creates have been prominent features of
the financial crisis of 2007–2008. This paper presents a model of banks optimally
choosing the maturity structure of their debt.1 The banks interact in a competitive
equilibrium framework with endogenous interest rates and liquidation values.
The rollover risk for a given maturity structure arises endogenously from the co-
ordination problem of a bank’s creditors which is captured by a global game. The
model distinguishes between two exogenous sources of risk faced by an individual
bank, idiosyncratic risk specific to the bank’s assets as well as aggregate risk if
assets are correlated across banks. After receiving additional information about
the two sources of risk, the banks have a strong incentive to take excessive risks
and therefore use short-term debt as a disciplining device.

The main contribution of this paper is to show an important inefficiency that
arises from the use of short-term debt in the presence of aggregate risk. While
short-term debt acts as an effective disciplining device when banks only face
idiosyncratic risk, it is severely undermined when aggregate risk is added. The
problem is that the disciplining effect is too weak in good aggregate states and
too powerful in bad aggregate states. This leads to a two-sided inefficiency: In
good aggregate states the banks take excessive risks in the form of projects with
negative net present value. Bad aggregate states suffer from fire sales as projects
with positive net present value are liquidated. As a result, economic surplus is
destroyed in both situations.

In addition, the paper uses the competitive equilibrium framework with en-
dogenous liquidation values to highlight that the inefficiency reinforces itself through
a feedback effect. Given the presence of aggregate risk, even the first-best allocation
has liquidation values that vary across aggregate states. However, the use of
short-term debt – whose problem originates in this variation – further increases
the volatility of liquidation values and thereby amplifies the impact of aggregate
risk.

To be more specific, I model a group of banks, each with the opportunity to

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1Throughout the paper I mostly use the term “bank.” However, the paper applies not only
to traditional banks but any type of leveraged market-based financial institution.
invest in a project of its own. After the investment decision is made, additional information about each project’s expected payoff becomes available and a bank can decide whether to continue or liquidate its project. A key assumption of my model that differs from most of the literature on rollover risk is that liquidation is not inherently inefficient. Liquidated assets are employed in a secondary sector so the liquidation value reflects the true economic value of the assets in alternative uses. This implies that liquidation is only inefficient if the assets’ value in the secondary sector is less than their expected value in current use as the bank’s project. Importantly, this also implies that not liquidating is inefficient if the project’s expected payoff is less than the assets’ value in the secondary sector.

The secondary sector exhibits decreasing marginal productivity which implies that the liquidation value a bank receives is decreasing in aggregate asset sales. If projects are correlated across banks, leading to uncertainty about the equilibrium level of aggregate asset sales, this gives rise to aggregate risk with states of the world that differ in total asset sales and therefore liquidation values. Each individual bank then faces two sources of risk, idiosyncratic risk about its own project payoff and aggregate risk about the liquidation value determined by aggregate asset sales.

Unless a bank is fully equity financed, it has the wrong incentives when it comes to continuing or liquidating its project. Similar to a risk-shifting problem, the bank has an incentive to continue excessively risky projects at the cost of debt holders, i.e. projects whose expected payoff has turned out to be less than the liquidation value. Therefore a bank’s choice of maturity structure and the implied exposure to rollover risk play an important role for the realized economic surplus of the bank’s project.

A bank can choose any combination of long-term and short-term debt to finance its investment. While long-term debt has the same maturity as the project’s final payoff, short-term debt has to be rolled over after the additional information about the project’s expected payoff and the liquidation value becomes available. Rollover risk arises since it may not be possible to satisfy all withdrawals of short-term creditors, even by liquidating all of the bank’s assets.

I model the resulting coordination problem among short-term creditors as
a global game and derive a unique equilibrium with very intuitive properties. After bad news the short-term creditors withdraw their loans and the bank has to be liquidated while after good news the creditors roll over and the project is continued. Due to the two sources of risk there are two ways in which news can be bad. A creditor run can be triggered by bad idiosyncratic news about the bank itself or by bad aggregate news about the liquidation value. In addition, the two sources of risk interact in determining a bank’s rollover risk. A bank is more vulnerable to idiosyncratic news for bad aggregate news and more vulnerable to aggregate news for bad idiosyncratic news.

Since the global game equilibrium is unique and has continuous comparative statics, a bank’s initial choice of debt maturity structure directly translates into its exposure to rollover risk when the additional information becomes available. By choosing a greater fraction of short-term debt, the bank increases the risk that it suffers a run and has to liquidate its project.

To distinguish between the different effects of the two sources of risk on a bank’s maturity structure choice, I first analyze the benchmark case without aggregate risk, i.e. with projects that are uncorrelated across banks. Without uncertainty about the liquidation value, a bank has full control over the amount of rollover risk it exposes itself to. Therefore the bank chooses its financing exactly so as to implement the efficient liquidation policy where the project is liquidated if and only if the expected payoff turns out to be less than the liquidation value. Optimally exposing itself to rollover risk allows the bank to fully eliminate its incentive problem, maximizing its project’s ex-ante and interim net present value.

Adding aggregate risk in form of correlated projects and a random liquidation value has two important effects. First, the optimal liquidation policy now depends on the realization of the liquidation value. If the liquidation value turns out to be high, efficiency requires liquidating projects that should be continued if the liquidation value were low. At the same time, the bank’s rollover risk given the maturity structure chosen ex ante now varies with the realization of the liquidation value. If the liquidation value turns out to be high, creditors are less worried about the bank’s liquidity, making a run less likely.

The key problem is that these two effects go in opposite directions. For a high
liquidation value, *more* projects should be liquidated but the bank’s increased stability leads to *less* liquidation. For a low liquidation value, *less* projects should be liquidated but the bank’s reduced stability leads to *more* liquidation. Aggregate risk effectively drives a wedge between the efficient liquidation policy and the achievable liquidation policy. A bank optimally chooses its maturity structure but can no longer achieve the efficiency of the benchmark case without aggregate risk.

Given the optimal maturity structure, the disciplining effect of short-term debt is weaker than required in good aggregate states, allowing the bank to continue projects with negative net present value. This means that the bank is taking excessive risks with assets that have more valuable use elsewhere. As a mirror image, the disciplining effect is stronger than required in bad aggregate states, forcing the bank to liquidate projects with positive net present value. Here assets are sold at fire-sale prices which correspond to their actual value in alternative uses but are below their value in current use.

If projects are correlated then even the first-best allocation implies more asset sales and lower liquidation values in bad aggregate states than in good aggregate states. However, the competitive equilibrium always has *more* liquidation in bad states and *less* liquidation in good states than is efficient. This means that compared to the first-best allocation, liquidation values are higher in good states and lower in bad states, increasing volatility. Not only is the volatility of liquidation values causing the inefficiency in the first place, it is also further amplified in the competitive equilibrium. As a result, banks face greater aggregate risk than they would in the first-best allocation. Nevertheless, the competitive equilibrium is constrained efficient so there is no scope for policy intervention to improve welfare by changing banks’ use of short-term debt.

**Related Literature** The role of short-term debt as a disciplining device has been discussed in a literature going back to Calomiris and Kahn (1991).\(^2\) A common feature of this literature is that the benefit of a disciplining effect comes at the cost of inefficient liquidation and the choice of maturity structure has to trade

off the two. My paper differs, first, in the fact that liquidation is not per se inefficient and, second, in the distinction between two sources of risk. In particular, my model has an efficient outcome if only idiosyncratic risk is present. The new inefficiency in my model arises because of the inability of the disciplining mechanism to deal with two sources of risk. This leads to an inefficient outcome in good as well as bad aggregate states which the optimal maturity structure has to trade off. Another recent paper on optimal maturity structure choice is Brunnermeier and Oehmke (2010). Their model doesn’t have a disciplining problem so the optimal maturity structure is a corner solution of either all short-term or all long-term debt.

The paper is also related to Shleifer and Vishny (1992) who study the interaction of debt as a disciplining device with endogenous liquidation values. In their model, disciplining is only necessary in the good state and liquidation always happens at a (potentially inefficient) discount in the bad state. The focus of their paper is how equilibrium liquidation values limit debt capacity. In my model assets are always sold to outsiders but not necessarily at a discount. More importantly, the incentive problem in my model is present in all aggregate states so the optimal maturity structure has to trade-off the two inefficiencies of too much liquidation in one state and too little liquidation in the other.

Related from a technical point of view are several papers also using a global game setup to model the coordination problem among creditors, notably Morris and Shin (2004), Rochet and Vives (2004) and Goldstein and Pauzner (2005). In my paper, the global game is not as much front and center but rather used as a convenient modeling device. The convenience stems from the fact that under weak assumptions the global game has a unique equilibrium and that this equilibrium has continuous comparative statics. This allows me to study an ex-ante stage where the maturity structure is chosen optimally, taking into account the effect on the global-game equilibrium at a later stage. Finally, since the global game itself is restricted to a single time period, I avoid the complications in dynamic global games pointed out by Angeletos, Hellwig, and Pavan (2007).

