A model of time-varying risk premia with habits and production*

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Abstract

This paper develops a new utility specification that incorporates Campbell–Cochrane–type habits into the Epstein–Zin class of preferences. In a simple calibration of a real business cycle model with EZ-habit preferences, the model generates a strongly countercyclical equity premium, substantial equity return predictability, and a stable riskless interest rate, as in the data. Moreover, conditional on the average level of risk aversion, time-variation in risk aversion increases the volatility and mean return of equities. On the real side, the model matches the short and long-term variances of output, consumption, and investment growth. As an additional empirical test, I measure implied risk aversion and find that it has an $R^2$ of over 50 percent for 5-year stock returns in post-war data. Variables that predict stock returns in the data also predict returns in the model with a similar degree of explanatory power.

1 Introduction

Stock prices are more volatile than can be explained by movements in expected dividends. Moreover, excess returns on the aggregate stock market are predictable over time. The two phenomena are connected: changes in the discount rates applied to future dividends can induce excess volatility in asset prices. This paper develops a new preference specification with time-varying risk aversion that generates realistically predictable and volatile stock returns. When combined with a production framework, the model can match

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the short and long-run volatilities of output, consumption, and investment growth and at the same time generate a high and volatile price of risk. Simulated stock-return forecasting regressions are consistent with empirical results, and the model also delivers a new method for forecasting stock returns. The structural estimate of risk aversion has an $R^2$ for 5-year stock returns in the post-war period of over 50 percent.

The standard model of time-varying risk aversion is the habit specification of Campbell and Cochrane (1999). In their model, when an agent’s consumption falls close to her habit, her risk aversion rises. Using aggregate consumption data, they find that their implied risk aversion measure can explain a large proportion of the movements in the price-dividend ratio on the stock market. Campbell and Cochrane study an endowment economy, though, so they never test whether their utility function generates a realistic consumption process in equilibrium. In fact, Lettau and Uhlig (2000) and Rudebusch and Swanson (2008) find that Campbell–Cochrane preferences imply that consumers smooth consumption growth extremely and implausibly strongly following technology shocks in standard general-equilibrium models.

This paper embeds the intuition behind Campbell and Cochrane (1999)—that persistent external habits can induce time-varying risk aversion—into the framework developed by Kreps and Porteus (1978), Epstein and Zin (1989), and Weil (1989). The Epstein–Zin specification allows us to model risk aversion and intertemporal substitution separately, while the Campbell–Cochrane intuition motivates time-variation in risk aversion. In particular, consumers are modeled as having a time-varying external habit, which is a benchmark to which they compare their own lifetime utility. When lifetime utility is farther above the benchmark, risk aversion over proportional shocks to future welfare is lower. By explicitly separating variation in risk aversion from intertemporal substitution, the Epstein–Zin framework eliminates the problems that arise when standard Campbell–Cochrane preferences are used in a production setting. I refer to the new preference specification as the EZ-habit model for its combination of these two frameworks.

The simple real business cycle (RBC) model with fixed labor supply provides a transparent laboratory in which to study the effects of time-variation in risk aversion on the macroeconomy in general equilibrium. I find that the dynamics of real variables and real interest rates under the EZ-habit specification are highly

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1 A partial selection of other early papers studying habit formation is Abel (1990), Constantinides (1990), Boldrin, Christiano, and Fisher (2001), and Jermann (1998).

For other papers that study return predictability in a production setting, see Gourio (2010), Campanale, Castro, and Clementi (2010), and Guvenen (2009), though note that the latter two papers do not match the degree of predictability observed in the data.

2 Melino and Yang (2003) study a utility specification that is highly similar to mine in reduced form. However, they do not discuss the inclusion of a habit, and they do not insert the preferences into a production setting.
similar to a model with Epstein–Zin utility and constant relative risk aversion. The model can match both the short and long-run variances of output, investment, and consumption.

Since consumption and wealth are cointegrated under balanced growth, their long-run variances must be the same. But empirically, the short-run (quarterly) variance of consumption growth is much smaller than the variance of changes in wealth. To match both the long and short-run moments, a model must have either mean-reversion in wealth or strong persistence in consumption growth. A number of recent asset-pricing papers (e.g. Bansal and Yaron, 2004, Kaltenbrunner and Lochstoer, 2010) have gone the route of choosing very strong persistence for consumption growth. In Kaltenbrunner and Lochstoer’s (2010) analysis of asset prices in the RBC model, for example, innovations to the permanent component of consumption have a standard deviation of 8 percent per year, which is at odds with the data. The EZ-habit model, on the other hand, implies that consumption is roughly a random walk—the short and long-run variances are nearly equal—but wealth is mean-reverting: declines in risk aversion raise current asset prices and lower expected returns. Whereas other papers in the production-based asset-pricing literature do not check the fit of their models to the long-run variance of consumption and output, I show that the EZ-habit model can match this moment along with the short-run variances.

In addition to matching macro moments, the EZ-habit model improves the fit of the RBC model to financial moments. Previous habit-based models designed to generate high or volatile risk premia tended to have implausibly volatile interest rates, a flaw not found here. The reasonable behavior of interest rates is an important innovation of this paper; the EZ-habit model is able to have stable interest rates but still generate substantial asset price volatility because it has variation in discount rates on risky assets that is driven by variation in risk aversion. Movements in discount rates imply that asset returns should be predictable, and extensive tests show that the degree of predictability in the model is similar to what is observed in the data. Variation in risk aversion not only raises the volatility of asset returns, I find that it also makes the equity premium roughly 1/3 larger on average than it would be otherwise. Countercyclical movements in risk aversion thus increase both the quantity and price of risk in financial markets: good times seem even better and bad times worse.


There are numerous empirical methods of forecasting stock returns, but the majority of them are not based on equilibrium theories. For example, regressions of stock returns on price-dividend ratios are motivated simply by an identity that links the price-dividend ratio to future returns and dividend growth. Under the EZ-habit model, though, it turns out to be possible to directly measure risk aversion. As is standard in the habit literature, I assume that positive innovations to household welfare reduce risk aversion. So if we can measure welfare, we can also measure risk aversion. Under Epstein–Zin preferences with a constant elasticity of intertemporal substitution, welfare is a function of current household wealth and consumption. And this result holds generally; it is not dependent on the RBC model I analyze. Using data on consumption and wealth, I construct an empirical estimate of risk aversion and show that it is a strong forecaster of aggregate stock returns: it outperforms the price-dividend ratio, Lettau and Ludvigson’s (2001) measure of the consumption-wealth ratio, and Campbell and Cochrane’s (1999) excess consumption ratio. This result differentiates my paper from models of time-varying disaster risk because it does not rely on an unobservable latent process to drive risk premia.6

The model also can match forecasting results for consumption growth. Lettau and Ludvigson (2001) find little ability to forecast consumption growth using their measure of the consumption-wealth ratio. Campbell and Shiller (1988) obtain similar results for the stock market. As in the empirical data, it is essentially impossible to forecast consumption growth in the EZ-habit model using the consumption-wealth ratio, but forecasts of risk premia are highly effective.

An alternative way to forecast consumption growth is with interest rates. Hall (1988) and Campbell and Mankiw (1989), in trying to estimate the elasticity of intertemporal substitution (EIS), essentially ask whether consumption growth can be forecasted with interest rates. They find little forecasting power, suggesting the representative household has a small or even zero EIS. In this paper, the EIS is set to 1.5, but I still replicate the regression results from Hall (1988) and Campbell and Mankiw (1989). The EZ-habit model explains the failure of those regressions through a time-varying precautionary-saving effect. When risk aversion is high, households want to save more to protect themselves against future shocks, which drives interest rates downward. This effect biases standard Euler-equation estimation based on models with constant relative risk aversion.

After testing the model’s fit to macro and asset pricing moments and the predictions for the EIS regressions and return forecasting, I consider two extensions to the model. First, I examine the effect of

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6See Gourio (2010), and Wachter (2010), for recent models with time-varying disaster risk.
time-varying risk aversion on labor supply. Following positive technology shocks, risk aversion falls, raising consumption (through a decline in precautionary saving demand). This effect also lowers the response of labor supply to technology shocks. Intuitively, intratemporal optimization means that when households are willing to spend more money to raise consumption, they are also willing to sacrifice in terms of opportunity costs to raise leisure. Endogenous labor supply has little effect on risk premia in the economy, though. The reason is simply that under Epstein–Zin preferences with a high elasticity of intertemporal substitution, the volatility of the stochastic discount factor is driven mainly by the permanent component of consumption; so even if households smooth consumption growth by varying labor supply, the total amount of risk in the economy is essentially unchanged.

The second extension is a log-linearization of the model using methods similar to Campbell (1994) and Lettau (2003). Unlike standard perturbation methods, the log-linearization used here does not impose certainty equivalence, so we can obtain expressions that take into account potentially time-varying risk premia even in the first-order approximation. I am able to derive explicit expressions for the Sharpe ratio in the model as a function of current risk aversion and the underlying parameters of the model and find that the results are highly similar to those from accurate numerical solutions. Much of the previous production-based asset-pricing literature has focused on simulations to study the implications of various models, so this paper introduces an important methodological contribution in extending and simplifying the analytic results of Campbell (1994) and Lettau (2003). Further, in the case where risk aversion is constant, I give an analytic characterization of how endogenous consumption smoothing generates long-run risks in a production setting (Bansal and Yaron, 2004; Kaltenbrunner and Lochstoer, 2010). The log-linearization thus provides an analytic explanation for results that were previously supported only with simulation-based evidence.

The log-linear solution returns a stochastic discount factor (SDF) that takes on the essentially affine form that is widely used in the empirical asset-pricing literature. This is possibly the first paper to derive an essentially affine SDF with a time-varying price of risk from a production-based model. It thus connects the standard modeling framework in macroeconomics with one of the most widely used asset-pricing specifications in empirical finance.

The paper is organized as follows. Section 2 discusses the preference specification and lays out the economic environment. Section 3 calibrates a production economy and compares its behavior to the data. Section 4 tests the empirical implications of the model for return forecasting, and section 5 studies
extensions to the basic framework. Section 6 concludes.

