The Expected Cost of Default

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Abstract

The sample of observed defaults understates the average firm’s expected cost of default due to a sample selection bias. Because credit markets price default costs, firms with higher costs optimally choose lower leverage, reducing the probability of default. To quantify this selection bias, I embed a dynamic capital structure model in an economy with heterogeneous firms and time-varying macroeconomic conditions. Using the model’s implications for observable moments, I estimate the unobserved distribution of expected default costs. I find that the average firm expects to lose 45% of firm value in default, a cost much higher than existing estimates from observed defaults. However, the average cost of default in the sample of simulated defaults is only 25%, a value consistent with the existing empirical estimates. The estimated model is successful in reconciling the levels of leverage, credit spreads, and default rates observed in the data. Additionally, I characterize the determinants of the estimated firm-specific default costs.

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1 Introduction

The cost of default is an essential component to understanding the joint behavior of credit spreads, default rates, and firms’ optimal financing decisions. A common view in the finance literature, supported by empirical studies of defaulted firms, maintains that the average cost of default is relatively low.\(^1\) This conclusion plays a central role in the challenge faced by existing models to simultaneously explain the levels of leverage, credit spreads, and default rates observed in the data.

Existing estimates typically use a sample of defaulted firms to conclude that the average cost of default is low. However, estimates obtained in this manner are subject to a sample selection bias and consequently understate the cost that the average firm expects to incur in default. Since firms and credit markets internalize default costs when choosing leverage and pricing debt, respectively, those firms with a higher cost of default optimally choose lower leverage, thus making default less likely. Furthermore, the firms that default ex post should be those for which default was more likely ex ante. All else equal, these are the firms with a low cost of default. This implies that using the sample of defaulted firms gives a biased estimate of the average cost of default.

In this paper I estimate firm-specific expected default costs, which are the costs used ex ante by firms in setting their leverage and credit markets in pricing debt. Using the estimated distribution of expected default costs, I am able to quantify the sample selection bias. I then consider how accounting for the bias can help to reconcile the patterns of leverage, default rates, and credit spreads observed in the data. In particular, I ask whether using the distribution of expected default costs resolves the so-called “underleverage puzzle.”\(^2\) Additionally, I characterize the determinants of the estimated firm-specific default costs.

\(^1\)Andrade and Kaplan (1998) estimate the costs of financial distress to be 10-23% of the pre-distress firm value. This measure is intended to capture both direct and indirect costs. Estimates of direct bankruptcy costs tend to be much smaller. Warner (1977), Weiss (1990), and Altman (1984) all find small direct costs of bankruptcy of 5.3%, 3.1%, and 6% of pre-bankruptcy firm value, respectively. When I use the term “default cost,” it is meant to refer to both direct and indirect costs.

\(^2\)The “underleverage puzzle” refers to the fact that given observed costs of default and tax benefits to debt, the tradeoff model of capital structure appears unable to explain the low leverage ratios observed for many firms in the data.
I find that existing estimates of default costs, which are based on a small sample of defaulted firms, are subject to a severe sample selection bias and, as a result, significantly underestimate the magnitude of default costs faced by most firms. In my sample of 2,500 firms, the mean estimated cost of default is 45% of firm value, which is substantially higher than estimates obtained from the sample of defaulted firms. However, the average estimated cost for the subset of defaulted firms is only 25%. This latter value, which is the model counterpart to the observed sample of defaulted firms, is closely in line with existing empirical estimates for average default costs. Andrade and Kaplan (1998), a frequently cited set of estimates, find average costs of 10-23% for a sample of 31 highly leveraged transactions that became distressed. Similarly, Davydenko, Strebulaev, and Zhao (2010) find average default costs of 20.4% of firm value for a sample of 144 defaulted firms. In other words, my results suggest that the average firm faces an expected cost of default nearly twice as large as that inferred from the sample of defaulted firms. Equally important, I find substantial heterogeneity in default costs, which is consistent with the persistent cross-sectional differences in leverage observed in the data.

Taken together, these results have important implications. A number of conclusions relating to leverage, credit spreads, and the importance of default costs rely on the assumption that the low observed default costs accurately reflect the costs faced by the population of firms. An important message of this paper is that many of these conclusions should be revisited. As a primary example, I show that accounting for heterogeneous default costs, and the sample selection bias that they induce, goes a long way towards resolving the underleverage puzzle. Using the values reported in Andrade and Kaplan (1998) and the estimated tax benefits to debt, previous work has concluded that default costs are too low for a tradeoff

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3 The estimated range of 10-23% in Andrade and Kaplan (1998) is for a sample of 31 highly leveraged transactions. Given that the sample selection problem is especially severe for such a sample and I look at all simulated defaults, not just a subset analogous to their HLTs, a value of 25% seems to be closely in line with their estimates. Furthermore, Andrade and Kaplan (1998) note this, saying “It is possible that the firms that undertook HLTs were those that, ex ante, expected to have low costs of financial distress. If this is true, our estimates of the costs of financial distress underestimate the costs of financial distress for firms in general” (p. 1489).

4 Lemmon, Roberts, and Zender (2008) document persistent cross-sectional variation in leverage ratios which they argue can’t be explained by previously identified determinants.
model of leverage to explain the data.\footnote{Graham (2000) estimates the tax benefits to debt. He concludes that the estimated tax benefits of debt are too large, given the low observed default costs to explain the low leverage chosen by many firms.} My results overturn this conclusion. The estimated model is not only consistent with observed default rates and credit spreads, but is also able to match the cross-section of leverage, including firms with very low leverage, while still replicating the low observed default costs seen in the data.\footnote{The model does not explain zero leverage firms, however, as this would likely require a fixed cost of initial debt issuance, which is excluded from the model for tractability.} Due to the sample selection bias, low observed default costs can be reconciled with low observed leverage ratios in a tradeoff model of leverage. Additionally, heterogeneity in default costs implies that firms with identical default probabilities have very different credit spreads. Alternatively, observed differences in credit spreads should not be attributed solely to differences in the likelihood of default.

The implications of a sample selection bias in default costs extend far beyond the environment studied in this paper. Because default costs directly affect the losses suffered by bondholders, observed recovery rates give an inflated estimate of expected recovery rates. This suggests that the sample selection bias has implications for any model that considers default risk, not just the framework used in this paper. For example, reduced-form models of default risk, which are typically calibrated to match observed default and recovery rates, should also account for this selection bias.

I now discuss my approach to estimating expected default costs and measuring the sample selection bias. I embed a dynamic capital structure model in an economy with ex ante heterogeneous firms and time-varying macroeconomic conditions. The structure of the model is very similar to recent work by Bhamra, Kuehn, and Strebulaev (2009a,b), Chen (2010), and Hackbarth, Miao, and Morellec (2006).\footnote{These papers feature contingent claims models of capital structure, in the spirit of Merton (1974) and Leland (1994), in environments with macroeconomic risk. While the framework is very similar, these papers do not study cross-sectional heterogeneity in default costs or the selection bias that it induces. Although time-varying macroeconomic conditions are not central to the selection bias, these papers show that this channel is quantitatively important. Since I intend to estimate the unobserved default costs, using a model that is quantitatively consistent with the data is essential.} A key difference is that I consider a cross-section of ex ante heterogeneous firms, whereas in those papers firms are ex ante identical.\footnote{In those models firms still differ ex post, however, due to differences in their idiosyncratic shocks.}
This allows me to study the effects of heterogeneity in unobserved expected default costs on observed quantities and prices. Furthermore, it means the model can be estimated to obtain a cross-sectional distribution of expected default costs and an estimate of the sample selection bias.

In the model, firms choose financial leverage optimally by trading off the expected benefit of the interest tax shield with the expected cost of default. Differences in firms’ cost of default are given exogenously in the model, but this heterogeneity may be due to differences in capital specificity, industry competition, or other firm characteristics.\(^9\) In the model, firms with lower default costs optimally choose higher leverage and consequently have a higher probability of default. Thus, in simulated data from the model, the sample of defaults consists disproportionately of firms with low default costs.

The structural model allows me to investigate how variation in default costs affects the joint behavior of leverage, credit spreads, and default rates in a unified framework without having to fix one or more of these quantities exogenously. Specifically, the model determines optimal leverage, default probabilities, credit spreads, and equity values jointly in equilibrium. An increase in the cost of default leads credit markets to demand a higher yield on the debt, all else equal. As an optimal response, equityholders choose to issue less debt, resulting in a lower optimal leverage ratio and default probability. These adjustments are all determined endogenously in the model as the result of equityholders seeking to maximize firm value and credit markets pricing the debt competitively.

In the general model, firms differ ex ante in their cost of default as well as the parameters of their cash flow process. This is an important distinction from most related work, which considers economies of ex ante identical firms. This ex ante heterogeneity adds a substantial computational burden, however, as it means a separate firm problem has to be solved for each firm in the economy, instead of just once for a representative firm. This is essential, however, if one wishes to consider the effect of differences in default costs. Furthermore, this

\(^9\)After estimating firm-specific expected default costs, I regress the estimates on a number of firm characteristics. I find that the determinants of these costs are largely consistent with previously identified characteristics. For more details, see Section 6.

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allows me to estimate firm-specific default costs.

Before estimating the distribution of default costs, I consider a special case of the model in which firms differ ex ante only in their default costs. This allows me to investigate the effects of variation in default costs separately from cross-sectional differences in the parameters of the cash flow process. In this version of the model, which is calibrated to match several features of the data, I show that the sample selection bias in default costs is substantial. As a result, from a tradeoff view of capital structure, the model generates a counterfactual underleverage puzzle, which is consistent with what is observed in the data. That is, firms choose leverage optimally in the model, but the econometrician, observing only the default costs from the sample of defaulted firms, mistakenly concludes that default costs are low and many firms are underleveraged. For these firms, it appears that value could be increased by issuing additional debt when, in fact, this is not the case.

I then use the general model to estimate firm-specific default costs and quantify the magnitude of the sample selection bias. Using a simulated method of moments procedure, I estimate expected default costs, as well as other firm-specific parameters of the model, for each of the 2,500 firms in my sample. Next, I characterize these estimates by examining how they relate to firm characteristics, industry, and credit rating. I find that the estimated default costs are consistent with previously identified determinants.

Finally, I simulate the model under the cross-sectional joint distribution of firm-specific parameters estimated from the SMM procedure. Simulating a model economy under the estimated cross-sectional distribution, I am able to compare the unconditional average cost of default with the conditional average from the sample of simulated defaults, thus obtaining an estimate of the sample selection bias. As already mentioned, I find the selection bias to be large and economically significant. The average estimated default cost is 45% of firm value, which is much larger than the existing estimates from ex post defaults. However, simulations of the estimated model produce average observed default costs of only 25%, thus replicating the findings of Andrade and Kaplan (1998). Though the distribution of expected default costs is ultimately unobservable, this suggests that the estimated distribution is consistent
with the conditional distribution of observed default costs.

**Related Literature**

In a broad sense, my work is related to a growing body of literature that considers the interactions of corporate financing decisions and asset prices.\(^{10}\) Additionally, my approach to estimate expected default costs is related to a number of papers estimating a structural model.\(^{11}\) For example in a very different framework than the one I consider here, Hennessy and Whited (2007) use a structural model to estimate financing costs. They estimate bankruptcy costs equal to 10.4% of capital. A relatively novel aspect of this paper is the allowance for firm heterogeneity. In contrast, for tractability reasons, most related work estimates the parameters of a single representative firm. While I do not explicitly model the sources of default costs, a number of papers have considered these sources. Examples of indirect default costs include asset fire sales (Shleifer and Vishny (1992)), asset substitution (Jensen and Meckling (1976)), and deterioration of supplier and customer relationships (Titman (1984)).

More specifically, my work is related to two distinct strands of existing literature. The first primarily consists of empirical work that seeks to measure the cost of distress or default. Examples of work in this literature include Andrade and Kaplan (1998), Pulvino (1998), Franks and Torous (1989), Opler and Titman (1994), Gilson (1997), Ofek (1993), Asquith, Gertner, and Scharfstein (1994), Acharya, Bharath, and Srinivasan (2007), and Davydenko, Strebulaev, and Zhao (2010). Miller (1977) notes that default and distress costs appear far too small, given estimated tax benefits to debt, to explain empirical leverage ratios. Graham (2000) estimates the tax benefits of debt to up to 5% of firm value and concludes that many firms are, on average, under levered. Almeida and Philippon (2007) note that default is more likely to occur in bad states when marginal utility is high. Using risk-neutral

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\(^{10}\)Recent examples in this literature include Gomes and Schmid (2010), Garlappi and Yan (2010), Berk, Green, and Naik (1999), Carlson, Fisher, and Giammarino (2004), and Gomes, Kogan, and Zhang (2003), as well as the previously mentioned Bhamra, Kuehn, and Strebulaev (2009a,b), Chen (2010), and Hackbarth, Miao, and Morellec (2006).

\(^{11}\)Recent examples include Hennessy and Whited (2007), Morellec, Nikolov, and Schuerhoff (2010), Nikolov and Whited (2010), and Taylor (2010).
probabilities and the estimates of Andrade and Kaplan (1998), they conclude that firms are not, on average, under levered. This conclusion refers to average leverage and credit spreads, however, and thus does not explain the observed cross-sectional heterogeneity. Elkamhi, Parsons, and Ericsson (2010) note that this calculation does not filter out economic shocks, which are unrelated to leverage, that drive the firm to default or distress. They argue that once the economic shocks are accounted for separately, the default cost estimates of Andrade and Kaplan (1998) are too low to account for the observed leverage ratios. The structural model that I use avoids this issue. Using the marginal tax benefit estimates of Graham (2000), van Binsbergen, Graham, and Yang (2010) estimate firm-specific costs of debt under the assumption that firms are optimally levered. Based on the estimates of Almeida and Philippon (2007), they conclude that approximately half of their estimated cost of debt is due to default or distress costs.

