The same bond at different prices: identifying search frictions and selling pressures

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Abstract

I model how corporate bond prices are affected by search frictions and occasional selling pressures and test my predictions empirically. A key prediction is that in a distressed market with more sellers than buyers, prices paid by large traders decrease more than those of small traders. Using a structural estimation, the model is able to identify liquidity crises (i.e. high number of forced sellers) based on the relative prices of small and large traders. New light is shed on two crises, the downgrade of GM and Ford in 2005 and the subprime crisis. The model can also explain why the spread between corporate bond yields and Treasury yields is so large, the so-called credit spread puzzle.
1 Introduction

The U.S. corporate bond market is a principal source of financing for firms. It is bigger than the U.S. Treasury market measured in amount outstanding, but trading volume is more than thirty times lower.\(^1\) An investor sequentially contacts one or several dealers over the telephone to trade a corporate bond. Dealers typically do not ”make a market” and a price quote is firm for only a short period of time, limiting the ability to obtain multiple quotations before committing to a trade.\(^2\) Hence, prices are outcomes of a bargaining game, determined in part by the ease with which investors find counterparties and the relative number of investors currently looking to buy or sell.

I model the search for counterparties and price bargaining in the corporate bond market. A key prediction is that when there is a sudden need for many investors to sell, prices facing large traders fall more than those facing small traders. Thus, price differences in small and large trades in the same bond on a given day identify possible selling pressure. I propose an approach to estimate parameters of the model using maximum likelihood and carry out the estimation using transactions data for the period October 2004 to June 2009. There is a growing literature on search models, but to my knowledge no one has structurally estimated a model before. The estimation approach also allows me to empirically identify periods of liquidity shortage as captured by large numbers of selling investors. Liquidity shortage is identified by the distribution of small vs large trade prices.

There are two periods with strong selling pressure according to the empirical results. In May 2005 S&P downgraded GM and Ford to speculative grade causing strong selling pressure in their bonds. In the preceding months selling pressure intensified as a downgrade became more likely, consistent with findings in Acharya, Schaefer, and Zhang (2008). My results show that the selling pressure was largely isolated to GM and Ford bonds, and that the time pattern of selling pressure in GM bonds was different from that in Ford bonds. Selling pressure in GM bonds peaked in May, likely because GM was downgraded by both S&P and Fitch and dropped out of the important Lehman

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\(^{1}\)Principal outstanding volume by the end of 2008 was $6,205bn in the U.S. corporate bond market and $6,082bn in the U.S. Treasury market while average daily trading volume in 2008 was $14.3bn in the U.S. corporate bond market and $553.1bn in the U.S. Treasury market. Source: Securities and Financial Markets Association (www.sifma.org).

\(^{2}\)See Bessembinder and Maxwell (2008) for further details about the U.S. corporate bond market.
investment grade index. In contrast, selling pressure in Ford bonds decreased in May as Ford was only downgraded by S&P and remained in the Lehman index. The second period with selling pressure takes off at the beginning of the subprime crisis in summer 2007. During the crisis there are three peaks in the estimated forced number of sellers; when Bear Sterns is taken over, when Lehman Brothers defaults, and in March 2009 when stock markets lost 30% in two months.

The credit spread puzzle refers to the finding that spreads between corporate bond yields and Treasury yields are too high to be explained by the credit risk of corporate bond issuers. For example, Huang and Huang (2003) find a historical average 4-year A-Treasury spread of 96 basis points out of which credit risk can explain around 10 basis points. I provide a search-based explanation for the puzzle. The premium due to search costs is calculated as the midyield for an average corporate bond investor minus the midyield for an investor who can instantly find a trading partner. The premium represents average yields in the corporate bond market versus average yields in the ultra-liquid Treasury market. Empirically, the premium is large for short bond maturities and modestly sized for long maturities. The premium due to selling pressure is calculated as the midyield for an average investor under an average liquidity shock minus the midyield in absence of a shock. This premium has a significant effect for all but very short maturities. The search and selling pressure premia combined have a variety of possible shapes as function of maturity and the size is similar to the reported unexplained component in credit spreads. For example, the average total liquidity premium for a 4-year bond is 88 basis points matching the unexplained 96-10=86 basis points for an A-rated bond in Huang and Huang (2003).

My model is a variant of the search model in Duffie, Gărleanu, and Pedersen (2005) (DGP05). Some investors own a corporate bond. Trading occurs because investors switch randomly between needing liquidity or not; liquidity-constrained bond-owners are sellers while non-constrained non-owners are buyers. Investors trade through a dealer whom they find at random with different search intensities. Sophisticated investors find a dealer fast while unsophisticated investors are slow to find one. Dealers trade continuously with each other through the interdealer market and carry no inventory. Once an investor meets a dealer they bargain, and the resulting price reflects their alternatives to immediate trade. There are two differences between my model and that
of DGP05. In their model investors can trade with each other directly or through a dealer, while investors in my model trade only through dealers. Also, the asset is infinitely lived in DGP05 while the asset here has finite maturity. The modifications reflect that corporate bond investors trade through financial intermediaries and that bonds have finite maturity.

In the model I study price reactions to a liquidity shock, defined as an increased number of forced sellers. In particular, I study reactions in prices paid by small traders versus those paid by large traders on the same bond. One can think of small traders as retail investors and large traders as institutional investors, since trades smaller than $100,000 are predominantly retail trades and larger trades are mainly institutional trades (Goldstein, Hotchkiss, and Sirri (2007) and Bessembinder, Kahle, Maxwell, and Xu (2009)). If there is a shock in funding needs of investors such that there are more sellers than buyers, the model predicts that prices paid by large traders decrease more than those paid by small traders. Buyers can choose to look for a new trading partner instead of trading with their current partner, and the negotiated price reflects this outside option of the buyer. Assuming that large traders are more sophisticated than small traders, large buyers find new trading partners more easily. This strong outside option enables large buyers to negotiate larger price discounts during a liquidity shock than small buyers.

Figure 1 illustrates how the relation in prices between small and large trades identify selling pressure. The left-hand graph shows that prices for large trades are higher than those for small trades in a normal market. In the middle graph large transaction prices are lower than small transaction prices because there are more sellers than buyers in the market. The right-hand graph shows how prices of large trades are markedly lower than prices of small trades under a severe liquidity crisis.

Figure 2 shows transactions for a GM bond in a period before and after the downgrade in spring 2005. In the period preceding the downgrade prices in large trades tend to be lower than prices in small trades indicating selling pressure. In a period following the downgrade prices in large trades tend to be higher than prices in small trades indicating a normal market. Buys and sells by insurance companies from the NAIC database are

\footnote{For an in-depth discussion of funding liquidity, and a model that links market liquidity and trader’s funding liquidity, see Brunnermeier and Pedersen (2009).}
Figure 1: Small and large trades on a normal day, day of distress, and day of crisis.
This graph shows three examples of all trades for a bond during a day smaller than $100,000 (marked with crosses) and trades of at least $1,000,000 (marked with circles).

also plotted. Insurance companies are mainly sellers in both periods and selling prices relative to all trades are predominantly low in the pre-downgrade period and high in the post-downgrade period.\(^4\) Before the initiation of TRACE many papers relied on the NAIC database, and the Figure illustrates how rich the TRACE data set is compared to the NAIC database.\(^5\)

A final example of the identification of selling pressure is given in Figure 3 where all transaction prices in a Citigroup bond on March 11-12 2009 are graphed with time stamps. For each transaction the transaction type (customer buying from dealer/customer selling to dealer/interdealer) and size (small/medium/large) is indicated. Transaction type information just recently became available in TRACE. At a given point in time the bond trades at multiple prices reflecting that bond trading is over-the-counter with bilateral negotiation. Large traders buy or sell the bond for around $70 while small trades buy or sell the bond at prices averaging $75. That large trade price are significantly

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\(^4\) The net amount insurance companies sell in the period October 1, 2004-March 1, 2005 is $39,7 million while it is $9 million in the period December 1, 2005-May 1, 2006.

Figure 2: Transactions in a GM bond. This graph shows in two periods small and large transactions in a GM bond with coupon 7.40% and maturity September 1, 2025. In the first period there was selling pressure in the bond while the second period was a normal period. The light-colored squares mark transactions with a nominal dollar value of less than 100,000 while the red-colored crosses mark transactions with a dollar value of one million or more. The upward pointing dark triangles mark a sell by an insurance company to a dealer while downward pointing triangles mark buys. Insurance company transactions are from the NAIC database.

lower than small trade prices show strong selling pressure.

I estimate parameters of the model and periods of selling pressure by maximum likelihood using more than two million corporate bond transactions from the TRACE database in the period October 2004 to June 2009. It is difficult to empirically disentangle liquidity risk from credit risk since they are typically correlated and have a similar price effect; prices decrease both when there is increased liquidity risk and when credit risk increases. To make sure that my results on selling pressure are not driven by changes in credit risk, I use a novel approach and "filter out" credit risk by looking at demeaned prices. I can still identify selling pressure by looking at demeaned prices of large trades.
Figure 3: Transactions under strong selling pressure. Customer buy from a dealer, customer sell to a dealer, and interdealer trades on March 11 and 12 2009 for the Citigroup bond with coupon 7.25% and maturity October 1, 2010. Average transaction price during the two days is 74.8 for trades with a notional below $100K, 73.8 for trades with a notional in the range $100-$999K, and 70.2 for trades of $1,000K or more.

versus small trades. Therefore, I demean bond prices and estimate parameters by fitting demeaned model prices to actual demeaned prices. In a given bond on a given day, I define a demeaned bid (ask) price as a transaction bid (ask) price minus the average price across all bid and ask prices on that day. Until October 2008 TRACE does not have buy/sell indicators on transactions, so bid and ask prices are filtered out using a measure I call ‘unique roundtrip trades’ (URT). URTs are based on the observation that a corporate bond often does not trade for a while and then two or three trades occur within a short period of time. This is likely because a dealer matches a buyer and a seller and collects the bid-ask spread as a fee. When a dealer has found a match, a trade between seller and dealer along with a trade between buyer and dealer are carried out. Possibly, the matching occurs through a second dealer in which case there is also a transaction between the two dealers. Thus, I assume that if two or three trades in a
given bond with the same volume occurs within 15 minutes and there are no other trades with the same volume on that day, the transactions are part of a URT. The maximal price in a URT is the ask price and the minimal price the bid price.

The outline of the paper is as follows. A literature review is given in Section 2. Section 3 contains the model and its theoretical predictions. Section 4 describes the transaction data and the estimation methodology. Section 5 reports the estimated model and my main empirical findings. Section 6 checks the robustness of the main findings and Section 7 concludes.

2 Literature review

The early treatment of search frictions in economic models dates back at least to Stigler (1961) but have until recently mainly been used in labor economics to study among other things the coexistence of unemployment and jobless individuals and the randomness of unemployment (see for example Diamond (1982)). One notable exception in the finance literature is Garbade and Silber (1976). They propose a simple investor search model to explain why price dispersion exists at a given point in time and find empirical support in the US Treasury market.

The work of Duffie, Gârleanu, and Pedersen (2005) (DGP05) ignited new interest in models of search in finance. In their model, there are two types of risk-neutral agents, investors and dealers. A selling investor searches for a buying investor or a dealer and once he finds a counterparty, the price is set through a bargaining process that reflects both buyer’s and seller’s alternatives to immediate trade. In an extension of their model, they study bid-ask spreads when there are two types of investors, sophisticated and unsophisticated, with different speeds at which they find counterparties. An important implication of their model is that investors who find counterparties more easily trade at lower bid-ask spreads. Their model has been extended in various directions. Biais and Weill (2009) study order book dynamics by letting investors place an order in a limit order book instead of trading directly with each other, but as in the DGP05 model investors make contact with the market at random times. Vayanos and Weill (2008), Vayanos and Wang (2007) and Weill (2008) extend the number of assets in the economy and derive cross-sectional implications for expected returns. Gârleanu (2009) and
Lagos and Rocheteau (2009) ease the restriction in DGP05 that asset holdings are 0 or 1 and find that trading volume increases when counterparties find each other more easily. Lagos, Rocheteau, and Weill (2009) furthermore allows dealers to hold inventory and study dealers’ liquidity provision under a liquidity shock. In slightly different models, Miao (2006) and Yin (2005) extend the number of markets and study prices and transaction costs when investors can trade in both a centralized and decentralized market. Duffie, Gârleanu, and Pedersen (2007) (DGP07) study two variants of their original model. They strip out dealers and let agents be risk-averse. They find that a model with risk-neutral agents is a first-order approximation of a model with risk-averse agents. Also, they examine the impact of a liquidity shock on prices in different markets; that is the price reaction when a large number of investors suddenly need to sell. They find the price impact of a liquidity shock to be larger in markets where investors are slower to find each other.

My model is similar to those in DGP05 and DGP07 with deviations that reflect the structure of the corporate bond market (and most over-the-counter markets). Corporate bonds have a fixed maturity date, so the traded asset in my model has finite maturity. This has implications for the bid-ask spread at which investors trade the bond and the bond price sensitivity to a liquidity shock. Also, investors trade through financial intermediaries in the corporate bond market, so in my model investors trade through dealers and not directly with each other (see Bessembinder and Maxwell (2008) for details on how investors trade in the corporate bond market). Finally, I examine the impact of a liquidity shock on prices paid by different investors in the same market. In contrast, DGP07 and Lagos, Rocheteau, and Weill (2009) study price impact in different markets, where each market has one investor type.

While there is a large recent theoretical literature on the effect of search frictions on asset prices, empirical evidence in fixed income markets is rarer. Ashcraft and Duffie (2007) study intra-day variations of overnight loans of federal funds traded in the inter-bank market. They find the rate that a bank negotiates on a loan to be less attractive than average rates if the bank has more to gain from trade than its counterparty, and if the bank is less active in the market. Newman and Rierson (2004) document a temporary decrease in prices in response to supply shocks in the European telecommunication-sector bond market. Mitchell, Pedersen, and Pulvino (2007) find a price decrease over
several months in the convertible bond market in response to selling pressure linked to redemption of capital in hedge funds. Key implications in most search-based models are that prices at a given point in time are dispersed and that bid-ask spreads decrease in trade size and overwhelming support is found for the municipal bond market in Green, Hollifield, and Schürhoff (2007b), Green, Hollifield, and Schürhoff (2007a), Harris and Piwowar (2006), the corporate bond market in Edwards, Harris, and Piwowar (2007), Schultz (2001), and Jotikasthira (2007), and for the Treasury market in Garbade and Silber (1976) and Dunne, Hau, and Moore (2009).

From an empirical perspective, the contribution of my paper is to present and carry out a structural estimation of a model of search frictions. That is, I estimate by ML the parameters of my model, pricing implications of the model are examined and the size of the parameters of the model are interpreted.

The standard structural model of default by Merton (1974) assumes that firm value evolves according to a diffusion-type stochastic process and if the value hits a default boundary, the firm defaults. Only a fraction of the nominal value of a bond issued by the firm is recovered in case of default. The price of the bond is therefore lower than a bond free of default, and the promised yield is higher than the yield on a riskless bond. However, the spread between the promised yield on corporate bonds and riskless Treasury bonds - particular at short maturities and for high rated firms - is too large to be explain by the default risk of firms, the so-called credit spread puzzle. Jones, Mason, and Rosenfeld (1984) and more recently Huang and Huang (2003) (HH03) document the puzzle. For example, HH03 find the average 4-year AAA-Treasury spread to be 55 basis points while the calculated spread in the Merton model is 1.1 basis points. HH03 also show that proposed extensions of the Merton model such as jumps in firm value, stochastic interest rates, endogenously determined default boundary, and mean-reverting leverage ratios do not come close to solving the puzzle once they are calibrated with empirically reasonable parameters. More recent papers such as Hackbarth, Miao, and Morellec (2006), Chen, Collin-Dufresne, and Goldstein (2009), Chen (2009), and Bhamra, Kuehn, and Strebulaev (2009) link firm decisions, earnings, risk premia, and/or default rates to the business cycle. They show that realistic credit spreads are generated at longer maturities and for firms with a rating of typically BBB. However, none of the models are able to generate realistic spreads for highly-rated short-maturity debt.
There is strong evidence that part of the spread between corporate bonds and Treasury bonds is due to factors not related to credit risk. Comparing rates on arguably liquid credit default swaps and less liquid corporate bonds, Longstaff, Mithal, and Neis (2005) find a significant non-default component. For example, the AAA 5-year non-default component is estimated to be 53 basis points. Similar evidence of a large non-default component at both short and long maturities is found in Longstaff (2004) and Feldhütter and Lando (2008). Despite the evidence for a non-default component in credit spreads few papers have tried to explain the credit spread puzzle as arising from the difference in liquidity between corporate bonds and Treasury bonds. A notable exception is Ericsson and Renault (2006). They develop a structural bond valuation model capturing both liquidity and credit risk. Bond holders are randomly hit by a liquidity shock - a sudden need to sell - and when they need to sell, their selling price is lower than the perfectly liquid price. Their liquidity premium is related to the premium due to search costs in this paper with a similar size and shape as a function of bond maturity. In contrast to their paper, I model the liquidity premium as arising from two components, one due to search costs and one due to occasional selling pressure. The selling pressure premium is important in order to generate empirically plausible liquidity premia not only at short maturities but also at long maturities.