2 The model

2.1 Household preferences

For households with a constant elasticity of intertemporal substitution (EIS), Epstein–Zin (1989) utility can be expressed as

\[ V_t = \left\{ (1 - \exp(-\beta)) C_t^{1-\rho} + \exp(-\beta) \left[ G_t^{-1}(E_t[G_t(V_{t+1})]) \right]^{1-\rho} \right\}^{1/(1-\rho)} \]  (1)

for some function \( G_t \), where \( C_t \) is household consumption and \( E_t \) is the expectation operator conditional on information available at date \( t \).\(^7\) The term \( G_t^{-1}(E_t[G_t(V_{t+1})]) \) is a certainty equivalent. When there is no uncertainty about \( V_{t+1} \), \( G_t^{-1}(E_t[G_t(V_{t+1})]) = V_{t+1} \). The usual choice for \( G_t \) (going back to Weil, 1989, and Epstein and Zin, 1991) is power utility,

\[ G_t^{Power}(V_{t+1}) = V_{t+1}^{1-\alpha} \]  (2)

Epstein and Zin (1989) show that the coefficient of relative risk aversion for a household with preferences of the form (1) is equal to the coefficient of relative risk aversion for \( G_t \), while the EIS is equal to \( 1/\rho \).

Now consider a habit-formation utility function for \( G_t \),

\[ G_t^{Habit}(V_{t+1}; H_t) = (V_{t+1} - H_t)^{1-\alpha} \]  (3)

Value functions involving \( G_t^{Habit} \) are related to those using \( G_t^{Power} \) in the same way that usual habit specifications, e.g. Constantinides (1991), are related to time-separable power utility. Rather than caring only about the absolute level of their continuation value, \( G_t^{Habit} \) says that households care about the spread between tomorrow’s value and a benchmark \( H_t \). Since the utility function adds a habit to Epstein–Zin, I refer to it as the EZ-habit specification.\(^8\) I refer to the version of \( V_t \) using \( G_t^{Power} \) for the certainty

\(^7\)The preferences can be further generalized to study alternative time aggregators, instead of the constant elasticity of substitution form.

\(^8\)Other papers, for example Rudebusch and Swanson (2010) and Yang (2008), incorporate consumption habits into Epstein–Zin preferences. That is, the \( C_t^{1-\rho} \) term is replaced by \((C_t - X_t)^{1-\rho}\) where \( X_t \) is the habit. Rudebusch and Swanson (2008) show that in general equilibrium this does not lead to a time-varying Sharpe ratio because households endogenously smooth
equivalent as canonical Epstein–Zin in deference to its popularity in the literature.

The coefficient of relative risk aversion for $G_t^\text{Habit}$ is equal to $\alpha \frac{V_{t+1}}{V_{t+1} - H_t}$. As the spread between value and habit rises, the coefficient of relative risk aversion falls. Intuitively, when the continuation value falls close to its benchmark, proportional shocks to $V_{t+1}$ loom much larger than when the household has a cushion between its continuation value and $H_t$.

In principle it is possible to analyze a model with $G_t^\text{Habit}$, but it has three important drawbacks. First, if the support of the shocks to $V_{t+1}$ is sufficiently wide, there is a non-zero probability that $V_{t+1}$ will fall below $H_t$, leaving the certainty equivalent undefined.\(^9\) Second because $G_t^\text{Habit}$ is not log-linear in $V_{t+1}$, obtaining simple analytic results with it is difficult or impossible. Third, because $G_t^\text{Habit}$ is not log-linear, standard arguments for the existence of a representative agent do not apply.\(^10\)

For the remainder of the paper I therefore replace $G_t^\text{Habit}$ with the alternative

$$G_t^{TV}(V_{t+1}) = V_{t+1}^{1-\alpha_t}$$

$$\alpha_t = \alpha \frac{V_t}{V_t - H_t} \quad (4)$$

$G_t^{TV(-1)}(E_tG_t^{TV}(V_{t+1}))$ (where TV stands for time-varying) is a second-order approximation to $G_t^{\text{Habit}(-1)}(E_tG_t^\text{Habit}(V_{t+1}))$ around the non-stochastic version of the model.\(^11\) Moreover, the appendix shows that in the continuous-time limit (i.e. under stochastic differential utility), preferences with $G_t^{TV}$ are exactly equivalent to preferences using $G_t^\text{Habit}$.\(^12\) $G^{TV}$ is locally equivalent to $G^\text{Habit}$ in terms of risk preferences, but it solves the problems of integrability inside the certainty equivalent and the existence of a representative agent.

As in Campbell and Cochrane (1999), I assume that households take the excess value ratio, $\frac{V_t}{V_t - H_t}$, and consumption to reduce their overall risk exposure. That said, the specification in Rudebusch and Swanson (2008) is meant to generate smooth consumption growth rather than a high risk premium. In principle, there is no reason that this type of habit formation could not be added to the EZ-habits model to help generate smoother consumption (e.g. to help explain the excess smoothness puzzle of Campbell and Deaton, 1989). Dew-Becker (2011) studies preferences with both time-varying risk aversion and consumption habits in a medium-scale DSGE model.

\(^9\)This issue also arises in other habit specifications. When models are solved with standard perturbation methods, the problem is simply ignored. I use a more precise global numerical solution technique that forces me to grapple with the problem.

\(^10\)A representative agent may exist, but their preferences need not actually look like the preferences of any particular agent. Ideally, if every agent has identical preferences, the representative agent will also have those preferences.

\(^11\)More precisely, the second-order approximation also assumes no growth. Adding a constant growth rate $\mu$ to $V$ would change the result to $\alpha_t = \alpha \frac{(1+\mu)V_t}{(1+\mu)V_t - H_t}$. The remainder of the analysis is identical.

\(^12\)Melino and Yang (2003) study a utility function with the same form as $G^{TV}$, but they take $\alpha_t$ as a latent variable and give no theoretical motivation for its variation. This paper is original for proposing inserting habits into the certainty-equivalent part of Epstein–Zin preferences to motivate movements in $\alpha_t$. 

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hence the coefficient of relative risk aversion, $\alpha_t$, as external to their own decisions. The final step, then, is to specify a dynamic process for risk aversion. I assume a simple log-linear process, which we will find to be highly tractable,

$$\alpha_{t+1} = \phi \alpha_t + (1 - \phi) \bar{\alpha} + \lambda (\Delta v_{t+1}^A - E_t \Delta v_{t+1}^A)$$

(5)

where $v_t^A$ is the log of $V_t$ for the representative agent. Intuitively, when value unexpectedly rises, it moves away from the habit and risk aversion falls, so $\lambda < 0$. Movements in the habit, and hence risk aversion, depend on aggregate value so that they are not affected by an individual household’s decisions. The AR(1) specification for risk aversion is approximately equivalent to a specification where $\log H_t$ is a geometrically weighted moving average of past values of $v_t^A$.

The appendix shows how to derive the marginal rate of intertemporal substitution (the stochastic discount factor, or SDF) for the general form of Epstein–Zin preferences in (1). In the case of $G^{TV}$, we end up with the expression,

$$M_{t+1} \equiv \frac{\partial V_t}{\partial C_{t+1}} = \exp(-\beta) \frac{V_t^{\rho-\alpha_t}}{(E_t V_t^{\rho-\alpha_t})^{\frac{1}{\rho-\alpha_t}}} \frac{C_t^{\rho}}{C_{t+1}^{\rho}}$$

(6)

with the only difference from the SDF under canonical Epstein–Zin preferences being the subscript on $\alpha_t$. The SDF is a critical piece of the model since its volatility determines the price of risk in the economy. As usual, changes in expected consumption growth or volatility will affect the SDF through their effects on $V_{t+1}$. Changes in $\alpha_{t+1}$ (or $H_{t+1}$) will also affect the SDF in the same way. Specifically, when the habit rises and households are more risk averse, they penalize consumption uncertainty more, driving $V_{t+1}$ down. High risk-aversion states thus have high Arrow-Debreu prices.

It is also straightforward to derive the standard result that

$$W_t = V_t^{1-\rho} C_t^\rho / (1 - \exp(-\beta))$$

(7)

where $W_t$ is the equilibrium price of a claim on the household’s consumption stream, which I refer to as the aggregate wealth portfolio. This formula holds regardless of whether risk aversion varies over time. Intuitively, the market price of the consumption stream is equal to the utility value that a household places

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13It is straightforward to derive the actual process that $H_t$ must follow in order for risk aversion to follow the process in (5).

14Hansen and Jagannathan (1991) show that the maximum Sharpe ratio (expected excess return divided by standard deviation) attained by any asset in the economy is equal to the standard deviation of the SDF divided by its mean.
on it, $V_t$, divided by the marginal utility of consumption, $V_t^\rho C_t^{-\rho} (1 - \exp (- \beta))$. This leads to the familiar result from Epstein and Zin (1991),

$$M_{t+1} = \exp (- \beta)^{\frac{1-\alpha_t}{1-\rho}} \left( \frac{C_{t+1}}{C_t} \right)^{-\rho \frac{1-\alpha_t}{1-\rho}} \frac{\rho - \alpha_t}{R_{w,t+1}} \tag{8}$$

where $R_{w,t+1}$ is the return on the wealth portfolio.

2.2 Discussion

The model is motivated as an extension of habit-based preferences. Rather than consumers having a habit level of consumption that they target, I assume they have a habit level of value. Since equation (7) shows that there is a direct link between value and wealth, we could also think of the model as saying that households have a benchmark level of wealth. The house-money effect of Thaler and Johnson (1990) has a somewhat similar intuition. They find that when subjects in lab experiments have recently gained money in betting games, they play more aggressively.\footnote{Barberis, Huang, and Santos (2001) embed the house-money effect in a full asset-pricing model. See Gertner (1993) and Post et al. (2008) for evidence on the house-money effect from game shows.}

Abel (1990) interprets habits in consumption as a "keeping up with the Joneses" effect. That intuition extends to the EZ-habit model. What households try to keep up with in this model, though, is fundamentally different. For example, consider a college senior who is trying to decide between following her friends into consulting or getting a law degree. With the J.D., she knows that in the short run her consumption will be lower than that of her friends, but in the long run she will likely be better off. In a model with an external consumption habit, three years of consumption below that of her friends looks painful. But when the habit appears as a function of value, the student is comfortable giving up consumption in the short run as long as she knows she will do well compared to her friends in the long run. Since the habit appears only in the risk aggregator, an agent with EZ-habit preferences is willing to substitute consumption over time in a way that an agent with standard habit-forming preferences is not. For the same reason, the EZ-habit model is not inconsistent with the mixed evidence on the effects of classic consumption habits at the micro level (e.g. Dynan, 2000, and Ravina, 2007).