Korteweg (2009) estimates the net benefits to leverage. Consistent with previous work, he concludes that many firms are under levered. Using firms at or near distress, he identifies the cost of distress as the computed (negative) net benefit to leverage. He estimates distress costs of 15-30%. The set of firms at or near distress, however, are likely to be those for which default costs were low ex ante.

The second strand of literature is a class of structural models of capital structure and credit risk. Examples of this work include the seminal papers of Merton (1974) and Le-land (1994) as well as more recent work by Chen (2010), Bhamra, Kuehn, and Streibulaev (2009a,b), and Hackbarth, Miao, and Morellec (2006). These models are primarily concerned with matching aggregate facts regarding credit spreads, default frequency, and leverage. Furthermore, the models in Chen (2010) and Bhamra, Kuehn, and Streibulaev (2009a,b) are consumption-based asset pricing models featuring the long-run risk framework of Bansal and Yaron (2004). Chen (2010) seeks to explain the observed credit spreads and leverage ratios while Bhamra, Kuehn, and Streibulaev (2009b) focus on a levered equity premium. Bhamra, Kuehn, and Streibulaev (2009a), focus on the dynamics of leverage in an economy.

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12 Other similar models include Chen, Collin-Dufresne, and Goldstein (2009), Fischer, Heinkel, and Zechner (1989), Goldstein, Ju, and Leland (2001)
with macroeconomic risk. In their model, all firms are identical ex ante, but differ ex post due to idiosyncratic shocks. In contrast, I focus on computing a firm-specific measure of the cost of default and quantifying the magnitude of the sample selection bias. To that end, the economy I consider features a cross-section of firms which are ex ante heterogeneous, differing in the parameters of their cash flow process as well as default costs.

The remainder of the paper is organized as follows. In Section 2 I introduce the model framework. Section 3 discusses the implications of heterogeneity in default costs. In Section 4, I present results from a calibrated version of the model in which firms differ ex ante only in their cost of default. I illustrate how the model is able to produce a counterfactual underleverage puzzle when in fact all firms are optimally levered. Additionally, I am able to quantify these effects in the model. In Section 5, I estimate firm-specific expected default costs using the simulated method of moments and in Section 6 I examine how the estimated default costs relate to firm characteristics. In Section 7, I simulate the model again under the estimated joint cross-sectional distribution of the firm-specific parameters, which gives an estimate for the bias in observed default costs. Section 8 concludes.

2 Model

I construct a partial equilibrium model featuring a cross-section of ex ante heterogeneous firms and time-varying macroeconomic conditions. The model setup is very similar to the models of Chen (2010) and Bhamra, Kuehn, and Strebulaev (2009a,b), with a key difference being that I consider heterogeneity at the firm level.\footnote{These papers show that by introducing macroeconomic risk and a countercyclical price of risk, a dynamic capital structure model is able to match the observed average levels of leverage and credit spreads. Hackbarth, Miao, and Morellec (2006) and Chen, Collin-Dufresne, and Goldstein (2009a,b) also consider time-varying macroeconomic conditions in models of credit risk, though in slightly different frameworks than the one used in this paper.}

Specifically, I use a structural tradeoff model of the firm’s dynamic capital structure decision in which the cash flows are specified exogenously.\footnote{This follows in the tradition of a long line of literature, with seminal papers by Merton (1974), Black and Cox (1976), and Leland (1994). The dynamic aspect of the model, allowing for a firm to adjust leverage at a later date, follows Fischer, Heinkel, and Zechner (1989) and Goldstein, Ju, and Leland (2001).} Firms are exposed to both
systematic and idiosyncratic cash flow shocks in an environment with time-varying macro-economic risk. Firms choose leverage ratios by weighing the tax benefits of debt against the deadweight losses incurred in default. Critically, leverage and credit spreads are determined endogenously in the model with equityholders choosing optimal leverage to maximize the initial total firm value. Conditional on not defaulting, a firm can restructure upwards by issuing additional debt at any point in time.\footnote{The option to restructure downwards is excluded for tractability. While perhaps limiting, this assumption is common to other dynamic capital structure models, such as, Goldstein, Ju, and Leland (2001), Chen (2010), and Bhamra, Kuehn, and Streubulaev (2009a,b).} Restructuring is assumed to entail a cost, however, which implies that restructuring occurs only once firm cash flows exceed an optimally chosen restructuring boundary. In trading off the benefits of a tax shield with the costs of default, the model gives optimal endogenous leverage choices.\footnote{Note, however, that due to fluctuations in firm cash flows and economic conditions and the assumed cost of restructuring, the firm’s actual leverage will drift away from its optimal target. In the model, the firm is at its optimally chosen leverage ratio only at time 0 and subsequent restructuring dates.}

Time is continuous and the firm’s investment policy is fixed. The state of the economy is determined by the state variable $\nu_t$, which evolves according to a 2-state time-homogeneous Markov chain. That is, $\nu_t \in \{H, L\}$, where the switching between regimes follows a Poisson arrival process. Changes in the aggregate state are assumed to be observable by all agents in the economy and given $\nu_t$, the state-dependent parameters are known constants.

The aggregate earnings of the economy, denoted by $X_{A,t}$, evolve according to a Markov-modulated geometric Brownian motion:

$$
\frac{dX_{A,t}}{X_{A,t}} = \mu_A(\nu_t) dt + \sigma_A(\nu_t) dW^A_t
$$

where $W^A_t$ is a standard Brownian motion. As indicated by the notation, the expected growth rate, $\mu_A(\nu_t)$, and volatility, $\sigma_A(\nu_t)$, of aggregate earnings depend on the aggregate state of the economy, $\nu_t$.

In the model, a firm’s earnings growth depends on aggregate earnings shocks as well as idiosyncratic shocks specific to the firm. Firms generate a perpetual stream of cash flows which are taxed at rate $\tau_c$ and full loss offset is assumed. Firm $i$’s before-tax earnings, $X_{i,t}$,
evolve according to\footnote{I use the terms “earnings” and “cash flows” interchangeably. Both are meant to refer to the firm’s earnings before interest and taxes (EBIT).}

\[
\frac{dX_{i,t}^l}{X_{i,t}^l} = \left(\mu_i + \mu_A(\nu_t)\right)dt + \beta_i \sigma_A(\nu_t)dW_A^t + \sigma_i,F dW_{i,F}^t.
\]

This implies that firm $i$’s expected earnings growth in state $\nu_t$ is given by $\left(\mu_i + \mu_A(\nu_t)\right)$, where $\mu_i$ represents a state-invariant firm fixed-effect and $\mu_A(\nu_t)$ is the state-dependent expected growth rate of aggregate earnings. Thus, the expected earnings growth rate for all firms is assumed to depend, in part, on the aggregate state of the economy. Additionally, $\beta_i$ parameterizes firm $i$’s exposure to aggregate earnings shocks generated by the Brownian motion $W_A^t$. Note that the volatility of aggregate earnings shocks, $\sigma_A(\nu_t)$, is assumed to be state-dependent, but the firm’s exposure to these shocks, $\beta_i$, is constant. Finally, firm $i$ is exposed to idiosyncratic earnings shocks with volatility $\sigma_{i,F}$ generated by the firm-specific Brownian motion $W_{i,F}^t$. Thus, firms are exposed to three types of shocks: aggregate earnings shocks generated by $W_A^t$, idiosyncratic earnings shocks generated by $W_{i,F}^t$, and changes in the aggregate state of the economy, $\nu_t$.

\section{Pricing Kernel, Risk Neutral Measure}

I assume markets are complete and that there exists a default-risk-free asset that pays a state-dependent interest rate, $r(\nu_t)$. The model is partial equilibrium and I take the pricing kernel as exogenous. Specifically, the pricing kernel is assumed to evolve according to

\[
\frac{d\pi_t}{\pi_t} = -r(\nu_t)dt - \varphi(\nu_t)dW_A^t.
\]

In this economy, $\varphi(\nu_t)$ is the state-dependent market Sharpe ratio and the risk premium for firm $i$’s cash flows in state $\nu_t$ is given by $\beta_i\sigma_A(\nu_t)\varphi(\nu_t)$. This functional form is consistent with the pricing kernel in a complete markets economy where the representative agent has CRRA preferences. With this specification, a change in the aggregate state does not induce a jump in the pricing kernel. Consequently, the risk-neutral probability of a change in the
aggregate state is the same as the physical probability.\footnote{This is not true for alternative pricing kernels. For example, in an economy where the representative agent has recursive preferences, a change in the aggregate state can result in a jump in the pricing kernel, implying that the generator matrix is different under the physical and risk-neutral measures ($\tilde{\Lambda} \neq \Lambda$). For examples of this, see the consumption-based asset pricing models of Chen (2010) and Bhamra, Kuehn, and Strebulaev (2009a,b). While I do not use such a specification, the model could readily accommodate a pricing kernel with those features.}

Given the specification for the pricing kernel, I can derive the risk-neutral probability measure, $Q$, which will be used for pricing assets.\footnote{Details of the derivation of the risk-neutral measure and risk-neutral cash flow dynamics are provided in Appendix A.} Under the risk-neutral measure, firm $i$’s cash flow process evolves according to

$$\frac{dX_{i,t}}{X_{i,t}} = \tilde{\mu}_i(\nu_t)dt + \sigma_{i,X}(\nu_t)d\tilde{W}_i^t. \quad (4)$$

where $\tilde{\mu}_i(\nu_t)$ represents the cash flow growth under the risk-neutral measure, $\tilde{W}_i^t$ is a $Q$-Brownian motion, and

$$\sigma_{i,X}(\nu_t) = \sqrt{(\beta_i \sigma_A(\nu_t))^2 + (\sigma_{i,F})^2}.$$ represents the total earnings volatility for firm $i$.

### 2.2 Unlevered Firm Value and Price of a Risk-free Consol Bond

The unlevered value of the firm is the value if the firm were to never issue any debt, which is simply the value of a claim to the firm’s perpetual cash flow stream.\footnote{Details of the derivation of unlevered firm value are provided in Appendix B.} The firm’s earnings are taxed at rate $\tau_c$ and full loss offset is assumed. At time $t$ in state $\nu_t$, the value before taxes of unlevered firm $i$ is given by

$$V_i^U(X_{i,t}, \nu_t) = E_t \left[ \int_t^\infty \tilde{\pi}_s X_{i,s} ds \bigg| \nu_t \right] \quad (5)$$

That is, the value of the unlevered firm is simply a claim to its perpetual stream of cash flows. Note that this value is state-conditional but time-independent. Alternatively, the before-tax unlevered value of firm $i$ in state $\nu_t$ at time $t$ can be expressed as

$$V_i^U(X_{i,t}, \nu_t) = \frac{X_{i,t}}{r_i^U(\nu_t)} \quad (6)$$
where $r_i^U(\nu_t)$ is the discount rate applied to firm $i$'s unlevered cash flows in state $\nu_t$. For current state $H$,

$$r_i^U(H) = \frac{[\lambda_{HL} + r_f(H) - \hat{\mu}_i(H)][\lambda_{LH} + r_f(L) - \hat{\mu}_i(L)] - \lambda_{HL}\lambda_{LH}}{r_f(L) - \hat{\mu}_i(L) + \lambda_{HL} + \lambda_{LH}}$$  \hspace{0.5cm} (7)

where $r_f(H)$ is the instantaneous risk-free rate in state $H$, $\hat{\mu}_i(H)$ is firm $i$'s risk-neutral cash flow growth rate in state $H$, and $\lambda_{HL}$ is the probability of switching from state $H$ to $L$. This expression shows that the discount rate applied to the firm’s cash flows accounts for the possibility of a change in the aggregate state. An analogous expression holds for $r_i^U(L)$ by exchanging $H$ and $L$. With no regime-switching, the expression for unlevered firm value collapses to a familiar Gordon growth formula:

$$V_i^U(X_{i,t}, \nu_t) = \frac{X_{i,t}}{r_f - \hat{\mu}_i}.$$  

The value in state $\nu_t$ of a consol bond, $b(\nu_t)$, that has no default risk and pays a constant coupon rate of 1 can be computed similarly and is given by\(^{21}\)

$$b(\nu_t) = \frac{1}{r_P(\nu_t)}$$  \hspace{0.5cm} (8)

where $r_P(\nu_t)$, is the interest rate in state $\nu_t$ on a default-risk-free perpetuity. For current state $H$,

$$r_P(H) = r_f(H) + \frac{\lambda_{HL}(r_f(L) - r_f(H))}{\lambda_{HL} + \lambda_{LH} + r_f(L)}.$$  

with an analogous expression holding for $r_P(L)$.

### 2.3 Financing Decision

Firms make their leverage and default decisions by balancing the benefit of the interest tax shield against the cost of default, with the objective of maximizing the value of equity. The firm can issue debt in the form of a perpetuity that pays a constant coupon rate of $C$. This rate is chosen at issuance and paid to bondholders until equityholders choose to default or

\(^{21}\)Since the bond pays a coupon of 1 and has a zero growth rate, by plugging in 1 for $X_t$ and setting $\mu_1 = \mu_2 = 0$ in the expression for unlevered firm value, with some rearranging, we get the value of the default-risk-free consol bond.
restructure by issuing additional debt. In the case of restructuring, the firm is assumed to call the outstanding debt and issue a new perpetuity with a new coupon rate.

The firm is assumed to distribute all earnings after the coupon payment and corporate taxes to equity holders in the form of a dividend, which is taxed at rate \( \tau_d \). In the event that current earnings are less than the coupon payment owed, \( X_t - C < 0 \), the firm can issue additional equity. Due to limited liability, equity holders are not obligated to inject additional funds to pay the bondholders. However, failure to do so results in default at which point the bondholders receive ownership rights to the firm. Consequently, equity holders will optimally choose to raise additional funds only in the event that the value of equity in the current state is positive. Thus, under the assumption that the absolute priority rule holds, equity holders optimally choose to default once the equity value is 0.