3 A search model

This section sets up a search model similar to the model in Duffie, Gârleanu, and Pedersen (2005) with two differences. First, the traded asset has a finite maturity. Since bonds mature and become liquid at maturity, this reflects an important feature of corporate bonds. Second, investors trade through financial intermediaries, not directly with each other, as is the case in the corporate bond market. I first derive equilibrium bid and ask prices facing investors with different levels of sophistication in finding trading partners. Next, I define a liquidity shock as a shock increasing the number of sellers relative to the number of buyers and find how prices are affected by such a shock. Finally, it is shown that a liquidity shock leads to a larger decrease in prices faced by sophisticated investors relative to those faced by unsophisticated investors.
3.1 Model

The economy is populated by two kinds of agents, investors and dealers, who are risk-neutral and infinitely lived.\(^6\) They consume a nonstorable consumption good used as numeraire and their time preferences are given by the discount rate \(r > 0\).

Investors have access to a risk-free bank account paying interest rate \(r\). The bank account can be viewed as a liquid security that can be traded instantly. To rule out Ponzi schemes, the value \(W_t\) of an investor’s bank account is bounded from below. In addition, investors have access to an over-the-counter corporate bond market for a credit-risky bond paying coupons at the constant rate of \(C\) units of consumption per year. The bond has maturity \(T\) and face value \(F\), meaning that it matures randomly with constant intensity \(\lambda_T = 1/T\) and pays \(F\) at maturity. The bond defaults with intensity \(\lambda_D\) and pays a fraction \((1−f)F\) of face value in default. A bond trade occurs when an investor finds a dealer in a search process that will be described in a moment.

Investors hold at most 1 unit of the bond and cannot short-sell. Because agents are risk-neutral, investors hold either 0 or 1 unit of the bond in equilibrium. An investor is of type "high" or "low". The "high" type has no holding cost when owning the asset while the "low" type has a holding costs of \(\delta\) per time unit. The holding cost can be interpreted as a funding liquidity shock that hits the investor. An investor switches from "low" to high” with intensity \(\lambda_u\) and from "high" to "low” with intensity \(\lambda_d\) and the switching processes are for all investors pair-wise independent. Investors differ in the ease with which they find counterparts to trade with. A sophisticated investor quickly finds a trading partner while an unsophisticated investor spends considerable time finding someone to trade with.

I assume that there is a unit mass of independent non-atomic dealers who maximize profits. An investor with level of sophistication \(i, i \in \{1, 2, ..., I\}\) meets a dealer with intensity \(\rho^i\), which can be interpreted as the sum of the intensity of dealers’ search for investors and investors’ search for dealers. The search intensity is observable to every one. Without loss a generality assume that \(\rho^i < \rho^j\) when \(i < j\), implying that investors with intensity \(\rho^1\) are the most unsophisticated and those with intensity \(\rho^I\) are the most

\(^6\)As shown by Duffie, Gârleanu, and Pedersen (2007) the model with risk-neutral agents can be viewed as a first-order approximation to a model with risk-averse agents where the asset yields risky dividends and they find the approximation to work well.
sophisticated. When an investor and a dealer meet, they bargain over terms of trade. The bargaining will be described in the next section. Dealers immediately unload their positions in an interdealer market, so they have no inventory.

The set of investors is \( \Gamma = \{ h_0^i, h_n^i, l_o^i, l_n^i \}_{i=1}^L \) where \( h/l \) refers to the ”high”/”low” type and \( o/n \) to owner/non-owner. There is a continuum of investors and for investors with sophistication level \( i \), \( \mu^i_{\sigma}(t) \) denotes the fraction at time \( t \) of type \( \sigma \). Since the fractions of ”\( i \)”-investors add to 1 at any point in time we have

\[
\mu^i_{h_0}(t) + \mu^i_{h_n}(t) + \mu^i_{l_o}(t) + \mu^i_{l_n}(t) = 1
\]

for every \( t \) and any \( i = 1, ..., I \). There is also a continuum of credit-risky firms who issue bonds. If a firm defaults it is replaced by an identical new firm. They issue bonds through dealers when there is an excess demand in the market and the amount they issue is limited by the speed with which dealers meet investors.

The model is solved in two steps. First, asset allocations are determined. This is possible without reference to prices because only low-type owners are sellers and high-type non-owners are buyers, and when a low-type owner or high-type non-owner meets a dealer bargaining theory tells us that trade occurs immediately [Rubinstein (1982)]. This is done in Appendix A where explicit expressions for the allocations are derived. Second, prices are derived. A low-owner investor ”\( i \)” is willing to sell to a dealer if the price is at least his reservation price \( \Delta V^i_l \) while a high-nonowner ”\( i \)” is willing to buy from a dealer if the price is no more than his reservation price \( \Delta V^i_h \). Likewise, the dealer is willing to buy if the price is no more than the interdealer price \( M \) (at which he immediately unloads the bond in the interdealer market), and willing to sell if the price is no less than \( M \). Nash bargaining between investor and dealer leads to bid and ask prices

\[
A^i = \Delta V^i_h z + M (1-z) \quad (1)
\]
\[
B^i = \Delta V^i_l z + M (1-z) \quad (2)
\]

where \( z \) is the bargaining power of the dealer. Duffie, Gårleanu, and Pedersen (2007) show that a bargaining outcome of this kind can be justified by an explicit bargaining procedure. Any interdealer price between \( \Delta V^i_l \) and \( \Delta V^i_h \) can be an outcome in the model.
If we define $\Delta V_l$ to be the lowest price at which all investors, $i = 1, ..., I$, are willing to sell at and $\Delta V_h$ the highest at which they are willing to buy at, we let $M = \tilde{q}\Delta V_h + (1 - \tilde{q})\Delta V_l$ where $0 \leq \tilde{q} \leq 1$. If $\tilde{q}$ is close to one, the ask price is close to the reservation value of buyers and the market can be interpreted as a sellers’ market, while a $\tilde{q}$ close to zero can be interpreted as a buyers’ market. An example in the Appendix illustrates the impact of $z$ and $\tilde{q}$ on prices.

The following theorem states equilibrium bid and ask prices in the economy and a proof is given in the Appendix. The top graph in Figure 4 illustrates how trading occurs in equilibrium; there are more buyers than sellers finding dealers and firms issue new bonds.

**Theorem 3.1. (Prices in equilibrium).** In equilibrium, the bid $B^i$ and ask $A^i$ prices for investor $i$ with search intensity $\rho^i$ are given as

$$A^i = \Delta V^i_h z + M(1 - z)$$
$$B^i = \Delta V^i_l z + M(1 - z)$$

where

$$\Delta V^i_l = \frac{\rho^i(1 - z)M + \lambda_T F + \lambda_D(1 - f)F + C}{r + (1 - z)\rho^i + \lambda_T + \lambda_D} - \frac{\delta(r + \lambda_T + \lambda_D + \lambda_u + \rho^i(1 - z) + r + \lambda_T + \lambda_D)}{(r + \lambda_T + \lambda_D + \rho^i(1 - z))(\lambda_u + \lambda_T + \lambda_D + \rho^i(1 - z) + r + \lambda_T + \lambda_D)}$$
$$\Delta V^i_h = \frac{C + \lambda_T F + \lambda_D(1 - f)F}{r + \lambda_T + \lambda_D} - \frac{\delta(\lambda_u + \rho^i(1 - z) + r + \lambda_T + \lambda_D)}{(r + \lambda_T + \lambda_D)(r + (1 - z)\rho_0 + \lambda_T + \lambda_D)}$$
$$M = \frac{C + \lambda_T F + \lambda_D(1 - f)F}{r + \lambda_T + \lambda_D} - \frac{\delta(\lambda_u + \rho^i(1 - z) + r + \lambda_T + \lambda_D)}{(r + \lambda_T + \lambda_D)(r + (1 - z)\rho_0 + \lambda_T + \lambda_D)}$$

and $\rho_0 \leq \min(\rho_1, ..., \rho_I)$ and $0 \leq \tilde{q} \leq 1$.

The parameter $\rho_0$ reflects the minimum level of sophistication required to participate in the corporate bond market. As an obvious consequence of the theorem we have the following corollary.

**Corollary 3.1. (Bid-ask spreads).** The bid-ask spread for investor $i$ with search intensity $\rho_i$ is given as

$$A^i - B^i = \frac{z\delta}{\lambda_u + \lambda_T + \lambda_D + \rho^i(1 - z) + r + \lambda_T + \lambda_D}$$ (3)

We see that sophisticated investors trade at tighter bid-ask spreads than unsophisticated investors because bid-ask spreads decrease in $\rho$. The price buyers and sellers

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7In the empirical section $\rho_0$ is set to 10.
negotiate with dealers reflect their alternatives to immediate trade. An alternative is to cut off negotiations and find a new dealer. Since sophisticated investors find a new dealer more easily, their alternative to trade is stronger and they negotiate better bid and ask prices (see also Duffie, Gârleanu, and Pedersen (2005)).

We also see that bid-ask spreads decrease in the maturity of the bond $1/\lambda_T$. A seller can let a bond mature instead of selling, giving him a strong alternative to trade in case of a short-maturity bond. A buyer is aware that a short-maturity bond will mature soon and that he will only receive coupons for a short period. Thus, neither buyer nor seller are willing to give large price concessions to the dealer, leading to tight bid-ask spreads. Also, the bid-ask spread is decreasing in the default probability of the issuer. The reason for this is the same as that of maturity; as default intensity increases, the firm is more likely to default and a fraction of the notional of the bond is repayed. Thus, the bond becomes “liquid” at default and is similar to a bond with short maturity. This effect is likely to be small except for very credit-risky bonds.

Next, a liquidity shock to investors is defined. I assume that the fractions of investors are in steady state and a sudden liquidity shock occurs. If a shock of size $0 \leq s \leq 1$ occurs a ”high”-investor (no liquidity need) becomes a ”low”-investor (liquidity need) with probability $s$:

**Definition 3.1. (Liquidity shock).** Assume that the fractions of types of investors with search intensity $\rho$ are in steady state, denoted $\mu_{ho}^{ss}$, $\mu_{hn}^{ss}$, $\mu_{lo}^{ss}$, and $\mu_{ln}^{ss}$. When a liquidity shock of size $0 < s \leq 1$ occurs, any high-investor becomes a low-investor with probability $s$. The fractions of types immediately after the shock are $\mu_{ho}(s) = (1-s)\mu_{ho}^{ss}$, $\mu_{hn}(s) = (1-s)\mu_{hn}^{ss}$, $\mu_{lo}(s) = \mu_{lo}^{ss} + s\mu_{ho}$, and $\mu_{ln}(s) = \mu_{ln}^{ss} + s\mu_{hn}$.

Prices following a liquidity shock are given in the following theorem and a proof is in the Appendix. Goldstein, Hotchkiss, and Sirri (2007) find that dealers do not split trades and perform a matching/brokerage function in illiquid bonds. And according to market participants, risk limits often prohibit dealers from taking bonds on the book and splitting trades when there is a liquidity shock. To capture this, I assume that

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8 According to conversations with market participants, a corporate bond trade is often carried out as follows, especially during a crisis. If an investor wants to sell a significant amount of a corporate bond, he contacts a sales person from a given bank. The sales person asks the marketmaker in the bank if he wants to buy it. Often and in particular during a crisis, the marketmaker cannot take the bond on the book due to the risk. The sales person therefore searches directly for a buyer, and once there is a match, the transaction is carried out. Typically, the bid-ask fee is collected by the sales person, not the marketmaker. Consistent with this, Bessembinder and Maxwell (2008) notes that "(i)n interviews,
investors are segmented, meaning that after a liquidity shock an investor “i” only trade with other investors of the same type (through the dealer). Trading after a liquidity shock is illustrated in the bottom graph in Figure 4.

**Theorem 3.2. (Prices after a liquidity shock).** Assume that a liquidity shock of size \(0 < s \leq 1\) occurs to all investors and that \(\rho^i + \lambda_T + \lambda_D > \lambda_d + \lambda_u\). If \(s \leq \frac{\lambda_T + \lambda_D}{\rho^i + \lambda_T + \lambda_D}\) bid and ask prices do not change after the liquidity shock. If \(s > \frac{\lambda_T + \lambda_D}{\rho^i + \lambda_T + \lambda_D}\) bid and ask prices immediately after the shock are

\[
B^i(s) = e^{-t^i_s(\lambda_d + \lambda_T)} \Delta V^i_l + (1 - e^{-t^i_s(\lambda_d + \lambda_T)}) \Delta V^i_{imb}
\]

\[
A^i(s) = B + \frac{z \delta}{\lambda_u + \lambda_d + \rho^i(1 - z) + r + \lambda_T + \lambda_D}
\]

where \(\Delta V^i_l\) is given in Theorem 3.1,

\[
\Delta V^i_{imb} = \frac{C + \lambda_T F + \lambda_D (1 - f) F}{r + \lambda_T + \lambda_D} - \frac{\delta (r + \lambda_T + \lambda_D + \lambda_d + (1 - z) \rho^i)}{(r + \lambda_T + \lambda_D) (\lambda_u + \lambda_d + \rho^i (1 - z) + r + \lambda_T + \lambda_D)}
\]

and \(t^i_s\) is the unique solution to

\[
0 = 1 - s e^{-(\lambda_d + \lambda_T) t^i_s} - \frac{\rho^i}{\rho^i + \lambda_T + \lambda_D} e^{-(\lambda_d + \lambda_T) t^i_s}.
\]

Prices decrease because sellers arriving at dealers outnumber buyers arriving at dealers for a while. During this period, sellers sell at their reservation price \(\Delta V_l\), the price at which they are indifferent between selling or not. If any seller sold at \(\Delta V_l + \epsilon\), investors not being able to sell would be willing to underbid at \(\Delta V_l + \frac{1}{2} \epsilon\). Therefore the selling price is \(\Delta V_l\) to balance the number of sellers and buyers. The term \(\Delta V^i_{imb}\) is the reservation price of an “i”-investor in a situation where there are more sellers than buyers permanently. The theorem shows that seller’s reservation price is a linear combination of this reservation price and that in equilibrium where sellers are not constrained. The weight on \(\Delta V^i_{imb}\) depends on the amount of time that sellers are constrained after the shock. The larger the shock \(s\) is, the longer the period of constrained sellers is, and the lower the prices following the shock are. As the next theorem shows, bid and ask prices facing sophisticated investors decrease *more* than bid and ask prices facing unsophisticated investors.

numerous corporate bond market participants(...)told that, post-TRACE, bond dealers no longer hold large inventories of bonds for some of the most active issues; for less-active bonds, they now serve only as brokers”.

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Theorem 3.3. (Relation between prices after a liquidity shock). If a liquidity shock of size \(0 < s \leq 1\) occurs to any two investors of type \(i\) and \(j\) where \(\rho_i < \rho_j\), the following holds:

1. Bid-ask spreads of investors are not affected by the liquidity shock.

2. Assume that \(\lambda_u > \frac{1-z}{z} \lambda_d\) and \(\rho_i\) and \(\rho_j\) are sufficiently high. For any shock size \(\frac{\lambda_T + \lambda_D}{\min\{\rho_i, \rho_j\} + \lambda_T + \lambda_D} < s \leq 1\) prices at which investors trade satisfy that \(M_i(s) - M_j(s)\) is increasing in \(s\), where \(M\) can be either the bid or ask price.

The theorem shows that the difference between the price of unsophisticated investors and the price of sophisticated investors is a monotonously increasing function of the shock size, where the price can be either the bid or ask. The reason is that prices are outcomes of bargaining, and they reflect investors’ alternatives to immediate trade. Buying investors have the alternative to search for a new counterparty and this alternative is strong for sophisticated investors since they find new counterparties quickly. This is illustrated in Figure 4. One might think that sophisticated sellers have an equally strong outside option. However, since sellers temporarily outnumber buyers, they sell at their reservation value and their outside options are irrelevant while the shock lasts.

Theorem 3.3 also shows that bid-ask spreads do not react to a liquidity shock. The result does not imply that there is no relation between bid-ask spreads and corporate bond prices, a relation found by Chen, Lesmond, and Wei (2007) among others. For example, an "illiquid" bond traded mostly by investors with low search intensities will have a high bid-ask spread and likely have low prices relative to a similar "liquid" bond traded mostly by investors with high search intensities (this is explored in Section 5.3). However, bid-ask spreads do not change when there is selling pressure. While the response of corporate bond prices to supply/demand imbalances has been studied in a number of papers (Chen, Lookman, Schürhoff, and Seppi (2009), Ellul, Jotikasthira, and Lundblad (2009), Ambrose, Cai, and Helwege (2009), and Newman and Rierson (2004)) the behavior of bid-ask spreads has not. As an exception, Newman and Rierson (2004) look at both price and bid-ask spread reaction of European telecommunication bonds to an increased supply in the number of bonds. Consistent with Theorem 3.3 they find a significant decrease in prices and an almost negligible impact on bid-ask spreads.