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In a related model not using a global game setup, He and Xiong (2010) study the inter-temporal coordination problem among creditors with different maturity dates and derive very similar comparative statics.
The rest of the paper is structured as follows. In Section 2 I lay out the model and highlight the important features. I then proceed according to backwards induction and first derive the endogenous rollover risk for a given maturity structure in Section 3. Then I analyze the optimal maturity structure choice and competitive equilibrium in Section 4 for the efficient benchmark case without aggregate risk. In Section 5 I add aggregate risk and illustrate the two-sided inefficiency this causes. To quantify the effects of the model, I run a numerical simulation in Section 6. Finally, in Section 7 I discuss the robustness of the main results as well as some extensions. Section 8 concludes.

2 Model

There are three time periods $t = 0, 1, 2$ and all agents are risk neutral with a discount rate equal to the risk-free rate of zero. There is a continuum of identical banks $i \in [0, 1]$, each with the opportunity to invest in a project.

**Project** Bank $i$’s project requires an investment of 1 in the initial period $t = 0$ and has a random payoff in the final period $t = 2$ given by

$$
\begin{cases} 
X & \text{with probability } \theta_i, \\
0 & \text{with probability } 1 - \theta_i.
\end{cases}
$$

In the interim period $t = 1$, the project can still be abandoned and any fraction of its assets can be sold off to alternative uses at a liquidation value of $\ell < 1$. At the time of investment in $t = 0$, there is uncertainty about both the project’s expected payoff $\theta_iX$ and the liquidation value $\ell$, which is not resolved until additional information becomes available in the interim period $t = 1$. The structure of bank $i$’s project and its time-line is illustrated in Figure 1.

Importantly, in $t = 1$ the liquidation value $\ell$ is not directly linked to the expected payoff $\theta_iX$ of bank $i$’s project. In this model, liquidating a project entails taking the assets out of their current use and selling them to be used for a different purpose – an actual reallocation of capital. Therefore, liquidation is not inherently inefficient: Efficiency requires that a project be abandoned and that its assets be
liquidated whenever the expected payoff turns out to be less than the liquidation value and vice versa:

\[ \theta_i X \leq \ell \Rightarrow \text{abandon} \]
\[ \theta_i X > \ell \Rightarrow \text{continue} \]

**Incentive Problem** A debt-financed bank faces a basic incentive problem when it comes to continuing or liquidating its project which is similar to the risk-shifting problem of Jensen and Meckling (1976). Suppose that in the initial period \( t = 0 \) a bank has \( \eta \in [0, 1] \) of equity and raises \( 1 - \eta \) in some form of debt. Denote by \( D_t \) the face value of this debt at \( t = 1, 2 \). After learning about \( \theta_i \) and \( \ell \) in \( t = 1 \), the bank wants to continue its project whenever the expected equity payoff from continuing is greater than the equity payoff from liquidating:

\[ \theta_i (X - (1 - \eta) D_2) > \max \{0, \ell - (1 - \eta) D_1\} \]
\[ \Leftrightarrow \theta_i > \begin{cases} 0 & \text{for } 1 - \eta \leq \frac{\ell}{D_1} \\ \frac{\ell - (1 - \eta) D_1}{X - (1 - \eta) D_2} & \text{for } 1 - \eta > \frac{\ell}{D_1} \end{cases} \]

Unless the bank is fully equity financed (\( \eta = 1 \)), its decision doesn’t correspond to the efficient one of continuing whenever \( \theta_i X > \ell \Leftrightarrow \theta_i > \ell / X \). In particular, as long as \( D_1 X > D_2 \ell \), i.e. \( X \) sufficiently larger than \( \ell \), the bank wants to take excessive risks in the interim period by continuing projects with negative net
present value. Since this incentive problem is present for any \( \eta < 1 \) I consider the worst case and assume that banks have no initial equity.\(^4\)

**Uncertainty**  There are two possible aggregate states \( s \in \{H, L\} \) in the interim period \( t = 1 \), with probabilities \( p \) and \( 1 - p \) for the high state and the low state, respectively. Conditional on the aggregate state \( s \), the banks’ success probabilities \( \{\theta_i\} \) are i.i.d. with cumulative distribution function \( F_s \) on \([0, 1]\). The difference between the high state and the low state is that the distribution \( F_H \) strictly dominates the distribution \( F_L \) in terms of first-order stochastic dominance:

\[
F_H(\theta) < F_L(\theta) \quad \text{for all} \quad \theta \in (0, 1)
\]

This means that higher success probabilities are more likely in state \( H \) than in state \( L \) and therefore that banks’ projects are positively correlated through their success probabilities \( \{\theta_i\} \).

Both the aggregate state \( s \) and each bank’s success probability \( \theta_i \) are realized at the beginning of the interim period \( t = 1 \), before the continuation decision about the project, but after the investment decision in \( t = 0 \).

**Liquidation Value**  The liquidation value for the banks’ assets is determined endogenously from a downward-sloping aggregate demand for liquidated assets. The assets are reallocated to a secondary sector of the economy where they are employed with decreasing marginal productivity. For a total mass \( \phi \in [0, 1] \) of assets sold off by all banks, the liquidation value \( \ell(\phi) \) is given by a continuous and strictly decreasing function \( \ell : [0, 1] \rightarrow [0, 1] \) which corresponds to the assets’ marginal product in the secondary sector. Due to the exogenous correlation in banks’ \( \theta_i \)s there are fluctuations in equilibrium asset sales \( \phi \) across the aggregate states \( H \) and \( L \) which implies volatility in the endogenous liquidation value with two different liquidation values \( \ell_H = \ell(\phi_H) \) and \( \ell_L = \ell(\phi_L) \) in the two states.

Given the model setup so far, each bank \( i \) is exposed to two sources of risk. It faces aggregate risk in terms of the state \( s \) which determines the distribution

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\(^4\)This assumption abstracts from the choice of leverage to focus on the choice of maturity structure. Section 7 discusses implications of allowing for equity financing.
$F_s$ as well as the liquidation value $\ell_s$ and it faces idiosyncratic risk in terms of its success probability $\theta_i$ drawn from $F_s$. The first source of risk is “aggregate” in the sense that its outcome affects all banks in the same way, the second source is “idiosyncratic” in the sense that its outcome affects only the particular bank itself.

**Financing** Each bank has to raise the entire investment amount of 1 through loans from competitive investors in $t = 0$. A bank can choose any combination of long-term debt and short-term debt to finance its project.\(^5\) Long-term debt matures in the final period $t = 2$ at a face value of $D_{iLT}$. Short term debt, on the other hand, has to be rolled over in the interim period $t = 1$ and, if rolled over, has a face value of $D_{iST}$ in the final period $t = 2$. Instead of rolling over in $t = 1$ a short-term creditor has the right to withdraw the principal of his loan.\(^6\) This creates the possibility of the bank becoming illiquid in $t = 1$ since it may face more withdrawals from short-term creditors than it can satisfy by liquidating even the entire project.

Denoting by $\alpha_i \in [0, 1]$ the fraction of bank $i$’s project financed by short-term debt, the bank’s choice of debt maturity structure in the initial period $t = 0$ amounts to a combination of short-term debt and long-term debt $(\alpha_i, 1 - \alpha_i)$. The face values $D_{iST}$ and $D_{iLT}$ are then determined endogenously, taking into account both the idiosyncratic and aggregate risk, as well as the the rollover risk arising from the bank’s maturity structure.

**Competitive Equilibrium** A competitive equilibrium consists of a maturity structure $\alpha_i$ for every bank $i$ and liquidation values $\ell_H$ and $\ell_L$ in the two aggregate states such that (i) each bank chooses its maturity structure optimally given the equilibrium liquidation values, and (ii) the liquidation values result from the asset sales induced by the equilibrium maturity structure choices.

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\(^5\)Allowing only for short-term and long-term debt is essentially equivalent to assuming that $\theta_i$ and $\ell$ are not contractible. Section 7 discusses the implications of allowing more state-contingent contracting.

\(^6\)The assumption that the interim face value equals the principal is just a normalization and without loss of generality.
To reduce notational clutter I will drop the bank index $i$ in the following sections that deal only with an individual bank.

## 3 Endogenous Rollover Risk

Denoting the fraction of a bank’s short-term creditors who withdraw their loans in $t = 1$ by $\lambda$, the bank has to liquidate enough of the project to raise $\alpha \lambda$ for repayment. Since the bank can raise at most $\ell$ by liquidating the entire project, it can become illiquid if $\alpha > \ell$ and it will be illiquid whenever\(^7\)

$$\lambda > \frac{\ell}{\alpha}.$$

If the bank becomes illiquid in $t = 1$, there will be nothing left in $t = 2$ to repay the long-term creditors and, more importantly, any short-term creditors who decided to roll over their loan. The short-term creditors therefore face a coordination problem which I model as a global game by assuming a small amount of noise in each creditor’s information. The bank’s rollover risk is then derived from the equilibrium of the creditors’ coordination game.