### 2.4 Default Event and the Cost of Default

In the event of default, debtholders take over the firm with equity holders receiving nothing. Firms incur a cost in the event of default, which reduces debtholders’ recovery rate. In particular, if firm \( i \) defaults at time \( t \), bondholders receive \( (1 - \alpha_i) V_i^U(X_t, \nu_t) \) where \( V_i^U(X_t, \nu_t) \) is the unlevered value of firm \( i \) given in equation (6). Thus, \( \alpha_i \) represents the deadweight loss incurred by firm \( i \) in default. As indicated by the notation, these costs are assumed to vary across firms but are constant across aggregate states.\(^{22}\) While I do not specifically model the nature of this loss, it may be due to a variety of factors such as reputation costs, asset fire sales, loss of customer or supplier relationships, legal and accounting fees, and costs of changing management.

I refer to \( \alpha_i \) as firm \( i \)’s cost of default, which is the fraction of the unlevered firm value that is lost in the event that firm \( i \) defaults. Note that this cost is represented as a fraction

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\(^{22}\)The model readily allows for state dependence in \( \alpha_i \) and there is empirical evidence that recovery rates vary with economic conditions, which would motivate such a feature. For the results presented, I choose to shut down state-dependence in \( \alpha \) to be consistent with the estimation in Section 5. Because I do not have sufficiently rich data to identify firm-specific time variation in \( \alpha_i \), the estimation requires this parameter to be fixed through time. Thus to present simulation results consistent with the model that I estimate, I set \( \alpha_i(\nu_t) = \alpha_i \). I have simulated the model allowing for state dependence in \( \alpha_i \) and the results are very similar to the version presented.
and not a dollar amount. The dollar loss incurred in default by firm $i$ in state $\nu_t$ is given by $a_i V_i^U(X_t, \nu_t)$.

### 2.5 Overview of the Firm’s Problem

In order to solve for the firm’s optimal capital structure, the values of debt and equity must first be computed. Given the specified cash flow process and pricing kernel, I use a contingent claims approach to solve for the values of these securities and then find the optimal coupon that maximizes initial firm value. The solution technique follows that of Chen (2010) which is based on a method of pricing options on securities with Markov-modulated dynamics presented in Jobert and Rogers (2006). The solution procedure is as follows. First, I solve for the unlevered value of the firm. Then, I solve for the values of debt and equity for arbitrary coupon rate and set of restructuring thresholds with the optimally chosen default thresholds determined by the smooth-pasting conditions. Given these security values, I solve for the optimal default thresholds chosen by the equity holders as a function of an arbitrary coupon. Finally, I solve for the optimal coupon rate and set of upward restructuring thresholds subject to the smooth pasting conditions for the default thresholds.

### 2.6 Valuing Debt and Equity

Debt and equity are contingent claims on the firm’s cash flows that pay a continuous dividend rate while the firm is solvent and a lump sum payment in the event of default. As time-homogeneous contingent claims, the value of these two securities at time $t$ depend only on the present cash flows, $X_t$, and the current state, $\nu_t$. Thus the debt and equity values can be solved for in a manner analogous to the technique used to solve for the unlevered firm value. Specifically, the values for debt and equity can be characterized as systems of ordinary differential equations.

Once the firm has issued debt, default becomes a possibility and the firm must choose a cash flow threshold at which it defaults. Since the value of the firm’s cash flows (as well as a contingent claim on the cash flows) is different in the two states, there will be a different
default threshold for each state. I denote the threshold at which the firm defaults in state \( i \) as \( X_{D,i} \). Since equity holders receive nothing in default, the default threshold for a given state will always be less than the coupon payment.

Debt is a contingent claim on firm \( i \)'s cash flows that pays the constant coupon payment \( C \) while the firm is solvent and pays \((1 - \alpha_i)V_i^U(X_t, \nu_t)\) in the event of default by firm \( i \) at time \( t \) in state \( \nu_t \). That is, debt holders receive a fraction \((1 - \alpha_i)\) of the unlevered firm value in the event of default, where the size of the fraction as well as the unlevered firm value depend on the state. As previously mentioned, \( \alpha_i V_i^U(X_t, \nu_t) \) is the cost of default for firm \( i \) when default occurs at time \( t \) in state \( \nu_t \).

In what follows, I suppress the firm-specific subscript for notational convenience. Thus the values presented apply to a given firm, but are not fixed to be constant across firms. In the event of restructuring, the debt is called and the bondholders receive \( D(X_0; \nu_0) \). When default occurs at time \( t \) in state \( \nu_t \), the bondholders receive a payment of \((1 - \alpha)V(X_t, \nu_t)\).

For current cash flow \( X_t \), debt issued when the state was \( \nu_0 \) has current value given by

\[
D(X_t; \nu_0) = \sum_{j=1}^{k} w_{k,j}(\nu_0)g_{k,j}X_t^{\psi_{k,j}} + \xi_k(\nu_0)X + \zeta_k(\nu_0), \quad X_t \in \mathcal{R}_k, \quad k = 1, ..., 3
\]  

(9)

where \( \mathcal{R}_k \) represents the current cash flow region. The \( \psi \)'s are the eigenvalues and \( g \) represent the eigenvectors of the firm’s eigenvalue problem discussed in the Appendix. The terms \( \xi_k(\nu_0) \) and \( \zeta_k(\nu_0) \) represent solutions to the inhomogeneous equation.

Similarly, equity is a contingent claim that pays a dividend \((X_t - C)\) until default or restructuring occurs. In the event of restructuring, the equity holders have a claim to the newly levered firm value. As previously mentioned, in default, equity holders receive nothing. Thus, for current cash flow \( X_t \) and initial debt issuance occurring in state \( \nu_0 \), the value of equity is given by

\[
E(X_t; \nu_0) = \sum_{j=1}^{k} w_{k,j}(\nu_0)g_{k,j}X_t^{\psi_{k,j}} + \xi_k(\nu_0)X + \zeta_k(\nu_0), \quad X \in \mathcal{R}_k, \quad k = 1, ..., 3
\]  

(10)

With these expressions, we can solve for the firm’s optimal capital structure, which consists of choosing a coupon rate and default and restructuring boundaries.
2.7 Firm’s Problem

The firm faces a dynamic capital structure decision at time $t = 0$. In choosing its capital structure, the firm balances the tax benefits of debt with the costs of default. The debt issued is a perpetuity and the firm is able to restructure upwards in the future by issuing additional debt, subject to a proportional cost of debt issuance, $\phi_D$. Given the initial state, $\nu_0$, the firm chooses the coupon rate and 2 state-dependent default and upward restructuring boundaries, $\{X_D(\nu_0), X_U(\nu_0)\}$, to maximize the initial value of equity. At time 0, the initial value of the firm for initial cash flow level $X_0$ and initial state $\nu_0$ is given by

$$E(X_0, \nu_0; \nu_0) + (1 - \phi_D)D(X_0, \nu_0; \nu_0)$$

(11)

where $\phi_D$ is a proportional cost of debt issuance. Note that even at a later date, $t$, the equity and debt value depend on the initial state, $\nu_0$, and well as the current state, $\nu_t$. The firm’s problem is given by

$$\max_{C(\nu_0),X_U(\nu_0)} E(X_0, \nu_0; \nu_0) + (1 - \phi_D)D(X_0, \nu_0; \nu_0) \text{ s.t.}$$

(12)

$$\frac{\partial}{\partial X} E(X, k, C; \nu_0) \bigg|_{X \downarrow X_{D}(\nu_0)} = 0, \ k = 1, 2$$

where $X_U(\nu_0) = \{X_1^U(\nu_0), X_2^U(\nu_0)\}$.

The initial optimal leverage ratio is given by

$$L_0(X_0, \nu_0, Y) = \frac{D(X_0, \nu_0; C^*)}{E(X_0, \nu_0; C^*) + D(X_0, \nu_0; C^*)}$$

(13)

where $C^*$ is the optimally chosen coupon rate.

2.8 Comment on Distress vs. Default Costs

In the model, firms do not incur distress costs prior to declaring default, at which point the equity holders no longer have a claim to the firm. In reality, firms typically incur distress costs prior to the event of default and some firms may incur distress costs without ever

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23Similarly, the coupon rate and thresholds chosen will depend on the initial state.
declaring default. Moreover, the costs of distress outside of default are borne directly by equity holders (though debtholders may suffer losses as a result), whereas default in the model occurs when equity value is zero, with the subsequent default costs coming out of the bondholders’ recovery.

Despite this simplification, the effect on a firm’s capital structure decision is similar to a model with explicit distress costs. In the model, equityholders do not explicitly incur distress costs, but they behave as if they did insofar as the costs borne by the debtholders in default are internalized by the equityholders. In the model, a higher cost of default ($\alpha$) results in a lower recovery rate for debtholders, all else equal. Recognizing this, the debtholders demand a higher credit spread, for a given level of leverage and default probability. This leads a firm with larger default costs to issue less debt and thus have a lower default probability than an otherwise identical firm with smaller default costs. That is, while the default costs are not directly borne by the equityholders, they internalize these costs as they adjust their optimal level of leverage. Thus, even without explicit distress costs incurred prior to default, equityholders still behave similarly to a case with such explicit costs. This underscores the importance of using a modeling framework in which leverage choices and debt prices are determined jointly and endogenously. The equilibrium pricing of the firm’s debt ensures that default costs, which are borne directly by debtholders, are internalized by the equityholders when choosing leverage.

3 Model Implications

As a starting point, I calibrate a model economy of ex ante heterogeneous firms to match both aggregate and cross-sectional features of the data. This allows me to examine the quantitative impact of variation in default costs on a firm’s optimal leverage decision, default probability, credit spread, and price of debt and equity. The model generally matches the salient features of the data and thus provides an environment in which the quantitative magnitude of the bias in default costs can be examined. Before discussing the model calibration and simulation results, I first present some comparative statics for optimal firm policies and credit spreads
with respect to variation in a firm’s default cost, $\alpha_i$.

Figure 1 shows how variation in $\alpha$ affects optimal firm policies, credit risk, and firm value. Each plot shows variation in $\alpha$ between 0 and 1, fixing all other parameters at the values given in Table I. In all cases, the initial state is the high state and the initial cash flow, $X_0$, is normalized to 1. Panel A of Figure 1 displays a firm’s optimal initial book leverage as a function of $\alpha$. As expected, optimal leverage is decreasing in the magnitude of default costs. Intuitively, higher default costs make a firm’s cost of debt higher while leaving the tax benefits unchanged, thus leading to a lower optimal leverage ratio.

Similarly, Panel C indicates that the optimal coupon is decreasing in the default costs. That is, a firm with higher default costs, $\alpha$, optimally chooses to issue less debt. It is important to note that this is an optimal choice with respect to the objective of maximizing the initial total firm value. Since the firm is initially all equity, this is the same as saying the equityholders are maximizing the initial value of equity. Panel D shows that for a fixed level of earnings, the total firm value is decreasing in default costs. Issuing debt gives the firm a tax shield that increases its value. For a given face value, the debt of a firm with high default costs is worth less.

Panel B of Figure 1 shows the optimal default thresholds for each state as a function of default costs. The solid (dashed) line indicates the default threshold when the current state is high (low). As indicated by the graph, for all values of $\alpha$, firms choose to default sooner in the low state than in the high state. Under the parameter specification of Table I, the value of equity is higher in the high state.

Additionally, Panel B shows that firms with low costs of default optimally choose to default sooner than those with high costs. As indicated by the plot, this relationship is nonlinear and indicates that in the model firms with low costs of default are more likely to default, all else equal.
4 Simulation Results

In this section, I simulate the model fixing all firm-specific parameters except for the default costs, $\alpha$, which are assumed to be distributed $U[0, 1]$ in the cross-section. This provides a benchmark environment in which the effects of heterogeneity in default costs can be studied, separate from heterogeneity in other firm-specific parameters. In Section 5, I estimate the firm-specific parameters for a sample of firms and in Section 7, I present results from the model simulated under the estimated parameter distribution. As will be shown in Section 5, there is nontrivial cross-sectional correlation between the estimated parameters. Thus, I use this section to isolate the effect of heterogeneity in default costs and provide a sense of whether this heterogeneity alone can produce a significant bias.

To generate results from the model, I perform 5,000 simulations of a panel of 4,000 firms. The model is simulated at a quarterly frequency for 40 years and simulated sample statistics are averaged across the 5,000 simulations. To illustrate the effects of ex ante heterogeneity in default costs distinct from variation in other firm specific parameters, I only allow for cross-sectional variation in $\alpha$. That is, all firm-specific parameters except for $\alpha$ are identical across firms.\footnote{The model can accommodate cross-sectional heterogeneity in these other parameters, however, to clearly illustrate the effect of heterogeneity in $\alpha$ I fix all other parameter values. This should not affect the quantitative inference unless $\alpha$ is correlated in the cross-section with another parameter that would affect the leverage decision. While there is no economic reason to believe this to be the case \textit{a priori}, this is an empirical issue that is addressed in the estimation of Section 5.} It is important to note, however, that firms are still exposed to idiosyncratic earnings shocks and thus will differ ex post. Table I presents the parameter values used in the benchmark model calibration. I assume in the cross-section $\alpha_i \sim U[0, 1]$ and it is constant across states.\footnote{Empirically, there is evidence that recovery rates are lower in times of bad economic conditions, when there are a greater number of defaults. See, for example, Altman et al. (2005). To illustrate the effect of cross-sectional heterogeneity in $\alpha$, distinct from time variation, I do not allow $\alpha$ to vary across economic states. This is done for expositional clarity and the results do not depend on this assumption.} To implement this, I discretize the distribution into 40 grid points for $\alpha$, each with equal mass and solve the model for each of these firm “types.”