\(^{9}\)The precise condition for \(\rho_i\) and \(\rho_j\) being "sufficiently high" is that \((1-z)\rho \lambda_u (\rho + \lambda_T + \lambda_D) - (C + \lambda_u + \lambda_d)^2 (\lambda_u + \lambda_d) > 0.\) For realistic parameters this is trivially satisfied except for very short-term bonds (say a maturity of one month or less).
Corporate bond transactions data only recently became available on a large scale. Since January 2001 FINRA\textsuperscript{10} members are required to report their secondary over-the-counter corporate bond transactions through TRACE (Trade Reporting and Compliance Engine). Because of the uncertain benefit to investors of price transparency not all trades reported to TRACE where initially disseminated at the launch of TRACE July 1, 2002. The first research papers using TRACE transactions data focus on the effect of enhanced price transparency and find that dissemination of prices lowered transaction costs for investors (Edwards et al. (2007), Goldstein et al. (2007), and Bessembinder et al. (2006)). The dissemination starts in July 1, 2002 with dissemination of a small subset of trades and from October 1, 2004 all trades are disseminated. Trades must be reported within 15 minutes as of July 1, 2005\textsuperscript{11}. TRACE covers all trades in the secondary over-the-counter market for corporate bonds and accounts for more than 99\% of the total secondary trading volume in corporate bonds. The only trades not covered by TRACE are trades on NYSE which are mainly small retail trades.

I use a sample of non-callable, non-convertible, straight coupon bullet bonds with maturity less than 30 years. Their trading history is collected from TRACE covering the period from October 1, 2004 to June 30, 2009 and after filtering out erroneous trades 10,050,090 trades are left. Error trades are filtered out using the approach in Dick-Nielsen (2009).

To estimate the search model outlined in the previous section an estimate of roundtrip costs in the dealer market is needed, i.e. the difference between the price at which a dealer sells a bond to a costumer and the price at which a dealer buys a bond from a customer. Two main approaches to estimate roundtrip costs exist in the literature. The first is on a given day to average sell prices and subtract average buy prices (Hong and Warga (2000) and Chakravarty and Sarkar (2003)). The second is a regression-based methodology where each transaction price is regressed on a benchmark price and a buy/sell indicator (Schultz (2001), Bessembinder et al. (2006), Goldstein et al. (2007),

\textsuperscript{10}The Financial Industry Regulatory Authority formerly named National Association of Security Dealers (NASD).

\textsuperscript{11}This requirement has gradually been tightened from 1 hour and 15 minutes to 15 minutes. In practice 80\% of all transactions are reported within 5 minutes.
and Edwards et al. (2007)). However, both approaches require a buy/sell indicator for each trade, which is not publicly available for most of the sample. I imply out bid-ask spreads from the data set of transactions by a different procedure, which I describe below. Beginning in November 2008 buy/sell indicators are available, which allows me to check the accuracy of the procedure for this subsample in Section 6.

The methodology for estimating roundtrip costs in this paper is based on unique roundtrip trades (URT). For a given bond on a given day, if there are exactly 2 or 3 trades for a given volume, I define them to be part of a URT. The intuition is that either 1) a customer sells a bond to a dealer who sells it to another customer, or 2) a customer sells a bond to a dealer, who sells it to another dealer, who ultimately sells it to a customer. For a URT the roundtrip cost is defined as the maximal price minus the minimal price. I delete URTs with a zero roundtrip cost from the sample. URTs are closely related to Green, Hollifield, and Schürhoff (2007b)’s ”immediate matches”. An ”immediate match” is a pair of trades where a buy from a customer is followed by a sale to a customer in the same bond for the same par amount on the same day with no intervening trades in that bond. However, since there is no information about the sides in the transactions in the TRACE database, ”immediate trades” cannot be calculated. In the model in Section 3 dealers do not have an inventory and therefore I restrict the sample to URTs where the trades occur within 15 minutes.

Of the 10,050,090 trades in the full sample, 2,159,447 are part of a 15 minutes URT resulting in a 973,600 URTs. Summary statistics of trading costs using URTs are given in Table 1. Average transaction cost is 59 cents, and panel A shows decreasing transaction costs as a function of trade size as documented in several papers (Schultz (2001), Chakravarty and Sarkar (2003), and Edwards, Harris, and Piwowar (2007)). Transaction costs are increasing in maturity as panel B shows consistent with Chakravarty and Sarkar (2003), with a transaction cost that is up to four times as large at long maturities compared to short maturities. Panel C and D addresses whether URTs occur mostly in the liquid or illiquid segment of the corporate bond market. Panel C shows that most of the URT trades are in bonds that have few trades each day. However, panel D shows that the total fraction of trade volume that is captured by URTs is almost the same across bonds that trade frequently and infrequently, on average 13% of volume. Thus, URTs capture transaction costs evenly across liquid and illiquid bonds.
Liquidity risk and credit risk are hard to empirically disentangle, since prices decrease in response to an increase in either of them. Assuming that large traders are more sophisticated than small traders, my model predicts that prices of large traders react stronger to selling pressure than those of small traders. Therefore, a liquidity shock can be identified through the relation of small trades versus large trades. and allows me to use a novel approach and fit the model to demeaned prices. By demeaning, effects due to credit risk are "filtered out", while cross-sectional differences in trade prices identify liquidity effects. Any bid or ask prices for a given bond on a given day are demeaned with the average of all bid and ask prices for this bond on this day. All prices refer to trades that are part of URTs. That is, if there are \( N_{tb} \) URTs on bond \( b \) on day \( t \), and \( A_{tbi} \) is the \( i \)'th ask price and \( B_{tbi} \) the corresponding bid price, the demeaned ask price is defined as \( A_{tbi} - \overline{AB}_{tb} \) and demeaned bid price as \( B_{tbi} - \overline{AB}_{tb} \) where \( \overline{AB}_{tb} = \frac{1}{2N_{tb}} \sum_{i=1}^{N_{tb}} (A_{tbi} + B_{tbi}) \).

For day \( t \) and bond \( b \) all demeaned bid and ask prices are denoted \( P_{tb}^1, P_{tb}^2, \ldots, P_{tb}^{2N_{tb}} \) (the sorting does not matter). The demeaned fitted prices are denoted \( \hat{P}_{tb}^1, \hat{P}_{tb}^2, \ldots, \hat{P}_{tb}^{2N_{tb}} \) and are calculated using Theorem 3.1. I assume that fitting error are independent and normally distributed with zero mean and a standard deviation that depends on the maturity of the bond

\[
P_{tb}^i - \hat{P}_{tb}^i \sim N(0, w_{tb}\sigma^2),
\]

\[
w_{tb} = \max(1, T_{tb}),
\]

where \( T_{tb} \) is the maturity of bond \( b \) on day \( t \). The choice of \( w_{tb} \) is motivated by the fact that pricing errors tend to increase with maturity, while at the same time I wish to avoid excessive influence of prices for bonds with maturity close to zero. Therefore, errors decrease as maturity decreases but is held constant for maturities less than one year such that short-maturity errors do not "blow up". In Appendix E I show that results are robust to alternative specifications of the errors. With this error specification, we have that

\[
\epsilon_{tb}^i = \frac{P_{tb}^i - \hat{P}_{tb}^i}{\sqrt{w_{tb}}} \sim N(0, \sigma^2).
\]

Define \( \Theta \) as a vector with the parameters of the model. The likelihood function is given
as

$$L(\Theta, \sigma|Y) = \prod_{t=1}^{T} \prod_{b=1}^{N_b} \prod_{i=1}^{2N_{tb}} \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{(c_{tb}^i)^2}{2\sigma^2}\right)$$

where $N_b$ is the number of bonds in the sample. We have that

$$-2\log L(\Theta, \sigma|Y) = \frac{1}{\sigma^2} \sum_{t=1}^{T} \sum_{b=1}^{N_b} \sum_{i=1}^{2N_{tb}} (c_{tb}^i)^2 + \sum_{t=1}^{T} \sum_{b=1}^{N_b} \sum_{i=1}^{2N_{tb}} [\log(\sigma^2) + 2\pi]$$

(4)

and maximizing the likelihood function therefore amounts to minimizing the sum of squared weighted errors $\sum_{t=1}^{T} \sum_{b=1}^{N_b} \sum_{i=1}^{2N_{tb}} (c_{tb}^i)^2$. Standard errors are calculated using the outer product of gradients estimator. Appendix D shows that if there is at most one URT observation per bond per day, minimizing the likelihood function is equivalent to minimizing the weighted squared errors between actual and fitted bid-ask spreads.

Investors in the search model presented in Section 3 have different speeds with which they meet a dealer. Unsophisticated investors have a low search intensity while sophisticated investors have a high search intensity. I use trade size as a proxy for investor sophistication. Goldstein, Hotchkiss, and Sirri (2007) and Bessembinder, Kahle, Maxwell, and Xu (2009) find that small trades are mainly retail trades and large trades are predominantly institutional trades, so one interpretation of a small trader is that of an unsophisticated retail investor while a large trader is a sophisticated institutional investor. Specifically, there are six investor classes who differ in their search intensity $\rho$, and they trade in par values of $0 - 10,000$, $10,000 - 50,000$, $50,000 - 100,000$, $100,000 - 500,000$, $500,000 - 1,000,000$, and more than $1,000,000$. I show in Appendix E that the results in the next section are robust to different specifications of investor classes.

The empirical analysis is done in two steps. First, the bond market is assumed to be in equilibrium and parameters are estimated. Since the subprime crisis has been a period of distress in the corporate bond market (along with other fixed income markets), I base the parameter estimation on the period October 2004-July 2007. The full sample is used when periods of selling pressure is estimated.

There are a number of parameters in the model and many of them are not empirically well identified. The following parameters are fixed before estimation. The riskless rate is set to $r = 0.05$, which is close to the average 10-year swap rate of 4.94% in the
estimation period. The bond coupon is set to 7 close to the average coupon rate in the sample period, face value to $F = 100$, holding cost to $\delta = 7$, and default intensity to $\lambda_D = 0.02$. The recovery rate on the bond in case of default is set to zero such that $f = 1$ and the bargaining power of the dealer is 0.9. Finally, $\lambda_d = 0.1$ and $\lambda_u = 1$ such that an investor is a "high" type 91% of the time.

5 Empirical results

In this section I discuss parameter estimates and the ability of the model to fit actual roundtrip costs. Then, time variations in shocks to funding liquidity are examined, and finally the impact of search costs on average yields at different maturities is assessed.

5.1 Parameter estimates and model fit

Table 2 shows the parameter estimates. We see that search intensities increase as trade size increases, so more sophisticated investors trade in larger sizes. The most unsophisticated investors (trading in sizes between 0 and $5,000$) have a search intensity of 69. This implies that they need 3-4 business days on average before they find a dealer with whom to trade with. This can be viewed as the time it takes a non-professional to learn how to trade in the corporate bond market, keep up-to-date about information relevant for trading, and finding an alternative dealer in case his preferred one gives him uncompetitive prices. The most sophisticated investors (trading sizes of more than $1,000,000$) have a search intensity of 474 implying that it takes half a day to complete trades of large size. The search intensities are identified mainly through different trading costs across trade sizes, and Panel A of Table 3 shows actual and fitted roundtrip costs. The model slightly underestimates roundtrip costs for the smallest trades and overestimates costs for the largest trades, but otherwise actual costs are matched well. In particular, the strong negative relation between trade size and trading costs is captured. In Panel B we see that roundtrip costs are fitted fairly well across maturity except for long-maturity bonds where costs are underestimated. Chakravarty and Sarkar (2003) point to increased interest rate risk as a possible explanation for the positive relation between trading costs and maturity. This paper offers a different explanation, namely the better outside options of investors trading short maturity bonds.
The parameter $\tilde{q}$ is estimated to be 0.89, which implies that in a trade intermediated by a dealer the gains of trade is larger for the seller compared to the buyer. Remember that $\tilde{q}$ does not appear in the determination of bid-ask spreads in equation (3). In addition, recall that if there is at most one URT observation for any bond on any day, minimizing the likelihood is equivalent to minimizing weighted squared errors of fitted minus actual bid-ask spreads. Therefore the identification of $\tilde{q}$ occurs through observations of bid and ask prices of different investors on the same bond on the same day. A high $\tilde{q}$ implies that midprices of (sophisticated) large traders are high relative to midprices of (unsophisticated) small traders as discussed in the appendix, and $\tilde{q} = 0.89$ implies that midprices of large traders are higher than those of small traders.

The liquid market for credit default swaps might be a reason for a $\tilde{q}$ above 0.5. There is an approximate arbitrage relation between corporate bonds and credit default swaps: one can approximately create a bond with no default risk by buying a corporate bond and insuring against default by buying protection through a credit default swap (Duffie (1999), Longstaff, Mithal, and Neis (2005), and Blanco, Brennan, and Marsh (2005)). If corporate bond prices are low relative to credit default swaps, one can buy the bond and buy protection through a credit default swap contract and earn an abnormal profit. If prices are high relative to credit default swaps, one needs to short the bond and sell protection through a credit default swap. It can be difficult and expensive to short the bond as shown in Nashikkar and Pedersen (2007), so the arbitrage is easier to carry out when corporate bond prices are low compared to when they are high.

The following calculations provide an estimate of the additional cost due to search that investors in the corporate bond market incur compared to that of the Treasury market. The average maturity in the data sample is 5.5 years, so a 5-year bond is the most representative bond for the corporate market. An estimate of the average bid-ask spread as a percentage of par value of a 5-year bond in the Treasury market is 0.000122 according to Fleming (2003). For an average investor, i.e. an investor with an average search intensity, the corresponding estimate for a 5-year bond in the corporate bond market is 0.00260 according to the parameter estimates and equation (3). Thus, an estimate of the cost of search on a roundtrip in the corporate bond market relative to the Treasury market is 0.00248. The yearly trading volume in the corporate bond market was $3646.5 billion in 2008, so an estimate of the additional yearly costs investors bear in
the corporate bond market compared to the Treasury market is $3646.5 \times 0.00248 = $9.0 billion\(^{12}\).

### 5.2 Liquidity shocks

Theorem 3.3 shows that liquidity shocks can be detected through an unusual high price difference between small trades and large trades. That is, for any given bond the cross-section of prices paid by different investors reveals the amount of selling pressure, if any, there is. In the following I estimate the monthly time variation in the aggregate amount of selling pressure in the US corporate bond market.

In the previous section model parameters in equilibrium are estimated, and the steady state fraction of investor-types (high/low and owner/nonowner) are a by-product of this estimation. To estimate an aggregate shock for month \(m\), I assume that the market is in equilibrium and suddenly a fraction \(s_m\) of high investors are hit by a liquidity shock as defined in Definition 3.1. Holding the parameters estimated in the previous section fixed, I estimate \(s_m\) by maximum likelihood using all demeaned prices in month \(m\) (prices are calculated according to Theorem 3.2). I do this for every month in the sample and obtain a time series for the liquidity shocks\(^{13}^{14}\).

In response to a liquidity shock, prices decrease and slowly return to their equilibrium level as time passes. Since prices are demeaned, a liquidity shock is not identified through this pattern. Instead, a shock is identified through the difference in bid and ask prices of different investors. For example, if the difference in bid prices in a bond between large trades and small trades in equilibrium is 20 cents, and it decreases to 10 cent one month there is a liquidity shock that month. If it decreases to, say, -10 cents there is a even larger liquidity shock. The same pattern in ask prices identify liquidity shocks, and the estimation procedure uses the information in both bid and ask prices. Note that

\(^{12}\)Average daily volume was $14.3 billion according to footnote 1, so yearly was 255*$14.3 billion.

\(^{13}\)In Appendix E I take time-variation in transaction costs into account and show that estimated liquidity shocks are not changed. Thus, possible time variation in transaction costs does not influence liquidity shocks.

\(^{14}\)A concern might be that results are influenced by possible heavy trading in firm A’s bond by large traders in response to bad news from firm A. This could lead to daily average of large trader prices being lower than daily average of small trader prices on “bad news” days. To address this concern, I estimate liquidity shocks on a subsample of the data that corresponds to “good news” days. A “good news” day for a bond is defined as a day where the average price is higher than the average price of the closest day back in time where transactions are recorded. Results on liquidity shocks are robust to using this subsample: Average shock size is 17% (relative to 19% in the full sample) and the time series correlation between liquidity shocks using the subsample and using the whole sample is 82%.
the shock size is only identified through multiple observations of bid and ask prices in a bond on a given day for investors with different search intensities. If investors were not sorted according to sophistication and there instead was a single representative investor, shocks could not be identified.