The short-term debt in this model is meant to represent market-based financing such as commercial paper. In these markets, most of the funds are allocated through intermediaries, e.g. money market funds in the commercial paper market. It turns out to me much more tractable to assume that the roll-over decision is taken by a fund manager on behalf of the actual investor.\(^8\) There is a continuum of fund managers with the following payoffs. If a fund manager withdraws his loan in $t = 1$ he receives a constant payoff of $w > 0$, a base salary. If the fund manager rolls over his loan in $t = 1$ the payoff depends on whether the bank repays the loan in $t = 2$: if the bank repays, the fund manager receives a payoff of $bw$, his base salary multiplied by a bonus factor $b > 1$; if the bank doesn’t repay, the

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\(^7\)The paper focuses on the case of $\alpha > \ell$ where the bank can become illiquid. Appendix C discusses the case of $\alpha \leq \ell$ and presents sufficient conditions for the optimal maturity structure to satisfy $\alpha > \ell$.

\(^8\)The assumption of intermediation in the supply of short-term funding is similar to Rochet and Vives (2004) but doesn’t affect the validity of the main results. Appendix B presents the model without fund managers.
fund manager receives a payoff of zero.\footnote{These simple payoffs are chosen in order to economize on exogenous parameters appearing in the model’s expressions. More complicated payoffs, e.g. a withdrawal payoff also depending on the rollover outcome can easily be accommodated.} A higher $b$ corresponds to higher-powered incentive structures and implies that the fund managers are willing to take greater risks. It can also be interpreted as a proxy for the risk tolerance of the short-term funding sector in general.\footnote{See Krishnamurthy (2010) for a discussion on the importance of lenders’ risk tolerance in the short-term funding markets.}

Each fund manager has to make his roll-over decision in the interim period $t = 1$ based on the following information. While the resolution of aggregate risk in the form of the liquidation value $\ell$ is perfectly observed by everyone and becomes common knowledge, the resolution of idiosyncratic risk in the form of the bank’s success probability $\theta$ is not perfectly observed. Instead, each fund manager $j$ receives a noisy signal $\tilde{\theta}_j = \theta + \varepsilon_j$, where the signal noise $\varepsilon_j$ is i.i.d. uniformly on $[-\varepsilon, \varepsilon]$ for some arbitrarily small $\varepsilon > 0$.

Since the bank can become illiquid should too many loans be withdrawn, each fund manager’s expected payoff of rolling over depends critically on the fraction $\lambda$ of other fund managers who withdraw. Given that he only receives the bonus $bw$ if the bank remains liquid in $t = 1$ \textit{and} the project succeeds in $t = 2$, the expected payoff of rolling over is

\[
\text{Pr} \left[ \text{liquid} \mid \ell, \tilde{\theta}_j \right] \cdot \text{Pr} \left[ \text{success} \mid \ell, \tilde{\theta}_j \right] \cdot bw
\]

while the payoff of withdrawing is $w$ for sure.

Using global game techniques I can derive the \textit{unique} equilibrium of the fund managers’ coordination game.\footnote{See Appendix A for the details of this global game. For a comprehensive discussion of the use of global games since the seminal papers of Carlsson and van Damme (1993a,b) see Morris and Shin (2003).} The equilibrium is symmetric in switching strategies around a signal threshold $\hat{\theta}$ such that each fund manager withdraws for all signals below the threshold and rolls over for all signals above. The equilibrium switching point $\hat{\theta}$ is determined by the fact that a fund manager exactly at the
switching point has to be indifferent between rolling over and withdrawing, given his belief about the fraction $\lambda$ of others withdrawing. Taking the limit as the signal noise $\varepsilon$ goes to zero, the distribution of $\lambda$ conditional on being at the switching point $\hat{\theta}$ becomes uniform on $[0, 1]$. The indifference condition for a fund manager at the switching point therefore simplifies to

$$\frac{\ell}{\alpha} \cdot \hat{\theta} \cdot bw = w,$$

which pins down the equilibrium switching point as

$$\hat{\theta} = \frac{\alpha}{\ell b}.$$ 

(1)

**Proposition 1** For $\varepsilon \to 0$, the unique equilibrium among short-term creditors is in switching strategies around the success probability threshold $\hat{\theta} = \frac{\alpha}{\ell b}$:

- For realizations of $\theta$ below $\hat{\theta}$, all short-term debt is withdrawn and the bank becomes illiquid.
- For realizations of $\theta$ above $\hat{\theta}$ all short-term debt is rolled over and the bank remains liquid.

The simple structure of the equilibrium highlights the key determinants of a bank’s rollover risk before the uncertainty about $\theta$ and $\ell$ is resolved. This ex-ante rollover risk, i.e. the probability that the bank will suffer a run in the interim period is given by

$$\Pr[\theta \ell < \frac{\alpha}{b}].$$

First, the rollover risk is increasing in the fraction of short-term debt $\alpha$. Having a balance sheet that relies more heavily on short-term debt makes the bank more vulnerable to runs since it increases the total amount of withdrawals the bank may face. By choosing its debt maturity structure, the bank can therefore directly influence its ex-ante rollover risk.

Second, once the maturity structure is in place, whether the bank suffers a run or not depends on both sources of risk, idiosyncratic and aggregate. A run can be triggered by bad news about the project’s expected payoff (low $\theta$), or by bad
news about the liquidation value for the project’s assets (low $\ell$). When deciding whether to roll over, creditors (or their fund managers) are worried about a low $\theta$ because it means they will less likely be repaid (or receive their bonus) in the final period. In addition, they are worried about a low $\ell$ because it means the bank is more likely to become illiquid in the interim period. The first corresponds to a fundamentals-based run while the second corresponds to a market-based run. These two effects are very similar to the ones derived by He and Xiong (2010).\footnote{The distinction is also similar to the concepts of “funding liquidity” and “market liquidity” in Brunnermeier and Pedersen (2009).}

Third, the two sources of risk interact in determining the bank’s rollover risk. In particular, the bank is more vulnerable to idiosyncratic risk for a low realization of the liquidation value. The destabilizing effect of a low liquidation value means that the bank suffers runs for idiosyncratic news that would have left it unharmed had the liquidation value been higher. If the liquidation value fluctuates with the aggregate state, a bank will be more vulnerable to runs in the low aggregate state than in the high aggregate state, for any ex-ante maturity structure. This effect will play a crucial role in the inefficiency result of this paper.

4 Equilibrium without Aggregate Risk

I first analyze the model for the benchmark case without aggregate risk when the distribution of success probabilities is the same across states, $F_H = F_L =: F$. In this case each bank is able to maximize its project’s net present value and the competitive equilibrium achieves full efficiency.

4.1 Optimal Maturity Structure

In the initial period $t = 0$, short-term and long-term creditors and the bank anticipate what will happen in the following periods. This means that the face values of short-term debt and long-term debt, $D^{ST}$ and $D^{LT}$, have to guarantee that investors break even. The bank, when choosing its debt maturity structure $(\alpha, 1 - \alpha)$, takes into account the effect of $\alpha$ on the face values $D^{ST}$ and $D^{LT}$, as
well as the effect of $\alpha$ on the creditor coordination in $t = 1$.

Without aggregate risk, the liquidation value $\ell$ for the bank’s assets in $t = 1$ is deterministic. The only uncertainty stems from the project’s payoff and this uncertainty is partially resolved in $t = 1$ when the success probability $\theta$ is drawn from its distribution $F$. Depending on the additional information received about the project’s expected payoff, it will be efficient to either continue with the project or to abandon it and put the liquidated assets to alternative use. Liquidation is efficient whenever the project’s expected payoff is less than the liquidation value, $\theta X \leq \ell$.