In order to maintain a balanced panel in the simulation, I assume that a new, ex ante identical firm replaces a defaulting firm in the same period that default occurs. This means
that the mass of firms as well as the cross-sectional parameter distribution is constant through each simulated time series. In Table II, I compare moments generated from model simulations with their counterparts in the data. Additionally, at each date in the simulation, I assign each firm to a credit rating according to its expected default probability. In Table III, I compare the credit spreads and default probabilities for the ratings groups in the model with their empirical counterparts. The model appears to do reasonably well in matching several salient features of the data, particularly the average default rate, leverage, and credit spreads.

4.1 Parameter Calibration and Estimation

First, I estimate the parameters of the regime-switching aggregate earnings process using quarterly aggregate earnings data from NIPA Table 1.14 provided by the BEA. In the model, aggregate earnings is assumed to evolve according to a Markov-modulated geometric Brownian motion:

\[
\frac{dX_t^A}{X_t^A} = \mu^A(\nu_t)dt + \sigma^A(\nu_t)dW_t^A. \tag{14}
\]

By Itô’s Lemma, the quarterly log earnings growth rate, \(x_{t+1}^A\), can be written as

\[
x_{t+1}^A \equiv \Delta\log(X_{t+1}^A) = \mu^A(\nu_t) - \frac{1}{2}\sigma^A(\nu_t) + \varepsilon_{t+1}^A \tag{15}
\]

where \(\varepsilon_{t+1}^A \sim \mathcal{N}(0, (\sigma^A(\nu_t))^2)\). The identifying assumption for the two regimes is a negative earnings growth rate in the low state. I estimate the six parameters of the aggregate earnings process, \((\mu_1^A, \mu_2^A, \sigma_1^A, \sigma_2^A, \lambda_{12}, \lambda_{21})\) via maximum likelihood. The estimates for the aggregate earnings process and the generator matrix, \(\Lambda\), are presented in Table I. Note that the low state, which is identified by the negative earnings growth, also has higher volatility. For details on the estimation procedure, see Appendix F.

The tax rate on corporate profits is set to 35%, the current top U.S. federal corporate marginal tax rate. The tax rate on equity distributions, \(\tau_d\), and interest income, \(\tau_i\), are set to 12% and 29.6%, respectively, which are the values computed in Graham (2000). The cost of debt issuance, \(\phi_D\) is set to match the costs found in Altinkilic and Hansen (2000).

\[^{26}\text{Additional details can be found in Appendix E}\]
In this version of the simulation, all firms have identical cash flow parameters, $\mu_i$, $\beta_i$, and $\sigma_i^F$. I set $\mu_i = 0$ so that all firms have a state-conditional expected earnings growth equal to the aggregate. Idiosyncratic volatility, $\sigma_i^F$, is set to match the cross-sectional variance of earnings growth in the data.

4.2 Bias in the Estimated $\alpha$

By simulating the model, I am able to compare the true distribution of $\alpha$, which I have specified, with the conditional distribution inferred from the sample of defaults in the simulated data. This allows me to quantify the magnitude of the bias in the estimated $\alpha$.

I assume that the econometrician can only observe firm $i$’s cost of default, $\alpha_i$, if the firm defaults. Thus, the econometrician can use the sample of historical defaults to construct a distribution of $\alpha$’s from the firms that default. I denote the mean of this conditional distribution as $\hat{\alpha}_{\text{Default}}$, which is defined as

$$\hat{\alpha}_{\text{Default}} = E[\alpha_i \mid \text{Firm } i \text{ Defaulted}]$$

I simulate the panel of 5,000 firms for 35 years and I repeat the simulation 5,000 times. For each simulation, I take the set of firms which defaulted during the 35 year period and compute the average $\alpha$ of the firms in this sample. Thus, each simulation has a computed value of $\hat{\alpha}_{\text{Default}}$. In Figure 3, I plot a histogram of the 5,000 $\hat{\alpha}_{\text{Default}}$’s computed in each simulation. The vertical red dotted line represents the unconditional mean of the distribution of $\alpha$. As indicated by the figure, in every simulation $\hat{\alpha}_{\text{Default}}$ is substantially smaller than the unconditional average $\alpha$ represented by the vertical dotted line. In this sense, the existence of the bias in $\alpha$ is not a statement about the particular sample path observed in the data. As indicated by the variation shown in the histogram, the magnitude of the bias varies across sample paths but in all simulations the bias is quantitatively large. Put differently, in any of the model simulations the average defaults costs inferred from the sample of defaults is much less than the true average.

This bias does not just affect the estimated mean value of $\alpha$. Figure 4 plots the inferred and true distributions for $\alpha$. The inferred distribution is taken by averaging over all 5,000
simulations. As shown, low cost firms are over-represented and high cost firms underrepresented in the sample of defaults, leading one to infer a distribution for $\alpha$ that is very different from its true distribution.

The intuition for this effect is straightforward. If two firms differ in their default costs but not the tax benefits to issuing debt, then the firm for which default is more costly should endogenously choose a lower leverage ratio such that it is less likely to default. Firms trade off the expected benefits of the tax shield with the expected costs of default. A firm which incurs higher costs conditional on default reduces its expected costs by issuing less debt, thus making default less likely. This naturally results in lower $\alpha$ firms having a higher default probability.

4.3 Underleverage

Having established that the model produces ex post estimates of default costs, $\alpha$, that are substantially smaller than the unconditional average, I now illustrate how this generates what appears to be an underleverage puzzle. Throughout this section, I use the parameters given in Table I.

In Panel C of Figure 1, I plot the interest coverage ratio at time 0 for firms that differ in their $\alpha$’s but are otherwise identical.\textsuperscript{27} In the framework of Graham (2000), these firms have identical tax benefit functions with “kinks” at the same level of interest expense.\textsuperscript{28} In that respect, the high $\alpha$ firms, which optimally choose lower leverage, appear to be underlevered. That is, it seems that these firms could increase value by issuing additional debt. Furthermore, Figure 2 appears to confirm the idea that the lower leverage firms could increase value by issuing more debt. The figure shows the initial total firm value as a function of $\alpha$, again with all other parameters fixed across firms. Note that earnings process for all of these firms is identical. While earnings will differ ex post due to differences in the realizations of firm-specific shocks, all firms in Figure 2 have identical ex ante tax benefits to

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\textsuperscript{27}Since firms in the model issue perpetuities which pay no principal, the interest coverage coverage ratio, measure as the ratio of earnings to interest expense, is higher in the model than in the data.

\textsuperscript{28}Graham (2000) defines the “kink” in a firm’s marginal tax benefit function as the point at which the function becomes downward sloping.
debt. These firms differ in their default costs, however, which leads them to optimally select different levels of leverage. Furthermore, differences in the default costs, $\alpha$, produce large differences in optimal leverage. Since the default costs, $\alpha$, are unobserved prior to default, these firms appear identical ex ante, with the exception of their chosen leverage ratio and credit spread.

However, examining the credit spreads does not change the conclusion that some firms are underlevered. The bottom panel of the figure illustrates this by plotting the time-0 credit spreads against the initial leverage. Note that the credit spreads are increasing in leverage. This relation is completely consistent with a case in which firms do not differ in their $\alpha$’s but simply use debt with differing degrees of conservatism. All else equal, one should expect credit spreads to be increasing in leverage. Furthermore, if one assumed that firms were ex ante identical but chose different level of leverage (for some arbitrary reason), then one would conclude that Figure 2 represents the cost of debt function faced by each firm.

Note, however, that this is not the case. Each firm has chosen its debt optimally to maximize the initial equity value. Thus, issuing more debt for any firm here would result in a *reduction* in value.\(^{29}\) Thus, the unobserved heterogeneity in $\alpha$ makes some firms appear to use debt too conservatively, when in fact this is not the case.

None of this rules out the possibility that there are firms in the data which are, in fact, underlevered. However, failure to account for the effects of heterogeneity in default costs has the effect of making the underleverage puzzle appear more severe than it is. Moreover, in the model I show that this effect is quantitatively large, and thus can potentially account for a sizeable portion of the underleverage puzzle observed in the data.

Figure 3 displays the distribution of the average ex post $\alpha$ from the 1000 simulations. In truth, $\alpha \sim U[0, 1]$, with a mean value of 0.5. As we can see from the histogram, the ex post estimated $\alpha$ from the simulations is consistently biased and by a large magnitude.

\(^{29}\) The comparisons here are made using time-0 values. The results do not depend on this, however, and the same conclusions would hold at a later time for firms which experience the same realization of shocks.
5 SMM Estimation

In this section, I estimate firm-specific default costs and cash flow parameters via the simulated method of moments. I construct a sample of firms from the Compustat Fundamentals Quarterly file merged with equity data from CRSP. Details and variable definitions are provided in Appendix E. The dataset consists of firm-specific moments for 2,505 firms. The aggregate parameters are set to the values used in the model simulation of Section 4.

The method of moments estimator selects the vector of parameters for each firm that minimizes the distance between a firm’s moments in the data and moments from simulated data produced by the model. Intuitively, it selects the set of model parameters for each firm that “best” explain that firm’s data moments. Recall that in the model firm $i$’s cash flows evolve according to

$$\frac{dX_{i,t}}{X_{i,t}} = (\mu_i + \beta_i \mu_A(\nu_t))dt + \beta_i \sigma_A(\nu_t)dW^A_t + \sigma_{i,F} dW^i_{t,F}$$  \hspace{1cm} (17)

This gives three firm-specific cash flow parameters $(\mu_i, \beta_i, \sigma_{i,F})$, in addition to the cost of default parameter, $\alpha_i$, to be estimated for each of the 2,505 firms in my sample. For each firm $i$ in the sample, I estimate a firm-specific vector of parameters, $\theta_i$, where

$$\theta_i = [\alpha_i \quad \mu_i \quad \beta_i \quad \sigma_{i,F}]$$

Let $M^i$ denote the $K \times 1$ vector of data moments for firm $i$. Given a parameter vector $\theta$, for each simulation $s = 1, ..., S$, I simulate a time series of length $T$ and compute a vector of moments from the simulated data, $\tilde{m}_s(\theta)$, that serves as an analog to the data moments, $M^i$. The method of moments estimator for the parameters of firm $i$ is defined as

$$\hat{\theta}_i = \arg\min_{\theta} \left( M^i - \frac{1}{S} \sum_{s=1}^{S} \tilde{m}_s(\theta) \right)' W_i \left( M^i - \frac{1}{S} \sum_{s=1}^{S} \tilde{m}_s(\theta) \right)$$  \hspace{1cm} (18)

where $W_i$ is a positive semidefinite weighting matrix for firm $i$. For the results presented, I set $W_i$ to be the inverse of the estimated covariance matrix of firm $i$’s moments in the data. The covariance matrix for each firm’s data moments is estimated by bootstrapping from the sample of observations for each firm. I repeat the SMM procedure for each firm in the sample, obtaining a vector of parameter estimates, $\hat{\theta}_i$, for each firm $i = 1, ..., N$. 24
5.1 Selection of Moments

The selection of moments used in the estimation is important to ensure that the four parameters are identified. I select a set of seven firm-specific moments that are informative in that they are sensitive to the parameter values. In particular, I include moments that are informative about both prices and quantities. The moments used are the firm’s mean book leverage, mean excess equity return, median price-earnings ratio, mean earnings growth rate, volatility of earnings growth, and mean and volatility of quasi-market leverage. The quasi-market leverage measure is the ratio of the book value of debt to the sum of the book value of debt and market value of equity. I briefly discuss the moments identifying each of the parameters.

The cost of default parameter, $\alpha_i$, is determined by the book and quasi-market leverage measures. As shown in Panel A of 1, the firm’s optimal leverage choice is sensitive to the value of default costs. Additionally, since quasi-market leverage contains the market value of equity, it contains information independent of book leverage. As a result, both are informative regarding the default cost parameter $\alpha_i$.

As one would expect, the firm-specific component to expected earnings growth, $\mu_i$, is pinned down primarily by the earnings growth rate, however, other moments are informative as well. The price-earnings ratio, for example, is increasing in the rate of earnings growth, all else equal. Intuitively, controlling for the discount rate, a firm with a higher expected earnings growth has a larger value of equity and thus a higher price-earnings ratio. Due to the volatility of a firm-level price earnings ratio, I choose to use the median value of the firm’s price-earnings ratio in the sample period. This appears to be somewhat more informative than the mean value.

The firm’s risk exposure, which is parameterized by $\beta_i$, impacts the mean excess equity return, price-earnings ratio, and quasi-market leverage values. A larger value of $\beta_i$ implies greater exposure to systematic risk, which translates to a higher expected return. Similarly, this higher expected return results in a lower present value of equity, which all else equal, means a lower price-earnings ratio. While an increase in $\beta_i$ does increase the volatility of
quasi-market leverage and earnings, the impact on these moments is substantially smaller. This is because most of the variation in the volatility measures is driven by differences in the idiosyncratic volatility, not differences in exposure to aggregate shocks. Additionally, an increase in $\beta_i$ increases the mean quasi-market leverage ratio in that it reduces the market value of equity, all else equal. However, again, this effect is small compared to the impact of other parameters on the quasi-market leverage ratio.

Finally, the idiosyncratic volatility, $\sigma_{i,F}$ is determined primarily by the earnings growth and quasi-market leverage volatilities. Again, this is straightforward as these volatility measure are monotonically increasing in $\sigma_{i,F}$. However, the volatility also impacts the levels of book and quasi-market leverage as a higher volatility, all else equal, implies a greater default probability. At the same time, this effect is somewhat mitigated by the fact that, all else equal, higher idiosyncratic volatility increases the equityholders’ option to delay default. I now turn to the estimation results.