There are a number of papers that use investment choices to measure variation in risk aversion. Carroll (2002) finds that households with higher wealth tend to tilt their investment portfolios towards more risky
assets. Brunnermeier and Nagel (2008), though, argue that there is little evidence that changes in wealth affect portfolio choices in household data. Rather, they find that inertia is the dominant characteristic of household portfolio choice. Calvet, Campbell, and Sodini (2009), after controlling for the inertia studied by Brunnermeier and Nagel, find a strong and significant relationship between innovations to wealth and the riskiness of a household’s portfolio.\textsuperscript{16} Furthermore, they show that weakness in the instruments for wealth shocks can cause a researcher to erroneously find that wealth does not affect risk-taking. Calvet and Sodini (2010) show that higher past income, controlling for current wealth and genetic differences in risk attitudes, is also negatively related to the share of household portfolios invested in risky assets. On net, with the notable exception of Brunnermeier and Nagel (2008), the empirical literature supports the idea that increases in wealth reduce risk aversion.

\subsection*{2.3 Production}

Aggregate output is a function of the capital stock, $K_t$, and productivity $A_t$

$$Y_t = A_t^{1-\gamma} K_t^\gamma$$

In section 5.3 I add endogenous labor supply and show that it does not substantially change the dynamics of the model. The production function (9) can be thought of Cobb–Douglas with labor supply held fixed at unity.

The aggregate resource constraint is

$$K_{t+1} = (1 - \delta) K_t + Y_t - C_t$$

where $\delta$ is the depreciation rate of capital.

\textsuperscript{16}See also Tanaka, Camerer, and Nguyen (2010), who find that income, both its raw level and instrumented for with exogenous shocks, has a negative impact on loss aversion, and Guiso, Sapienza, and Zingales (2011) who find that following the financial crisis of 2008, households both reduced the risky shares of their portfolios and became more averse to gambles in survey questions.
For the benchmark calibration, productivity follows a random walk in logs,\(^{17}\)

\[
\log A_{t+1} = \log A_t + \mu + \sigma_a \varepsilon_{t+1} \\
\varepsilon_{t+1} \sim \mathcal{N}(0, 1)
\]

The drawback of using random-walk technology is that it is difficult to generate the degree of volatility for output and investment that is observed in the data.\(^{18}\) I therefore also consider a dual-shock version of the model that can match both the short and long-run variances of output,

\[
\begin{align*}
A_t &= \bar{A}_t X_t \\
\log \bar{A}_{t+1} &= \log \bar{A}_t + \mu + \sigma_a \varepsilon_{t+1} \\
\log X_{t+1} &= \phi_a \log X_t + \sigma_x \varepsilon_{x,t+1} \\
\varepsilon_{t+1}, \varepsilon_{x,t+1} &\sim i.i.d. \mathcal{N}(0, 1)
\end{align*}
\]

\(\bar{A}_t\) here is the permanent component of output, while \(X_t\) can be interpreted as a simple method of trying to capture forces that drive short-run fluctuations in output and consumption, e.g. shocks to monetary policy or energy prices. I refer to the version of the model with random-walk technology as the benchmark model, while the model with permanent and temporary technology shocks is the dual-shock model.

\section*{3 Calibration and simulation}

I solve the model with projection methods, which entails fitting a polynomial approximation to the decision rule and searching for coefficients so that the equilibrium conditions hold exactly at certain specified points

\(^{17}\)An alternative is a trend-stationary process for productivity. Alvarez and Jermann (2005) argue that permanent shocks to the level of productivity (more generally, to the level of state prices) are necessary to explain asset-pricing facts. Also, in models with Epstein–Zin preferences, because the SDF depends not only on current consumption but also on the level of the value function itself, an I(1) process for productivity tends to increase the volatility of the SDF compared to models with trend-stationary productivity, which helps explain the equity premium. Kaltenbrunner and Lochstoer (2010) find that in order to match the empirical equity premium in a model with trend-stationary productivity, their model needs an implausibly small EIS (0.05). With difference-stationary productivity they are able to choose a more reasonable value (1.5).

\(^{18}\)In particular, without a mean-reverting component, it is impossible for the model to replicate the result from Cochrane (1994) that the long-run variance of output is smaller than the unconditional variance. In the RBC model, output does not overshoot its long-run trend following a permanent increase in technology: it does not have a mean-reverting component to its dynamics. Because they rely only on permanent shocks in the RBC model, Kaltenbrunner and Lochstoer (2010) have to set the annual standard deviation of technology shocks to an implausibly high 8.2 percent per year to match the unconditional standard deviation of output growth.
in the state space. The Euler equation errors in the simulations imply households misprice a claim on capital by uniformly less than 1/100th of 1 basis point (i.e. one part in one million) over the range of the state space that the simulations visit, and the median simulated error is an order of magnitude smaller.

The model is parameterized to match quarterly data. Table 1 lists the parameter values and the target moments. Many of the parameters, e.g. the exponent on capital in the production function, take standard values. I discuss here the parameters that are unique to this paper or do not have standard and agreed-upon values.

I set $\rho = 2/3$ as in Bansal and Yaron (2004), for an EIS of 1.5. Bansal and Yaron note that an EIS greater than 1 is necessary for increases in volatility to lower asset prices (specifically, the wealth-consumption ratio) in an endowment economy. In a production economy this result does not hold exactly (because consumption is endogenous), but it is approximately true. Similarly, an EIS greater than 1 ensures that increases in risk aversion increase the expected return on the wealth portfolio and lower its current price. Many studies attempting to estimate the EIS have obtained values much smaller than 1 (Hall, 1988; Campbell and Mankiw, 1989). An important test of the model will be whether it can match that result even though the calibrated EIS is larger than 1.

I choose the variance of permanent innovations to technology to match the long-run variance of consumption growth in the data. Since technology and consumption are cointegrated in the model, the long-run variance of consumption growth is equal to the variance of the permanent technology shocks. I estimate the empirical long-run variance (i.e. the spectral density at frequency zero) of consumption growth with a third-order univariate AR model (where the lag length was selected with the Bayesian information criterion) and obtain a value of 0.0088. That is, the quarterly innovations to the permanent component of consumption have a standard deviation of 0.88 percent. For the dual-shock model, I select the parameters $\sigma_x$ and $\phi_x$ to match the short-run volatility of consumption and output growth. The parameters imply that the temporary component of technology has an unconditional standard deviation of 2.7 percent.

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19 See Caldara et al. (2009) for a good description of the method as applied to models with recursive utility. When solving the RBC model with Epstein–Zin preferences, they find that projection methods are orders of magnitude more accurate than the perturbation methods used in the majority of the macro literature.

20 Intuitively, an increase in risk aversion or volatility has two effects – it lowers the risk-free rate and raises the excess return on the wealth portfolio. Which of these effects dominates depends on the EIS.

21 By choosing a smaller value for the long-run variance than the long-run risks literature, I only make the task of matching the equity premium harder. I also make the model consistent with the point estimate of the long-run variance of consumption, rather than choosing a value in the upper end of the confidence interval.

Empirically, I measure consumption as real per-capita nondurable and service consumption from the BEA.

22 Smets and Wouters (2007) estimate that the 1-quarter autocorrelation of stationary technology shocks is 0.95. On the other hand, the 1-quarter autocorrelation of detrended real GDP is 0.85. I take $\phi_x = 0.90$ as the midpoint between these two values.
The persistence of risk aversion, $\phi$, is set to match the empirical persistence of the price-dividend ratio for the aggregate stock market, as in Campbell and Cochrane (1999). The mean and volatility of risk aversion ($\bar{\alpha}$ and, implicitly, $\lambda$) are chosen to match the average Sharpe ratio for the stock market in the post-war sample and the degree of predictability observed using the price-dividend ratio to forecast stock returns. Mean risk aversion is 14 and the standard deviation is set to 6.2.\textsuperscript{23}

### 3.1 Comparisons across models

Table 2 reports basic moments from the three models. The first column gives the moments from the data while the second column gives results from the canonical Epstein–Zin model with constant relative risk aversion (EZ-CRRA). Columns 3 and 4 give results for the EZ-habit model under the benchmark calibration and with temporary technology shocks added.

The first row simply shows that all three models are calibrated to match the long-run variance of consumption exactly, which, under balanced growth, means they also match the long-run variances of output and investment growth. Rows 2 through 4 give the standard deviations of quarterly output, consumption, and investment growth. Both the EZ-CRRA and single-shock EZ-habit models have volatilities for output and investment growth that are well below the empirical values. The dual-shock model rectifies this problem, matching both the short-run and long-run variances well. Both versions of the EZ-habit model match the empirical variance of consumption growth.

Row 5 reports the correlation between the risk-free rate and the next period’s consumption growth. Empirically, the real risk-free rate is measured as the 3-month nominal interest rate minus an inflation forecast.\textsuperscript{24} In the EZ-CRRA model, the risk-free rate has a substantial amount of forecasting power for consumption growth, while in the data interest rates and consumption growth seem essentially unrelated. The two EZ-habit calibrations come much closer to matching that fact.

Rows 1 through 5 show that the EZ-habit model can capture the basic unconditional moments of output, consumption, and investment. Rows 6 through 12 of table 2 summarize the financial side of the model. We can begin by looking at a measure of the price of risk. The Sharpe ratio on an asset is the ratio of its

\textsuperscript{23}When $\alpha_t < 0$, I still use the standard Euler equation even though the household’s optimization problem is convex. In the simulations, $\alpha_t < 0$ only 1.5 percent of the time. Treating households as if they are risk-neutral in periods when $\alpha_t < 0$ (i.e. censoring $\alpha_t$ at zero) has no discernible effect on the results.