To set up the bank’s maximization problem it is instructive to first derive the endogenous face values $D^{ST}$ and $D^{LT}$. Since the liquidation value is deterministic, so is the threshold determining the outcome of the creditor coordination in $t = 1$:

$$\hat{\theta} = \frac{\alpha}{\ell b}$$

For realizations of $\theta$ below $\hat{\theta}$, there will be a creditor run on the bank. In this case, each short-term creditor receives an equal share of the liquidation proceeds, $\ell/\alpha$, while long-term creditors don’t receive anything. For realizations of $\theta$ above $\hat{\theta}$, all short-term creditors roll over their loans and the bank continues to operate the project. In this case, all creditors receive the face value of their loan in $t = 2$ if the project is successful. Note that we are now dealing with the payoffs of the actual investors whose money is at stake, not the payoffs of the fund managers.\(^{13}\)

For a short-term creditor this implies an ex-ante expected payoff given by

$$F(\hat{\theta}) \frac{\ell}{\alpha} + \int_{\hat{\theta}}^{1} \theta D^{ST} dF(\theta).$$

With probability $F(\hat{\theta})$ the realization of the success probability is $\theta \in [0, \hat{\theta}]$ and there is a run on the bank leading to full liquidation; in this case the short-term creditor receives an equal share $\ell/\alpha$ of the liquidation value. Otherwise the realization of the success probability is $\theta \in (\hat{\theta}, 1]$ and there is no run on the bank;

\(^{13}\)It is natural to assume that the payments to the fund manager, the bonus $bw$ and base salary $w$, have to be paid by the investor. For simplicity I assume that these payments are negligible as a fraction of total investment and focus on the limiting case of $w \rightarrow 0$ but holding $b$ constant.
in this case the short-term creditor receives the face value of his loan $D_{ST}$ if the project is successful which happens with probability $\theta$.

A long-term creditor, on the other hand, only receives a payment if (i) there is no run in the interim period and (ii) the project is successful in the final period. The ex-ante expected payoff of a long-term creditor therefore is

$$\int_{\tilde{\theta}}^{1} \theta D_{LT} dF(\theta).$$

Since all creditors have to break even at the risk-free rate of zero, their expected payoff has to equal their investment of 1 so the endogenous face values for short-term debt and long-term debt are given by

$$D_{ST} = \frac{1 - F(\tilde{\theta})_{\ell}}{\int_{\tilde{\theta}}^{1} \theta dF(\theta)} \quad \text{and} \quad D_{LT} = \frac{1}{\int_{\tilde{\theta}}^{1} \theta dF(\theta)}.$$ 

Due to the effective seniority of short-term debt in the interim period, the face values satisfy $D_{ST} < D_{LT}$, i.e. the interest rate on short-term debt is lower than the interest rate on long-term debt – an upward sloping yield curve.

Given the rollover risk and the face values for a given maturity structure $(\alpha, 1 - \alpha)$, it remains to derive the bank’s ex-ante payoff. For realizations $\theta \leq \tilde{\theta}$ there is a run by short-term creditors in the interim period and the bank’s payoff is zero. For realizations $\theta > \tilde{\theta}$ there is no run in $t = 1$ and with probability $\theta$ the project is successful in $t = 2$. In this case the bank receives the project’s cash flow $X$ and has to repay the face value of its liabilities $\alpha D_{ST} + (1 - \alpha) D_{LT}$. The ex-ante expected payoff of the bank therefore is

$$\int_{\tilde{\theta}}^{1} \theta [X - \alpha D_{ST} - (1 - \alpha) D_{LT}] dF(\theta).$$

Substituting in the face values from (2) and rearranging, the bank’s ex-ante expected payoff becomes

$$F(\tilde{\theta})\ell + \int_{\tilde{\theta}}^{1} \theta XDdF(\theta) - 1.$$ 

(3)
Due to the rational expectations and the competitive creditors, the bank receives the entire economic surplus of its investment opportunity, given the rollover-risk threshold $\hat{\theta}$. The first term in (3) is the economic value realized in the states where the project is liquidated. The second term is the expected economic value realized in the states where the project is continued. The third term is the initial cost of investment. Since it receives the entire economic surplus, the bank fully internalizes the effect of its maturity structure choice on the efficiency of the rollover outcome.

Recalling the expression for $\hat{\theta}$ from the creditor coordination game in (1), the bank chooses $\alpha$ to solve the following problem:

$$\max \left\{ F(\hat{\theta})\ell + \int_{\hat{\theta}}^{1} \theta X dF(\theta) - 1 \right\} \quad \text{subject to} \quad \hat{\theta} = \frac{\alpha}{\ell b}$$

In choosing its maturity structure $\alpha$, the bank effectively chooses a rollover-risk threshold $\hat{\theta}$ since the creditor coordination results in a one-to-one mapping from maturity structure to rollover risk. The first order condition to the bank’s problem is

$$f(\hat{\theta}) \frac{1}{\ell b} (\ell - \hat{\theta} X) = 0,$$

which implies the following result.

**Proposition 2** Without aggregate risk, a bank chooses its maturity structure to implement the efficient liquidation policy:

$$\alpha^* = \frac{\ell^2 b}{X} \quad \text{resulting in} \quad \hat{\theta}^* = \frac{\ell}{X}.$$ 

The bank uses short-term debt as a disciplining device to implement a liquidation threshold $\hat{\theta}$ maximizing its payoff. Optimally exposing itself to rollover risk allows the bank to fully eliminate its incentive problem and maximize the project’s economic surplus. The perfect match between implemented and efficient liquidation policy is illustrated in Figure 2. Efficiency requires that projects with success probabilities $\theta \leq \ell/X$ be liquidated and that projects with success probabilities $\theta > \ell/X$ be continued. Since the bank has full control over its rollover-risk
threshold \( \hat{\theta} \) it chooses a maturity structure so that creditors withdraw and force liquidation for \( \theta \leq \ell/X \) and that they roll over and allow continuation for \( \theta > \ell/X \).

This result has important implications for the comparative statics of the bank’s rollover risk. While the rollover-risk threshold \( \hat{\theta} \) for a given maturity structure \( \alpha \) is decreasing in the liquidation value \( \ell \), the efficient liquidation threshold \( \ell/X \) is increasing in the liquidation value \( \ell \). As discussed in Section 3 above, for a given maturity structure, a higher liquidation value has a stabilizing effect on the bank and therefore reduces rollover risk. In terms of efficiency, however, a higher liquidation value means that there are better alternative uses for the project’s assets which raises the bar in terms of expected project payoff to justify continuing. Since the bank is able to implement the optimal liquidation policy, a higher liquidation value will cause it to increase rollover risk by choosing a maturity structure more reliant on short-term debt. This is reflected in the fact that \( \alpha^* \) is increasing in \( \ell \).

### 4.2 Competitive Equilibrium

In the previous section I derive a bank’s optimal maturity structure, taking the equilibrium liquidation value as given. In this section I derive the competitive equilibrium with an endogenous liquidation value. Making the dependence of the liquidation value on the aggregate asset sales \( \phi \) explicit, the optimization of the previous section results in

\[
\alpha^*(\phi) = \frac{\ell(\phi)^2 b}{X} \quad \text{and} \quad \hat{\theta}^*(\phi) = \frac{\ell(\phi)}{X}
\]
Since all banks are identical ex ante, the competitive equilibrium is symmetric with $\alpha^*_i = \alpha^*_j$ and $\hat{\theta}^*_i = \hat{\theta}^*_j$ for all $i, j$.

With a continuum of banks $i \in [0, 1]$ and the success probabilities $\{\theta_i\}$ i.i.d. with distribution $F$, the total mass $\phi$ of assets sold off in $t = 1$ is equal to the fraction of banks with $\theta_i \leq \hat{\theta}^*(\phi)$ who experience a run by their short-term creditors and have to liquidate their assets. The competitive equilibrium value $\phi^{CE}$ is therefore the solution to the fixed point equation

$$\phi^{CE} = F\left(\hat{\theta}^*(\phi^{CE})\right).$$

(4)

This is a fixed point condition for a continuous function mapping the unit interval onto itself so by Brouwer’s fixed point theorem there exists a solution. Since the right-hand side of the condition (4) is decreasing in $\phi$, the fixed point is unique.

**Proposition 3** The competitive equilibrium without aggregate risk is characterized by a mass $\phi^{CE}$ of assets liquidated, implicitly defined by

$$\phi^{CE} = F\left(\frac{\ell(\phi^{CE})}{X}\right),$$

as well as optimal maturity structures $\{\alpha_i^{CE}\}$ and resulting liquidation thresholds $\{\hat{\theta}_i^{CE}\}$ given by

$$\alpha_i^{CE} = \frac{\ell(\phi^{CE})^2 b}{X} \quad \text{and} \quad \hat{\theta}_i^{CE} = \frac{\ell(\phi^{CE})}{X} \quad \text{for all } i \in [0, 1].$$

Note that the competitive equilibrium is efficient since it equalizes the marginal productivity of assets used in the banking sector to the marginal productivity of assets used in the secondary sector:

$$\hat{\theta}^{CE} X = \ell(\phi^{CE})$$

This efficiency breaks down in the case with aggregate risk discussed next.
5 Equilibrium with Aggregate Risk

I now analyze the model with aggregate risk. The state is either high, $s = H$ with probability $p$, in which case each project’s success probability is drawn from the distribution $F_H$ or the state is low, $s = L$, with distribution $F_L$. Banks are no longer able to implement the efficient liquidation policy, as a two-sided inefficiency arises: Negative NPV projects are continued in the high state and positive NPV projects are liquidated in the low state.