5.2 Estimation Results

The results from the estimation are presented in Figure 6, which shows the cross-sectional distribution for each of the four firm-specific parameters. Panel A of Figure 6 shows the cross-sectional distribution of the estimated default cost parameter, $\alpha_i$. Note that the estimated values of $\alpha_i$ show considerable cross-sectional dispersion, with a standard deviation of 27%. This suggests that applying a single cost of default to the entire cross-section of firms is likely to give misleading results.

In Table IV, I present summary statistics and correlations for the estimated parameters. As indicated in the table, the mean estimated default cost is 44.5% of firm value, with substantial standard deviation of 27%. Additionally, Panel B shows the correlation matrix for the estimated parameters. It is interesting to note that the estimated default costs, $\alpha$, have nontrivial correlation with the estimated cash flow parameters. This underscores the importance of estimating all four firm-specific parameters jointly. Finally, Panel C displays Spearman rank correlations for the estimated parameters and data moments.
6 Characterizing the Estimated Default Costs

In this section I characterize the estimated firm-level default costs by examining how they relate to firm characteristics, credit ratings, and industry. Using data that was not included in the estimation procedure allows me to check whether the estimates are consistent with previously identified determinants of leverage and default costs. This not only gives an external validation check of the estimates obtained from the structural model, but also provides insights into the determinants of a firm’s default costs.

In Table V, I display summary statistics for the estimated default costs by industry. The industries are grouped according to the Fama-French 17 industry classification, with utilities and financials excluded from the sample.\textsuperscript{30} Note that while there is some variation in the mean default cost across industry, there is substantial intraindustry variation. In Table VI, I present the mean value for all four firm-specific parameters by industry.

Table VII displays the average estimated parameters by credit rating. Note that the average estimated default costs are increasing in the quality of credit rating. Since firms with high default costs choose leverage such that their probability of default is low, these firms are likely to be those that have a high credit rating, at least at their optimal financing date. This implies that firms which at one point had a high credit rating and later defaulted, the so-called fallen angels, should have higher than average default costs. This prediction is confirmed in a recent working paper, Davydenko, Strebulaev, and Zhao (2010), who find that fallen angels have realized default costs 45% higher than those of original-issue junk issuers.\textsuperscript{31}

In Table VIII, I present regressions of the estimated default costs on various firm characteristics. I present six different specifications in which I compare the results with and without controls for leverage and industry fixed effects. While not all statistically significant, the re-

\textsuperscript{30} The Fama-French industry classification is according to Standard Industrial Classification (SIC) codes, which are available for the firms in the Compustat database. Details of the classification are provided on Ken French’s website: \url{http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html}.

\textsuperscript{31} The original-issue junk issuers are those firms rated speculative grade at the time when the bonds are issued.
relationship between default costs and firm characteristics generally appear consistent with intuition. Firms with higher market-to-book ratios and investment rates appear to have higher default costs, even after controlling for leverage. Additionally, while not statistically significant, the R&D/Sales ratio appears positively correlated with default costs. Additionally, firms with higher cash to asset ratios appear to be those with higher default costs, consistent with a hedging motive.

7 Estimation of the Selection Bias in Default Costs

In the simulation of Section 4, I assumed $\alpha_i \sim U[0, 1]$ and fixed the three firm-specific cash flow parameters, $\mu_i, \beta_i, \sigma_F$, to be identical across firms. Consequently, the bias computed in $\alpha$, while quantitatively significant, is not independent of the parameter distribution chosen. I have repeated the simulation exercise of Section 4 for different assumed distributions for $\alpha$ and in all cases a bias results. The magnitude of this bias, however, obviously depends on the distribution chosen.

Thus, the bias calculation of Section 4, while illustrative, does not provide an actual estimate of the magnitude of the bias. To address this, I use the firm-specific parameters estimated in the SMM of Section 5 to simulate the model again, but under the estimated joint cross-sectional distribution of the firm-specific parameters. Aggregating the firm-specific estimates obtained from the SMM, I have an estimated four-dimensional joint distribution over the cross-section. I am now able to simulate the model under this joint distribution and estimate the bias in measured default costs.

As in the simulation of Section 4, I simulate a panel of 5,000 firms at a quarterly frequency for a 35 year period and repeat the simulation 5,000 times. In each simulation, I collect the firms that defaulted in the sample period and compute an average $\alpha$ for this conditional sample. Thus, I obtain 5,000 mean values for $\alpha$. Figure 7 displays the distribution of these conditional mean $\alpha$’s across simulations. The red vertical line indicates the true unconditional mean $\alpha$ of the estimated distribution obtained from the SMM. As indicated by the graph, in none of the 5,000 sample simulations is the conditional average $\alpha$ computed
from the sample of defaulted firms as large as the true unconditional mean. Similarly, Table IX compares the mean across the 5,000 simulations of 0.246 with the true mean of the distribution, 0.445. Thus, the estimated bias is quantitatively large. Using the sample of ex post defaults from the simulation leads one to conclude that the average default costs are 24.6% of the remaining firm value left at default. In contrast, the true mean of the distribution of these costs is 44.5% of the remaining firm value. In other words, the average firm expects to incur costs in default that are nearly twice as large as what is inferred by estimating these costs from the sample of defaulted firms.

Furthermore, the bias affects the entire distribution, not just the mean value. Figure 8 compares the estimated distribution of expected default costs (Panel A) with the distribution of observed default costs from the sample of defaulted firms generated by simulating the model (Panel B). As shown in the figure, the selection bias has the effect of shifting the entire distribution. While defaults of high default cost firms are observed, these are rare and infrequent. As a result, the observed sample of default costs is a biased sample that understates the magnitude of default costs.

8 Conclusion

This paper argues that ex ante heterogeneity in firms’ expected default costs have important implications for the levels of leverage, credit spreads, and default rates observed in the data. In particular, I show that since firms internalize their expected default costs, those firms with higher costs optimally choose lower levels of leverage, all else equal. As a result, these firms are less likely to default than those firms with lower costs. The estimates of default costs from a historical sample of defaulted firms, is therefore biased, understating the true expected costs faced by the average firm. Since it is the latter that determines a firm’s optimal leverage, many firms appear underleveraged when, in fact, they simply have high expected default costs. Furthermore, this bias is quantitatively significant and accounting for it is shown resolve the underleverage puzzle. That is, the low levels of leverage adopted by many firms in the data can be reconciled with a tradeoff model of capital structure.
Table I:  
Parameters

This table reports the parameter values used in simulating the model described in Section 4. Where applicable, values are quarterly. All parameters are identical across firms except for $\alpha$, which is distributed cross-sectionally according to $U[0, 1]$. The aggregate earnings parameters and probability of a regime change are estimated via maximum likelihood using aggregate earnings data. See the appendix for details on the estimation procedure.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Symbol</th>
<th>State 1</th>
<th>State 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Default Costs</td>
<td>$\alpha$</td>
<td>–</td>
<td>–</td>
</tr>
<tr>
<td>Aggregate Earnings Growth Rate</td>
<td>$\mu$</td>
<td>0.0192</td>
<td>-0.0076</td>
</tr>
<tr>
<td>Aggregate Earnings Volatility</td>
<td>$\sigma_A$</td>
<td>0.0366</td>
<td>0.0770</td>
</tr>
<tr>
<td>Idiosyncratic Earnings Volatility</td>
<td>$\sigma_f$</td>
<td>0.130</td>
<td>0.130</td>
</tr>
<tr>
<td>Market Sharpe Ratio</td>
<td>$\varphi$</td>
<td>0.140</td>
<td>0.238</td>
</tr>
<tr>
<td>Instantaneous Risk-free Rate</td>
<td>$r_f$</td>
<td>0.006</td>
<td>0.009</td>
</tr>
<tr>
<td>Probability of Regime Change</td>
<td>$\lambda$</td>
<td>0.07</td>
<td>0.10</td>
</tr>
<tr>
<td>Tax Rate on Corporate Earnings</td>
<td>$\tau_\pi$</td>
<td>0.35</td>
<td>0.35</td>
</tr>
<tr>
<td>Tax Rate on Dividends</td>
<td>$\tau_d$</td>
<td>0.12</td>
<td>0.12</td>
</tr>
<tr>
<td>Tax Rate on Interest Income</td>
<td>$\tau_i$</td>
<td>0.296</td>
<td>0.296</td>
</tr>
<tr>
<td>Proportional Debt Issuance Cost</td>
<td>$\phi_D$</td>
<td>0.015</td>
<td>0.015</td>
</tr>
</tbody>
</table>
Table II:  
Simulation Results

This table displays various moments from the simulated model and their empirical counterparts. Model moments are produced by simulating a panel of 5,000 firms at quarterly frequency for 35 years. The simulation is repeated 5,000 times. The simulation is performed at a quarterly frequency and all values are annualized. The leverage statistics are computed by taking the cross-sectional mean or standard deviation at each quarter of the simulation. A time series average of this statistic is computed for each simulation. The value reported is the mean across the 5,000 simulations. The model parameters used for the simulation can be found in Table I. Aggregate earnings data is from NIPA, leverage moments and the cross-sectional dispersion of earnings are computed from Compustat data. Recovery rate and cumulative default probability data are from Moody’s. Book leverage is (book debt)/(book debt + book equity), quasi-market leverage is (book debt)/(book debt + market equity).

<table>
<thead>
<tr>
<th>Moment</th>
<th>Model</th>
<th>Data</th>
</tr>
</thead>
<tbody>
<tr>
<td>Excess Equity Ret</td>
<td>0.049</td>
<td>0.063</td>
</tr>
<tr>
<td>Aggregate Earnings Growth</td>
<td>0.039</td>
<td>0.034</td>
</tr>
<tr>
<td>Earnings Cross-Sectional Std</td>
<td>0.130</td>
<td>0.152</td>
</tr>
<tr>
<td>Book Leverage Mean</td>
<td>0.207</td>
<td>0.228</td>
</tr>
<tr>
<td>Book Leverage Cross-Sectional Std</td>
<td>0.153</td>
<td>0.184</td>
</tr>
<tr>
<td>Quasi-Market Leverage Mean</td>
<td>0.297</td>
<td>0.280</td>
</tr>
<tr>
<td>Quasi-Market Leverage Cross-Sectional Std</td>
<td>0.218</td>
<td>0.260</td>
</tr>
<tr>
<td>Recovery Rate Mean</td>
<td>0.423</td>
<td>0.375</td>
</tr>
<tr>
<td>Recovery Rate Cross-Sectional Std</td>
<td>0.137</td>
<td>0.257</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Cumulative Default Probability</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1 year</td>
<td>0.014</td>
<td>0.016</td>
</tr>
<tr>
<td>5 year</td>
<td>0.069</td>
<td>0.071</td>
</tr>
<tr>
<td>10 year</td>
<td>0.132</td>
<td>0.111</td>
</tr>
<tr>
<td>15 year</td>
<td>0.188</td>
<td>0.142</td>
</tr>
</tbody>
</table>
Table III:
Simulated Default Probabilities and Credit Spreads by Credit Rating

This table displays 10-year default probabilities and credit spreads by credit rating from the simulated model of Section 4. At each date in the simulation, each firm’s current 10-year default probability and credit spread on a 10-year bond are computed via Monte Carlo. The firms are then assigned a credit rating at each date according to their current default probability.

Panel A: 10 Year Credit Spread Over Aaa (bps)

<table>
<thead>
<tr>
<th>Rating</th>
<th>Model</th>
<th>Data</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>15</td>
<td>36</td>
</tr>
<tr>
<td>Baa</td>
<td>73</td>
<td>81</td>
</tr>
<tr>
<td>Ba</td>
<td>222</td>
<td>253</td>
</tr>
<tr>
<td>B</td>
<td>415</td>
<td>386</td>
</tr>
</tbody>
</table>

Panel B: 10 Year Cumulative Default Probability by Credit Rating (%)

<table>
<thead>
<tr>
<th>Rating</th>
<th>Model</th>
<th>Data</th>
</tr>
</thead>
<tbody>
<tr>
<td>Aaa</td>
<td>0.69</td>
<td>0.50</td>
</tr>
<tr>
<td>A</td>
<td>2.63</td>
<td>2.05</td>
</tr>
<tr>
<td>Baa</td>
<td>8.93</td>
<td>4.85</td>
</tr>
<tr>
<td>Ba</td>
<td>22.04</td>
<td>19.96</td>
</tr>
<tr>
<td>B</td>
<td>36.37</td>
<td>44.38</td>
</tr>
</tbody>
</table>
Table IV: Cross-sectional Statistics for Firm-Specific Parameter Estimates

This table reports summary statistics for the firm-specific parameter estimates obtained from the SMM of Section 5. The firm-specific parameters consist of three cash flow parameters ($\mu_i$, $\beta_i$, and $\sigma_F^i$) and the cost of default parameter $\alpha_i$. Firm $i$'s earnings in the model evolve according to

$$\frac{dX_{i,t}}{X_{i,t}} = (\mu_i + \beta_i \mu_A(\nu_t))dt + \beta_i \sigma_A(\nu_t)dW^A_t + \sigma_i,F dW^i,F_t$$

where $\mu_i$ represents a firm fixed effect for the expected earnings growth rate, $\beta_i$ is the loading of the firm's cash flows on the aggregate earnings shock, and $\sigma_F^i$ is the volatility of the firm's idiosyncratic earnings shocks. The fraction of unlevered firm value lost in default for firm $i$ is given by $\alpha_i$, where the unlevered value is defined in equation (5). Panel A displays cross-sectional moments for the firm-specific parameter estimates. The cross-sectional correlation of the parameter estimates are shown in Panel B. In Panel C, I display the Spearman rank correlation for the estimated parameters with the firm data moments. The sample consists of 2,505 firms from the merged Compustat and CRSP databases. See Appendix E for further details regarding the data.