Figure 5 graphs the estimated liquidity shocks. A 95% confidence interval is bootstrapped according to Bradley (1981). In the first part of the sample period there is one shock occurring. GM and Ford was downgraded to junk bond status in May 2005 causing a major test for the corporate bond market because of the amount of GM/Ford debt outstanding, and the figure shows that selling pressure grows strong in March 2005, peaks in May 2005, and thereafter disappears within a few months. To examine in more detail the effect of the downgrade on the corporate bond market, Figure 6 shows the selling pressure for Ford bonds, GM bonds, and the rest of the corporate bond market around this period. In October 2004 S&P downgrade GM to BBB-, the last rating notch before a junk rating, and the graph shows considerable selling pressure already at this period consistent with evidence in Acharya, Schaefer, and Zhang (2008). Many bond investors and asset managers required to invest in only investment grade bonds started to sell off GM bonds anticipating the future downgrade to junk. In fact, BIS (2005) write "the downgrade had long been anticipated and so asset managers had ample opportunity to adjust their portfolios. Since mid-2003, the auto makers’ spreads had been trading closer to speculative grade issuers than those on other BBB-rated issuers." As it became increasingly likely that especially GM would be downgraded, selling pressure increases in the beginning of 2005. Interestingly, the selling pressure temporarily decreases in February 2005. On January 24, 2005 Lehman announced that it would change methodology for computing their index rating for bonds where the rating agencies disagree on whether it is investment grade or junk (see Chen, Lookman, Schürhoff, and Seppi (2009)). Before the announcement it was the lower of S&P and Moody’s rating whereas beginning July 1, 2005 it would be the middle rating of S&P, Moody’s, and Fitch. For many investment grade investors, the Lehman investment grade index is an important benchmark, and this move made it less likely that Ford/GM bonds would drop out of the index since a downgrade from one of the two major rating agencies was now not sufficient to cause such a drop. Likely, this caused temporary relief in the selling pressure in February.

\footnote{For each month the bootstrapped standard errors are based on 500 simulated data sets.}
However, a steep profit warning from GM on March 16 reintensified the selling pressure and it peaked in May when GM was downgraded to junk by both Fitch and S&P. We see a different selling pressure pattern for Ford bonds with a peak in April and a decrease in May. In contrast to GM bonds, Ford bonds were only downgraded by S&P and were still classified as investment grade under the new Lehman index rule leading to a decrease in selling pressure. The last signs of selling pressure is on August 2005 when GM and Ford were downgraded to junk by Moody’s. Finally, we see from the Figure that there is at best a very moderate selling pressure in other bonds underpinning that the sell-off was concentrated in GM and Ford bonds.

The second period with a large number of forced sellers according to Figure 5 takes off in August 2007 when interbank markets froze and the ”credit crunch” began. However, the first signs of a liquidity shock appears already in April 2007 when the subprime mortgage crisis spills over into the corporate bond market (Brunnermeier (2009)). There is a large shock in March 2008. BIS (2008) writes that ”(t)urmoil in credit markets deepened in early March...tightening repo haircuts caused a number of hedge funds and other leveraged investors to unwind existing positions. As a result, concerns about a cascade of margin calls and forced asset sales accelerated the ongoing investor withdrawal from various financial markets. In the process, spreads on even the most highly rated assets reached unusually wide levels, with market liquidity disappearing across most fixed income markets.” A liquidity squeeze on Bear Sterns caused a take-over by JPMorgan on March 17. The Federal Reserve cut the policy rate by 75 basis points, and ”(t)hese developments appeared to herald a turning point in the market...with investors increasingly adopting the view that various central bank initiatives aimed at reliquefying previously dysfunctional markets were gradually gaining traction” (BIS (2008)). According to the model, the liquidity shock in May 2008 is very low compared to a few months earlier with the percentage of bond owners with a sudden need to sell down to around 10%. According to BIS (2008), ”(b)y the end of the period in late May, the process of disorderly deleveraging had come to a halt, giving way to more orderly credit market conditions. Market liquidity had improved and risk appetite increased, luring investors back into the market”. However, this rebound of the corporate bond market was short-lived and the model-implied liquidity shocks peak again in September and October 2008 as a consequence of Lehman Brothers filing for bankruptcy on September 2008, one of the biggest
credit events in history, and a trigger for a new and intensified stage of the credit crisis.
At the end of 2008 there is a brief halt in selling pressure but in the first three months of 2009 selling pressure intensifies again. In this period stock markets lost more than 30% in value. This loss likely worsened funding conditions leading to a loss in liquidity across asset classes as predicted by Brunnermeier and Pedersen (2009). Finally, the second quarter of 2009 sees a decrease in the selling pressure consistent with credit spreads tightening in the period.

5.3 The credit spread puzzle

One of the most widely employed frameworks of credit risk, structural models, was developed in the seminal work of Merton (1974). Structural models take as given the dynamics of the value of a firm and value corporate bonds as contingent claims on the firm value. In structural models the spread between the yield on a corporate bond and the riskless rate goes to zero as maturities shortens. However, yield spreads are typically positive, also at very short maturities, and this has given rise to the "credit spread puzzle", namely that corporate yield spreads, particularly at very short maturities, are too high to be explained by the corporate bond issuer’s default risk (see for example Huang and Huang (2003) and Chen, Collin-Dufresne, and Goldstein (2008)). This paper offers an explanation for the credit risk puzzle, namely costs due to search frictions and occasional selling pressure\(^{16}\).

I define the search premium for an investor as the midyield paid by this investor minus the yield of an investor who can instantly find a trading partner \((\rho = \infty)\) in which case the bid-ask spread is zero.\(^{17}\) This mimics a trade in the corporate bond market versus a trade in the ultra-liquid Treasury market. I do this for an ”average” corporate bond investor, where the search intensity is the average of all the estimated search intensities in Table 2. For the same ”average” investor I define the selling pressure premium as the midyield on trades occurring under a liquidity shock of 19% (matching

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\(^{16}\)The model in this paper is related to reduced-form models of credit risk, where there is an intensity process governing the risk of default. Thus, it does not predict a near zero contribution of default risk to spreads at very short maturities as structural models, but nevertheless the implications of search costs can be examined in the model.

\(^{17}\)It is easy to show that the discounted present value of the promised payments of a bond using discount rate \(y\) is \(E(\int_0^{\tau_T} C e^{-y t} dt + e^{-y \tau_T} F) = \frac{C + \lambda_T F}{y + \lambda_T}\) where \(\tau_T\) is the (stochastic) maturity. Therefore the yield \(y\) on a bond with price \(P\) is \(y = \frac{C + \lambda_T F}{P} - \lambda_T\), which is the formula used to convert prices to yields.
the average liquidity shock during the sample) minus the midyield on trades occurring in the absence of a liquidity shock.

Figure 7 graphs the term structure of search premia and selling pressure premia. The figure shows that search costs affect primarily the short end of the yield curve with a premium of more than 100 basis points for bonds with very short maturities (less than say two weeks). As maturity goes to zero the premium goes to 245 basis points. For bonds with longer maturities (a year or more) the effect of search costs is in the single-digit range. Thus, although search-induced bid-ask spreads decrease as maturity decrease, the effect of search costs on yields increases as maturity decreases. The credit spread puzzle is most severe at very short maturities and search frictions have the largest impact precisely here. Search frictions are therefore important to take into account when examining short-term credit spreads. Covitz and Downing (2007) examine very short-term spreads (maturity of 1-35 days) in the commercial paper market for the roles of credit risk and liquidity. The average trade size in their data set is USD15 million, so trades are a magnitude larger than in the corporate bond market, and their results can be interpreted in the framework of this paper with a $\rho$ that is larger than those estimated here. Consistent with the downward-sloping effect of search on yields as a function of maturity, they find that the effect of trade size on commercial paper yield spreads decreases with maturity. The following back-of-the-envelope calculation shows how their results can be interpreted in the model in this paper. Their results imply that for an average trade size, the liquidity effect on commercial paper yields is 49.5 basis points smaller for the 35-day yield compared to the 1-day yield\textsuperscript{18}. Holding other parameters constant and fitting the search intensity $\rho$ such that a difference between the 1-day and 35-day search premium is 49.5 basis points gives $\rho = 9.655$. This implies that it takes a little less than a quarter of an hour to carry out a trade in the commercial paper market, which historically has been a very liquid market.

Turning to the impact of selling pressure, Figure 7 shows that the average effect has a hump-shape at short maturities and decreases moderately for long maturities. For a

\textsuperscript{18}Equations (2) and (3) in Covitz and Downing (2007) give the spread effect of total trading volume and number of trades. Using that the log-effect of average trade size equals the log-effect of total volume minus the log-effect of number of trades and using the regression coefficients in equation (3) to be conservative, the difference in impact of liquidity on the 1-day and 35-day segment is $\exp(0.147*\log(15))-\exp((0.147-0.042*\log(35))*\log(15))=0.495$. 

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ten-year bond the effect is around 80 basis points. However, the effect is quite different from the early sample period, where there was a moderate amount of selling pressure, and the later part of the sample, where the selling pressure has been strong. Figure 8 shows the total liquidity premium - the sum of premia due to search and due to selling pressure - as a function of maturity before and after August 2007, the beginning of the "credit crunch". We see that the liquidity premium for a 10-year bond is around 30 basis points before the "credit crunch" and 120 basis points after. The size of the premium is comparable to the non-default component of corporate bond spreads found in other papers. For example, both Huang and Huang (2003) and Longstaff, Mithal, and Neis (2005) find the average non-default component of the 5-year AAA-Treasury spread to be 50-55 basis points. Such comparisons should be interpreted with care due to differences in sample periods and estimation methodologies, but they do show that the estimated premia in this paper is of a size that explains the credit spread puzzle.

Figure 9 shows the 20-year BBB-Treasury spread along with the 20-year estimated total liquidity premium over time. Remember that the size of the yield spread is not used in the estimation, because prices are demeaned before any estimation of model parameters and periods of selling pressure. Despite of this, the estimated liquidity premium is strongly correlated with the corporate bond spread with a correlation of 80%. The average liquidity premium is 55 basis points out of an average spread of 212 basis points. The difference between the corporate bond spread and the estimated liquidity premium is also positively correlated with the liquidity premium with a correlation of 51%. This shows that credit risk and liquidity risk are correlated and illustrates the importance of "filtering" out credit risk by demeaning prices before estimating liquidity risk.

Search costs and selling pressure provide a rich explanation for the credit spread puzzle. Search costs dominate short-term premia while selling pressure dominates long-term premia, and the term structure of total premia can obtain a variety shapes depending on conditions in the corporate bond market - such as downward-sloping, upward-sloping, and hump-shaped - and long-term premia can largely move independently of short-term premia.
6 Robustness checks

URTIs are implicit measures of roundtrip costs, the difference in price between a buy and a sell, and used because TRACE provide no buy/sell indicators for most of the data sample. For the last eight months buy/sell/interdealer indicators are available, and I use this subsample to examine URTIs closer.

Table 4 Panel A shows the percentage of URTIs that include a buy and sell transaction, a buy and interdealer transaction, etc. Overall 4% of URTIs include a buy and sell, 89.7% include an interdealer transaction together with an investor buy or sell transaction, and 6.3% include only sells, buys, or interdealer trades. This evidence suggests that a more appropriate interpretation of URTIs is that they represent the half-spread, since most URTIs reflect either buy-interdealer or interdealer-sell transactions. Panel A also shows that the percentage URTIs representing full roundtrip costs increases in trade size. The increase is accompanied by a corresponding increase in purely one-sided URTIs (only buys, sells, or interdealer trades), so URTIs are reasonable measures of the half-spread for both small and large trade sizes.

I assume in the empirical analysis that URTIs represent full roundtrip costs. Since half-spread is a more appropriate interpretation of URTIs, estimated search intensities $\rho_i$’s are upward biased. An alternative explanation is that the holding cost $\delta$ is downward biased, since a higher $\delta$ yields higher bid-ask spreads as equation (3) shows. Most importantly, however, the relation between estimated search intensities of different investors are unlikely to be influenced by the bias in URT, since URTs of different trade sizes have similar biases.

The identification of liquidity shocks is not affected by the fact that URTIs can be interpreted as a measure of the half-spread. The following example illustrates this. We can find the size of a liquidity shock by looking at, say, the ‘difference-in-bidprice-differences’ $\gamma_*$, where

$$\gamma_*(s) = [B_*(s)^S - B_*(s)^L] - [B^S - B^L]$$

where $S$ denotes 'small trader', $L$ denotes 'large trader', $*$ marks prices under a liquidity shock of size $s$, and an absence of stars marks prices in equilibrium. Theorem 3.3 tells us that $\gamma_*$ is increasing in $s$. Assume we are mistakenly looking at sell transactions for
the large trader and buy for the small trader, so instead of $\gamma_*$ we are investigating

$$
\gamma_{**}(s) = [B_*(s)^S + \omega_S^*(s) - B_*(s)^L] - [B_S^* + \omega^S - B_L^*]
$$

$$
= [B_*(s)^S - B_*(s)^L] - [B_S^* - B_L^*] + [\omega_S^*(s) - \omega^S]
$$

where $\omega^S$ is the bid-ask spread for a small trader in equilibrium and $\omega_S^*(s)$ is the bid-ask spread under a shock of size $s$. According to Theorem 3.3 $\omega_S^*(s)$ and $\omega^S$ are the same, so $\gamma_* = \gamma_{**}$ for every $s$. Therefore, liquidity shocks can be identified by looking at either $\gamma_*$ or $\gamma_{**}$. In fact, any combination of bid, ask, of interdealer prices of small versus large traders can be used to identify liquidity shocks.

Liquidity shocks can be identified by looking at either bid or ask prices. To test separately on bid and ask prices, I do the following. I sort all bid prices in the period November 2008-June 2009 into small and large bid prices. In this robustness check, I use all available straight coupon bullet bond prices with a maturity less than 30 years (I have bid/ask indicators and therefore do not need to filter bid and ask prices out of the data using unique roundtrip trades). For robustness, I use three different cutoffs between small and large prices. Median trade size of 24,000$ is one cutoff, cutoff of 100,000$ to distinguish between institutional and retail investors is another, and prices smaller than 24,000$ and larger than 100,000$ is a third (throwing in-between prices away). For a given bond on a given day, if I have both a small and a large bid, I have an observation of the difference in bids (if I have several small respectively large bids, I take the average). I average all the differences in bids during a month to get a monthly average and find the correlation between monthly averages and estimated monthly liquidity shocks. I repeat this for ask prices to get a correlation between monthly average ask differences and liquidity shocks. Finally, I repeat this exercise on a weekly basis to provide a further robustness check. Panel B in Table 4 shows the results. Across different specifications, average correlation between estimated liquidity shocks and bid differences is 49% while the corresponding average correlation is 58% for ask differences. Thus, differences in prices of small versus large trades, whether it is bid or ask prices, are correlated with estimated liquidity shocks.
I study a model capturing the search-and-bargaining features in the corporate bond market. When a liquidity shock occurs, the model predicts that prices of large traders decrease more than those of small traders. I structurally estimate the model using US corporate bond transactions data from October 2004 to June 2009. The estimation provides new insights into two periods of selling pressure; the downgrade of GM/Ford and the subprime crisis. Liquidity premia generated by the model provide an explanation for the credit spread puzzle at both short and long maturities.

The analysis raises a number of questions. In this paper, the US corporate bond market is examined on an aggregate level. An extension is to understand the cross-sectional variation in selling pressure across bonds. The US municipal bond market is a possibly more illiquid over-the-counter market with transactions data available, and the nature of its illiquidity can be studied using the approach in this paper. Even in highly liquid markets search frictions matter as Ashcraft and Duffie (2007) show for the Fed Funds market, and the Treasury market can be viewed through the lens of a search model. In late 2008 and early 2009 there was selling pressure in the corporate bond market while short-term yields in the Treasury market were close to and on a few occasions below zero indicating strong demand. The model in this paper addresses selling pressure but with a few modifications buying pressure in the Treasury market can be examined.
References


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Appendix A  Equilibrium allocations and prices

In this Appendix asset allocations are determined. The model setup is as described in Section 3. It is possible to determine asset allocations without reference to prices because only low-type owners are sellers and high-type non-owners are buyers, and when a low-type owner or high-type non-owner meets a dealer trade occurs immediately. Bargaining theory tells us that trade occurs immediately [Rubinstein (1982)]. I first derive equilibrium allocations in the case where all investors have the same search intensity, and thereafter argue that these allocations are the same in the case where investors have different search intensities.

Assume now that all investors have the same search intensity. The rate of change of mass \( \mu_{lo}(t) \) of low-type owners is

\[
\dot{\mu}_{lo}(t) = -\rho \mu_m(t) - (\lambda_T + \lambda_D)\mu_{lo}(t) - \lambda_u \mu_{lo}(t) + \lambda_d \mu_{ho}(t)
\]

where \( \mu_m(t) = \min\{\mu_{hn}(t), \mu_{lo}(t)\} \). Dealers buy from \( lo \) investors and sell to \( hn \) investors instantly through the interdealer market. If a \( lo \) investor meets a dealer with intensity \( \rho \) and if \( \mu_{lo}(t) \leq \mu_{hn}(t) \) all meetings lead to a trade and the \( lo \) investor becomes a \( ln \) investor. However, if \( \mu_{lo}(t) > \mu_{hn}(t) \) not all meetings result in a trade. The first term \(-\rho \mu_m(t)\) in (5) reflects this fact. A \( lo \) investor becomes a \( ln \) investor if the owned bond either matures or defaults, and the second term in (5), \(-(\lambda_T + \lambda_D)\mu_{lo}(t)\), reflects this. The third term is present because \( lo \) investors switch type to \( ho \) with intensity \( \lambda_u \), and the last term is due to investors switching from type \( ho \) to \( lo \).