\textsuperscript{24}Expected inflation is measured as a forecast of quarterly inflation based on lagged levels of inflation and the nominal risk-free interest rate.
expected excess return over the risk-free rate divided by its standard deviation, so it measures the risk–return tradeoff. Hansen and Jagannathan (1991) show that the maximum Sharpe ratio obtained by any asset in the economy is equal to the standard deviation of the SDF divided by its mean. Recall that all three calibrations have the same average coefficient of relative risk aversion. The Hansen–Jagannathan bound and the mean Sharpe ratio for the consumption claim are roughly 1/3 higher in the two EZ-habit models than the EZ-CRRA case. The reason for this is that the household’s value, $V_t$, a component of the SDF (equation 6), is more volatile in the EZ-habit models. In all the models, a technology shock permanently raises expected consumption and hence $V_t$. In the EZ-habit case, the coefficient of relative risk aversion also falls. Households become less averse to future uncertainty, so $V_t$ rises even more. Countercyclical variation in risk aversion thus makes good times even better and bad times even worse, raising the volatility of the SDF. This effect allows the model to explain the equity premium (or at least the Sharpe ratio on equities) with a lower coefficient of relative risk aversion than we would need in the EZ-CRRA model.

To test whether the models can match the degree of predictability for stock returns that is observed in the data, I regress simulated quarterly excess returns on the consumption claim on its lagged price-dividend ratio. I then estimate the standard deviation of the conditional Sharpe ratio as the standard deviation of the fitted returns divided by the unconditional standard deviation of returns (i.e. assuming a constant volatility). Row 7 reports the median standard deviation from 5,000 simulations of 228 quarters of data, while row 8 reports the proportion of the simulations that have a standard deviation as high as observed empirically (0.22).

In column 2, we can see that there is actually a nontrivial amount of implied predictability on average in the EZ-CRRA model due to small-sample overfitting, but only 16 percent of the simulations match the variability observed in the data. For the EZ-habit model, the predictability observed in the data is calibrated to be exactly the median value in the simulations.

Rows 9 and 10 report the mean and standard deviation of the excess return on a levered consumption claim in the model. For comparability to past results, I follow Abel (1999) and Gourio (2010) in assuming a leverage ratio of 2.74 (i.e. the asset pays a dividend of $C_t^{2.74}$). The two EZ-habit models are able to generate means and volatilities for returns that are far closer to the equity return observed in the data than the EZ-CRRA model can. Part of the reason for this success is that consumption growth, and hence

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25 The fitted Sharpe ratio is measured empirically by forecasting the CRSP value-weighted aggregate excess return with the aggregate price/dividend ratio.
dividend growth, is more volatile in the EZ-habit models than in the EZ-CRRA case, and part of the reason is that discount rates are more volatile. Following a positive technology shock, not only do dividends rise, but discount rates fall, thus making the returns on the wealth portfolio and the levered consumption claim more volatile.26

Rows 11 and 12 show that the means and standard deviations of the real risk-free rate in the three models are all reasonably close to the data. The volatility of interest rates is similar across all three models, and somewhat lower than in the data. The real risk-free rate is measured empirically as the nominal 3-month Treasury yield minus a forecast of inflation. Errors in the inflation forecast will make the estimated real risk-free rate more volatile than the true real risk-free rate, which explains some of the divergence between the empirical and simulated volatilities.

A common problem in early attempts to generate a high equity premium (e.g. Constantinides, 1990; Boldrin, Christiano, and Fisher, 2001, and Jermann, 1998), is a highly volatile risk-free rate. The EZ-habit specification replaces movements in discount rates coming from the risk-free rate with movements coming from risk premia.

To summarize, table 1 shows that the EZ-habit model can match a broad array of features of the economy—the short and long-run variances of output growth, the relative volatilities of investment and consumption growth, and the mean and standard deviation of the Sharpe ratio on equities. The model also helps generate a larger premium on a levered consumption claim, closing roughly half the gap in the equity premium between the EZ-CRRA model and the data. Finally, the behavior of the risk-free rate is reasonably similar to the data, unlike previous general-equilibrium attempts at generating a high and volatile Sharpe ratio.

### 3.2 Predictability in the simulated model

#### 3.2.1 The magnitude of return predictability

Figure 1 plots $R^2$s from univariate regressions of excess aggregate stock returns over various horizons on the log price-dividend ratio on the CRSP value-weighted portfolio (e.g. Campbell and Shiller, 1988, among many others), Lettau and Ludvigson’s (2001) measure of the consumption-wealth ratio, $cay$, Campbell and Cochrane’s (1999) excess consumption ratio, and an estimate of risk aversion derived from the EZ-habit

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model in section 4. For the four different variables used in the empirical sample, the $R^2$s generally rise as the sample length grows, and estimated risk aversion outperforms $cay$, excess consumption, and the price-dividend ratio.

The gray line labeled "Simulated mean" gives the mean $R^2$ from 5000 regressions of excess returns on a consumption claim on the price-dividend ratio (equivalently, the wealth-consumption ratio) over 228-quarter spans in the benchmark simulation of the single-shock model (the same length as the empirical sample). The upper gray line gives the 95th percentile of the simulations. As in the data, the simulated $R^2$s rise as the horizon lengthens. The model compares favorably with the price-dividend and excess-consumption ratios, with the simulated mean tracking the empirical values closely (the median follows almost the same path). The empirical $R^2$s for $cay$ are at or below the 95th percentile in the simulations. The only variable that the simulations cannot match is estimated risk aversion, but raising the volatility of risk aversion in the calibration would solve this problem.

The $R^2$s generated here are substantially higher than those obtained in production models such as Campanale, Castro, and Clementi’s (2010) model of time-varying first-order risk aversion and Guvenen (2009) and De Graeve et al.’s (2010) studies of limited participation. The population $R^2$s are also essentially identical to those found by Wachter (2010) and Gourio (2010) in endowment-economy and production-based models, respectively, with time-varying disaster risk.

The top panel of table 3 reports the percentage of simulated samples in which the simulated $R^2$ is as high as we observe in the data for $cay$ and the price-dividend ratio (results for excess consumption are similar to the price-dividend ratio), and where a high price-dividend ratio forecasts low returns. The table reports values for horizons of one quarter and one through five years. The EZ-CRRA model matches empirical $R^2$s for $cay$ less than 5 percent of the time at horizons shorter than 16 quarters, but can match the $R^2$s for the price-dividend ratio 15 to 25 percent of the time. The habit model substantially raises the likelihood of the simulations of matching the data, by a factor of three or more at every horizon, and it never matches less than 5 percent of the time except for $cay$ at the one-quarter horizon.

As an alternative to the $R^2$, I also consider the test statistic suggested by Kiefer, Vogelsang, and Bunzel (KVB, 2000) based on Newey–West standard errors with the lag window equal to the sample size. At various horizons, I calculate the $t$-statistic on the coefficient in a regression of stock returns on the price-dividend ratio in the simulated samples. The bottom panel of table 3 repeats the analysis from the top half, but with the KVB test statistics. In every case, the habit model matches the empirical $t$-statistics.
at least five percent of the time. The EZ-CRRA model again has trouble matching the results for \( cay \), and only replicates the statistics for the price-dividend ratio in 5 to 20 percent of the samples, compared to 20 to 50 percent of the samples for the habit model.

### 3.2.2 Other return predictors

Table 4 reports the simulated correlation between five-year excess returns on the aggregate wealth portfolio and a variety of return predictors. The first row gives the correlation for actual risk aversion, which we would expect would be highest of all of the variables. The second row shows that the predictive power of the price-dividend ratio is nearly as high as that of \( \alpha_t \) in the benchmark model, but somewhat lower in the dual-shock calibration (though still not as much lower as in the data).

Fama and Schwert (1977) and Campbell (1987) find that short term interest rates negatively predict future stock returns.\(^{27}\) In table 4, I do not replicate the result that the real interest rate negatively forecasts returns, but the risk-free rate minus its 4-quarter moving average (denoted RREL as in Campbell, 1987), does weakly negatively forecast returns. Table 4 shows that the correlation of the five-year excess stock return with the real risk-free rate and RREL is substantially negative and nearly as large as that of \( \hat{\alpha} \). In the model, two effects cause interest rates to forecast stock returns. First, positive technology shocks raise interest rates and lower risk aversion. Second, even if risk aversion were driven by shocks unrelated to technology, interest rates might still forecast stock returns since a decline in risk aversion lowers the precautionary saving effect, raising interest rates. Intuitively, there is a flight-to-quality effect in interest rates, linking them to expected stock returns.

Table 4 reports the mean and standard deviation of the real term spread for the EZ-habit model. For the sake of simplicity, I follow the literature in modeling long-term debt as an asset that has a constant probability of paying its principal of one unit of the consumption good and retiring.\(^{28}\) If the bond does not retire and pay out, the holder retains the bond for another period. I assume that the quarterly probability of payout is \( 1 - 0.9^{1/4} \) so that the expected maturity of the bond is ten years. The term spread is the yield to maturity on this bond minus the one-quarter riskless yield. The term spread in the model is on average negative, whereas the nominal Treasury yield curve is almost always upward-sloping in the data. The

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\(^{27}\) Campbell (1991) subtracts the 12-month moving average of the nominal risk-free rate from itself as a way to detrend the short term interest rate, since the nominal rate may be nonstationary if there are changes in trend inflation. Detrending in that way should be unnecessary in the model since interest rates are stationary, but I still check this variable.

\(^{28}\) See, e.g. Rudebusch and Swanson (2008) and Miao and Wang (2010).
reason we have a negative term spread in the model is that in good times the marginal product of capital, and hence the risk-free rate, is above average. So in good times, short-term bonds have low prices, and hence they are a hedge and have a negative risk premium.

Fama and French (1989) show that the term spread forecasts stock returns. Interestingly, table 4 shows that even though the term spread is negative on average in the model, it still positively predicts future stock returns as in the data. This essentially comes through an expectations-hypothesis effect. In periods when the risk premium is low, the risk-free rate is high and expected to fall. To the extent that long-term yields are just averages of expected future short yields, long yields will rise less than short yields. So in periods when risk aversion is low, the term spread falls, and the term spread thus positively predicts stock returns.