5.1 Optimal Maturity Structure

The additional source of risk with the resulting uncertainty in liquidation values has two main implications from the point of view of an individual bank. The first implication is that the optimal project continuation decision is affected by the realization of $\ell$. While in the case without aggregate risk there was a single critical value for the project’s expected payoff, there are now two. For the low liquidation value $\ell_L$ the project should only be continued if $\theta X > \ell_L$, while for the high liquidation value $\ell_H$ the condition is $\theta X > \ell_H$. In particular, for realizations of the project’s success probability $\theta$ in the interval $[\ell_L/X, \ell_H/X]$, efficiency calls for liquidation if the assets have a high liquidation value and for continuation if the assets have a low liquidation value.

The second implication of aggregate risk is that the creditor coordination game is different depending on the aggregate state. There are now two equilibrium switching points, $\hat{\theta}_H$ and $\hat{\theta}_L$, one for each realization of $\ell$:

$$\hat{\theta}_H = \frac{\alpha}{\ell_H b} \quad \text{and} \quad \hat{\theta}_L = \frac{\alpha}{\ell_L b}$$

If the liquidation value turns out to be high, each creditor is less concerned about the other creditors withdrawing their loans and therefore more willing to roll over his own loan than when the liquidation value turns out to be low. Therefore the bank will be more stable and less likely to suffer a run by its short-term creditors if the liquidation value is high, which is reflected in the rollover-risk threshold.
being lower:

\[ \hat{\theta}_H < \hat{\theta}_L \]

As in the case without aggregate risk, the bank receives the entire economic surplus of its project, given the liquidation resulting from its maturity structure. The bank therefore chooses \( \alpha \) to solve the following problem

\[
\max \left\{ p \left( F_H(\hat{\theta}_H)\ell_H + \int_{\hat{\theta}_H}^{1} \theta X dF_H(\theta) \right) + (1-p) \left( F_L(\hat{\theta}_L)\ell_L + \int_{\hat{\theta}_L}^{1} \theta X dF_L(\theta) \right) \right\} \\
\text{subject to } \hat{\theta}_H = \frac{\alpha}{\ell_H b} \text{ and } \hat{\theta}_L = \frac{\alpha}{\ell_L b},
\]

which gives a first order condition

\[
p \left( f_H(\hat{\theta}_H)\frac{1}{\ell_H b} \left( \ell_H - \hat{\theta}_H X \right) \right) + (1-p) \left( f_L(\hat{\theta}_L)\frac{1}{\ell_L b} \left( \ell_L - \hat{\theta}_L X \right) \right) = 0. \quad (5)
\]

Although it cannot be solved explicitly for the optimal maturity structure, with \( \ell_H > \ell_L \) the first order condition implies \( \ell_H - \hat{\theta}_H X > 0 \) and \( \ell_L - \hat{\theta}_L X < 0 \) which gives the following result.

**Proposition 4** With aggregate risk and \( \ell_H > \ell_L \) a bank chooses its maturity structure resulting in

\[ \hat{\theta}_H < \frac{\ell_H}{X} \text{ and } \hat{\theta}_L > \frac{\ell_L}{X}. \]

There is a two-sided inefficiency:

- For \( s = H \), negative-NPV projects are continued whenever \( \theta \in (\hat{\theta}_H, \ell_H/X) \).
- For \( s = L \), positive-NPV projects are liquidated whenever \( \theta \in (\ell_L/X, \hat{\theta}_L) \).

The key effect of aggregate risk is that it drives a wedge between the efficient liquidation policy and the achievable liquidation policy. The effectiveness of using the maturity structure to eliminate the incentive problem and to implement an efficient liquidation policy is undermined when aggregate risk is added to the bank’s idiosyncratic risk. It is important to note that there are efficiency losses for both realizations of the liquidation value. When the liquidation value is high,
excessively risky projects that should be liquidated because they have negative net present value are continued. This first effect is illustrated in Figure 3. When the liquidation value is low on the other hand, valuable projects that should be continued because they have positive net present value are liquidated at fire-sale prices. This second effect is illustrated in Figure 4.

The two-sided inefficiency comes from the ambivalent role played by the liquidation value of the bank’s assets. A high liquidation value in good aggregate states makes the bank less vulnerable to runs by its short-term creditors but at the same time, the high liquidation value raises the bar in terms of alternate uses for the bank’s assets which worsens the incentive problem. Exactly the opposite happens in bad aggregate states where the liquidation value is low. This means that the disciplining effect of short-term debt is weak in the states where it is needed more and is strong in the states where it is needed less.
5.2 Competitive Equilibrium

In the competitive equilibrium, each bank $i \in [0, 1]$ chooses its maturity structure optimally according to the first order condition (5) of the previous section, taking the liquidation values $\ell_H$ and $\ell_L$ as given. As in the case without aggregate risk, the banks’ ex-ante symmetry implies that they all choose the same maturity structure. Making the dependence of the liquidation value on the asset sales $\phi$ explicit I denote the optimal maturity structure from the first order condition (5) by $\alpha^*(\phi_H, \phi_L)$. The resulting liquidation thresholds are then given by

$$\hat{\theta}_H(\phi_H, \phi_L) = \frac{\alpha^*(\phi_H, \phi_L)}{\ell(\phi_H)b} \quad \text{and} \quad \hat{\theta}_L(\phi_H, \phi_L) = \frac{\alpha^*(\phi_H, \phi_L)}{\ell(\phi_L)b}.$$

Since the success probabilities $\{\theta_i\}$ are i.i.d. conditional on the aggregate state $s$, the total mass $\phi_s$ of assets sold off in state $s$ is equal to the fraction of banks with $\theta_i \leq \hat{\theta}_s(\phi_H, \phi_L)$. As in the case without aggregate risk, the competitive equilibrium is characterized by a fixed point condition for the asset sales $(\phi_H, \phi_L)$, except that it is now two-dimensional:

$$\phi_{CE}^H = F_H\left(\hat{\theta}_H(\phi_{CE}^H, \phi_{CE}^L)\right) \quad \text{and} \quad \phi_{CE}^L = F_L\left(\hat{\theta}_L(\phi_{CE}^H, \phi_{CE}^L)\right).$$

This is a fixed point condition for a continuous function mapping the unit square onto itself so by Brouwer’s fixed point theorem there exists a solution.

**Proposition 5** The competitive equilibrium with aggregate risk is characterized by asset sales $(\phi_{CE}^H, \phi_{CE}^L)$ implicitly defined by

$$\phi_{CE}^H = F_H\left(\frac{\alpha^*(\phi_{CE}^H, \phi_{CE}^L)}{\ell(\phi_{CE}^H)b}\right) \quad \text{and} \quad \phi_{CE}^L = F_L\left(\frac{\alpha^*(\phi_{CE}^H, \phi_{CE}^L)}{\ell(\phi_{CE}^L)b}\right),$$

where $\alpha^*(\phi_{CE}^H, \phi_{CE}^L)$ is the optimal maturity structure defined by equation (5).

Due to the nesting of endogenous variables with the exogenous functions it is hard to specify general conditions to guarantee that the competitive equilibrium is unique and satisfies $\phi_{CE}^H < \phi_{CE}^L$ and therefore $\ell_H > \ell_L$. For the purposes of this
paper I restrict attention to cases where this is true. The functional forms and parameters in the numerical simulation of Section 6 show that such cases exist and don’t require unreasonable parameter assumptions.

To highlight the equilibrium effect of the two-sided inefficiency it is instructive to compare the competitive equilibrium to the first-best allocation with efficient liquidation thresholds:

\[ \hat{\theta}^{FB}_H = \frac{\ell_H}{X} \quad \text{and} \quad \hat{\theta}^{FB}_L = \frac{\ell_L}{X} \]

Using the efficient liquidation thresholds, the first-best allocation is characterized by asset sales \((\phi^{FB}_H, \phi^{FB}_L)\) implicitly defined by

\[ \phi^{FB}_H = F_H\left(\frac{\ell(\phi^{FB}_H)}{X}\right) \quad \text{and} \quad \phi^{FB}_L = F_L\left(\frac{\ell(\phi^{FB}_L)}{X}\right). \]

Since by strict first order stochastic dominance \(F_H(\theta) < F_L(\theta)\) for any \(\theta \in (0, 1)\), the first-best allocation satisfies \(\phi^{FB}_H < \phi^{FB}_L\) and therefore \(\ell^{FB}_H > \ell^{FB}_L\). This means that even in the first-best allocation the liquidation values vary across aggregate states.

From Proposition 4 we know that \(\hat{\theta}^{CE}_H < \hat{\theta}^{FB}_H\) and \(\hat{\theta}^{CE}_L > \hat{\theta}^{FB}_L\) which means that compared to the first-best allocation the competitive equilibrium has less liquidation in the high state and more liquidation in the low state. This implies the following result.