Derivations of the rates of change of mass of the other investor types are very similar. The only slight difference is that although sellers might be rationed, buyers are not. If there are not enough sellers, firms step in and issue more debt. The rates of change are therefore given as

\[
\begin{align*}
\dot{\mu}_{ho}(t) &= \rho \mu_{hn}(t) - (\lambda_T + \lambda_D)\mu_{ho}(t) + \lambda_u \mu_{lo}(t) - \lambda_d \mu_{ho}(t) \\
\dot{\mu}_{hn}(t) &= -\rho \mu_{hn}(t) + (\lambda_T + \lambda_D)\mu_{ho}(t) + \lambda_u \mu_{in}(t) - \lambda_d \mu_{hn}(t) \\
\dot{\mu}_{ln}(t) &= \rho \mu_{m}(t) + (\lambda_T + \lambda_D)\mu_{lo}(t) - \lambda_u \mu_{in}(t) + \lambda_d \mu_{hn}(t).
\end{align*}
\]

If \( \mu_m = \mu_{hn} \) in steady state then the sum of (5) and (6) yields \((\lambda_T + \lambda_D)(\mu_{lo} + \mu_{ho}) = 0\)
which cannot be the case, so \( \mu_m = \mu_{lo} \) in steady state. Inserting \( \mu_{hn} = 1 - (\mu_{ho} + \mu_{ln} + \mu_{lo}) \) into (8) and letting \( \dot{\mu}_{ln}(t) = 0 \) yields

\[
-\lambda_d = (\rho + \lambda_T + \lambda_D - \lambda_d)\mu_{lo} - (\lambda_u + \lambda_d)\mu_{ln} - \lambda_d\mu_{ho}
\]

so in steady state we have

\[
\begin{pmatrix}
\mu_{lo} \\
\mu_{ho} \\
\mu_{hn} \\
\mu_{ln}
\end{pmatrix}
= A^{-1}
\begin{pmatrix}
0 \\
0 \\
0 \\
-\lambda_d
\end{pmatrix}
\]

(9)

where

\[
A =
\begin{pmatrix}
-(\rho + \lambda_T + \lambda_D + \lambda_u) & \lambda_d & 0 & 0 \\
\lambda_u & -(\lambda_T + \lambda_D + \lambda_d) & \rho & 0 \\
0 & \lambda_T + \lambda_D & -(\lambda_d + \rho) & \lambda_u \\
\rho + \lambda_T + \lambda_D - \lambda_d & -\lambda_d & 0 & -(\lambda_u + \lambda_d)
\end{pmatrix}
\]

(10)

I now consider the case where there are investors with different search intensities. Assume some sellers are constrained from selling. In this case firms do not issue bonds because the supply is larger than the demand. The total amount of bonds outstanding decreases because some bonds mature. This is inconsistent with being in equilibrium, so in equilibrium no sellers are constrained. For investor type "i" equations (5)-(8) remain unchanged apart from the term \( \rho_i \mu_{m}(t) \) (and a superscript 'i' on all \( \mu \)'s). In equilibrium, no sellers are constrained from selling and the term is equal to \( \rho^i \mu^i_{lo}(t) \). Thus, the steady state asset allocations are still given by (9) and (10) with a superscript "i" on the \( \mu \)'s and \( \rho \).

Next, I determine the prices that prevail in steady state: a) the bid price \( B_t \) at which investors sell to dealers, b) the ask price \( A_t \) at which investors buy from dealers, and c) the interdealer price. Each investor’s utility for future consumption depends only on his current type \( \sigma(t) \in \Gamma \) and wealth \( W_t \) in his bank account. Lifetime utility is

\[
U(W_t, \sigma(t)) = \sup_{\zeta, \theta} E_t \int_t^\infty e^{-rs} d\zeta_{t+s}
\]

(11)
subject to

\[ dW_t = rW_t dt - d\zeta_t + \theta_t (C - \delta 1_{\sigma_t(t) = t_0}) dt - \hat{P}_t d\theta_t \]  

(12)

where \( \zeta \) is a cumulative consumption process, \( \theta_t \in \{0,1\} \) is a feasible holding process, \( \sigma^\theta \) is the type process induced by \( \theta \), and at the time of a possible holding change, \( \hat{P}_t \in \{A_t, B_t, F, (1-f)F\} \) is the ”trade price”. From (11) and (12) we have that lifetime utility is \( W(t) + V_{\tau(t)} \) where

\[ V_{\tau(t)}(t) = \sup_\theta E_t \left[ \int_t^\infty e^{-r(t-s)} \theta_s (C - \delta 1_{\sigma^\theta(s) = t_0}) ds - e^{-r(t-t)} \hat{P}_s d\theta_s \right]. \]

In order to calculate \( V_{\tau} \) and the bid/ask prices, we consider a particular agent and a particular time \( t \). Let \( \tau_t \) be the next stopping time at which the agent’s type changes, \( \tau_m \) the next time a dealer is met, (in case of an owner) \( \tau_T \) the time at which the bond matures and \( \tau_D \) the time at which the bond defaults. Furthermore, let \( \tau = min\{\tau_t, \tau_m\} \), \( \bar{\tau} = min\{\tau_t, \tau_m, \tau_T, \tau_D\} \), and \( \tau = min\{\tau_t, \tau_m, \tau_T, \tau_D\} \). Then

\[
\begin{align*}
V_{ln}(t) &= E_t [e^{-r(t-\bar{\tau})} V_{hn}(\bar{\tau}_t)] \\
V_{hn}(t) &= E_t [e^{-r(t-\bar{\tau})} V_{ln}(\bar{\tau}_t) 1_{\{\bar{\tau}_t = \tau\}} + e^{-r(t-\tau_m-\bar{\tau})} (V_{ho}(\tau_m) - A_{\tau_m}) 1_{\{\tau_m = \tau\}}] \\
V_{lo}(t) &= E_t \left[ \int_t^\tau e^{-r(u-t)} (C - \delta) du + e^{-r(t-\bar{\tau})} V_{ho}(\bar{\tau}_t) 1_{\{\bar{\tau}_t = \tau\}} \right. \\
&+ e^{-r(t-\tau_m-\bar{\tau})} (V_{ln}(\tau_m) + B_{\tau_m}) 1_{\{\tau_m = \tau\}} \\
&+ e^{-r(t-\tau_T-\bar{\tau})} (V_{ln}(\tau_T) + F) 1_{\{\tau_T = \tau\}}] \\
&+ e^{-r(t-\tau_D-\bar{\tau})} (V_{ln}(\tau_D) + (1-f)F) 1_{\{\tau_D = \tau\}}] \\
V_{ho}(t) &= E_t \left[ \int_t^{\bar{\tau}} C e^{-r(u-t)} du + e^{-r(t-\bar{\tau})} V_{lo}(\bar{\tau}_t) 1_{\{\bar{\tau}_t = \bar{\tau}\}} \right. \\
&+ e^{-r(t-\bar{\tau})} (V_{hn}(\tau_T) + F) 1_{\{\tau_T = \bar{\tau}\}}] \\
&+ e^{-r(t-\tau_D-\bar{\tau})} (V_{hn}(\tau_D) + (1-f)F) 1_{\{\tau_D = \bar{\tau}\}}] \\
\end{align*}
\]

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Suppressing the dependence on time, the value functions satisfy the (HJB) equations

\[ \begin{align*}
\dot{V}_{ln} &= rV_{ln} - \lambda_u(V_{hn} - V_{ln}) \quad (13) \\
\dot{V}_{hn} &= rV_{hn} - \lambda_d(V_{ln} - V_{hn}) - \rho(V_{ho} - V_{hn} - A) \quad (14) \\
\dot{V}_{lo} &= rV_{lo} - \lambda_u(V_{ho} - V_{lo}) - \rho(B + V_{ln} - V_{lo}) - \lambda_T(F + V_{ln} - V_{lo}) \\
&\quad - \lambda_D((1 - f)F + V_{ln} - V_{lo}) - (C - \delta) \quad (15) \\
\dot{V}_{ho} &= rV_{ho} - \lambda_d(V_{lo} - V_{ho}) - \lambda_T(F + V_{hn} - V_{ho}) \\
&\quad - \lambda_D((1 - f)F + V_{hn} - V_{ho}) - C \quad (16)
\end{align*} \]

To see this, we explicitly derive equation (16) and note that the derivation of (13)-(15) is very similar. We have that

\[
E_t\left[ \int_t^{\tilde{\tau}} e^{-r(u-t)} du \right] = \int_t^{\infty} \int_0^{x-t} e^{-ru} dud\tilde{\tau}
\]
\[= \int_t^{\infty} \int_0^{\infty} \frac{1}{\lambda_d + \lambda_T + \lambda_D} e^{-x(\lambda_d + \lambda_T + \lambda_D)} dx \]
\[= \frac{1}{\lambda_d + \lambda_T + \lambda_D} \left[ -1 \right]_{0}^{\infty} \]
\[= \frac{1}{r + \lambda_d + \lambda_T + \lambda_D} \left( 1 \right) \]
and for $\tau = \min\{\tau_1, \tau_2\}$ that

$$
E_t[e^{-r(\tau_1 \land t)}1_{\{\tau_1 = \tau\}}V(\tau_1)] = \int_t^\infty \int_t^\infty e^{-r(x-t)}V(x)\lambda_1 e^{-\lambda_1(x-t)}\lambda_2 e^{-\lambda_2(y-t)}1_{\{x < y\}}dxdy
$$

$$
= \int_t^\infty \int_t^\infty e^{-r(x-t)}V(x)\lambda_1 e^{-\lambda_1(x-t)}\lambda_2 e^{-\lambda_2(y-t)}dydx
$$

$$
= \int_t^\infty e^{-r(x-t)}V(x)\lambda_1 e^{-\lambda_1(x-t)}[-e^{-\lambda_2(y-t)}]_x^\infty dx
$$

$$
= \int_t^\infty e^{-r(x-t)}V(x)\lambda_1 e^{-\lambda_1(x-t)}e^{-\lambda_2(x-t)}dx
$$

$$
= \int_t^\infty \lambda_1 e^{-(r+\lambda_1+\lambda_2)(x-t)}V(x)dx
$$

such that

$$
\dot{V}_{ho} = \frac{\partial}{\partial t} E_t[\int_t^\tau C e^{-r(u-t)}du + e^{-r(\tau-t)}V_{ho}1_{\{\tau_1 = \tau\}}
+ e^{-r(T-t)}(V_{ho} + F)1_{\{\tau_2 = \tau\}} + e^{-r(T-t)}(V_{ho} + (1 - f)F)1_{\{\tau_3 = \tau\}}]
$$

$$
= \frac{\partial}{\partial t} E_t[e^{-r(\tau-t)}V_{ho}1_{\{\tau_1 = \tau\}} + e^{-r(T-t)}(V_{ho} + F)1_{\{\tau_2 = \tau\}}
+ e^{-r(T-t)}(V_{ho} + (1 - f)F)1_{\{\tau_3 = \tau\}}]
$$

$$
= \int_t^\infty \lambda_1 (r + \lambda_d + \lambda_T + \lambda_D)e^{-(r+\lambda_d+\lambda_T+\lambda_D)(x-t)}V_{ho}dx - \lambda_d V_{ho}
$$

$$
+ \int_t^\infty \lambda_T (r + \lambda_d + \lambda_T + \lambda_D)e^{-(r+\lambda_d+\lambda_T+\lambda_D)(x-t)}(V_{ho} + F)dx - \lambda_T(V_{ho} + F)
$$

$$
+ \int_t^\infty \lambda_D (r + \lambda_d + \lambda_T + \lambda_D)e^{-(r+\lambda_d+\lambda_T+\lambda_D)(x-t)}(V_{ho} + (1 - f)F)dx
$$

$$
\quad - \lambda_D(V_{ho} + (1 - f)F)
$$

$$
= (r + \lambda_d + \lambda_T + \lambda_D)E_t[e^{-r(\tau-t)}V_{ho}1_{\{\tau_1 = \tau\}}] - \lambda_d V_{ho}
$$

$$
+ (r + \lambda_d + \lambda_T + \lambda_D)E_t[e^{-r(T-t)}(V_{ho} + F)1_{\{\tau_2 = \tau\}}] - \lambda_T(V_{ho} + F)
$$

$$
+ (r + \lambda_d + \lambda_T + \lambda_D)E_t[e^{-r(T-t)}(V_{ho} + (1 - f)F)1_{\{\tau_3 = \tau\}}]
$$

$$
\quad - \lambda_D(V_{ho} + (1 - f)F)
$$

$$
= (r + \lambda_d + \lambda_T + \lambda_D)[V_{ho} - \frac{C}{r + \lambda_d + \lambda_T + \lambda_D}] - \lambda_d V_{ho} - \lambda_T(V_{ho} + F)
$$

$$
\quad + \lambda_D(V_{ho} + (1 - f)F)
$$

$$
= (r + \lambda_d + \lambda_T + \lambda_D)V_{ho} - \lambda_d V_{ho} - \lambda_T(V_{ho} + F) - \lambda_D(V_{ho} + (1 - f)F) - C.
$$

In steady state $\dot{V}_{\sigma} = 0$ and hence the HJB equations imply the following equations

---

19 According to Leibnitz's rule $\frac{\partial}{\partial t} \int_t^\infty F(x, \alpha)dx = \int_t^\infty \frac{\partial F}{\partial \alpha} dx - F(\alpha, \alpha)$. 

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for the value functions and prices:

\[
V_{ln} = \frac{\lambda_u V_{hn}}{r + \lambda_u} \tag{17}
\]

\[
V_{hn} = \frac{\lambda_d V_{ln} + \rho V_{ho} - \rho A}{r + \lambda_d + \rho} \tag{18}
\]

\[
V_{lo} = \frac{\lambda_u V_{ho} + \rho B + \lambda_T F + \lambda_D (1 - f) F + (\rho + \lambda_T + \lambda_D) V_{ln} + C - \delta}{r + \lambda_u + \rho + \lambda_T + \lambda_D} \tag{19}
\]

\[
V_{ho} = \frac{\lambda_d V_{lo} + (\lambda_T + \lambda_D) V_{hn} + \lambda_T F + \lambda_D (1 - f) F + C}{r + \lambda_d + \lambda_T + \lambda_D} \tag{20}
\]

A low-owner investor is willing to sell to a dealer if the price is at least \(\Delta V_l = V_{lo} - V_{ln}\) while a high-nonowner is willing to buy from a dealer if the price is no more than \(\Delta V_h = V_{ho} - V_{hn}\). Likewise, the dealer is willing to buy if the price is no more than the interdealer price \(M\) (at which he immediately unloads the bond in the interdealer market), and willing to sell if the price is no less than \(M\). If sellers are not constrained the bid price \(B\) can be any price between seller’s reservation value \(\Delta V_l\) and the interdealer price \(M\). In contrast, if sellers are constrained the bid price is seller’s reservation value, since sellers must be indifferent to trading or not if some trade and others do not. Buyers are per construction not constrained since firms issue bonds if there is a demand, and the ask price \(A\) is between buyer’s reservation value \(\Delta V_h\) and the interdealer price \(M\). If sellers are not constrained, Nash bargaining between investors and dealers in which the outside option of the dealer is to trade in the interdealer market results in the bid and ask prices

\[
A = \Delta V_h z + M (1 - z) \tag{21}
\]

\[
B = \Delta V_l z + M (1 - z) \tag{22}
\]

where \(z\) is the bargaining power of the dealer. Duffie, Gárleanu, and Pedersen (2007) show that a bargaining outcome if this kind can be justified by an explicit bargaining
procedure. Thus
\((r + \lambda_u) V_{ln} = \lambda_u V_{hn}\)
\((r + \lambda_d) V_{hn} = \lambda_d V_{ln} + \rho \Delta V_h = \rho [\Delta V_h z + M (1 - z)]\)
\((r + \lambda_u) V_{lo} = \lambda_u V_{ho} + \rho [\Delta V_l z + M (1 - z)] + \lambda_T F + \lambda_D (1 - f) F - (\rho + \lambda_T + \lambda_D) \Delta V_l + C - \delta\)
\((r + \lambda_d) V_{ho} = \lambda_d V_{lo} - (\lambda_T + \lambda_D) \Delta V_h + \lambda_T F + \lambda_D (1 - f) F + C.\)

These equations reduce to
\((r + \lambda_u + (1 - z) \rho + \lambda_T + \lambda_D) \Delta V_l = \lambda_u \Delta V_h + \rho (1 - z) M + \psi_c - \delta\)
\((r + \lambda_d + (1 - z) \rho + \lambda_T + \lambda_D) \Delta V_h = \lambda_d \Delta V_l + \rho (1 - z) M + \psi_c\)

where \(\psi_c = \lambda_T F + \lambda_D (1 - f) F + C\). This implies that
\[
\begin{pmatrix}
\Delta V_l \\
\Delta V_h
\end{pmatrix} \frac{\rho (1 - z) M + \psi_c (1)}{K} - \frac{\delta}{K (K + \lambda_d + \lambda_u)} \begin{pmatrix} 1 \\ K \end{pmatrix} \lambda_d
\] (23)

where \(K = r + (1 - z) \rho + \lambda_T + \lambda_D\). The agent faces a bid-ask spread of
\[z (\Delta V_h - \Delta V_l) = \frac{z \delta}{r + (1 - z) \rho + \lambda_d + \lambda_u + \lambda_T + \lambda_D}\]

For agents with different search intensities the same arguments can applied. Next, we now find the range of interdealer prices for which all investors trade. Define \(\rho_i = \min_{i=1,...,I} \rho^i\).