In the model, the equity premium is a nearly a constant multiple of $\alpha_t$. The variables that forecast returns in table 4 are all correlated with $\alpha_t$, but imperfectly. For example, the price-dividend ratio also depends on expected consumption growth and interest rates. The fourth column of table 4 shows that the dual-shock model can qualitatively, if not quantitatively, match the empirical result in column 1 that estimated risk aversion is a more powerful forecaster of excess stock returns than any of the other variables, since it is uncontaminated by factors like expected consumption growth.

### 3.2.3 Consumption growth predictability

The aggregate price-dividend and wealth-consumption ratios may be driven by either movements in expected dividend (consumption) growth or movements in discount rates. For the aggregate stock market, Campbell and Shiller (1988) and Cochrane (2008) find that the price-dividend ratio has at best weak forecasting power for dividend growth. Similarly, Lettau and Ludvigson (2001) find that the wealth-consumption ratio has little forecasting power for consumption growth.

Figure 2 shows that the EZ-habit model is consistent with those results. First, to get a general sense of the dynamic properties of consumption growth, the top panel of figure 2 plots the autocorrelations of consumption growth against their empirical counterparts. The shaded region is the 95-percent confidence interval for the empirical estimates using the Newey–West method with a lag window of 12 quarters. In the model, the autocorrelations are near zero at all horizons. The data suggests that the first three autocorrelations are positive, which the model does not match. At longer lag lengths, though, there is no

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29This result is exact in the log-linear approximation.
evidence for persistence in consumption growth, consistent with the model.

The bottom panel of figure 2 simulates 228-quarter samples as in figure 1 and calculates correlations between consumption growth between dates $t$ and $t+k$ and the consumption-wealth ratio at date $t$. What we see is that while many of the simulated correlations are far from zero, the mean sample correlation between the wealth-consumption ratio and future consumption growth is nearly zero. The figure also plots empirical correlations between $cay$ and future consumption growth at various horizons, and they are similar to the simulated mean. Figure 2 thus shows that the EZ-habit model not only matches the short and long-run variances of consumption growth, but it also replicates relevant features of the dynamics of consumption.

3.3 Impulse response functions

Figure 3 plots impulse response functions (IRFs) in the EZ-CRRA and benchmark EZ-habit models for four variables: consumption, household value, the risk-free rate, and the Sharpe ratio on the consumption claim. The lines give log deviations from steady-state, except for the risk-free rate, for which I report the absolute change in annualized percentage points. The shock is a unit standard deviation (88 basis-point) permanent increase in the level of technology, which will lead to an identical long-run increase in consumption, capital, and output.

The top-left panel shows the response of household value. For the EZ-CRRA model, value immediately jumps to a point just below its new steady state, and then slowly rises as households accumulate capital. For the EZ-habit model, though, value actually overshoots its new steady state. The reason is that the positive shock to productivity drives risk aversion down. When households are less risk-averse, they place a higher value on their future consumption stream because they penalize uncertainty less strongly. This effect helps increase the volatility of the SDF (equation 6), raising the Hansen–Jagannathan bound. The top-right panel shows that on the impact of a shock, the Sharpe ratio in the EZ-habit model falls by 12.5 percent (as a fraction of its mean), and then gradually rises again, with a half-life of 12 quarters.

The bottom-left panel shows the dynamics of the risk-free rate. The initial response is essentially identical for the two models. The reason for this is that the risk premium on an unlevered claim on capital is very small in the model, so the return on capital is roughly equal to the risk-free rate. Since the size of the capital stock is essentially fixed in the short-run, an increase in productivity directly increases the return on capital and hence the risk-free rate.
The final panel of figure 3 shows the response of consumption in the two models. The EZ-habit model shows a larger initial response of consumption, with lower expected consumption growth going forward. To see why this is, we can write consumption growth in equilibrium as

\[ E_t \Delta c_{t+1} = \bar{c} + \rho^{-1} r_{f,t+1} + \alpha_t \times \text{vol} \]  

(15)

where \( r_{f,t+1} \) is the risk-free interest rate between dates \( t \) and \( t + 1 \), \( \text{vol} \) represents a measure of the total volatility in the model, and \( \bar{c} \) is a constant. \( \alpha_t \times \text{vol} \) represents the precautionary saving effect and is a function of the current level of risk aversion and the variances of the shocks in the model. The standard interpretation in an endowment economy is that conditional on consumption growth, a strong precautionary saving motive leads to a low risk-free rate. In a production setting, though, it is the risk-free rate that is held roughly fixed since it is tied to the marginal product of capital, which is hard to change quickly through investment. Conditional on the risk-free rate, then, a small precautionary saving motive leads to lower expected consumption growth (more consumption today, saving less for tomorrow). In the EZ-habit model, a positive technology shock lowers risk aversion, and hence consumption rises more than in the canonical EZ case. This effect also serves to increase the volatility of the SDF, just as the higher response of value does.

Given the results in figure 3, it is straightforward to see what would happen in this economy if there were a pure shock to the coefficient of relative risk aversion. Since the risk-free rate is tied to the marginal product of capital, it would not move on the impact of a shock. The only effect on real variables of a pure decline in risk aversion then is that households would want a smaller buffer stock of savings, so they would raise consumption and lower investment: shocks to risk aversion look like simple demand shocks.

### 3.4 Estimating the EIS from interest rate regressions

The value of the elasticity of intertemporal substitution is controversial. Regressions based on aggregate consumption and asset returns often find a very small EIS (Hall, 1988; Campbell and Mankiw, 1989). Campbell (2003) reviews the literature and estimates the EIS using a variety of specifications and data from a broad range of countries, finding values generally less than 0.5 and often less than 0.2.\(^{30}\) This
result is in conflict with the calibration used here and in other recent production-based asset pricing studies (Kaltenbrunner and Lochstoer, 2010; Gourio, 2010), which assume that the EIS is greater than 1. The question is whether the EZ-habit model generates small EIS estimates in regressions similar to those estimated in Campbell (2003).

The standard aggregate EIS regressions start from a model in which the risk-free rate takes the form

\[ r_{f,t+1} = b_0 + \rho E_t \Delta c_{t+1} \]  

(16)

where \( r_{f,t+1} \) is the riskless interest rate between periods \( t \) and \( t + 1 \). \( b_0 \) is a parameter depending on the discount rate and underlying volatility in the model (which are taken to be constant). This relationship is straightforward to derive in an endowment economy with homoskedastic consumption growth and where households have a constant EIS and coefficient of relative risk aversion. It is also obtained in a log-linearization of the standard RBC model with homoskedastic technology shocks.

In principle, the EIS can be estimated from a regression of interest rates on consumption growth or vice versa. However, since the reduced-form relationship between consumption growth and interest rates is nearly zero in the EZ-habit model, in some of the simulations regressing \( E_t \Delta c_{t+1} \) on \( r_{f,t+1} \) produces explosive estimates for \( \rho^{-1} \) (since we have to invert the coefficient estimate). Moreover, consumption in the EZ-habit model nearly follows a random walk, so it is essentially unpredictable and there are serious weak-instruments problems in an IV regression of interest rates on consumption growth. I therefore focus on the regression of consumption growth on interest rates,

\[ E_t \Delta c_{t+1} = b'_0 + \rho^{-1} r_{f,t+1} \]  

(17)

In the simulations of the model in section 3, we have the ability to directly measure \( E_t \Delta c_{t+1} \). The first row of table 5 reports the population estimate of \( \rho^{-1} \) in regression (17) under the EZ-CRRA and EZ-habit models. In the EZ-CRRA case, the regression identifies \( \rho^{-1} \) exactly. On the other hand, the estimate of \( \rho^{-1} \) is biased substantially downwards in the EZ-habit specifications.

The bias comes from the fact that the time-varying precautionary saving effect (equation 15) is omitted from the regression. Since precautionary saving is correlated with both expected consumption growth earned by households.
and interest rates, omitting it biases the simple IS-curve regression usually used to identify the EIS. An alternative way to see the source of the bias is to go back to the IRFs in figure 3. In both models the risk-free rate rises by the same amount following a shock. In the EZ-habit specification, though, because of the decline in precautionary saving, expected consumption growth is lower following a shock than in the EZ-CRRA case. That means that the estimate of $\rho^{-1}$ will fall.\footnote{In Bansal and Yaron (2004), time-variation in the volatility of shocks in principle causes EIS regressions to be biased. However, Beeler and Campbell (2010) show that their calibration generates almost no actual bias—the median sample EIS estimates are well above 1. This paper thus represents an improvement in being able to generate a substantial bias in aggregate regressions without large movements in the conditional volatility of consumption.}

The regression in the first row of table 5 is in some sense ideal, but it is not the regression that we are actually able to run in the data since $E_t \Delta c_{t+1}$ is unobservable.\footnote{In principle, the real risk-free rate, $r_{f,t+1}$, is also unobservable in the data. As above, I form $r_{f,t+1}$ as the difference between the nominal 3-month interest rate and a forecast of inflation based on lagged inflation and nominal interest rates. Errors in the estimate of the true real-risk-free rate would bias the estimate of $\rho^{-1}$ towards zero. Instrumental-variables methods can theoretically eliminate this bias.} Rows 2 through 4 report results for estimates of $\rho^{-1}$ from regressions of actual consumption growth, $\Delta c_{t+1}$, on the risk-free rate, $r_{f,t+1}$. Row 2 gives the population estimates, while rows 3 and 4 give the median and 95-percent range of the estimates from 228-quarter simulations. With constant relative risk aversion, the population regression in row 2 estimates the EIS exactly. The median estimate from the small-sample regressions in row 3 is 1.16. The 95-percent range is wide, and it only just barely contains the estimate from the data. So it is in principle possible for the EZ-CRRA model to generate an estimate of the EIS as small as what we observe in the data, but the probability is small (less than 10 percent).

In the EZ-habit models, the bias is far larger. The population estimate in the single-shock case is 0.56, and the median sample estimate is 0.03. For the dual shock model, the estimates are only slightly better—0.71 in population and a median of 0.35 in small samples. The reason for this slight improvement is that the temporary shocks have little effect on risk aversion, so they induce variation in consumption and the risk-free rate that is closer to the usual EZ-CRRA case. The upper end of the 95 percent range for the simulated estimates is well below the true value of the EIS.