**Proposition 6** In the competitive equilibrium with aggregate risk and \(\ell_H > \ell_L\) the two-sided inefficiency has a self-reinforcing effect by amplifying the volatility in liquidation values:

\[ \ell^{CE}_H > \ell^{FB}_H \quad \text{and} \quad \ell^{CE}_L < \ell^{FB}_L. \]

The two-sided inefficiency originates in the fact that the liquidation values vary across aggregate states which is true even in the first-best allocation. Then the inefficiency causes too little liquidation in the high state and too much liquidation in the low state. This further increases the volatility of liquidation values, reinforcing the inefficiency in a feedback effect.
Nevertheless, it is important to note that the competitive equilibrium is still constrained efficient. Since each bank maximizes the economic surplus of its investment opportunity it has the same objective function as a social planner who is constrained to choosing a debt-maturity structure. The banks fully internalize the effect of their maturity structure when trading off the inefficiencies in the two aggregate states. Therefore, a policy intervention such as a tax on the use of short-term debt would reduce efficiency. It would lead to an increase of the inefficiency due to excessive risk-taking which more than outweighs the reduction of the inefficiency due to excessive liquidation.

6 Numerical Simulation

To quantify the two-sided inefficiency of this paper and to illustrate some of the comparative statics I now run a numerical simulation with specific functional forms. I assume a very simple functional form for the distributions of success probabilities in the two states:

\[
F_H(\theta) = \begin{cases} 
0 & \text{for } \theta < 0 \\
(1-q)\theta & \text{for } 0 \leq \theta < 1 \\
1 & \text{for } \theta \geq 1 
\end{cases}, \quad F_L(\theta) = \begin{cases} 
0 & \text{for } \theta < 0 \\
q + (1-q)\theta & \text{for } 0 \leq \theta < 1 \\
1 & \text{for } \theta \geq 1 
\end{cases}
\]

The distributions \(F_H\) and \(F_L\) are both uniform on the interval \((0,1)\) but have probability mass \(q\) at one of the endpoints, \(F_H\) at \(\theta = 1\) and \(F_L\) at \(\theta = 0\).

This boils down the nature of aggregate risk to two parameters, the probability \(p\) of the high state and the probability mass \(q\) in the state-contingent distributions of success probabilities. The two parameters have straightforward interpretations. The probability \(p\) is a proxy for the negative skew of aggregate risk since for higher values of \(p\) the low aggregate state is less likely. The probability mass \(q\) is a proxy for the correlation of the banks’ projects.\(^\text{14}\) I assume that the liquidation value is given by \(\ell(\phi) = 1 - \phi\) and that the payoff of a successful project in the final period \(t = 2\) is given by \(X = 2\).

\(^\text{14}\)The actual correlation coefficient between any two success probabilities \(\theta_i, \theta_j\) is given by 
\[\rho = 3q^2/(4q(3p-1)+1)\] which is strictly increasing in \(q\) for all \(p, q \in [0,1]\).
Without Aggregate Risk  In the case without aggregate risk, i.e. $q = 0$, the equation characterizing the competitive equilibrium simplifies to\(^{15}\)

$$\phi^{CE} = F\left(\frac{\ell(\phi^{CE})}{X}\right) = 1 - \frac{\phi^{CE}}{2},$$

which implies $\phi^{CE} = \hat{\theta}^{CE} = \frac{1}{3}$ and an equilibrium liquidation value of $\ell^{CE} = \frac{2}{3}$.

The expected profit of an individual bank given these values is

$$F(\hat{\theta}^{CE})\ell^{CE} + \int_{\hat{\theta}^{CE}}^{1} \theta X dF(\theta) - 1 = \frac{1}{9},$$

which corresponds to a return of 11.1% on the initial investment of 1.

With Aggregate Risk  In the case with aggregate risk, i.e. $q > 0$, the optimal maturity structure for given liquidation values $\ell_H, \ell_L$ implies liquidation thresholds

$$\hat{\theta}_H = \frac{1}{2} \frac{\ell_H \ell_L^2}{p \ell_L^2 + (1 - p) \ell_H^2} \quad \text{and} \quad \hat{\theta}_L = \frac{1}{2} \frac{\ell_H^2 \ell_L}{p \ell_H^2 + (1 - p) \ell_L^2}.$$ 

Substituting these into the fixed point condition characterizing the competitive equilibrium and using the functional forms for $F_H, F_L$ and $\ell$ yields

$$\phi_H = (1 - q) \frac{1}{2p(1 - \phi_L)^2 + (1 - p)(1 - \phi_H)^2} \frac{(1 - \phi_H)(1 - \phi_L)^2}{(1 - \phi_H)^2 + (1 - p)(1 - \phi_H)^2},$$

and

$$\phi_L = q + (1 - q) \frac{1}{2p(1 - \phi_L)^2 + (1 - p)(1 - \phi_H)^2} \frac{(1 - \phi_H)^2 (1 - \phi_L)}{(1 - \phi_H)^2 + (1 - p)(1 - \phi_H)^2}. \quad (6)$$

I computationally derive the competitive equilibrium for five different values each of $p$ and $q$. The probability $p$ of the high state varies from 0.5 to 0.9 and the probability mass $q$ varies from 0 to 0.1.\(^{16}\) The fixed point condition (6) has a unique solution $(\phi_H, \phi_L)$ with $\phi_H < \phi_L$ for each of the combinations $(p,q)$ I

\(^{15}\)Note that for $q = 0$, there is no difference between the states $H$ and $L$ so the probability $p$ doesn’t matter.

\(^{16}\)These values for $p$ and $q$ span a range of correlation coefficients between two success probabilities $\theta_i, \theta_j$ from 0% to 2.5%.
Figure 5: Percentage of ex-ante surplus destroyed by the inefficiency consider, with $\phi_H$ ranging from 0.25 to 0.33 and $\phi_L$ ranging from 0.33 to 0.54.

Figure 5 shows the impact of the two-sided inefficiency on economic surplus for the different combinations of $p$ and $q$. The figure displays the percentage of expected economic surplus lost in the competitive equilibrium relative to the first-best allocation. We see that the inefficiency cost is exponentially increasing in the correlation of projects as captured by $q$. In addition, the effect is strongest if aggregate risk is negatively skewed, i.e. the probability $p$ of the good state is high and the low state is unlikely to occur. In the worst case almost 50% of ex-ante economic surplus is lost due to the inefficient liquidation policy.

The amplification effect of the two-sided inefficiency is illustrated in Figure 6 for the intermediate case of $p = 0.7$ and $q = 0.05$. The figure starts at the liquidation values in the first-best allocation which are $\ell_{FB}^H = 0.678$ and $\ell_{FB}^L = 0.644$. It then iterates between (i) the red dashed curves, representing asset sales implied by the banks’ optimal reaction for given liquidation values and (ii) the blue solid curve, representing liquidation values implied by given asset sales. The iteration ends at the competitive equilibrium, where $\ell_{CE}^H = 0.700$ and $\ell_{CE}^L = 0.600$. Due to the amplification, the standard deviation of liquidation values increases by a factor of three from 0.015 in the first-best allocation to 0.046 in the competitive equilibrium while the mean changes by only 0.002. We see that the two-sided inefficiency has a strong self-reinforcing effect, significantly amplifying the magnitude.
of aggregate risk faced by each bank.

7 Discussion

For purposes of exposition this paper presents a very stylized model. The main results, however, are robust. The first building block of the model is that the incentive problem of a bank’s equity holders is worse when asset liquidity is high. This applies whenever there is still an upside possible in the bank’s project and this upside is greater than the liquidation value. In this case equity holders stand to gain more by keeping the project running instead of liquidating. If the liquidation value reflects, at least in part, the assets’ value in alternative uses, it can be higher than the expected payoff in current use which implies that the equity holders’ decision is inefficient. This incentive problem is more severe, the higher the liquidation value, i.e. the more valuable the assets are in alternative uses.

The second building block is the fact that a bank’s rollover risk is decreasing in its asset liquidity. This is a very basic comparative static with a strong intuition: When creditors decide whether to roll over their loans in a situation where illiquidity is possible, their decision will depend on how vulnerable the bank is. The key factor determining the bank’s vulnerability is how many withdrawals it
can satisfy given the liquidation value of its assets before it runs out of funds. Therefore higher asset liquidity means less jittery creditors means lower rollover risk. If asset liquidity varies across aggregate states, so will the rollover risk a bank faces.

If the liquidation value is deterministic as in the case without aggregate risk, a bank can expose itself to exactly so much rollover risk as to implement the optimal liquidation policy. However, if the liquidation value is random as in the case with aggregate risk, then the incentive problem is worse when the disciplining device is weaker and vice versa, causing the two-sided inefficiency.