Panel A: Parameter Estimate Summary Statistics

<table>
<thead>
<tr>
<th></th>
<th>Mean</th>
<th>Median</th>
<th>Std</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha_i$</td>
<td>0.445</td>
<td>0.368</td>
<td>0.270</td>
</tr>
<tr>
<td>$\sigma_F^i$</td>
<td>0.132</td>
<td>0.147</td>
<td>0.055</td>
</tr>
<tr>
<td>$\beta_i$</td>
<td>1.278</td>
<td>1.000</td>
<td>0.577</td>
</tr>
<tr>
<td>$\mu_i$</td>
<td>-0.004</td>
<td>-0.010</td>
<td>0.007</td>
</tr>
</tbody>
</table>

Panel B: Correlation of Parameter Estimates

<table>
<thead>
<tr>
<th></th>
<th>$\alpha_i$</th>
<th>$\sigma_F^i$</th>
<th>$\beta_i$</th>
<th>$\mu_i$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha_i$</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\sigma_F^i$</td>
<td>-0.270</td>
<td>1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\beta_i$</td>
<td>0.320</td>
<td>0.023</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>$\mu_i$</td>
<td>0.358</td>
<td>-0.125</td>
<td>0.567</td>
<td>1</td>
</tr>
</tbody>
</table>

Panel C: Spearman Rank Correlations

<table>
<thead>
<tr>
<th></th>
<th>$\alpha_i$</th>
<th>$\sigma_F^i$</th>
<th>$\beta_i$</th>
<th>$\mu_i$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean Book Lev</td>
<td>-0.862</td>
<td>0.095</td>
<td>-0.357</td>
<td>-0.326</td>
</tr>
<tr>
<td>Mean Earnings Growth</td>
<td>0.046</td>
<td>-0.085</td>
<td>0.274</td>
<td>0.315</td>
</tr>
<tr>
<td>Std Earnings Growth</td>
<td>-0.144</td>
<td>0.660</td>
<td>0.191</td>
<td>0.159</td>
</tr>
<tr>
<td>Mean P/E Ratio</td>
<td>0.288</td>
<td>-0.131</td>
<td>0.286</td>
<td>0.562</td>
</tr>
<tr>
<td>Mean Quasi-Market Lev</td>
<td>-0.743</td>
<td>0.292</td>
<td>-0.484</td>
<td>-0.504</td>
</tr>
<tr>
<td>Mean Excess Ret</td>
<td>0.135</td>
<td>-0.059</td>
<td>0.207</td>
<td>0.154</td>
</tr>
</tbody>
</table>
### Table V: 
**Default Cost Estimates by Industry**

This table reports summary statistics by industry for the estimated α’s obtained in the firm-level SMM. Industries correspond to one of the 17 Fama-French industry index, based on SIC codes. Financials and utilities are excluded.

<table>
<thead>
<tr>
<th>Industry</th>
<th>Mean</th>
<th>Std</th>
<th>Q1</th>
<th>Median</th>
<th>Q3</th>
<th>N</th>
</tr>
</thead>
<tbody>
<tr>
<td>Food</td>
<td>0.389</td>
<td>0.255</td>
<td>0.211</td>
<td>0.316</td>
<td>0.474</td>
<td>128.000</td>
</tr>
<tr>
<td>Mining and Minerals</td>
<td>0.463</td>
<td>0.275</td>
<td>0.263</td>
<td>0.421</td>
<td>0.632</td>
<td>36.000</td>
</tr>
<tr>
<td>Oil</td>
<td>0.364</td>
<td>0.262</td>
<td>0.211</td>
<td>0.316</td>
<td>0.421</td>
<td>158.000</td>
</tr>
<tr>
<td>Clothing</td>
<td>0.452</td>
<td>0.273</td>
<td>0.263</td>
<td>0.368</td>
<td>0.632</td>
<td>110.000</td>
</tr>
<tr>
<td>Cons Durable</td>
<td>0.422</td>
<td>0.267</td>
<td>0.263</td>
<td>0.368</td>
<td>0.579</td>
<td>113.000</td>
</tr>
<tr>
<td>Chemicals</td>
<td>0.435</td>
<td>0.248</td>
<td>0.316</td>
<td>0.368</td>
<td>0.474</td>
<td>62.000</td>
</tr>
<tr>
<td>Drugs, Perfume, Tobacco</td>
<td>0.532</td>
<td>0.259</td>
<td>0.368</td>
<td>0.474</td>
<td>0.684</td>
<td>88.000</td>
</tr>
<tr>
<td>Construction</td>
<td>0.374</td>
<td>0.206</td>
<td>0.211</td>
<td>0.368</td>
<td>0.474</td>
<td>143.000</td>
</tr>
<tr>
<td>Steel</td>
<td>0.369</td>
<td>0.191</td>
<td>0.263</td>
<td>0.342</td>
<td>0.421</td>
<td>64.000</td>
</tr>
<tr>
<td>Fabricated Products</td>
<td>0.350</td>
<td>0.203</td>
<td>0.211</td>
<td>0.316</td>
<td>0.421</td>
<td>46.000</td>
</tr>
<tr>
<td>Machinery</td>
<td>0.489</td>
<td>0.250</td>
<td>0.316</td>
<td>0.421</td>
<td>0.632</td>
<td>411.000</td>
</tr>
<tr>
<td>Automobiles</td>
<td>0.397</td>
<td>0.205</td>
<td>0.263</td>
<td>0.368</td>
<td>0.474</td>
<td>58.000</td>
</tr>
<tr>
<td>Transportation</td>
<td>0.413</td>
<td>0.181</td>
<td>0.303</td>
<td>0.368</td>
<td>0.487</td>
<td>41.000</td>
</tr>
<tr>
<td>Retail Stores</td>
<td>0.442</td>
<td>0.266</td>
<td>0.263</td>
<td>0.368</td>
<td>0.526</td>
<td>251.000</td>
</tr>
<tr>
<td>Other</td>
<td>0.474</td>
<td>0.302</td>
<td>0.263</td>
<td>0.421</td>
<td>0.684</td>
<td>752.000</td>
</tr>
</tbody>
</table>

34
Table VI:
Parameter Estimates by Industry

This table reports the mean firm-specific parameter estimates by industry. Estimates are obtained from the SMM procedure described in Section 5. Note that $\mu$ is not an absolute growth rate, but is relative to the aggregate earnings growth rate. Industries correspond to one of the 17 Fama-French industry index, based on SIC codes. Financials and utilities are excluded.

<table>
<thead>
<tr>
<th>Industry</th>
<th>$\alpha$</th>
<th>$\sigma^F$</th>
<th>$\beta$</th>
<th>$\mu$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Food</td>
<td>0.389</td>
<td>0.135</td>
<td>1.195</td>
<td>-0.005</td>
</tr>
<tr>
<td>Mining and Minerals</td>
<td>0.463</td>
<td>0.141</td>
<td>1.319</td>
<td>-0.003</td>
</tr>
<tr>
<td>Oil</td>
<td>0.364</td>
<td>0.134</td>
<td>1.246</td>
<td>-0.003</td>
</tr>
<tr>
<td>Clothing</td>
<td>0.452</td>
<td>0.137</td>
<td>1.251</td>
<td>-0.007</td>
</tr>
<tr>
<td>Cons Durable</td>
<td>0.422</td>
<td>0.140</td>
<td>1.209</td>
<td>-0.007</td>
</tr>
<tr>
<td>Chemicals</td>
<td>0.435</td>
<td>0.121</td>
<td>1.131</td>
<td>-0.006</td>
</tr>
<tr>
<td>Drugs, Perfume, Tobacco</td>
<td>0.532</td>
<td>0.112</td>
<td>1.430</td>
<td>-0.004</td>
</tr>
<tr>
<td>Construction</td>
<td>0.374</td>
<td>0.146</td>
<td>1.146</td>
<td>-0.007</td>
</tr>
<tr>
<td>Steel</td>
<td>0.369</td>
<td>0.147</td>
<td>1.128</td>
<td>-0.007</td>
</tr>
<tr>
<td>Fabricated Products</td>
<td>0.350</td>
<td>0.150</td>
<td>1.141</td>
<td>-0.007</td>
</tr>
<tr>
<td>Machinery</td>
<td>0.489</td>
<td>0.133</td>
<td>1.338</td>
<td>-0.003</td>
</tr>
<tr>
<td>Automobiles</td>
<td>0.397</td>
<td>0.140</td>
<td>1.162</td>
<td>-0.006</td>
</tr>
<tr>
<td>Transportation</td>
<td>0.413</td>
<td>0.136</td>
<td>1.085</td>
<td>-0.007</td>
</tr>
<tr>
<td>Retail Stores</td>
<td>0.442</td>
<td>0.131</td>
<td>1.235</td>
<td>-0.005</td>
</tr>
<tr>
<td>Other</td>
<td>0.474</td>
<td>0.125</td>
<td>1.363</td>
<td>-0.003</td>
</tr>
</tbody>
</table>

Table VII:
Parameter Estimates by Credit Rating

This table reports the mean firm-specific parameter estimates by credit rating for those firms in the sample for which a credit rating is available. The parameter estimates are obtained in the SMM of Section 5. Note that $\mu$ is not an absolute growth rate, but is relative to the aggregate earnings growth rate.

<table>
<thead>
<tr>
<th>Credit Rating</th>
<th>$\alpha$</th>
<th>$\sigma^F$</th>
<th>$\beta$</th>
<th>$\mu$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Aaa</td>
<td>0.568</td>
<td>0.094</td>
<td>0.840</td>
<td>-0.006</td>
</tr>
<tr>
<td>Aa</td>
<td>0.585</td>
<td>0.093</td>
<td>1.128</td>
<td>-0.005</td>
</tr>
<tr>
<td>A</td>
<td>0.445</td>
<td>0.102</td>
<td>1.031</td>
<td>-0.006</td>
</tr>
<tr>
<td>Baa</td>
<td>0.419</td>
<td>0.112</td>
<td>1.100</td>
<td>-0.005</td>
</tr>
<tr>
<td>Ba</td>
<td>0.313</td>
<td>0.125</td>
<td>1.234</td>
<td>-0.004</td>
</tr>
<tr>
<td>B</td>
<td>0.305</td>
<td>0.137</td>
<td>1.296</td>
<td>-0.004</td>
</tr>
<tr>
<td>Caa-C</td>
<td>0.189</td>
<td>0.167</td>
<td>1.400</td>
<td>0.000</td>
</tr>
</tbody>
</table>
Table VIII:
Regressions of Estimated Default Costs on Firm Characteristics

This table reports regressions of the estimated firm-specific default costs, \( \alpha \), on firm characteristics. The firm-specific default costs, \( \alpha \), are estimated using the SMM procedure described in Section 5. Unless indicated otherwise, independent variables are a time series mean of the data available for each firm. In the regressions, all independent variables are normalized by their (cross-sectional) standard deviation, thus the coefficient can be interpreted as the absolute change in \( \alpha \) for a one standard deviation change in the independent variable. Regressions (4), (5), and (6) include industry fixed effects for the 15 Fama-French industries included in the sample. Robust standard errors are in parentheses. For more details on the data construction see the appendix.

<table>
<thead>
<tr>
<th>VARIABLES</th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
<th>(5)</th>
<th>(6)</th>
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<td>-0.216***</td>
<td>-0.202***</td>
<td>-0.221***</td>
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<td></td>
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<tr>
<td></td>
<td>(0.004)</td>
<td>(0.008)</td>
<td>(0.004)</td>
<td>(0.008)</td>
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<tr>
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<td></td>
<td>0.012</td>
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<tr>
<td></td>
<td>(0.007)</td>
<td></td>
<td>(0.007)</td>
<td></td>
<td></td>
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</tr>
<tr>
<td>R&amp;D/Sales</td>
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<td></td>
<td>0.018*</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.009)</td>
<td></td>
<td>(0.010)</td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>PPE/Assets</td>
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<td>0.005</td>
<td>0.019***</td>
<td>-0.041***</td>
<td>0.008*</td>
<td>0.016**</td>
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<tr>
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<td>(0.006)</td>
<td>(0.004)</td>
<td>(0.007)</td>
<td>(0.007)</td>
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<td>0.021**</td>
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<td>(0.004)</td>
<td>(0.008)</td>
<td>(0.007)</td>
<td>(0.004)</td>
<td>(0.009)</td>
</tr>
<tr>
<td>Earnings/Assets</td>
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<td>0.030***</td>
<td>0.033***</td>
<td>0.074***</td>
<td>0.030***</td>
<td>0.033**</td>
</tr>
<tr>
<td></td>
<td>(0.006)</td>
<td>(0.004)</td>
<td>(0.007)</td>
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<td>(0.008)</td>
</tr>
<tr>
<td>M/B</td>
<td>0.016**</td>
<td>0.024***</td>
<td>0.034***</td>
<td>0.014**</td>
<td>0.024***</td>
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<td>(0.007)</td>
<td>(0.007)</td>
<td>(0.005)</td>
<td>(0.008)</td>
</tr>
<tr>
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<td>-0.007*</td>
<td>-0.008</td>
<td>-0.017***</td>
<td>-0.006</td>
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<td>(0.008)</td>
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<td>log(Assets)</td>
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<td>0.000</td>
<td>-0.010**</td>
<td>-0.002</td>
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<td>(0.003)</td>
<td>(0.005)</td>
<td>(0.005)</td>
<td>(0.003)</td>
<td>(0.006)</td>
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<tr>
<td>Constant</td>
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<td>0.443***</td>
<td>0.437***</td>
<td>0.445***</td>
<td>0.442***</td>
<td>0.433**</td>
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<tr>
<td></td>
<td>(0.005)</td>
<td>(0.003)</td>
<td>(0.007)</td>
<td>(0.005)</td>
<td>(0.003)</td>
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<td>Observations</td>
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<td>670</td>
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<td>2,340</td>
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<td>0.185</td>
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<td>0.723</td>
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<td>Industry FE</td>
<td>N</td>
<td>N</td>
<td>N</td>
<td>Y</td>
<td>Y</td>
<td>Y</td>
</tr>
</tbody>
</table>

Robust standard errors in parentheses
*** p<0.01, ** p<0.05, * p<0.1
Regression of Estimated \( \alpha \)'s on Firm Characteristics
Table IX:
Estimated Bias in Default Costs

This table the inferred mean default costs from the sample of defaulted firms in the simulated data, using the
parameter distributions estimated in the SMM of Section 5. That is, I take the 4-dimensional joint cross-
sectional distribution for the firm-specific parameters and simulate a panel of firms with this parameter
distribution. The panel consists of 5,000 firms simulated at a quarterly frequency for 35 years for each
simulation. 1,000 simulations are performed. For each simulation, the $\alpha$’s for the firms which defaulted are
collected and a mean value is computed for each simulation. The Mean of the “Ex Post Mean $\hat{\alpha}$” is the
mean across simulations of these computed means. The true mean $\alpha$ indicates the unconditional mean value
of $\alpha$ from the cross-sectional distribution estimated in the SMM of Section 5.