For all \(\rho \geq \rho_0\) the interdealer price has to satisfy
\[M \leq \Delta V_h = \frac{\rho (1 - z) M + \psi_c}{K} - \frac{\delta \lambda_d}{K (K + \lambda_d + \lambda_u)}\]

according to (23) and rearranging this inequality yields
\[M \leq \frac{\psi_c}{r + \lambda_T + \lambda_D} - \frac{\delta \lambda_d}{(r + \lambda_T + \lambda_D) (r + (1 - z) \rho_0 + \lambda_d + \lambda_u + \lambda_T + \lambda_D)}\].

If this holds for all \(\rho \geq \rho_0\) then
\[M \leq \frac{\psi_c}{r + \lambda_T + \lambda_D} - \frac{\delta \lambda_d}{(r + \lambda_T + \lambda_D) (r + (1 - z) \rho_0 + \lambda_d + \lambda_u + \lambda_T + \lambda_D)}\].
Likewise we find that

\[ M \geq \frac{\psi_c}{r + \lambda_T + \lambda_D} - \frac{\delta (K + \lambda_d)}{(r + \lambda_T + \lambda_D)(K + \lambda_d + \lambda_u)} \]

which holds for all \( \rho \) if

\[ M \geq \frac{\psi_c}{r + \lambda_T + \lambda_D} - \frac{\delta (r + (1 - \tilde{z}) \rho_0 + \lambda_d + \lambda_T + \lambda_D)}{(r + \lambda_T + \lambda_D)(r + (1 - \tilde{z}) \rho_0 + \lambda_d + \lambda_u + \lambda_T + \lambda_D)}. \]

A.1 The effect of \( z \) and \( \tilde{q} \)

Consider a buyer who is willing to buy at a maximal price of \( \Delta V_h = 103 \), a seller who is willing to sell at a minimal price of \( \Delta V_l = 100 \), and a dealer who is intermediating the trade. The gains of trade of 103-100=3 is to be split between the three agents. Assume that \( z = 0.5 \) and \( \tilde{q} = 0.75 \). The interdealer price is 102.25, the bid price is 101.125, and the ask price is 102.625. The dealer gains the bid-ask spread \( z * 3 = 1.5 \), so \( z \) measures the part of the total gain the dealer is receiving. The seller gains 1.125 while the buyer gains 0.375, so \( \tilde{q} \) determines how the rest of the gain is split between buyer and seller. Consider now \( \tilde{q} = 0.25 \). In this case the bid price is 100.375 and the ask price 101.875. The bid-ask spread is the same as before, but bid and ask prices are lower.

Remember that \( \tilde{q} \) does not appear in the determination of bid-ask spreads in equation (3). The identification of \( \tilde{q} \) occurs through observations of bid-ask spreads of different investors on the same bond on the same day. In order to understand the effect of \( \tilde{q} \) consider \( \tilde{q} \) close to 1. In this case, both small and large buyers are almost indifferent between trading or not because the interdealer price is high, and they trade at similar prices. The outside option of sellers to cut off negotiations and find a new dealer to trade with is stronger for large sellers, and they negotiate a higher price compared to small sellers. Overall, this means that midprices of small traders are lower than midprices of large traders. If \( \tilde{q} \) is close to 0 the reverse is the case and midprices of small traders are typically higher than those of large traders. Figure 10 shows how the difference in midprice between large and small traders increases as \( \tilde{q} \) increases. Except for low values of \( \tilde{q} \) the difference is positive, so in equilibrium the difference in midprices of large versus small traders is typically positive.
Appendix B  Proof of Theorem 3.2

This section provides a proof of Theorem 3.2. First I find the length of time with more sellers than buyers following a liquidity shock. Then bid and ask prices prevailing immediately after a liquidity shock are found.

If we define $\mu_l = \mu_{lo} + \mu_{ln}$ and $\mu_h = \mu_{ho} + \mu_{hn}$ we have according to equations (5)-(8) that

$$\begin{pmatrix} \dot{\mu}_l \\ \dot{\mu}_h \end{pmatrix} = \begin{bmatrix} -\lambda_u & \lambda_d \\ \lambda_u & -\lambda_d \end{bmatrix} \begin{pmatrix} \mu_l \\ \mu_h \end{pmatrix}.$$  

The solution to these ODEs is

$$\begin{pmatrix} \mu_l(t) \\ \mu_h(t) \end{pmatrix} = (1 - e^{-t(\lambda_u+\lambda_d)}) \begin{pmatrix} \frac{\lambda_d}{\lambda_d+\lambda_u} \\ \frac{\lambda_u}{\lambda_d+\lambda_u} \end{pmatrix} + e^{-t(\lambda_u+\lambda_d)} \begin{pmatrix} \mu_l(0) \\ \mu_h(0) \end{pmatrix}.$$  

This result will be useful in a short moment. We now look at $\mu_{hn} - \mu_{lo}$ since sellers are constrained if this difference is negative. We have that

$$\dot{\mu}_{hn} - \dot{\mu}_{lo} = \rho(\mu_m - \mu_{hn}) + (\lambda_T + \lambda_D)(\mu_{ho} + \mu_{lo}) + \lambda_u(\mu_{ln} + \mu_{lo}) - \lambda_d(\mu_{hn} + \mu_{ho})$$

$$= \rho(\mu_m - \mu_{hn}) - (\lambda_T + \lambda_D)(\mu_{hn} - \mu_{lo}) + (\lambda_T + \lambda_D - \lambda_d - \lambda_u)\mu_h + \lambda_u$$

If sellers are constrained we have $\mu_m = \mu_{hn}$ and

$$\dot{\mu}_{hn} - \dot{\mu}_{lo} = -(\lambda_T + \lambda_D)(\mu_{hn} - \mu_{lo}) + (\lambda_T + \lambda_D - \lambda_d - \lambda_u)\mu_h + \lambda_u.$$  

Assume that $\mu_{hn}(0) - \mu_{lo}(0) < 0$. Using the result in equation (24), the solution for $\mu_{hn}(t) - \mu_{lo}(t)$ for any $t$ where $\mu_{hn} - \mu_{lo}$ has not yet been positive is

$$\mu_{hn}(t) - \mu_{lo}(t) = \frac{\lambda_u}{\lambda_d+\lambda_u} e^{-t(\lambda_u+\lambda_d)}(\mu_{h}(0) - \frac{\lambda_u}{\lambda_d+\lambda_u})$$

$$- (\mu_{ho}(0) + \mu_{lo}(0)) e^{-(\lambda_T+\lambda_D)t}.$$  

This equation has a unique $t_s$ where $\mu_{hn}(t_s) - \mu_{lo}(t_s) = 0$. This implies that if sellers are constrained at time 0 they become unconstrained at time $t_s$. What remains to show is that they stay unconstrained after time $t_s$. If sellers are not constrained we have $\mu_m = \mu_{lo}$ and

$$\dot{\mu}_{hn} - \dot{\mu}_{lo} = -(\rho + \lambda_T + \lambda_D)(\mu_{hn} - \mu_{lo}) + (\lambda_T + \lambda_D - \lambda_d - \lambda_u)\mu_h + \lambda_u$$

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The solution is (assuming that $\mu_{hn}(0) - \mu_{lo}(0) = 0$

$$\mu_{hn}(t) - \mu_{lo}(t) = \frac{\lambda_u}{\lambda_d + \lambda_u} \frac{\lambda_T + \lambda_D}{\rho + \lambda_T + \lambda_D}$$

$$+ \frac{\lambda_T + \lambda_D - \lambda_d - \lambda_u}{\rho + \lambda_T + \lambda_D - \lambda_d - \lambda_u} (\mu_h(0) - \frac{\lambda_u}{\lambda'_d + \lambda_u}) e^{-(\lambda_u + \lambda_d)t}$$

$$- \left[ (\mu_h(0) - \frac{\lambda_u}{\lambda_d + \lambda_u}) (\rho + \lambda_T + \lambda_D - \lambda_d - \lambda_u) + \frac{\lambda_u}{\lambda_d + \lambda_u} (\lambda_T + \lambda_D) \right]$$

$$\times e^{-(\rho + \lambda_T + \lambda_D)t}.$$

Assume that $\mu_h(0) = (1-s)\frac{\lambda_u}{\lambda_d + \lambda_u}$. Then we have

$$\mu_{hn}(t) - \mu_{lo}(t) = \frac{\lambda_u}{\lambda_d + \lambda_u} \left[ \frac{\lambda_T + \lambda_D}{\rho + \lambda_T + \lambda_D}$$

$$- s \frac{\lambda_T + \lambda_D - \lambda_d - \lambda_u}{\rho + \lambda_T + \lambda_D - \lambda_d - \lambda_u} e^{-(\lambda_u + \lambda_d)t}$$

$$- \left[ (1-s)(\lambda_T + \lambda_D)(\rho + \lambda_T + \lambda_D - \lambda_u - \lambda_d) + s\rho(\lambda_T + \lambda_D) \right] e^{-(\rho + \lambda_T + \lambda_D)t} \right].$$

If $\rho + \lambda_T + \lambda_D > \lambda_d + \lambda_u$ we have that $\mu_{hn}(t) - \mu_{lo}(t) \geq 0$ for all $t$. What I have now shown is that if the fractions of investors are such that sellers are constrained, the sellers are constrained a period of time $t_s$ and unconstrained thereafter. Next, I find $t_s$. The steady state value of $\mu_{hn} - \mu_{lo}$ is $\frac{\lambda_u}{\lambda_d + \lambda_u} \frac{\lambda_T + \lambda_D}{\rho + \lambda_T + \lambda_D}$. This, along with equations (5) and (6) yielding

$$\dot{\mu}_{lo} + \dot{\mu}_{ho} = \rho(\mu_{hn} - \mu_{lo}) - (\lambda_T + \lambda_D)(\mu_{ho} + \mu_{lo})$$

gives the steady state value of $\mu_{lo} + \mu_{ho}$ as $\frac{\lambda_u}{\lambda_d + \lambda_u} \frac{\rho}{\rho + \lambda_T + \lambda_D}$. Since the steady state value of $\mu_h$ is $\frac{\lambda_u}{\lambda_d + \lambda_u}$ we have according to (25) that the time, $t$, an investor is constrained after a shock of $s$ solves

$$0 = 1 - s e^{-(\lambda_u + \lambda_d)t} - \frac{\rho}{\rho + \lambda_T + \lambda_D} e^{-(\lambda_T + \lambda_D)t}.$$

If $s \leq \frac{\lambda_T + \lambda_D}{\rho + \lambda_T + \lambda_D}$ the investor is not constrained at any time after the shock and prices do not change.

Next, I find bid and ask prices immediately after the liquidity shock. Rewriting
equations (13)-(16) we have that

\[
\begin{align*}
\dot{\Delta V}_h &= (C_1 + \lambda_d + \rho) \Delta V_h - \lambda_d \Delta V_l - \psi_C - \rho A \\
\dot{\Delta V}_l &= (C_1 + \lambda_u + \rho) \Delta V_l - \lambda_u \Delta V_h - (\psi_C - \delta) - \rho B \\
C_1 &= r + \lambda_D + \lambda_T \\
\psi_C &= \lambda_T F + \lambda_D (1 - f) F + C.
\end{align*}
\]

If sellers are constrained we have that the interdealer price is \( M = \Delta V_l \), so \( A = \Delta V_h z + \Delta V_l (1 - z) \) and \( B = \Delta V_l \). Thus,

\[
\begin{align*}
\dot{\Delta V}_h &= (C_1 + \lambda_d + (1 - z)\rho) \Delta V_h + (-\lambda_d - (1 - z)\rho) \Delta V_l - \psi_C \\
\dot{\Delta V}_l &= (C_1 + \lambda_u) \Delta V_l - \lambda_u \Delta V_h - (\psi_C - \delta)
\end{align*}
\]

which can be rewritten as

\[
\begin{pmatrix}
\dot{\Delta V}_h \\
\dot{\Delta V}_l
\end{pmatrix} =
\begin{pmatrix}
C_1 + \lambda_d + (1 - z)\rho & -\lambda_d - (1 - z)\rho \\
-\lambda_u & C_1 + \lambda_u
\end{pmatrix}
\begin{pmatrix}
\Delta V_h \\
\Delta V_l
\end{pmatrix} -
\begin{pmatrix}
\psi_C \\
\psi_C - \delta
\end{pmatrix}
\tag{27}
\]

The reservation values immediately after the shock is found be solving these ODEs backwards from the steady state reservation values for the period of time sellers are constrained. That is, the reservation values after the shock is the time \( t_s \) solution to

\[
\begin{pmatrix}
\Delta V_h(0) \\
\Delta V_l(0)
\end{pmatrix} =
\begin{pmatrix}
\Delta V_h^{ss} \\
\Delta V_l^{ss}
\end{pmatrix}.
\]

The steady state solution of this system is

\[
\begin{pmatrix}
\Delta V_h^{imb} \\
\Delta V_l^{imb}
\end{pmatrix} =
\begin{pmatrix}
-C_1 - \lambda_d + (1 - z)\rho & \lambda_d + (1 - z)\rho \\
\lambda_u & -(C_1 + \lambda_u)
\end{pmatrix}^{-1}
\begin{pmatrix}
\psi_C \\
\psi_C - \delta
\end{pmatrix}
\]

where \( \sqrt{\gamma} = \lambda_d + \lambda_u + (1 - z)\rho \). If sellers were always constrained, the reservation values
of investors would be \( \begin{pmatrix} \Delta V^\text{imb}_h \\ \Delta V^\text{imb}_l \end{pmatrix} \). Tedious calculations and the use of Corollary 11.3.3 in Bernstein (2005) show that

\[
\begin{pmatrix} \Delta V_h(t) \\ \Delta V_l(t) \end{pmatrix} = e^{-tC_1} \begin{pmatrix} \Delta V^\text{ss}_h \\ \Delta V^\text{ss}_l \end{pmatrix} + (1 - e^{-tC_1}) \begin{pmatrix} \Delta V^\text{imb}_h \\ \Delta V^\text{imb}_l \end{pmatrix}
\]
Appendix C  Proof of Theorem 3.3

This section provides a proof of Theorem 3.3. Part 1 of the theorem - that bid-ask spreads are unaffected by a liquidity shock - follows immediately from Theorem 3.1 and 3.2. Consequently, any relation between ask prices of investors with different levels of sophistication can be shown by showing the relation for bid prices.