The first four rows of table 5 show that in general, regressions of interest rates on expected consumption growth are not a very good way to estimate the EIS, and in the EZ-habit model they are biased and inconsistent. It is worth noting, though, that if we could observe $\alpha_t$, we could completely eliminate the bias in the EIS regressions. Empirically, this suggests that regressions designed to estimate the EIS could be improved by including a control for risk aversion, such as the price-dividend ratio on the stock market.
The final three rows of table 5 try to estimate the EIS including a control for risk aversion. In the data, I use a measure of risk aversion derived from the EZ-habit model below in section 4, denoted $\alpha_t$. The empirical estimate of the EIS is essentially unchanged from when $\alpha_t$ is not included. In population, when $\alpha_t$ is included in the simulated regressions, the EIS is estimated exactly. In small-sample regressions, though, the estimate of the EIS in the model is still biased downward. In the single-shock model, the median estimate is 0.07. and in the dual-shock model 0.94. Row 7 shows, though, that the 2.5 percentile of the small-sample estimates is -3.08 in the dual shock model, while the 97.5 percentile is 3.11. So even though the median estimate in the dual-shock model is not enormously biased, the empirical value of 0.18 is well within the simulated range. In the end, while controlling for risk aversion should, in principle, allow us to estimate the EIS consistently, in small samples the regressions still do not seem to provide useful estimates because of weak-identification problems.

As an alternative to these regressions, the EIS could be consistently estimated if we had an instrument for the risk-free rate that was uncorrelated with risk aversion. Standard aggregate instruments like lagged interest rates and consumption growth (e.g. Campbell, 2003) will certainly not be valid instruments under the EZ-habit model since the precautionary saving effect is persistent. Household-level instruments would work better if my model is correct in treating risk aversion as being driven by aggregate factors. The EZ-habit model thus has the ability to explain the divergence between micro and macro estimates of the EIS if the micro instruments are valid and the macro instruments invalid.

### 4 Empirical return forecasting

This section shows how the model suggests we can directly estimate the coefficient of relative risk aversion in the data, and then demonstrates that the estimate is a powerful predictor of stock returns. Second, I present novel evidence that technology growth forecasts stock returns, just as it does in the production model. The results differentiate the EZ-habit paper from models with time-varying disaster risk. Gourio (2010) predicts that when there are changes in the probability of a large disaster occurring, price-dividend ratios will forecast returns, which is also true in the EZ-habit model. The EZ-habit model also predicts, though, that technology and estimated risk aversion will forecast stock returns, and that estimated risk

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33 Gruber (2005) discusses precisely these issues and tries to resolve them by using household-specific variation in tax rates as an instrument for consumption growth. Dynan (1993) explicitly controls for the precautionary savings effect at the household level by predicting the conditional volatility of consumption, but she does not deal with the possibility that risk aversion varies over time.
aversion will be the single most powerful forecaster of returns, which would not be true in the time-varying disaster model or models based on other forms of time-varying volatility (e.g. Bansal and Yaron, 2004, Bloom, 2009, or Fernandez-Villaverde et al., 2011).

4.1 Estimating risk aversion

If risk aversion follows the AR(1) process given in (5), then we can measure current risk aversion if we simply observe the history of aggregate value, $v_t^A$. For a given value of the EIS and observed data on wealth and consumption, it is possible to calculate $v_t^A$ by rearranging equation (7)

$$v_t^A = \frac{1}{1-\rho} w_t^A - \frac{\rho}{1-\rho} c_t^A + \frac{1}{1-\rho} \log (1 - \exp(-\beta))$$  \hspace{1cm} (18)

If we can measure household wealth and consumption, then we can measure value. We then simply plug the estimates of $v_t^A$ into equation (5) to obtain estimates of $\alpha_t$.

Lettau and Ludvigson (2001) study a cointegrating relationship between consumption and aggregate wealth. Their method is valid in my model since consumption and wealth are cointegrated under balanced growth. While their analysis was designed to estimate the consumption-wealth ratio, it also delivers, as a byproduct, a measure of aggregate wealth (since we can always add consumption to the wealth-consumption ratio to obtain wealth). The estimate of wealth derived using their method is a combination of asset wealth data obtained from the flow of funds accounts plus an estimate of human wealth. They treat labor income as the dividend from the stock of human wealth. Assuming the price-dividend ratio for human wealth is stationary, we can use labor income as a proxy for human wealth. Denoting asset wealth as $a_t$ and labor income as $y_t$, the appendix shows that we then have a cointegrating relationship,

$$c_t = \zeta \omega a_t + \zeta (1 - \omega) y_t + \xi'_t$$  \hspace{1cm} (19)

where $\zeta$ and $\omega$ are parameters and $\xi'_t$ is a stationary error term. Lettau and Ludvigson (2001) refer to the residual $\xi'_t$ as $cay$. This variable essentially represents an estimate of the consumption-wealth ratio. Since I want to estimate wealth, I define

$$ay_t \equiv \omega a_t + (1 - \omega) y_t$$  \hspace{1cm} (20)

which, under the assumptions above, will be a statistically unbiased estimate of total wealth, but will
include error due to the fact that we do not assume we directly measure human wealth.\footnote{In simulations with variable labor supply, the price/dividend ratio on human wealth does vary over time, but the variation is relatively small: risk aversion calculated using the method here (assuming a constant price-dividend ratio on human wealth) is over 95 percent correlated with actual risk aversion.}

With our measure of wealth $ay_t$, we estimate $v_t^A$ as

$$
\hat{v}_t^A = \frac{1}{1 - \rho} ay_t - \frac{\rho}{1 - \rho} c_t
$$

(21)

where we ignore constants, and a circumflex indicates an estimated variable. Note that since the parameters of the cointegrating relationship for $c_t$, $a_t$, and $y_t$ are estimated superconsistently we do not have to modify any standard errors in the subsequent analysis to take into account the fact that $ay_t$ is a generated regressor (which is why it does not receive a circumflex). That said, to the extent that there is measurement error in the consumption or wealth data, $\hat{v}_t^A$ will inherit that same error. When we use $\hat{v}_t^A$ to forecast market returns, this measurement error should only weaken the results. For measurement error to generate a spurious predictive relationship, it would have to be correlated with other predictors of returns.\footnote{One obvious source of measurement error is that human capital is not a perfect estimator of the value of human wealth. Suppose risk aversion rises above average and lowers the price-dividend ratio on human wealth below average. Labor income will then be overestimating human wealth (compared to its average). High levels of wealth drive out measure of $\hat{a}_t$ downward, so this measurement error should bias the results against correctly forecasting returns (high risk aversion in the data leads to low risk aversion in our estimates). In the simulations not reported here, though, this effect seems to be small.}

This definition of $\hat{v}_t^A$ is similar to Lettau and Ludvigson’s $cay_t$, except they have equal weights on $c_t$ and $ay_t$, whereas equation (21) uses a combination where the weights depend on the EIS. Also, $cay_t$ is stationary by construction, whereas $\hat{v}_t^A$ is growing over time (it is cointegrated with consumption and wealth).

In equation (21) a high EIS (low $\rho$) raises the weight on consumption relative to asset wealth. If the EIS is less than 1 ($\rho > 1$), the weight on wealth, $ay_t$, is actually negative, and the weight on consumption greater than 1. Bansal and Yaron (2004) and Kaltenbrunner and Lochstoer (2010) both find that an EIS of 1.5 allows their models to fit asset pricing facts, so I use the same value. This value is also consistent with the micro evidence of Vissing-Jorgensen and Attanasio (2003). The results reported below are broadly similar as long as the EIS is greater than 1.1 (at that level and below, $\hat{v}_t^A$ becomes very volatile). The appendix reports a sensitivity analysis for various values of the EIS.

Figure 4 plots $\hat{v}_t^A$ both in its raw levels and with a linear trend taken out. As we would expect, $\hat{v}_t^A$ follows a strong upward trend. There seem to be both low and high frequency components to detrended $\hat{v}_t^A$. In particular, there are long-run swings with peaks in the early 1970’s and 2000’s and a trough around
1994, generally consistent with movements in the aggregate price-dividend ratio and variation in average output growth. At the same time, there are business-cycle frequency movements, e.g. the troughs in 1973, 2001, and 2008.

I construct an estimate of $\alpha_t$, $\hat{\alpha}_t$, using the update process for risk aversion, equation (5), and the data on $\hat{v}_t^A$. In particular, we have

$$\hat{\alpha}_{t+1} = \phi \hat{\alpha}_t + (1 - \phi) \overline{\alpha} + \lambda (\Delta \hat{v}_{t+1}^A - E_t \Delta v_{t+1}^A)$$

(22)

As above, I assume that $\phi = 0.96$. $E_t \Delta v_{t+1}^A$ is estimated simply as the sample average of $\Delta \hat{v}_{t}^A$.37

The parameter $\lambda$ governs the volatility of $\alpha_t$, but it has only a multiplicative effect on $\hat{\alpha}_t$. That is, two estimates of $\hat{\alpha}_t$ will be perfectly correlated with each other, regardless of what values are chosen for $\lambda$. The same argument applies for $\overline{\alpha}$. As long as we are simply trying to forecast stock returns using a linear regression, we can ignore any additive or multiplicative shifts in $\hat{\alpha}_t$. Therefore, I set $\overline{\alpha} = 0$ and choose $\lambda$ so that $\hat{\alpha}_t$ has unit variance, normalizations that will have no effect on the regression-based measures of forecasting power (and I choose a negative value of $\lambda$ to match the habit-formation motivation of the model). In the first period of the sample I assume $\hat{\alpha} = \overline{\alpha}$.

An important feature of this method of forecasting is that it is based only on the preference specification. None of the assumptions we made about the production side of the economy are required for this method to be valid. We simply take advantage of the relationship between household value and changes in risk aversion and the relationship under Epstein–Zin preferences between household value and wealth.