<table>
<thead>
<tr>
<th>Ex Post Mean $\hat{\alpha}$</th>
<th>True Mean $\alpha$</th>
<th>Bias</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>0.246</td>
<td>0.445</td>
</tr>
<tr>
<td>SE</td>
<td>(.0015)</td>
<td>-</td>
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</table>
Figure 1: Optimal Policies as a Function of Default Costs, \( \alpha \). Panel A plots the optimally chosen initial book leverage for varying \( \alpha \). Panel B displays the equityholders’ optimal default threshold for the high (solid line) and low (dashed line) states. Panel C shows the optimal coupon on the firm’s perpetuity. Panel D shows the initial total firm value (sum of initial debt and initial levered equity), for optimally chosen leverage as a function of the default costs. In all cases, the initial state at time 0 is the high state. The time-0 cash flow, \( X_0 \), is normalized to 1. The other parameters can be found in Table I.
Figure 2: Initial Firm Value as a Function of Optimal Leverage. This figure shows the initial total firm value as a function of the optimal leverage chosen at time 0. The initial state is the high state. The optimal book leverage values displayed are for $\alpha \in [0, 1]$. 
Figure 3: Distribution of Mean Default Costs from the Sample of Defaulted Firms. A panel of 5,000 firms is simulated over 35 years and the simulation is repeated 5,000 times. For each simulation, a conditional distribution for \( \alpha \) is constructed from the sample of firms that defaulted in the simulation. The vertical dotted line represents the mean of the true, unconditional distribution of \( \alpha \) assumed in the simulation. The parameters used can be found in Table I.
Figure 4: Distribution of True and “Inferred” $\alpha$ and Leverage. A panel of 5,000 firms is simulated over 35 years and the simulation is repeated 5,000 times. The sample of defaulted firms is aggregated over all simulations and the default costs for this sample are plotted with the solid red line in Panel A. The horizontal blue line in Panel A indicates the true assumed distribution of default costs for the entire population of firms, which is $U \sim [0, 1]$. The econometrician observes the default costs and infers what the leverage distribution “ought” to be. This is plotted with the solid red line in Panel B, while the dotted blue line of Panel B shows the actual cross-sectional distribution of leverage selected by firms in the model.
Figure 5: Distribution of Median and Mean “Inferred” Optimal Leverage from the Sample of Defaulted Firms. A panel of 5,000 firms is simulated over 35 years and the simulation is repeated 5,000 times. For each simulation, a conditional distribution for $\alpha$ is constructed from the sample of firms that defaulted in the simulation. Given the other parameters, this distribution for $\alpha$ implies a distribution for optimal book leverage. For each simulation, I compute a mean and median of this “inferred” distribution for book leverage. Panel A displays the distribution over the 5,000 simulations for the median and Panel B does the same for the mean. The vertical dotted line represents the median (Panel A) and mean (Panel B) of the actual distribution of book leverage in the simulation. That is, this is the median and mean of the distribution of leverage according to the true, unconditional distribution for $\alpha$. The parameters used can be found in Table I.
Figure 6: Estimated Cross-Sectional Distributions for the Firm-Specific Parameters. This figure displays the cross-sectional distribution of the firm-specific parameter estimates from the SMM described in Section 5.
Figure 7: Distribution of Median and Mean Default Costs from the Sample of Defaulted Firms. A panel of 5,000 firms is simulated over 35 years and the simulation is repeated 1,000 times. For each simulation, a mean value of $\alpha$ is computed from the population of firms which defaulted during the simulation. The histogram indicates the distribution mean $\alpha$’s across the 1,000 simulations. The vertical red dashed lines indicates the estimated unconditional mean $\alpha$ from the SMM procedure of Section 5.
A. Distribution of Estimated $\alpha$

![Estimated Distribution for $\alpha_i$]

B. Distribution of $\alpha$ for Simulated Defaults

![Distribution of $\alpha_i$ for Simulated Defaults]

Figure 8: Estimated Distribution of Default Costs vs. Distribution for Simulated Defaults. This figure compares the estimated unconditional distribution of default costs, $\alpha$, with the conditional distribution from the sample of simulated defaults. The estimated distribution, which is displayed in Panel A, is obtained from the firm-specific SMM described in Section 5. I simulate 1,000 model economies under the estimated joint cross-sectional distribution of $\{\alpha_i, \mu_i, \beta_i, \sigma_i^F\}$ and collect the sample of firms which defaulted in each simulation. Panel B plots the distribution of default costs, aggregated over all simulations, of the firms which defaulted.
Appendices

A  Pricing Kernel, Risk-Neutral Measure

Given the exogenously specified process for the pricing kernel, the risk-neutral measure can be derived.\textsuperscript{32} The pricing kernel, $\pi_t$, evolves according to

$$\frac{d\pi_t}{\pi_t} = -r(\nu_t)dt - \varphi^m(\nu_t)dW^m_t$$

Define the density process for the risk-neutral measure by

$$\xi_t = E_t \left[ \frac{dQ}{dP} \right]$$

We know this density process and the pricing kernel are related by\textsuperscript{33}

$$\xi_t = B_t \pi_t$$

where

$$B_t = \exp \left\{ \int_0^t r(\nu_s)ds \right\}$$

is the time $t$ price of a bond paying the riskless rate and $B_0$ has been normalized to 1. Applying Itô’s Lemma gives

$$d\xi_t = B_t d\pi_t + \pi_t dB_t$$

Plugging in the expression for $d\pi_t$,

$$d\xi_t = B_t[-r(\nu_t)\pi_t dt - \varphi^m(\nu_t)\pi_t dW^m_t] + \pi_t dB_t$$

Replacing $\pi_t$ with $\frac{\xi_t}{B_t}$ and dividing through by $\xi_t$ gives

$$\frac{d\xi_t}{\xi_t} = -r(\nu_t) dt - \varphi^m(\nu_t)dW^m_t + \frac{1}{B_t}dB_t$$

\textsuperscript{32}Since the horizon is infinite, the risk-neutral measure, $Q$, that will be used for pricing contingent claims is not an equivalent probability measure to the physical measure, $P$. Still, the risk-neutral measure $Q$ will have the necessary properties for risk-neutral pricing. See Duffie (2001), Section 6N, for more details.

\textsuperscript{33}See Harrison and Kreps (1979).
Itô’s Lemma implies
\[ dB_t = r(\nu_t)dt \]
Thus the density process, \( \xi_t \), evolves according to
\[ \frac{d\xi_t}{\xi_t} = -\varphi^m(\nu_t)dW^m_t \]
Applying Girsanov’s Theorem, we have a new Brownian motion under the risk-neutral measure, given by
\[ d\hat{W}^m_t = dW^m_t + \varphi^m(\nu_t)dt \]
Note that the firm-specific Brownian motion, \( W^{f,n}_t \), that generates the idiosyncratic shocks to firm \( n \)’s cash flows is independent of the Brownian motion, \( W^m_t \) generating systematic shocks to the economy. Thus \( W^{f,n}_t \) is still a Brownian motion under the risk-neutral measure for all firms \( n \). Thus, under the risk-neutral measure, cash flows for firm \( n \) evolve according to
\[ \frac{dX^n_t}{X^n_t} = \hat{\mu}^n(\nu_t)dt + \sigma^m(\nu_t)d\hat{W}^m_t + \sigma^f dW^{f,n}_t \]
where \( \hat{\mu}^n(\nu_t) \) is the drift under the risk-neutral measure,
\[ \hat{\mu}^n(\nu_t) = \mu^n(\nu_t) - \sigma^m(\nu_t)\varphi^m(\nu_t). \]
The total volatility of the cash flows of firm \( n \) is given by
\[ \sigma^n_X(\nu_t) = \sqrt{(\sigma^n_m(\nu_t))^2 + (\sigma^n_f)^2} \]
Additionally, the two Brownian motions driving the idiosyncratic and systematic shocks to firm \( n \)’s cash flows under the risk-neutral measure can be aggregated into a single Brownian motion (under the risk-neutral measure) for firm \( n \) which is given by
\[ d\hat{W}^n_t = \frac{\sigma^m(\nu_t)}{\sigma^n_X(\nu_t)}d\hat{W}^m_t + \frac{\sigma^f}{\sigma^n_X(\nu_t)}dW^{f,n}_t. \]
So the evolution of firm $n$’s cash flows under the risk-neutral measure can be expressed as
\[
\frac{dX^n_t}{X^n_t} = \hat{\mu}_n(\nu_t)dt + \sigma^n_X(\nu_t)d\hat{W}^n_t
\]

### B Solving for Unlevered Firm Value

Here I show how to solve for the unlevered firm value. The pair of ODEs characterizing the unlevered firm value has an associated characteristic function given by:

\[
g_1(\beta)g_2(\beta) = \lambda_1\lambda_2
\]

where

\[
g_1(\beta) = \lambda_1 + r - (\mu_1 - \frac{1}{2}\sigma_1^2)\beta - \frac{1}{2}\sigma_1^2\beta^2
\]

\[
g_2(\beta) = \lambda_2 + r - (\mu_2 - \frac{1}{2}\sigma_2^2)\beta - \frac{1}{2}\sigma_2^2\beta^2
\]

This characteristic function has four distinct roots $\beta_1 < \beta_2 < 0 < \beta_3 < \beta_4$. The general form of the solution is given by

\[
A^1(X) = \phi_1(X) + \sum_{i=1}^{4} G_ix^{\beta_i}
\]

\[
A^2(X) = \phi_2(X) + \sum_{i=1}^{4} H_ix^{\beta_i}
\]

\[
H_i = l(\beta_i)G_i = \frac{g_1(\beta_i)}{\lambda_1}G_i = \frac{\lambda_2}{g_2(\beta_i)}G_i
\]

However boundedness conditions on the unlevered firm value need to be imposed. These are

\[
\lim_{x \to \infty} \frac{A^i(x)}{x} < \infty \quad \text{and} \quad \lim_{x \to 0} A^i(x) < \infty
\]

---

34The exposition follows Guo and Zhang (2004). See also Chen (2010) and Jobert and Rogers (2006).
These two conditions imply $\beta_i = 0, i = 1, \ldots, 4$. Thus the unlevered firm value has the form:

$$A^i(X) = \phi_i(X)$$

(26)

We conjecture that the unlevered firm value is affine in $X$. That is,

$$A^i(X) = c_iX + d_i$$

(27)

Furthermore, $d_i = 0, i = 1, 2$, since $A^i(0) = 0$

Thus the conjecture becomes

$$A^i(X) = c_iX$$

(28)

Plugging these expressions into the two ODEs characterizing the unlevered firm value and with some rearranging gives a linear system of two equations in two unknowns.

$$\mu_i c_i X - (\lambda_i + r)c_i X + X + \lambda_i c_j X = 0, \ j \neq i$$

(29)

Solving these two equations for $c_1, c_2$ gives the unlevered firm value in state $i$ as:

$$A^i(X) = \frac{(\lambda_1 + \lambda_2 + r - \mu_j)X}{\lambda_2(r - \mu_1) + (r - \mu_2)(\lambda_1 + r - \mu_1)}$$

(30)

Note that if $\mu_1 = \mu_2$ then the unlevered firm value is the same in both states and is given by

$$A(X) = \frac{X}{r - \mu}$$

(31)

C Eigenvalue Problem

This section describes the eigenvalue problem for the cash flow region in which neither default nor restructuring are immediate threats. Define the log cash flow process, $x_t = \log(X_t)$. By Itô’s Lemma, under the risk-neutral measure, the log cash flow process evolves according to

$$dx_t = \left[\tilde{\mu}(\nu_t) - \frac{1}{2}\sigma_X(\nu_t)^2\right]dt + \sigma_X(\nu_t)d\tilde{W}_t$$
Under the risk-neutral measure, the price process of any contingent claim on firm cash flows will be a martingale with the cash flows discounted by investors at the risk-free short rate, \( r(\nu_t) \). Thus, these contingent claims will be martingales of the form:

\[
M_t^f = \exp \left( - \int_0^t r(\nu_u) \, du \right) f(\nu_t, x_t)
\]

for some function \( f \) that depends on the payoffs of the given security.

Applying Itô’s Lemma gives

\[
dM_t^f = \exp \left( - \int_0^t r(\nu_u) \, du \right) \left[ (\Lambda - R)f + \frac{1}{2}\Sigma f_{xx} + \Theta f_x \right] dt
\]

\( R \) is the diagonal matrix of \( r_i \)'s. \( \Sigma \) is the diagonal matrix of \( \sigma_{iX}^2 \)'s. \( \Theta \) is the diagonal matrix of the risk-neutral drifts of the log cash flow process. \( \Lambda \) is the generator matrix of the Markov chain, \( \nu_t \).