I first assume that \( \frac{\lambda_T + \lambda_D}{\min(\rho_i, \rho_j) + \lambda_T + \lambda_D} < s \leq 1 \) and prove the second half of part 2 in the theorem by showing that the bid price of an unsophisticated investor minus that of a sophisticated investor, \( B_i(s) - B_j(s) \), is increasing in \( s \). I show this by proving that \( \frac{\partial B(s)}{\partial \rho \partial s} < 0 \). Implicit differentiation in equation (26) yields

\[
\frac{\partial t}{\partial s} = \frac{1}{s(\lambda_u + \lambda_d) + \rho(\lambda_T + \lambda_D) e^{(\lambda_u + \lambda_d - \lambda_T - \lambda_D)t}}
\]

so we have

\[
\frac{\partial B(s)}{\partial s} = (r + \lambda_D + \lambda_T) \frac{\partial t}{\partial s} e^{-t(r + \lambda_D + \lambda_T)}(\Delta V^{ss}_l - \Delta V^{imb}_l).
\]

Since

\[
\Delta V^{imb}_l - \Delta V^{ss}_l = \frac{\rho(1 - z)}{C(\lambda_T + \lambda_D)} \left[ \frac{(1 - \bar{q})C_0 + \lambda_d}{C_0 + \lambda_u + \lambda_d} - \frac{C + \lambda_d}{C + \lambda_u + \lambda_d} \right]
\]

\[
C_0 = r + (1 - z)\rho_0 + \lambda_T + \lambda_D
\]

\[
C = r + (1 - z)\rho + \lambda_T + \lambda_D
\]

we have that

\[
\frac{\partial B(s)}{\partial s} = \frac{\rho(1 - z)}{C} \left[ \frac{(1 - \bar{q})C_0 + \lambda_d}{C_0 + \lambda_u + \lambda_d} - \frac{C + \lambda_d}{C + \lambda_u + \lambda_d} \right] e^{-rt} \frac{s(\lambda_u + \lambda_d)e^{(\lambda_T + \lambda_D)t} + \rho(\lambda_T + \lambda_D)e^{(\lambda_u + \lambda_d)t}}{s(\lambda_u + \lambda_d)e^{(\lambda_T + \lambda_D)t} + \rho(\lambda_T + \lambda_D)e^{(\lambda_u + \lambda_d)t}}.
\]

Now

\[
\frac{\partial t}{\partial \rho} = \frac{\lambda_T + \lambda_D}{\rho(\lambda_T + \lambda_D)^2} e^{-t(\lambda_T + \lambda_D)}
\]

\[
\frac{\partial (\frac{\rho(C + \lambda_d)}{C(C + \lambda_u + \lambda_d)})}{\partial \rho} = \frac{\rho(1 - z)C\lambda_u + (r + \lambda_T + \lambda_D)(C + \lambda_d)(C + \lambda_d + \lambda_u)}{C^2(C + \lambda_u + \lambda_d)^2}
\]

\[
\frac{\partial (\frac{\rho(1 - z)}{C})}{\partial \rho} = \frac{(1 - z)(r + \lambda_T + \lambda_D)}{C^2}
\]
so the numerator in $\frac{\partial B(s)}{\partial \rho s}$ is

$$
\left[ (1-z)(r+\lambda_T+\lambda_D) \frac{(1-\tilde{q})C_{0}+\lambda_{d}}{C_0+\lambda_u+\lambda_d} - (1-z) \frac{\rho(1-z)C\lambda_u+(r+\lambda_T+\lambda_D)(C+\lambda_d)(C+\lambda_u+\lambda_d)}{C(C+\lambda_T+\lambda_D)} \right] \times 
$$

$$
\left[ s(\lambda_u + \lambda_d) e^{(\lambda_T+\lambda_D)t} + \frac{\rho(\lambda_T+\lambda_D)}{\rho+\lambda_T+\lambda_D} e^{(\lambda_T+\lambda_D)t} - \frac{\rho(1-z)}{C} \frac{(1-\tilde{q})C_{0}+\lambda_{d}}{C_0+\lambda_u+\lambda_d} - \frac{C\lambda_u+\lambda_d}{C+\lambda_T+\lambda_D} \right] 
$$

$$
\left[ (se^{(\lambda_T+\lambda_D)t})^2 + \frac{\rho}{\rho+\lambda_T+\lambda_D} e^{(\lambda_T+\lambda_D)t} \left( \frac{(\lambda_u+\lambda_d)(\lambda+\lambda_D)^2}{(\lambda_u+\lambda_d)e^{-(\lambda_T+\lambda_D)t}} + \frac{\rho(\lambda_T+\lambda_D)}{\rho+\lambda_T+\lambda_D} e^{-(\lambda_T+\lambda_D)t} \right) + \frac{(\lambda_T+\lambda_D)^2}{(\rho+\lambda_T+\lambda_D)^2} e^{(\lambda_T+\lambda_D)t} e^{(\lambda_T+\lambda_D)t} \right] = 
$$

$$
\left[ (1-z)\frac{C\lambda_u+\lambda_d}{C_0+\lambda_u+\lambda_d} - \frac{C\lambda_u+\lambda_d}{C+\lambda_T+\lambda_D} \right] \frac{1-z}{C} \times 
$$

$$
\left[ (se^{(\lambda_T+\lambda_D)t})^2 + \frac{\rho}{\rho+\lambda_T+\lambda_D} e^{(\lambda_T+\lambda_D)t} \left( \frac{(\lambda_u+\lambda_d)(\lambda+\lambda_D)^2}{(\lambda_u+\lambda_d)e^{-(\lambda_T+\lambda_D)t}} + \frac{\rho(\lambda_T+\lambda_D)}{\rho+\lambda_T+\lambda_D} e^{-(\lambda_T+\lambda_D)t} \right) + \frac{(\lambda_T+\lambda_D)^2}{(\rho+\lambda_T+\lambda_D)^2} e^{(\lambda_T+\lambda_D)t} e^{(\lambda_T+\lambda_D)t} \right] 
$$

$$
\rho \left[ (se^{(\lambda_T+\lambda_D)t})^2 + \frac{\rho}{\rho+\lambda_T+\lambda_D} e^{(\lambda_T+\lambda_D)t} \left( \frac{(\lambda_u+\lambda_d)(\lambda+\lambda_D)^2}{(\lambda_u+\lambda_d)e^{-(\lambda_T+\lambda_D)t}} + \frac{\rho(\lambda_T+\lambda_D)}{\rho+\lambda_T+\lambda_D} e^{-(\lambda_T+\lambda_D)t} \right) + \frac{(\lambda_T+\lambda_D)^2}{(\rho+\lambda_T+\lambda_D)^2} e^{(\lambda_T+\lambda_D)t} e^{(\lambda_T+\lambda_D)t} \right] 
$$

$$
\rho \left[ \frac{(1-z)^2}{C(\lambda_u+\lambda_d)^2} \left( \frac{\rho}{\rho+\lambda_T+\lambda_D} e^{(\lambda_T+\lambda_D)t} \left( \frac{(\lambda_u+\lambda_d)(\lambda+\lambda_D)^2}{(\lambda_u+\lambda_d)e^{-(\lambda_T+\lambda_D)t}} + \frac{\rho(\lambda_T+\lambda_D)}{\rho+\lambda_T+\lambda_D} e^{-(\lambda_T+\lambda_D)t} \right) + \frac{(\lambda_T+\lambda_D)^2}{(\rho+\lambda_T+\lambda_D)^2} e^{(\lambda_T+\lambda_D)t} e^{(\lambda_T+\lambda_D)t} \right] \right] 
$$

We have that $\frac{(1-z)\rho\lambda_u}{(C+\lambda_u+\lambda_d)^2} < 0$ and $\frac{r+\lambda_T+\lambda_D}{\rho+\lambda_T+\lambda_D} > \frac{\lambda_T+\lambda_D}{\rho+\lambda_T+\lambda_D}$. Also

$$
\frac{(1-z)\rho\lambda_u}{(C+\lambda_u+\lambda_d)^2} = \frac{\rho}{\rho+\lambda_T+\lambda_D} \left( \frac{(\lambda_u+\lambda_d)(\lambda+\lambda_D)^2}{(\lambda_u+\lambda_d)e^{-(\lambda_T+\lambda_D)t}} + \frac{\rho(\lambda_T+\lambda_D)}{\rho+\lambda_T+\lambda_D} e^{-(\lambda_T+\lambda_D)t} \right) 
$$

$$
= \frac{1}{(C+\lambda_u+\lambda_d)^2} \left[ \frac{(\rho+\lambda_T+\lambda_D)^2}{(\lambda_T+\lambda_D)^2} e^{(\lambda_T+\lambda_D-t)\lambda-a} + \frac{(\rho+\lambda_T+\lambda_D)}{\lambda_u+\lambda_d} \right] 
$$

A sufficient condition for $\frac{\partial B(s)}{\partial \rho s} < 0$ is that

$$
\frac{(1-z)\rho\lambda_u}{(C+\lambda_u+\lambda_d)^2} > \frac{1}{(\lambda_T+\lambda_D)^2} \left[ \frac{(\rho+\lambda_T+\lambda_D)^2}{(\lambda_T+\lambda_D)^2} e^{(\lambda_T+\lambda_D-t)\lambda-a} + \frac{(\rho+\lambda_T+\lambda_D)}{\lambda_u+\lambda_d} \right] 
$$

This is the case if $f(s) > 0$ for every $0 \leq s \leq 1$ where $f$ is defined as

$$
f(s) = \frac{(1-z)\rho\lambda_u}{(C+\lambda_u+\lambda_d)^2} \left[ \frac{(\rho+\lambda_T+\lambda_D)^2}{(\lambda_T+\lambda_D)^2} e^{(\lambda_T+\lambda_D-t)\lambda-a} + \frac{(\rho+\lambda_T+\lambda_D)}{\lambda_u+\lambda_d} \right] - 1.\]
Since $f(s) > f(0)$ for $s > 0$ we need to show that $f(0) > 0$. We have that

$$(1 - z)\rho \lambda_u (\rho + \lambda_T + \lambda_D) - (C + \lambda_u + \lambda_d)^2(\lambda_u + \lambda_d) = (1 - z)(z\lambda_u - \lambda_d(1 - z))\rho^2 - (1 - z)[(2\lambda_d + \lambda_u)(\lambda_T + \lambda_D) + 2(\lambda_u + \lambda_d)^2]\rho - (\lambda_u + \lambda_d)(\lambda_T + \lambda_D + \lambda_u + \lambda_d)^2.$$

The last expression is a second-order polynomial in $\rho$ and if this polynomial is larger than 0, then so is $f(0)$. If $(1 - z)(z\lambda_u - \lambda_d(1 - z)) > 0$ then the polynomial is positive for $\rho$ sufficiently large. If $\lambda_u > \frac{1 - z}{z}\lambda_d$ then $(1 - z)(z\lambda_u - \lambda_d(1 - z)) > 0$. Thus, if $\rho$ is sufficiently large and $\lambda_u > \frac{1 - z}{z}\lambda_d$ then $\frac{\partial B(s)}{\partial \rho \partial s} < 0$ as needed to be proven.

To illustrate the effect of liquidity shocks on prices, the top graph in Figure 11 shows bid and ask prices for a 10-year bond for a sophisticated and unsophisticated investor as a function of size of liquidity shock. We see that prices paid by both sophisticated and unsophisticated investors decrease in the size of the liquidity shock, and prices facing sophisticated investors decrease more than prices facing unsophisticated investors.

The middle graph in Figure 11 illustrates this by showing the difference in midprices between unsophisticated and sophisticated investors. As the previous theorem states this difference increases in the liquidity shock size.

The bottom graph in Figure 11 shows the percentage change in midprice as a function of maturity when 25% of investors experience a liquidity shock. The price impact of a liquidity shock is much higher for a long-maturity bond compared to a short-maturity bond, which illustrates the importance of taking into account maturity when modelling the impact of selling pressure in corporate bonds. The owner of a short-maturity bond with a liquidity need is not willing to sell at a much lower price because the bond will soon mature at par anyway, while the seller of a long-maturity bond accepts a sizeable price discount.
Appendix D  Additional details on estimation methodology

In the text $A_{tbi}$ is defined as the $i$’th ask price on bond $b$ and day $t$ and $B_{tbi}$ as the corresponding bid price. Fitted prices follow the same notation with an addition superscript $M$. The sum of squared errors in the likelihood in equation (4) equals

$$
\sum_{t=1}^{T} \sum_{b=1}^{N_b} \sum_{i=1}^{N_{tb}} \left[ ((A_{tbi} - \overline{A}_{tb}) - (A_{tbi}^M - \overline{A}_{tb}^M))^2 + ((B_{tbi} - \overline{B}_{tb}) - (B_{tbi}^M - \overline{B}_{tb}^M))^2 \right]
$$

where I use the notation

$$
\overline{A}_{tb} = \frac{1}{N_{tb}} \sum_{m=1}^{N_{tb}} A_{tbm} \\
\overline{B}_{tb} = \frac{1}{N_{tb}} \sum_{m=1}^{N_{tb}} B_{tbm} \\
\overline{AB}_{tb} = \frac{1}{2} (\overline{A}_{tb} + \overline{B}_{tb}).
$$

We have that

$$
\sum_{i=1}^{N_{tb}} [(A_{tbi} - \overline{A}_{tb}) - (A_{tbi}^M - \overline{A}_{tb}^M))^2
$$

$$
= \sum_{i=1}^{N_{tb}} [(A_{tbi} - A_{tbi}^M) - (A_{tb} - \overline{A}_{tb})] + N_{tb} [(A_{tb} - \overline{A}_{tb}) - (\overline{AB}_{tb} - \overline{AB}_{tb}^M)]^2
$$

and

$$
\sum_{i=1}^{N_{tb}} [(B_{tbi} - \overline{B}_{tb}) - (B_{tbi}^M - \overline{B}_{tb}^M))^2
$$

$$
= \sum_{i=1}^{N_{tb}} [(B_{tbi} - B_{tbi}^M) - (B_{tb} - \overline{B}_{tb})] + N_{tb} [(B_{tb} - \overline{B}_{tb}) - (\overline{AB}_{tb} - \overline{AB}_{tb}^M)]^2.
$$

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Since

\[ N_{tb} \left( \bar{A}_{tb} - \bar{A}_{tb}^M \right)^2 + N_{tb} \left( \bar{B}_{tb} - \bar{B}_{tb}^M \right)^2 \]

the squared errors equal

\[ \sum_{t=1}^{T} \sum_{b=1}^{N_{tb}} w_{tb} \left[ \sum_{i=1}^{N_{tb}} \left( A_{tb1}^i - \bar{A}_{tb} \right) - \left( A_{tb1}^M - \bar{A}_{tb}^M \right)^2 \right] + \sum_{i=1}^{N_{tb}} \left[ \left( B_{tb1} - \bar{B}_{tb} \right) - \left( B_{tb1}^M - \bar{B}_{tb}^M \right)^2 \right] \]

\[ + \frac{N_{tb}}{2} \left[ \left( \bar{A}_{tb} - \bar{B}_{tb} \right) - \left( \bar{A}_{tb}^M - \bar{B}_{tb}^M \right)^2 \right]. \]

This shows that fitting data to demeaned prices is equivalent to fitting to the sum of bid-ask spreads, demeaned ask prices, and demeaned bid prices. In particular, assume that there is at most one bond observation per day. Then the expression reduces to

\[ \sum_{t=1}^{T} \sum_{b=1}^{N_{tb}} \frac{w_{tb}}{2} \left[ \left( A_{tb1} - B_{tb1} \right) - \left( A_{tb1}^M - B_{tb1}^M \right)^2 \right] \]

which is the weighted sum of squared errors between actual and fitted bid-ask spreads.

### Appendix E Robustness checks

In this Appendix I conduct a series of tests to check that the main results are robust to a different specification of pricing errors, different number of groups of investor sophistication, and time-variation in transaction costs.

#### E.1 Variance on pricing errors

In the text I assume that pricing errors are normally distributed with mean 0 and variance \( \min(1,T_{tb})\sigma^2 \), where \( T \) is the maturity of bond \( b \) and day \( t \). If errors are assumed to be normally distributed with mean 0 and constant variance \( \sigma^2 \), estimated parameters are \( \tilde{q} = 0.72 \), \( \rho_1 = 56.1 \), \( \rho_2 = 80.5 \), \( \rho_3 = 105.5 \), \( \rho_4 = 161.6 \), \( \rho_5 = 338.6 \),
and $\rho_6 = 416.0$. We see that search intensities are 10-35% lower than in the main text. This is a consequence of putting more weight on pricing errors of long-maturity bonds: Table 3 shows that roundtrip costs for 5-30y bonds are somewhat underestimated, and lower search intensities lead to a better fit of those bonds. Estimated liquidity shocks in the corporate bond market are highly correlated with those in the main text with a correlation of 92%. The mean sample shock size is 9.4% which is lower than in the main text, partially because smaller search intensities lead to a greater impact of a shock. Regarding total liquidity premium it is 58 basis points on a 30y bond while the corresponding premium in the main text is 81. Overall, liquidity shocks are very similar and the liquidity premium is still of a size that explains the credit spread puzzle.

E.2 Degree of heterogeneity among investors

In the text investors are grouped into 6 different levels of sophistication. Trade size proxies for sophistication and they are grouped into investors trading in sizes < $10,000, $10,000–$50,000, $50,000–100,000, $100,000–500,000, $500,000–1,000,000, and ≥ $1,000,000. To verify the sensitivity of the results to the grouping of investors, I group investors into only 2 and 4 groups and report the results from these groupings.

First, investors are grouped into those trading in sizes less than $100,000 and those trading in sizes at least $100,000. The parameter estimates are $\tilde{q} = 0.58, \rho_1 = 100,$ and $\rho_2 = 288$. The estimated liquidity shocks in the corporate bond market are highly correlated with those in the main text with a correlation of 96%. The mean sample shock size is 23% which is slightly higher than the 19% in the main text. Total liquidity premium is 91 basis points on a 30y bond while the corresponding premium in the main text is 81 basis points.

Next, investors are sorted into four groups trading in sizes < $30,000, $30,000–$100,000, $100,000–700,000, and ≥ $700,000. Parameter estimates are $\tilde{q} = 0.70, \rho_1 = 91, \rho_2 = 131, \rho_3 = 226$ and $\rho_4 = 578$. Similar to the case of two investors, correlation between liquidity shocks in this case and shocks in the main text is very high at 97%
and the average liquidity shock is 23%. A 30y bond has a total liquidity premium of 91 basis points compared to 81 basis points in the text.

Overall, we see that the main results are insensitive to the grouping of investors: liquidity shocks are highly correlated with those in the main text, average shock sizes are fairly close, and liquidity premia are similar in magnitude.

E.3 Time-varying transaction costs

In the main text it is assumed that search intensities and thereby transaction costs are constant over time. If this is not the case, estimated liquidity shocks might be influenced by time-variation in transaction costs. To check whether this is the case, I do the following. The estimated equilibrium parameter $\tilde{\eta}$ is held fixed. For each month $m$ in the sample, I multiply all search intensities in Table 2 by $c_m$ and find the optimal value of $c_m$ according to the likelihood procedure explained earlier. If transaction costs are higher in month $m$ compared to average, $c_m$ will be lower than one and vice versa. Search intensities are now held fixed at $c_m$ times equilibrium search intensities. Then, the liquidity shock in month $m$ is estimated as explained in Section 5.2. This procedure takes into account time-variation in transaction costs when estimating liquidity shocks.