4.2 Forecasting market returns

The next question is to what extent the model-implied variation in expected returns is related to actual returns. Figure 5 plots $\hat{\alpha}_t$ and 5-year excess returns on the stock market (the value-weighted excess return from Kenneth French). The strong correlation between the two series (0.68) is immediately apparent. There are both high and low-frequency movements in $\hat{\alpha}_t$ associated with changes in growth in value. In the periods when value is growing quickly, e.g. the late 1990’s, risk aversion falls. At the same time, there

36Note, though, that linear detrending will tend to make the series look as if it is mean-reverting even if it follows a random walk. The linear trend is only used to make the graph legible; none of the results involve it.
37In principle, it is possible to forecast $\Delta \hat{v}_{t+1}^A$, but the amount of predictability in $\Delta \hat{v}_{t+1}^A$ is sufficiently small that the results are nearly identical to assuming that $\hat{v}_{t+1}^A$ simply follows a random walk.

The appendix shows that the results are robust to different choices for $\phi$. 26
are higher-frequency movements, such as the temporary increases in estimated risk aversion around the recessions in 1991 and 2001.

Figure 1 plots R²s from regressions of future stock returns on \( \hat{\alpha}_t, \ cay_t \), the price-dividend ratio (P/D), and the excess consumption ratio from Campbell and Cochrane (1999). Each line gives the R² from a univariate regression. The x-axis gives the horizon for the return in quarters. The \( n \)th point is the R² from a regression of \( \sum_{j=1}^{n} r_{t+j} \) on the predictor at time \( t \). The regressions are all run on quarterly data from 1952 to 1999 (to ensure we have data for the 40-quarter regression). Each regression uses the same sample for the predictors.

At every horizon, \( \hat{\alpha} \) is dominant. At the five-year horizon, the R² for estimated risk aversion peaks at more than twice that of the other variables. The R²s are impressively high for just a single variable: at the 5-year horizon, \( \hat{\alpha} \) explains 50 percent of the post-war variation in stock returns. Furthermore, in horse-race regressions (reported in the appendix), \( \hat{\alpha} \) dominates \( cay \) at all horizons.

An important consideration in long-horizon forecasting regressions is that the residuals are highly persistent. Kiefer, Vogelsang and Bunzel (2000) and Kiefer and Vogelsang (2005) show that by using Newey–West standard errors with a very long lag-window, we can obtain test statistics with better size properties than techniques that use a fixed (and usually short) lag window. I choose a lag window equal to half the sample size and use the critical values reported in Kiefer and Vogelsang (2005). For \( cay \), every regression except for those with horizons greater than 30 quarters is significant at the 5 percent level. For \( \hat{\alpha} \), the largest p-value is 0.0008. The price-dividend ratio is significant at the 5 percent level for forecasts of 14 quarters or longer. In other words, these regressions all imply that we have substantial ability to forecast stock returns in the post-war period, and \( \hat{\alpha} \) is the strongest of the predictors. Out-of-sample tests with both asymptotic and bootstrapped critical values give similar results (appendix D.3). Appendix D examines the sensitivity of the results in this section to the various parameters we had to calibrate (e.g. the EIS and the persistence of habits). The basic results hold across a broad range of parameter sets.

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38Kiefer and Vogelsang (2002) note that there is a size-power tradeoff. When the lag window is increased, the size of the test statistics gets closer to their nominal size, but there is a loss of power. I choose a lag window of half the sample to balance these considerations. The results are basically identical when using a lag window equal to the sample size as in Kiefer, Vogelsang, and Bunzel (2000), though the price-earnings is significant in more of the regressions.
4.3 Forecasts from estimates of technology

The method of estimating the level of risk aversion studied above does not rely on any assumptions about the structure of production in the economy, being derived purely from the preference specification. However, in the production model, changes in value are closely related to changes in productivity. If we can measure innovations to technology, then risk aversion should follow an AR(1) process where the innovations are equal to the shocks to the stochastic trend in technology.

There is a large literature that tries to estimate aggregate technology shocks. I consider two methods here. The first builds off of Solow (1957) and uses restrictions from a constant-returns production function:

\[ a_t = y_t - \gamma k_t - (1 - \gamma) l_t \]  

(23)

\( a_t \) measures technology if the economy has a Cobb–Douglas production function, with \( l \) denoting log labor supply. I also consider a simpler metric, labor productivity, \( lp_t = y_t - l_t \). Labor productivity does not take into account the effects of capital accumulation and simply models technology as the average product of labor. Capital can be difficult to measure, whereas the number of hours supplied in the economy is a fairly concrete quantity (though the quality of those hours is difficult to account for).\(^{39}\)

To extract the stochastic trend from the two productivity series, I estimate univariate ARMA models for each variable. The Bayesian information criterion implies that TFP growth is best fit with an MA(2), while labor productivity growth should be treated as i.i.d. \( \varepsilon_{t}^{TFP} \) is defined as the residual in the MA(2), while \( \varepsilon_{t}^{LP} \) is simply equal to labor productivity growth. That is, \( \varepsilon_{t}^{TFP} \) and \( \varepsilon_{t}^{LP} \) are innovations to the Beveridge–Nelson (1981) trends in productivity.

Section 5.1 shows that, at least in the case where log technology follows a random walk, risk aversion follows an AR(1) process of the form,

\[ \alpha_t = (1 - \theta) \tilde{\alpha}_t + \theta \alpha_{t-1} + \varepsilon_{t}^{X} \]  

(24)

where \( \varepsilon^{X} \) denotes a measure of technology growth. We then have two measures of \( \alpha_t \), which I denote

\(^{39}\) Furthermore, labor productivity determines the tradeoff that households face between consumption and leisure. If the capital stock rises because foreigners want to invest more in the US, household welfare will increase even if TFP does not. Similarly, a tax increase that reduced desired saving could lower welfare and labor productivity, without having any effect on TFP. And welfare is the relevant input in estimating \( \tilde{\alpha}_t \).
\( \hat{\alpha}^{TFP}, \hat{\alpha}^{LP} \), using \( \varepsilon_t^{TFP} \) and \( \varepsilon_t^{LP} \), respectively.\(^{40}\) The two measures turn out to be highly correlated (93 percent).

Figure 6 plots five-year excess returns against \( \hat{\alpha}^{TFP} \) and \( \hat{\alpha}^{LP} \). The two series are both clearly highly correlated with future excess returns. The p-values in regressions of quarterly excess returns on \( \hat{\alpha}^{TFP} \) and \( \hat{\alpha}^{LP} \) are 0.032 and 0.026, respectively (using Kiefer, Vogelsang, and Bunzel, 2000, t-type-statistics to account for autocorrelation). The relationship between the three series is most clear around the turning points. Productivity growth begins slowing down around 1970, driving risk aversion upwards. Forward-looking stock returns reach their trough at roughly the same point. Productivity growth rises again starting in the mid-1990’s, which is exactly when stock returns begin falling again.

The two measures of risk aversion in figure 6 clearly do not have the explanatory power of the variables studied above. The wealth and consumption data used above have a forward-looking component that is not present in instantaneous measures of technology. On the other hand, my measure of risk aversion is highly correlated with the consumption-wealth ratio. In almost any model with time-varying discount rates, the consumption-wealth ratio will forecast stock returns. It is not the case, though, that any model will predict that measures of technology should forecast stock returns. Figure 6 thus provides evidence in favor of the model presented here over other explanations of time-varying expected returns.

5 Extensions

5.1 Log-linearization

This section studies a log-linear version of the production economy from above. I use the solution to build a better understanding of the basic results reported in the previous sections. In particular, I derive analytic approximations for the consumption function, the risk-free rate and the conditional Sharpe ratio for the wealth portfolio. I also derive an essentially affine model of the term structure with a time-varying price of risk. This result connects the standard production theory in the macro literature to one of the most commonly used empirical asset-pricing frameworks.

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\(^{40}\)Note that \( \hat{\alpha}^{TFP} \) includes some forward-looking information since its construction requires the estimation of an MA(2) on the full sample. \( \hat{\alpha}^{LP} \) does not suffer from this flaw. It is true that in both cases we have to estimate mean productivity growth, but shifts in the estimated mean simply correspond to shifts in the mean of \( \hat{\alpha}_t \); they have no effect on its dynamics. In regressions of returns on \( \hat{\alpha}_t \), the constant will thus always absorb shifts in \( \hat{\alpha}_t \), so the estimation of the mean of productivity growth is irrelevant for forecasting returns.
5.1.1 Approximation method and solution

I derive a log-linear consumption function as the solution to a model that represents a log-linearization of the environment derived above with permanent technology shocks only. Specifically, if the capital accumulation equation, the return on capital, and the return on the wealth portfolio are log-linearized, then we are able to obtain an exact formula for the consumption function under EZ-habit utility (with canonical Epstein–Zin and power utility as special cases). The methods build on Campbell (1994) and Lettau (2003). Unlike the usual techniques in the macro literature, the solution is not based on certainty-equivalent or higher-order approximations to expectations.\footnote{\textsuperscript{41}} Rather, I take advantage of log-normality to calculate expectations exactly. That feature of the method is critical for accurately capturing risk premia and precautionary saving effects.

It is straightforward to show that the approximation technique delivers policy functions that are identical to those obtained from perturbation up to the first order (depending on where the derivatives are taken). The difference is that because I take advantage of formulas for log-normal expectations, a term involving risk aversion appears in the solution, meaning that the approximation captures the time-varying precautionary saving effect that is central to driving the difference in the consumption response between the EZ-habit model and the RBC model with canonical Epstein–Zin preferences.\footnote{\textsuperscript{42}}

The approximation method involves log-linear approximations to three components of the model: the budget constraint, the return on the wealth portfolio, and the return on capital,

\begin{align}
 k_{t+1} &\approx \lambda_0 + \lambda_k k_t + \lambda_a a_t + \lambda_c c_t \\
 r_{w,t+1} &\approx E_t r_{w,t+1} + \Delta E_{t+1} \sum_{j=0}^{\infty} \theta^j \Delta q_{t+j+1} - \Delta E_{t+1} \sum_{j=1}^{\infty} \theta^j r_{w,t+j+1} \\
 r_{k,t+1} &\approx r_0 + r_k (k_{t+1} - a_{t+1})
\end{align}

(25) (26) (27)

where \( \{\lambda_0, \lambda_k, \lambda_a, \lambda_c, \theta, r_0, r_k\} \) are linearization coefficients that I solve for in the appendix and lower-case letters denote logs.\footnote{\textsuperscript{43}}

\footnotetext{\textsuperscript{41}}The usual technique is the perturbation method of Judd (1999). See Woodford (2003) for a representative application and Rudebusch and Swanson (2009) for extensions to higher order approximations.