Two potential solutions to address the inefficiency problem come to mind. The first potential solution is equity financing. As discussed in the paper, a fully equity financed bank will not face an incentive problem so it doesn’t fall victim to the inefficiency. In addition, if all banks were fully equity financed, the amplification effect would be eliminated and the volatility of liquidation values would be much lower. I want to highlight that besides the usual argument that equity financing may not be used because it is more expensive, the amplification effect implies that banks do not internalize the social cost of using debt financing. Therefore, even if I allowed the banks in the model to choose equity financing, their decision would be distorted toward debt financing.

The second potential solution is state-contingent debt. Clearly, if the face value of short-term debt could be made contingent on the liquidation value, the inefficiency problem would be solved. By specifying different face values for different aggregate states, the bank gains an additional degree of freedom in its self-disciplining device. It can then tailor its exposure to rollover risk in exactly the right way, as is the case without aggregate risk. However, this will make short-term debt more risky for the creditors which runs against one of the main reasons for short maturities in the supply of credit.17

The source of aggregate risk in my model is the correlation of banks’ projects which is assumed to be exogenous. This raises the question if banks would choose correlated projects if this decision was endogenous.18 In my model a bank suf-

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17 In my model creditors are risk neutral and have no particular preference for the maturity of their loans. This focuses my model on the demand for short-term financing.

18 See Acharya (2009) for a model of banks choosing the correlation of their portfolios.
fers inefficiency costs in both aggregate states for intermediate realizations of its expected project payoff. This creates an incentive to choose projects with probability mass concentrated away from intermediate realizations. Given that other banks are choosing correlated projects, an individual bank would therefore want to choose a project that is either positively or negatively correlated with other banks. While the details of the parameterization will determine which way the decision goes, it is possible that banks would want to choose positively correlated projects, even absent considerations such as government bailouts.

8 Conclusion

In this paper I present a new model of debt-maturity structure choice and highlight an important inefficiency in the use of short-term debt. The benchmark model of banks facing only idiosyncratic risk establishes the mechanism of using the debt-maturity structure to overcome an incentive problem and implement a liquidation policy. By anticipating the coordination problem among short-term creditors, a bank can choose the right amount of rollover risk to maximize its economic surplus. The competitive equilibrium achieves the first-best allocation.

The addition of aggregate risk, however, severely undermines the disciplining mechanism of short-term debt and drives a wedge between desired and achievable liquidation policy. This implies a two-sided inefficiency where in good aggregate states there is excessive risk-taking, while in bad aggregate states there is excessive liquidation. The reason is that the disciplining effect of short-term debt is weakened in the states where it is needed more and strengthened in the states where it is needed less. The competitive equilibrium shows that this inefficiency causes sizeable losses of economic surplus and is self-reinforcing with a significant amplification of aggregate volatility.
Appendix

A Global Game

To apply the standard global games results summarized by Morris and Shin (2003) the payoffs have to satisfy certain properties. Using the fund manager payoffs of Section 3, the payoff difference between withdrawing and rolling over is:

\[ \Delta(\lambda, \theta) = \begin{cases} 
  w - \theta bw & \text{for } \lambda \leq \frac{L}{\alpha} \\
  w & \text{for } \lambda > \frac{L}{\alpha} 
\end{cases} \]

This payoff difference is monotone in \( \theta \) (state monotonicity) and there is a unique \( \theta^* \) that solves \( \int_0^1 \Delta(\lambda, \theta) d\lambda = 0 \) (strict Laplacian state monotonicity). In terms of limit dominance, for \( \theta < 1/b \) we have \( \Delta(\lambda, \theta) > 0 \) for all \( \lambda \) (lower dominance region). Taking the approach of Goldstein and Pauzner (2005) I assume that for sufficiently high \( \theta \) the bank cannot become illiquid, e.g. because the project matures early and pays off \( X \) for sure. This implies an upper dominance region \( [\overline{\theta}, 1] \) such that for \( \theta > \overline{\theta} \) we have \( \Delta(\lambda, \theta) < 0 \) for all \( \lambda \). The payoff difference \( \Delta(\lambda, \theta) \) is not monotone in \( \lambda \) but it satisfies the following single-crossing property: For each \( \theta \) there exists a \( \lambda^* \in \mathbb{R} \cup \{-\infty, +\infty\} \) such that \( \Delta(\lambda, \theta) < 0 \) for all \( \lambda < \lambda^* \) and \( \Delta(\lambda, \theta) > 0 \) for all \( \lambda > \lambda^* \). In addition, the signal about \( \theta \) with uniform noise satisfies the monotone likelihood ratio property. Given all these properties, there is a unique equilibrium and it is in symmetric switching strategies around a critical value \( \hat{\theta} \).

In equilibrium, a fund manager with signal \( \tilde{\theta}_j = \hat{\theta} \) has to be indifferent between rolling over and withdrawing:

\[ \Pr \left[ \lambda \leq \frac{L}{\alpha} \left| \hat{\theta} \right| \right] E[\theta|\hat{\theta}]bw = w \]

Given the signal structure, for a particular realization \( \theta \) the distribution of signals is uniform on \( [\theta - \varepsilon, \theta + \varepsilon] \) and for a particular signal realization \( \hat{\theta} \) the conditional

\(^{19}\)See Lemma 2.3 and the following discussion in Morris and Shin (2003) and Theorem 1 in Goldstein and Pauzner (2005) for details.
distribution of $\theta$ is

$$f(\theta|\hat{\theta}) = \begin{cases} \frac{f(\theta)}{F(\hat{\theta} + \varepsilon) - F(\hat{\theta} - \varepsilon)} & \text{for } \theta \in [\hat{\theta} - \varepsilon, \hat{\theta} + \varepsilon], \\ 0 & \text{otherwise.} \end{cases}$$

With the distribution of $\theta|\hat{\theta}$ we have an expression for $E[\theta|\hat{\theta}]$ so it remains to derive the distribution of $\lambda|\hat{\theta}$. We can derive the corresponding c.d.f. $G(\lambda|\hat{\theta})$ as follows: The probability that a fraction less than $\lambda$ receives a signal less than $\hat{\theta}$ (and therefore withdraws) equals the probability that $\theta$ is greater than $\theta'$ defined by

$$\frac{\hat{\theta} - (\theta' - \varepsilon)}{2\varepsilon} = \lambda$$

$$\Rightarrow \theta' = \hat{\theta} + \varepsilon - 2\varepsilon\lambda$$

We therefore have

$$G(\lambda|\hat{\theta}) = 1 - F(\hat{\theta} + \varepsilon - 2\varepsilon\lambda|\hat{\theta})$$

$$= 1 - \int_{\hat{\theta} - \varepsilon}^{\hat{\theta} + \varepsilon - 2\varepsilon\lambda} \frac{f(\theta)}{F(\hat{\theta} + \varepsilon) - F(\hat{\theta} - \varepsilon)}d\theta$$

$$= \frac{F(\hat{\theta} + \varepsilon) - F(\hat{\theta} + \varepsilon - 2\varepsilon\lambda)}{F(\hat{\theta} + \varepsilon) - F(\hat{\theta} - \varepsilon)}.$$
we have that \( \lim_{\varepsilon \to 0} E[\theta \mid \hat{\theta}] = \hat{\theta} \). Second, we have that 

\[
\lim_{\varepsilon \to 0} G(\lambda \mid \hat{\theta}) = \frac{F(\hat{\theta} + \varepsilon) - F(\hat{\theta} + \varepsilon - 2\varepsilon \lambda)}{F(\hat{\theta} + \varepsilon) - F(\hat{\theta} - \varepsilon)} \]

\[
= \lim_{\varepsilon \to 0} \frac{f(\hat{\theta} + \varepsilon) - f(\hat{\theta} + \varepsilon - 2\varepsilon \lambda) (1 - 2\lambda)}{f(\hat{\theta} + \varepsilon) + f(\hat{\theta} - \varepsilon)} \quad \text{by l'Hôpital's rule}
\]

\[
= \lim_{\varepsilon \to 0} \frac{f(\hat{\theta} + \varepsilon) - f(\hat{\theta} + \varepsilon - 2\varepsilon \lambda)}{f(\hat{\theta} + \varepsilon) + f(\hat{\theta} - \varepsilon)} + \lim_{\varepsilon \to 0} \frac{2\lambda f(\hat{\theta} + \varepsilon - 2\varepsilon \lambda)}{f(\hat{\theta} + \varepsilon) + f(\hat{\theta} - \varepsilon)}
\]

\[
= 0 + \frac{2\lambda f(\hat{\theta})}{2f(\hat{\theta})}
\]

\[
= \lambda
\]

So the distribution of \( \lambda \) conditional on being at the switching point becomes uniform as the signal noise goes to zero.