Since \( M_t^f \) is a martingale, it has zero drift, implying

\[
(\Lambda - R)f + \frac{1}{2}\Sigma f_{xx} + \Theta f_x = 0
\]

We seek a separable \( f \) of the form

\[
f(\nu_t, x_t) = g(\nu_t)\exp(-\beta x_t) = g(\nu_t)X_t^\beta
\]

This gives the following equation to be solved in \( \beta \) and \( g \):

\[
(\Lambda - R)g + \frac{1}{2}\beta^2\Sigma g - \beta\Theta g = 0.
\]

Premultiplying the above equation by \( 2\Sigma^{-1} \) gives

\[
2\Sigma^{-1}(\Lambda - R)g + \beta^2 g - 2\beta\Sigma^{-1}\Theta g = 0.
\]
This gives the following system of equations:

\[ \beta g = h \]
\[ \beta h = 2\Sigma^{-1}\Theta h - 2\Sigma^{-1}(\Lambda - R)g \]

This can be written as a standard eigenvalue problem of the form

\[
A \begin{pmatrix} g \\ h \end{pmatrix} = \begin{pmatrix} 0 & I \\ -2\Sigma^{-1}(\Lambda - R) & 2\Sigma^{-1}\Theta \end{pmatrix} \begin{pmatrix} g \\ h \end{pmatrix} = \beta \begin{pmatrix} g \\ h \end{pmatrix}
\]

(32)

If \((g, \beta)\) solve this eigenvalue problem, then

\[
M_t^f = \exp \left( -\int_0^t r(\nu_u) \, du - \beta x_t \right) g(\nu_t)
\]

is a martingale. The matrix A has exactly 2 eigenvalues with positive real parts and 2 with negative real parts.

D Solving for the \(w\) coefficients

For the case in which there are two aggregate states to the Markov chain, there are a total of 3 relevant cash flow regions and each security has a total of 16 \(w\) coefficients (8 for each initial state).

The cash flow regions are:

Region 1: \(X \in [X_D^1, X_D^2)\)
Region 2: \(X \in [X_D^2, X_U^{u(1)})\)
Region 3: \(X \in [X_U^{u(1)}, X_U^{u(2)})\)

Note that for \(X < X_D^1\) the firm is always in default regardless of the state and for \(X > X_U^{u(2)}\) the firm has already restructured upwards for any state.
Debt

For a given initial state, \( \nu_0 \), the 8 boundary conditions for debt are

\[
\lim_{X \uparrow X_D^{\nu}} D(X, 1, \nu_0) = \lim_{X \downarrow X_D^{\nu}} D(X, 1, \nu_0) \quad (33)
\]

\[
\lim_{X \uparrow X_D^{\nu}} D_X(X, 1, \nu_0) = \lim_{X \downarrow X_D^{\nu}} D_X(X, 1, \nu_0) \quad (34)
\]

\[
\lim_{X \uparrow X^{u(1)}_D} D(X, u(2), \nu_0) = \lim_{X \downarrow X^{u(1)}_D} D(X, u(2), \nu_0) \quad (35)
\]

\[
\lim_{X \uparrow X^{u(2)}_D} D(X, u(2), \nu_0) = \lim_{X \downarrow X^{u(2)}_D} D(X, u(2), \nu_0) \quad (36)
\]

\[
D(X_D^1, 1, \nu_0) = (1 - \alpha(1))V^U(X_D^1, 1) \quad (37)
\]

\[
D(X_D^2, 2, \nu_0) = (1 - \alpha(2))V^U(X_D^2, 2) \quad (38)
\]

\[
D(X^{u(1)}_D, u(1), \nu_0) = D(X_0, \nu_0) \quad (39)
\]

\[
D(X^{u(2)}_D, u(2), \nu_0) = D(X_0, \nu_0) \quad (40)
\]

Equations (33) and (35) are the value-matching conditions across cash flow regions and equations (34) and (36) are the smooth-pasting conditions across regions. Equations (39) and (40) are the value-matching boundary conditions for default and equations (39) and (40) are the value-matching boundary conditions for upward restructuring.

The initial (par value) of debt at time 0 is given by

\[
D(X_0, \nu_0; \nu_0) = w_{2,1}^D(\nu_0)g_{2,1}(\nu_0)e^{\beta_{2,1}x_0} + w_{2,2}^D(\nu_0)g_{2,2}(\nu_0)e^{\beta_{2,2}x_0} +
\]

\[
w_{2,3}^D(\nu_0)g_{2,3}(\nu_0)e^{\beta_{2,3}x_0} + w_{2,4}^D(\nu_0)g_{2,4}(\nu_0)e^{\beta_{2,4}x_0} + (1 - \tau_i)C(\nu_0)b(\nu_0)
\]

\[
D(X_0, \nu_0; \nu_0) = \sum_{j=1}^{4} w_{2,j}^D(\nu_0)g_{2,j}(\nu_0)e^{\beta_{2,j}x_0} + (1 - \tau_i)C(\nu_0)b(\nu_0) \quad (41)
\]

Note that \( g_{2,j}(\nu_0) \) is a scalar: it’s the \( \nu_0 \) element of the \( g_{2,j} \) eigenvector, where \( g_{2,j} \) is the \( j \)th eigenvector for the eigenvalue problem for the 2nd cash flow region. Thus, we have a system of 8 equations to solve for the 8 unknown \( w^D \) coefficients.
\[ G(X)_{LHS}W^D + \xi(X)_{LHS} + \zeta_{LHS} = G(X)_{RHS}W^D + \xi(X)_{RHS} + \zeta_{RHS} \]
\[ [G(X)_{LHS} - G(X)_{RHS}]W^D = \xi(X)_{RHS} + \zeta_{RHS} - \xi(X)_{LHS} - \zeta_{LHS} \]

Thus,
\[ W^D = [G(X)_{LHS} - G(X)_{RHS}]^{-1}(\xi(X)_{RHS} + \zeta_{RHS} - \xi(X)_{LHS} - \zeta_{LHS}) \quad (42) \]

**Equity**

For a given initial state, \( \nu_0 \), the 8 boundary conditions for equity are

\[ \lim_{X \uparrow X^2_D} E(X, 1, \nu_0) = \lim_{X \downarrow X^2_D} E(X, 1, \nu_0) \quad (43) \]
\[ \lim_{X \uparrow X^2_D} E_X(X, 1, \nu_0) = \lim_{X \downarrow X^2_D} E_X(X, 1, \nu_0) \quad (44) \]
\[ \lim_{X \uparrow X_U^{(1)}} E(X, u(2), \nu_0) = \lim_{X \downarrow X_U^{(1)}} E(X, u(2), \nu_0) \quad (45) \]
\[ \lim_{X \uparrow X_U^{(1)}} E_X(X, u(2), \nu_0) = \lim_{X \downarrow X_U^{(1)}} E_X(X, u(2), \nu_0) \quad (46) \]
\[ E(X_D^1, 1, \nu_0) = 0 \quad (47) \]
\[ E(X_D^2, 2, \nu_0) = 0 \quad (48) \]

\[ E(X_U^{(1)}, u(1), \nu_0) = \frac{X_U^{(1)}}{X_0}[(1 - q)D(X_0, u(1); u(1)) + E(X_0, u(1); u(1))] - D(X_0, \nu_0) \quad (49) \]
\[ E(X_U^{(2)}, u(2), \nu_0) = \frac{X_U^{(2)}}{X_0}[(1 - q)D(X_0, u(2); u(2)) + E(X_0, u(2); u(2))] - D(X_0, \nu_0) \quad (50) \]

Note that these conditions hold for an arbitrary coupon rate, \( C(\nu_0) \). For a given initial state, \( \nu_0 \), the optimal default thresholds (for an arbitrary coupon) satisfy the smooth-pasting conditions for equity such that

\[ \frac{\partial}{\partial X} E(X, 1; \nu_0) \bigg|_{X \downarrow X^1_D(\nu_0)} = 0 \quad (51) \]
\[ \frac{\partial}{\partial X} E(X, 2; \nu_0) \bigg|_{X \downarrow X^2_D(\nu_0)} = 0 \quad (52) \]
E Data

Aggregate Earnings

For the aggregate earnings series, I use the quarterly “Net Operating Surplus” series from NIPA Section 1, Table 1.14, Line 8. The quarterly series is available for the period 1947Q1-2010Q2. I construct the log earnings growth series and present summary statistics for the unconditional moments below (all values are quarterly).

| Unconditional Moments: Quarterly Aggregate Earnings Growth |
|---------------------------------|----------------|
| Mean                            | 0.017          |
| Std Dev                         | 0.053          |
| AC(1)                           | 0.111          |

Firm Data

I construct the sample of firms to be estimated from the Compustat Fundamentals Quarterly file merged with equity data from CRSP. I require firms to have at least 20 quarters of data in the Compustat and CRSP files.

Variable definitions:

- Book Leverage: \( \frac{dlccq + dltq}{atq} \)

- Earnings Growth:
  \[
  \tilde{\epsilon}_{t+1} = \frac{\sum_{j=0}^{K} e_{t+1-j}}{\sum_{j=0}^{K} e_{t-j}} - 1
  \]
  where \( e_t \) is Compustat item ‘oiadpq’ in quarter \( t \).

- Quasi-Market Leverage: \( \frac{dlccq + dltq}{dlccq + dltq + ME} \) where ME is constructed from CRSP as Price* (Shares Outstanding).

F Estimating Parameters of the Aggregate Earnings Process

The procedure I use to estimate the parameters of the aggregate earnings growth follows the exposition in Chapter 22 of Hamilton (1994) on estimating Markov chain regime-switching
processes. See also Hamilton (1989). In the model, aggregate earnings is assumed to evolve according to a Markov-modulated geometric Brownian motion:

\[
\frac{dX_t^A}{X_t^A} = \mu^A(\nu_t)dt + \sigma^A(\nu_t)dW_t^A.
\] (53)

By Itô’s Lemma, the quarterly log earnings growth rate, \(x_{t+1}\), can be written as

\[
x_{t+1} \equiv \Delta \log(X_{t+1}) = \mu^A(\nu_t) - \frac{1}{2} \sigma^A(\nu_t) + \varepsilon_{t+1}^A
\] (54)

where \(\varepsilon_{t+1}^A \sim \mathcal{N}(0, (\sigma^A(\nu_t))^2)\).

This gives six parameters to be estimated: \(\mu_1^A, \mu_2^A, \sigma_1^A, \sigma_2^A, \lambda_{12}\), and \(\lambda_{21}\). Stacking these parameters into a vector, \(\Theta\), the vector of conditional densities for each state can be expressed as

\[
\eta_t = \left[ f(x_t|\nu_{t-1} = 1, x_{t-1}; \Theta) \quad \frac{1}{\sqrt{2\pi\sigma_1^2}} \exp\left\{ \frac{-(x_t - \mu_1^A + \frac{1}{2}(\sigma_1^A))^2}{2(\sigma_1^A)^2} \right\} \right]
\]

Define the vector of optimal inferences for the current state at date \(t\), given the vector of observations up to and including date \(t\), \(X_t\), and the vector of population parameters, \(\Theta\) as

\[
\hat{\xi}_{t|t} = \begin{bmatrix} \mathbb{P}\{\nu_t = 1|X_t; \Theta\} \\ \mathbb{P}\{\nu_t = 2|X_t; \Theta\} \end{bmatrix}
\] (56)

Similarly, define the vector of optimal one period ahead forecasts for state \(\nu_{t+1}\) as

\[
\hat{\xi}_{t+1|t} = \begin{bmatrix} \mathbb{P}\{\nu_{t+1} = 1|X_t; \Theta\} \\ \mathbb{P}\{\nu_{t+1} = 2|X_t; \Theta\} \end{bmatrix}
\] (57)

The optimal inference and forecast can be defined recursively as

\[
\hat{\xi}_{t|t} = \frac{\hat{\xi}_{t|t-1} \odot \eta_t}{\mathbf{1}'(\hat{\xi}_{t|t-1} \odot \eta_t)}
\] (58)

\[
\hat{\xi}_{t+1|t} = P \hat{\xi}_{t|t}
\] (59)

where \(\odot\) denotes element by element multiplication and \(P\) is the discrete time transition matrix given by

\[
P = \begin{bmatrix} p_{11} & 1 - p_{11} \\ 1 - p_{22} & p_{22} \end{bmatrix}
\]
Starting with an initial guess for $\xi_{1|0}$ equal to the vector of unconditional probabilities and a vector of parameters, $\Theta$, the log likelihood function can be constructed by iterating on equations (58) and (59).

$$L(\Theta) = \sum_{t=1}^{T} \log f(x_t | x_{t-1}; \Theta) = \sum_{t=1}^{T} \log(1' (\hat{\xi}_{t|t-1} \odot \eta_t))$$

(60)

To estimate the parameter vector $\Theta$, I maximize the log likelihood function numerically. Finally, given the estimated discrete time transition matrix, $P$, the generator matrix, $\Lambda$, for the continuous time Markov chain can be computed as

$$\Lambda = \begin{bmatrix} \lambda_{11} & \lambda_{12} \\ \lambda_{21} & \lambda_{22} \end{bmatrix} = \begin{bmatrix} \log(p_{11}) & \frac{(1-p_{11}) \log(p_{11})}{p_{11}-1} \\ \frac{(1-p_{22}) \log(p_{22})}{p_{22}-1} & \log(p_{22}) \end{bmatrix}$$

(61)

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Note that this assumes that the probability of switching states more than once in a quarter is zero. See Jarrow, Lando, and Turnbull (1997) for more details.
References