Estimated liquidity shocks are almost indistinguishable from those in the main text with a correlation of 99.9%, which shows that time-variation in trading costs does not influence estimation of liquidity shocks. The correlation between the time series of search intensities $c_1, ..., c_T$ and liquidity shocks $s_1, ..., s_T$ is modest at 13% showing that liquidity shocks and transaction costs measure different aspects of liquidity.
Figure 4: The model in equilibrium and after a liquidity shock. The top graph illustrates how trading occurs in equilibrium. The bottom graph illustrates how trading occurs when there is a high number of sellers, i.e., a liquidity shock.
Figure 5: Selling pressure in the corporate bond market. A fraction of investors with high liquidity in steady state are shocked, bid and ask yields immediately after the shock are derived, and for each month in the sample the estimated shock size is found as explained in the text. The fraction in percentage of investors shocked is given on the y-axis. The y-axis A 95% confidence interval for the shock size is bootstrapped and shown as dashed lines.
Figure 6: Selling pressure in GM/Ford bonds around their downgrade to junk. A fraction of investors with high liquidity in steady state are shocked, bid and ask yields immediately after the shock are derived, and for each month in the sample the estimated shock size is found as explained in the text. This is done for GM bonds (GM and GMAC), Ford bonds (F and FMCC), and the remaining bonds in the corporate bond market. The fraction in percentage of investors shocked is given on the y-axis.
Figure 7: Premium in yields due to search costs and occasional selling pressures. This graph shows the premium in yields across bond maturity due to search costs and occasional selling pressures. The search premium for an investor for a given maturity is defined as the average yield of a buy and sell transaction for this investor minus the average yield of a buy and sell transaction for an investor that can instantly find a trading partner. In the latter case, the buy and sell yields are identical. The premium due to selling pressure for an investor for a given maturity is defined as the average yield of a buy and sell transaction for this investor right after a shock occurs minus the average yield of a buy and sell transaction in the absence of a shock. Investors are assumed to have a search intensity of $\rho = 230$ and the number of investors being hit by a liquidity shock is 19%. 
Figure 8: *Liquidity premium in basis points before and after the credit crunch.* This graph shows the liquidity premium in yields across bond maturity for the two subperiods October 2004-July 2007 and August 2007-June 2009. The liquidity premium is the sum of premia due to search and due to occasional selling pressure. Investors are assumed to have a search intensity of $\rho = 230$ and the number of investors being hit by a liquidity shock in the first subperiod is 4% and 42% in the second period.
Figure 9: *BBB-Treasury spread and estimated liquidity premium*. The BBB rate is the monthly Moody’s seasoned Baa rate from The Federal Reserve’s Statistical Release. The maturity of the bonds included is between 20 and 30 years (see http://research.stlouisfed.org/fred2/data/BAA.txt). The corresponding Treasury rate is the 20-year CMT rate. The estimated liquidity premium is the total 20-year liquidity premium; the sum of search premium and selling pressure premium.
Figure 10: The effect of $\tilde{q}$ on the difference in midprices of sophisticated and unsophisticated investors. This graph shows the difference in midprices faced by sophisticated and unsophisticated investors as a function of $\tilde{q}$. The bond is a 10-year bond. The parameters are $\rho_{\text{unsoph}} = 107, \rho_{\text{soph}} = 353, \lambda_u = 1, \lambda_d = 0.1, \delta = 7, C = 7, \lambda_D = 0.02, r = 0.05, F = 100, f = 1, \rho_0 = 10$ and $z = 0.9$. 
Figure 11: Prices and liquidity shocks. The top graph shows bid and ask prices for a 10-year bond for an unsophisticated and sophisticated investor as a function of size of liquidity shock. The middle graph shows the difference in midprice for an unsophisticated investor and midprice for a sophisticated investor for the same 30-year bond. The bottom graph shows the percentage change in midprice for an unsophisticated and sophisticated investor as a function of bond maturity when 25% of investors experience a liquidity shock. The parameters are \( \rho_{\text{unsophisticated}} = 107, \rho_{\text{sophisticated}} = 353, \lambda_u = 1, \lambda_d = 0.1, \delta = 7, C = 7, \lambda_D = 0.02, r = 0.05, F = 100, f = 1, z = 0.9, \tilde{q} = 0.89 \).
<table>
<thead>
<tr>
<th>Panel A: URT and trade size</th>
<th>all</th>
<th>0-5K</th>
<th>6-10K</th>
<th>11-20K</th>
<th>21-50K</th>
<th>51-100K</th>
<th>101-200K</th>
<th>201-500K</th>
<th>501-1000K</th>
<th>&gt;1000K</th>
</tr>
</thead>
<tbody>
<tr>
<td>URT (in cent)</td>
<td>59</td>
<td>82</td>
<td>68</td>
<td>68</td>
<td>65</td>
<td>57</td>
<td>48</td>
<td>35</td>
<td>23</td>
<td>16</td>
</tr>
<tr>
<td>obs. (in 1000)</td>
<td>974</td>
<td>61</td>
<td>109</td>
<td>214</td>
<td>271</td>
<td>120</td>
<td>72</td>
<td>49</td>
<td>23</td>
<td>54</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Panel B: URT and maturity</th>
<th>all</th>
<th>0-0.5y</th>
<th>0.5-1y</th>
<th>1-2y</th>
<th>2-3y</th>
<th>3-4y</th>
<th>4-5y</th>
<th>5-7y</th>
<th>7-10y</th>
<th>10-30y</th>
</tr>
</thead>
<tbody>
<tr>
<td>URT (in cent)</td>
<td>59</td>
<td>26</td>
<td>34</td>
<td>44</td>
<td>48</td>
<td>57</td>
<td>57</td>
<td>68</td>
<td>72</td>
<td>103</td>
</tr>
<tr>
<td>obs. (in 1000)</td>
<td>974</td>
<td>37</td>
<td>77</td>
<td>145</td>
<td>126</td>
<td>126</td>
<td>114</td>
<td>118</td>
<td>120</td>
<td>110</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Panel C: number of URT trades out of total number of trades</th>
<th># trades</th>
<th>all</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5-6</th>
<th>7-8</th>
<th>9-10</th>
<th>11-15</th>
<th>16-20</th>
<th>21-30</th>
<th>31-40</th>
<th>41-50</th>
<th>51-100</th>
<th>&gt;100</th>
</tr>
</thead>
<tbody>
<tr>
<td>% URT</td>
<td>22</td>
<td>0</td>
<td>46</td>
<td>43</td>
<td>33</td>
<td>32</td>
<td>29</td>
<td>27</td>
<td>24</td>
<td>21</td>
<td>18</td>
<td>15</td>
<td>13</td>
<td>11</td>
<td>6</td>
<td></td>
</tr>
<tr>
<td>bond days (in 1000)</td>
<td>1437</td>
<td>341</td>
<td>302</td>
<td>175</td>
<td>115</td>
<td>141</td>
<td>84</td>
<td>57</td>
<td>84</td>
<td>44</td>
<td>42</td>
<td>20</td>
<td>11</td>
<td>16</td>
<td>6</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Panel D: volume of URT trades out of total volume</th>
<th># trades</th>
<th>all</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5-6</th>
<th>7-8</th>
<th>9-10</th>
<th>11-15</th>
<th>16-20</th>
<th>21-30</th>
<th>31-40</th>
<th>41-50</th>
<th>51-100</th>
<th>&gt;100</th>
</tr>
</thead>
<tbody>
<tr>
<td>% URT</td>
<td>13</td>
<td>0</td>
<td>19</td>
<td>14</td>
<td>14</td>
<td>14</td>
<td>14</td>
<td>13</td>
<td>13</td>
<td>13</td>
<td>13</td>
<td>12</td>
<td>12</td>
<td>12</td>
<td>12</td>
<td>12</td>
</tr>
<tr>
<td>bond days (in 1000)</td>
<td>1437</td>
<td>341</td>
<td>302</td>
<td>175</td>
<td>115</td>
<td>141</td>
<td>84</td>
<td>57</td>
<td>84</td>
<td>44</td>
<td>42</td>
<td>20</td>
<td>11</td>
<td>16</td>
<td>6</td>
<td></td>
</tr>
</tbody>
</table>

Table 1: Summary statistics. Panel A shows average URT in cents as a function of trade size. For example, there are 61,000 observations of URTs for trade sizes between 0 and $5,000 and the average URT is 82 cents. Panel B shows average URT in cents as a function of bond maturity. In Panel C and D a bond day is defined as a day for a bond where there is at least one trade, and the bond days are sorted according to the number of trades occurring on that day. Out of the total number of bond trades on a bond day, Panel C shows the fraction of trades that is part of a URT. For example, there are 302,000 observations of a bond trading two times in a day and out of the 604,000 transactions 46% are part of a URT. Panel D shows the fraction of volume that is part of a URT. For example, there are 302,000 observations of a bond trading two times in a day, and out of the total volume of the 604,000 transactions 19% is part of a URT. The sample bonds are straight coupon bullet bonds and the sample period is October 1, 2004 to June 30, 2009.
Table 2: Parameter estimates for a search model for the corporate bond market. This table shows the estimated parameters of the search model presented in Section 3. The model parameters are estimated by maximum likelihood and standard errors are calculated using the outer product of gradients estimator. Corporate bond data used in estimation are actual transactions from TRACE for the subperiod October 1, 2004 to July 31, 2007.

<table>
<thead>
<tr>
<th>$\hat{q}$</th>
<th>$\rho_1$</th>
<th>$\rho_2$</th>
<th>$\rho_3$</th>
<th>$\rho_4$</th>
<th>$\rho_5$</th>
<th>$\rho_6$</th>
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<tbody>
<tr>
<td>0.889</td>
<td>69</td>
<td>108</td>
<td>142</td>
<td>208</td>
<td>378</td>
<td>474</td>
</tr>
<tr>
<td>(0.014)</td>
<td>(0.3)</td>
<td>(0.4)</td>
<td>(1.6)</td>
<td>(4.1)</td>
<td>(38.2)</td>
<td>(44.3)</td>
</tr>
</tbody>
</table>
Panel A: Trade size

<table>
<thead>
<tr>
<th></th>
<th>0-5K</th>
<th>6-10K</th>
<th>11-20K</th>
<th>21-50K</th>
<th>51-100K</th>
<th>101-200K</th>
<th>201-500K</th>
<th>501-1000K</th>
<th>&gt;1000K</th>
</tr>
</thead>
<tbody>
<tr>
<td>fitted (in bps)</td>
<td>73.2</td>
<td>56.9</td>
<td>50.7</td>
<td>48.1</td>
<td>35.1</td>
<td>28.1</td>
<td>25.3</td>
<td>14.2</td>
<td>12.8</td>
</tr>
<tr>
<td></td>
<td>(0.0)</td>
<td>(0.0)</td>
<td>(0.0)</td>
<td>(0.0)</td>
<td>(0.0)</td>
<td>(0.0)</td>
<td>(0.0)</td>
<td>(0.0)</td>
<td>(0.0)</td>
</tr>
<tr>
<td>actual (in bps)</td>
<td>87.1</td>
<td>68.6</td>
<td>67.0</td>
<td>59.3</td>
<td>49.0</td>
<td>34.9</td>
<td>22.3</td>
<td>14.2</td>
<td>7.5</td>
</tr>
<tr>
<td></td>
<td>(0.4)</td>
<td>(0.2)</td>
<td>(0.2)</td>
<td>(0.2)</td>
<td>(0.3)</td>
<td>(0.3)</td>
<td>(0.2)</td>
<td>(0.2)</td>
<td>(0.1)</td>
</tr>
</tbody>
</table>

Panel B: Maturity

<table>
<thead>
<tr>
<th></th>
<th>0-2m</th>
<th>2m-4m</th>
<th>4-6m</th>
<th>6m-1y</th>
<th>1-3y</th>
<th>3-5y</th>
<th>5-30y</th>
</tr>
</thead>
<tbody>
<tr>
<td>fitted (in bps)</td>
<td>26.3</td>
<td>36.9</td>
<td>41.3</td>
<td>44.7</td>
<td>48.1</td>
<td>47.6</td>
<td>48.0</td>
</tr>
<tr>
<td></td>
<td>(0.1)</td>
<td>(0.1)</td>
<td>(0.1)</td>
<td>(0.1)</td>
<td>(0.0)</td>
<td>(0.0)</td>
<td>(0.0)</td>
</tr>
<tr>
<td>actual (in bps)</td>
<td>23.4</td>
<td>25.1</td>
<td>27.4</td>
<td>32.5</td>
<td>45.7</td>
<td>55.8</td>
<td>76.6</td>
</tr>
<tr>
<td></td>
<td>(0.6)</td>
<td>(0.4)</td>
<td>(0.4)</td>
<td>(0.2)</td>
<td>(0.1)</td>
<td>(0.2)</td>
<td>(0.2)</td>
</tr>
</tbody>
</table>

**Table 3:** Estimated round-trip costs. This table reports the fitted roundtrip costs in basis points of par value for the search model presented in Section 3. Parameters in the model are those in Table 2. Below the fitted roundtrip costs are actual roundtrip costs. Corporate bond data used in estimation as well as in calculation of actual roundtrip costs are transactions from TRACE for the period October 1, 2004 to July 31, 2007.
### Panel A: URT vs buy/interdealer/sell

<table>
<thead>
<tr>
<th>Trade Size</th>
<th>B-S</th>
<th>B-D</th>
<th>D-S</th>
<th>B</th>
<th>S</th>
<th>D</th>
</tr>
</thead>
<tbody>
<tr>
<td>All</td>
<td>4.0%</td>
<td>19.2%</td>
<td>70.5%</td>
<td>0.2%</td>
<td>5.3%</td>
<td>0.8%</td>
</tr>
<tr>
<td>Trade size ≤ 10K</td>
<td>3.2%</td>
<td>19.1%</td>
<td>75.8%</td>
<td>0.2%</td>
<td>1.3%</td>
<td>0.5%</td>
</tr>
<tr>
<td>10K &lt; trade size ≤ 50K</td>
<td>2.0%</td>
<td>19.3%</td>
<td>76.0%</td>
<td>0.1%</td>
<td>1.9%</td>
<td>0.6%</td>
</tr>
<tr>
<td>50K &lt; trade size ≤ 100K</td>
<td>3.1%</td>
<td>19.7%</td>
<td>71.8%</td>
<td>0.1%</td>
<td>4.3%</td>
<td>0.9%</td>
</tr>
<tr>
<td>100K &lt; trade size ≤ 500K</td>
<td>5.9%</td>
<td>19.7%</td>
<td>58.0%</td>
<td>0.1%</td>
<td>14.8%</td>
<td>1.5%</td>
</tr>
<tr>
<td>500K &lt; trade size ≤ 1,000K</td>
<td>16.9%</td>
<td>18.3%</td>
<td>30.3%</td>
<td>0.4%</td>
<td>32.4%</td>
<td>1.7%</td>
</tr>
<tr>
<td>1,000K &lt; trade size</td>
<td>24.8%</td>
<td>15.6%</td>
<td>21.7%</td>
<td>0.5%</td>
<td>35.6%</td>
<td>1.8%</td>
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</tbody>
</table>

### Panel B: Correlation between price differences and liquidity shocks

<table>
<thead>
<tr>
<th></th>
<th>Small ≤ 100K</th>
<th>Small ≤ 24K</th>
<th>Small ≤ 24K</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bid weekly</td>
<td>35%</td>
<td>35%</td>
<td>43%</td>
</tr>
<tr>
<td>Ask weekly</td>
<td>60%</td>
<td>64%</td>
<td>62%</td>
</tr>
<tr>
<td>Bid monthly</td>
<td>53%</td>
<td>64%</td>
<td>65%</td>
</tr>
<tr>
<td>Ask monthly</td>
<td>27%</td>
<td>69%</td>
<td>63%</td>
</tr>
<tr>
<td>Observations</td>
<td>Bid 35,164</td>
<td>39,447</td>
<td>25,338</td>
</tr>
<tr>
<td></td>
<td>Ask 49,434</td>
<td>64,722</td>
<td>37,516</td>
</tr>
</tbody>
</table>

**Table 4:** *URT* and liquidity shocks versus bid/ask/interdealer transactions. The sample period in the paper is October 2004-June 2009. For the subperiod November 2008-June 2009 TRACE has a bid/ask/interdealer indicator for each transaction. In the subperiod, Panel A shows how many percent of URTs are buy-sell, buy-dealer, sell-dealer, sell-sell, buy-buy, interdealer-interdealer transactions. Panel B shows how estimated liquidity shocks are correlated with the difference between bid prices of small and large trades and ask prices of small and large trades. First column uses 100,000$ in face value as a cutoff between small and large prices. Second column uses 24,000$. In the third column small trades have face value smaller than 24,000$, large trades greater than 100,000$, and trades in-between are discarded. This is done both on a monthly (8 months) and weekly (35 weeks) basis.