**B Model without Fund Managers**

Instead of using fund manager payoffs, we can work with the real creditor payoffs. If enough short-term creditors roll over and the bank remains liquid, a creditor who rolls over receives an expected payoff of \( \theta D_{ST} \) while a creditor who withdraws receives 1. If too many short-term creditors withdraw and the bank becomes illiquid, a creditor who rolls over receives zero while a creditor who withdraws receives \( \ell/\alpha \). With the assumption of an upper dominance region, these payoffs satisfy the global game conditions of Appendix A guaranteeing that the equilibrium is unique and in switching strategies.

Indifference at the switching point between rolling over and withdrawing requires

\[
\frac{\ell}{\alpha} D_{ST} = \frac{\ell}{\alpha} + \left(1 - \frac{\ell}{\alpha}\right) \frac{\ell}{\alpha}
\]

so the critical value is given by

\[
\hat{\theta} = \frac{1}{D_{ST}} \left(2 - \frac{\ell}{\alpha}\right). \quad (7)
\]
As in the case with fund managers, the liquidation threshold from the creditor coordination game is decreasing in $\ell$. This implies that for a given maturity structure $\alpha$ and a given face value $D_{ST}$ the bank is more vulnerable to runs for lower liquidation values.

The main difference to the case with fund managers is that the critical value now depends on the face value $D_{ST}$. Through the ex-ante break-even condition, the face value $D_{ST}$ is endogenous and depends on $\hat{\theta}$:

$$F(\hat{\theta}) \frac{\ell}{\alpha} + \int_{\hat{\theta}}^{1} \theta D_{ST} dF(\theta) = 1. \quad (8)$$

We see that equations (7) and (8) jointly determine $\hat{\theta}$ and $D_{ST}$ for any given $\alpha$.

Combining the two equations gives us an implicit definition of $\hat{\theta}$

$$\hat{\theta} \left( 1 - F(\hat{\theta}) \frac{\ell}{\alpha} \right) - \int_{\hat{\theta}}^{1} \theta dF(\theta) \left( 2 - \frac{\ell}{\alpha} \right) = 0 \quad (9)$$

For $\alpha > \ell$ the left hand side of (9) is strictly increasing in $\hat{\theta}$ which implies that there is a one-to-one mapping between $\alpha$ and $\hat{\theta}$.

Now the bank chooses $\alpha$ to solve the following problem:

$$\max \left\{ F(\hat{\theta}) \ell + \int_{\hat{\theta}}^{1} \theta X dF(\theta) - 1 \right\} \quad \text{subject to} \quad (9)$$

As before, the bank maximizes the project’s economic surplus subject to a constraint which defines $\hat{\theta}$ as a function of $\alpha$ and the exogenous parameters.

C Case $\alpha \leq \ell$

This section considers the case where the mass of short-term creditors is small enough so they cannot cause the bank to fail. This corresponds to values of $\alpha \leq \ell$, such that withdrawals from all short-term creditors can be satisfied in $t = 1$ without liquidating the entire project. I assume that in $t = 2$ short-term debt is senior to long-term debt. As in the main part I start by deriving the endogenous face values and the bank’s expected payoff without aggregate risk.
Without rollover risk, the expected payoff to a fund manager from rolling over is $\theta bw$ regardless of the number of others withdrawing and the payoff to withdrawing is $w$ as before. The critical value for $\theta$ is therefore independent of $\alpha$ and given by

$$\tilde{\theta} = \frac{1}{b}.$$  

This implies that for realizations $\theta \leq \tilde{\theta}$ the bank has to liquidate a fraction $\alpha/\ell$ of its assets at a liquidation value of $\ell$ which raises a total of $\alpha$. The remaining fraction of assets $1 - \frac{\alpha}{\ell}$ remains in place. For realizations $\theta > \tilde{\theta}$ the bank doesn’t face withdrawals. Combining this with the bank’s expected payoff for $\alpha > \ell$ derived in Section 4.1 the complete expected payoff of the bank choosing $\alpha \in [0, 1]$ is

$$\begin{cases} F(\tilde{\theta}) \alpha + \int_0^{\tilde{\theta}} (1 - \frac{\alpha}{\ell}) \theta X dF(\theta) + \int_0^{1} \theta X dF(\theta) - 1 & \text{for } \alpha \leq \ell, \\ F(\tilde{\theta}) \ell + \int_0^{1} \theta X dF(\theta) - 1 & \text{for } \alpha > \ell. \end{cases}$$  

The payoff is continuous in $\alpha$ since the two expressions are the same for $\alpha = \ell$ but not differentiable at $\alpha = \ell$. It is either monotone or single-peaked. Due to the linearity of the bank’s expected payoff for $\alpha \leq \ell$, the optimal solution will be either $\alpha = 0$, $\alpha = \ell$, or we will be in the region $\alpha > \ell$ discussed in the main part of the paper.

To guarantee that the solution falls into the range of $\alpha > \ell$ we have to assume that the derivative of both pieces are positive at $\alpha = \ell$:

$$\begin{cases} F(\frac{1}{b}) - \frac{X}{\ell} \int_0^{\tilde{\theta}} \theta dF(\theta) > 0 \\ f(\frac{1}{b}) \frac{1}{b} (\ell - \frac{1}{b} X) > 0 \end{cases}$$

These conditions involve only exogenous parameters and can be satisfied.
With aggregate risk, the bank’s expected payoff is more complicated

\[
\begin{align*}
&\begin{cases}
p \left[ F_H(\hat{\theta}) \alpha + \int_{\hat{\theta}}^1 \left( 1 - \frac{\alpha}{\ell_H} \right) \theta X dF_H(\theta) + \int_{\hat{\theta}}^1 \theta X dF_H(\theta) - 1 \right] \\
+ (1 - p) \left[ F_L(\hat{\theta}) \alpha + \int_{\hat{\theta}}^1 \left( 1 - \frac{\alpha}{\ell_L} \right) \theta X dF_L(\theta) + \int_{\hat{\theta}}^1 \theta X dF_L(\theta) - 1 \right]
\end{cases} \\
&\begin{cases}
p \left[ F_H(\hat{\theta}) \alpha + \int_{\hat{\theta}}^1 \left( 1 - \frac{\alpha}{\ell_H} \right) \theta X dF_H(\theta) + \int_{\hat{\theta}}^1 \theta X dF_H(\theta) - 1 \right] \\
+ (1 - p) \left[ F_L(\hat{\theta}) \ell_L + \int_{\hat{\theta}}^1 \theta X dF_L(\theta) - 1 \right]
\end{cases}
\end{align*}
\]

for \( \alpha < \ell_L \)

\[
\begin{align*}
&\begin{cases}
p \left[ F_H(\hat{\theta}) \alpha + \int_{\hat{\theta}}^1 \left( 1 - \frac{\alpha}{\ell_H} \right) \theta X dF_H(\theta) + \int_{\hat{\theta}}^1 \theta X dF_H(\theta) - 1 \right] \\
+ (1 - p) \left[ F_L(\hat{\theta}) \ell_H + \int_{\hat{\theta}}^1 \theta X dF_L(\theta) - 1 \right]
\end{cases} \\
&\begin{cases}
p \left[ F_H(\hat{\theta}) \ell_H + \int_{\hat{\theta}}^1 \theta X dF_H(\theta) - 1 \right] \\
+ (1 - p) \left[ F_L(\hat{\theta}) \ell_L + \int_{\hat{\theta}}^1 \theta X dF_L(\theta) - 1 \right]
\end{cases}
\end{align*}
\]

for \( \ell_L < \alpha < \ell_H \)

\[
\begin{align*}
&\begin{cases}
p \left[ F_H(\hat{\theta}) \alpha + \int_{\hat{\theta}}^1 \left( 1 - \frac{\alpha}{\ell_H} \right) \theta X dF_H(\theta) + \int_{\hat{\theta}}^1 \theta X dF_H(\theta) - 1 \right] \\
+ (1 - p) \left[ F_L(\hat{\theta}) \ell_H + \int_{\hat{\theta}}^1 \theta X dF_L(\theta) - 1 \right]
\end{cases}
\end{align*}
\]

for \( \alpha > \ell_H \)

Again the payoff is continuous in \( \alpha \) but not differentiable at \( \alpha = \ell_L, \ell_H \). It is either monotone or single-peaked. For the optimal \( \alpha \) to be in the region \( \alpha > \ell_H \) we need

\[
\begin{align*}
&\begin{cases}
p \left[ F_H(\hat{\theta}) - \frac{1}{\ell_H} X \int_{\hat{\theta}}^1 \theta dF_H(\theta) \right] + (1 - p) f_L \left( \ell_H - \frac{\ell_H}{\ell_H - \ell_H} X \right) > 0 \\
p f_H \left( \frac{1}{\ell_H} X - \frac{\ell_H}{\ell_H} X \right) + (1 - p) f_L \left( \ell_H - \frac{\ell_H}{\ell_H} X \right) > 0
\end{cases}
\end{align*}
\]

Again these are conditions involving only exogenous parameters and can be satisfied.
References