\footnotetext{\textsuperscript{42}}In perturbation, the equilibrium equations are not only approximated with respect to the endogenous and exogenous variables, but also the volatility of the technology shock, \( \sigma_a \). The first-order perturbation solution therefore does not include any terms involving interactions of state variables with \( \sigma_a \). The approximation used here includes a term for \( \alpha_t \sigma_a^2 \). The solutions are otherwise identical.

\footnotetext{\textsuperscript{43}}(25) is a log-linearization of the resource constraint, \( K_{t+1} = (1 - \delta) K_t + A_{t+1}^{\gamma-1} K_t^{\gamma} - C_t \); (26) is the Campbell–Shiller (1988) approximation for the return on the wealth portfolio; (27) is a linearization of \( R_{k,t+1} = \gamma K_{t+1}^{\gamma-1} A_{t+1}^{\gamma-1} + 1 - \delta \).
Define \( \tilde{c}_t \equiv c_t - a_t \) and \( \tilde{k}_t \equiv k_t - a_t \). With the three log-linear approximations (25), (26), and (27), the appendix shows that we can obtain the following result:

**Result 1** Given the log-linear budget constraint, the log-linear return on the consumption claim, and the production function, the optimal consumption plan takes the form

\[
\tilde{c}_t = \eta_c \delta_t + \eta_{ck} \tilde{k}_t + \eta_{ca} \alpha_t 
\]

(28)

and the return on the wealth portfolio can be written as

\[
r_{w,t+1} = \eta_{w0} + \eta_{wa} \alpha_t + \rho E_t \Delta c_{t+1} + \kappa_r \sigma_{t+1} 
\]

(29)

where the coefficients \( \{ \eta_{c0}, \eta_{ck}, \eta_{ca}, \eta_{w0}, \eta_{wa}, \kappa_r \} \) are solved for in the appendix. Furthermore, the log price-dividend ratio of the consumption claim is linear in risk aversion and scaled capital.

The first important implication of this result is that even though risk aversion is time-varying, both consumption growth and returns on the wealth portfolio are homoskedastic. We did not assume the existence of a log-linear policy for consumption or homoskedastic wealth return. Variation in risk aversion induces variation in expected returns on risky assets, but not in their volatility. This same result is obtained in the numerical solution above.

**Remark 1** \( \eta_{ck} \) does not depend on the level or volatility of the coefficient of relative risk aversion (i.e. on \( \bar{\alpha} \) or \( \lambda \)).

The finding that \( \eta_{ck} \) is not affected by the time-variation in the coefficient of relative risk aversion helps us build intuition as to why the IRF for consumption changes when risk aversion varies. Consumption responds to a technology shock more strongly in the EZ-habit model than in the RBC model purely because the coefficient of relative risk aversion falls in response to positive technology shocks. If the economy experienced a hypothetical shock to the size of the capital stock holding the coefficient of relative risk aversion fixed, the behavior of consumption and saving would be identical under EZ-habit and EZ-CRRA preferences.
5.1.2 The risk-free rate and excess returns on the wealth portfolio

Denote the conditional standard deviation of a variable $x$ as $\sigma(x)$. We have the following formulas for the risk-free rate and Sharpe ratio,

**Result 2** *In the log-linearized model, the risk-free rate follows*

$$r_{f,t+1} = \eta_{f0} + \rho E\Delta c_{t+1} - \eta_{fa}\alpha_t$$  

(*30*)

and the Sharpe ratio of the consumption claim is

$$E_t r_{w,t+1} - r_{f,t+1} + \frac{1}{2} \sigma^2(r_w) = \rho \sigma(\Delta c) + (\alpha_t - \rho) \sigma(\Delta v)$$  

(*31*)

As usual, expected consumption growth affects the risk-free rate in proportion to the inverse of the EIS. There is an additional term $\eta_{fa}\alpha_t$ reflecting the time-varying precautionary saving motive. When risk-aversion is high, precautionary saving demand is high, driving the risk-free rate downwards, all else equal. It is immediately clear that a simple regression of consumption growth on interest rates will not identify the EIS, $1/\rho$, unless the instruments used for the interest rate are uncorrelated with current risk aversion or risk aversion is controlled for.

Note also that the Sharpe ratio is strictly increasing in $\alpha_t$. The terms $\sigma(\Delta c)$ and $\sigma(\Delta v)$ are the standard deviations of growth in household consumption and value, respectively (both of which are constant in equilibrium). Tallarini (2000) and Lettau (2003) show that in an RBC model with power utility, an increase in risk aversion need not increase the Sharpe ratio because consumers can endogenously smooth consumption.\(^{44}\) In this setting, though, risk aversion unambiguously increases the Sharpe ratio. The reason is that the preference for consumption smoothing comes from the EIS. An increase in risk aversion does not cause consumers to smooth consumption endogenously, and so the only effect is to raise the Sharpe ratio on the wealth portfolio.

An obvious question is what the term $\sigma(\Delta v)$ actually is. The appendix derives the following results,

**Result 3** *In equilibrium, the coefficient of relative risk aversion $\alpha_t$ follows the process,*

$$\alpha_t = \phi \alpha_{t-1} + (1 - \phi) \bar{\alpha} + \sigma_{aa} \varepsilon_t$$  

(*32*)

\(^{44}\)Lettau and Uhlig (2004) and Rudebusch and Swanson (2008) obtain similar results for Campbell–Cochrane preferences.
where \( \sigma_{aa} \) depends on the parameters \( \lambda, \theta, \phi, \) and \( \sigma_a. \)

**Result 4** The standard deviation of innovations to value is

\[
\sigma(\Delta v) = \frac{-1 + \sqrt{1 + 2\theta \phi \sigma_{aa} \sigma_a}}{1 - \phi / \theta \phi \sigma_{aa}}
\]  

(33)

\( \sigma(\Delta v) \) is increasing in \( \sigma_a \) for \( \sigma_a > 0 \) and \( \sigma_{aa} < 0. \)

Result 3 first shows that the coefficient of relative risk aversion follows an AR(1) process with innovations that are perfectly correlated with technology shocks. This result is a consequence of household value being a log-linear function of the level of technology, so that \( v_{t+1} - E_t v_{t+1} \) is a linear function of the technology shock.

Result 4 shows that the standard deviation of innovations to household value is constant. Furthermore, in the benchmark case where technology shocks drive risk aversion down, an increase in their volatility raises the volatility of innovations to value, as we would expect.

In the case where \( \sigma_{aa} = 0 \), which corresponds to power utility, we obtain a surprisingly simple formula:

**Remark 2** For the RBC model where technology follows a random walk and consumers have constant relative risk aversion, the Sharpe ratio on a consumption claim is approximately

\[
SR_t \approx \alpha \sigma_a (1 - \eta_{ck}) + (\alpha - \rho) \sigma_a \eta_{ck}
\]  

(34)

\[
\approx \alpha \sigma_a - \rho \sigma_a \eta_{ck}
\]  

(35)

This formula is similar to the formula obtained by Bansal and Yaron (2004) for the Sharpe ratio in the presence of long-run risks in an endowment economy. \( \sigma_a \) represents long-run shocks to consumption growth, since consumption eventually catches up to a technology shock. Of that total response, \( \sigma_a (1 - \eta_{ck}) \) comes in the first period, with \( \sigma_a \eta_{ck} \) in subsequent periods.\(^{45}\) We can thus think of the first component of the Sharpe ratio, \( \alpha \sigma_a (1 - \eta_{ck}) \), as Bansal and Yaron’s short-run risk term, and \( (\alpha - \rho) \sigma_a \eta_{ck} \) as long run risks. Kaltenbrunner and Lochstoor (2010) also show that production models generate long-run risk endogenously, but this simple formula for the Sharpe ratio has not been obtained elsewhere.

\(^{45}\)Recall that \( \eta_{ck} \) is the coefficient on scaled capital in the consumption function. A unit increase in the technology shock \( \varepsilon_{t+1} \) raises consumption by \( \sigma_a \); the associated decline in scaled capital of \( \sigma_a \) lowers consumption \( \eta_{ck} \sigma_a. \)
The second line shows that we can isolate the $\eta_{ck}$ term. If the EIS is large ($\rho$ is small) then the endogenous response of consumption in the model is unimportant and the Sharpe ratio is determined simply by the volatility of technology shocks and the coefficient of relative risk aversion.

5.2 Affine bond pricing

It turns out that the log-linear solution to the model allows us to connect the standard macro framework to the bond-pricing literature through the following result:

**Result 5** The log stochastic discount factor can be expressed as

\[ m_{t+1} = -r_{f,t+1} - \frac{1}{2} (\omega_0 + \omega_1 \alpha_t)^2 \sigma^2 + (\omega_0 + \omega_1 \alpha_t) \varepsilon_{t+1} \]  

(36)

The SDF takes the tractable essentially affine form studied in much of the recent bond-pricing literature (see Duffee, 2002, and Piazzesi, 2010, for a recent review). We have a production-based general-equilibrium affine model of the term structure with an endogenously varying price of risk. Not only is the one-period risk-free rate affine in the state variables, but so are the prices and yields for all longer-term zero-coupon bonds. The fact that the SDF is affine is convenient because it means that the model could be estimated using the Kalman filter, either through Bayesian or frequentist methods. I am not aware of an affine model of the term structure with a time-varying price of risk being derived in a production setting previously.

5.3 Labor supply

I model labor supply as in Gourio (2010) and van Binsbergen et al. (2010). The household’s value function takes the form

\[ V_t = \left\{ (1 - \exp(-\beta)) \left( C_t^{1-v} (1 - N_t)^{\alpha_t} \right)^{1-v} + \exp(-\beta) \left( E_t V_t^{1-\alpha_t} \right)^{1-v} \right\}^{1-\rho} \]  

(37)

where $N_t$ represents market labor and $\alpha_t$ follows the same process as above. The household’s labor supply condition is

\[ \frac{1}{1 - N_t} = \frac{1 - v \omega_t}{v C_t} \]  

(38)

where $\omega_t$ is the wage.
Note that risk aversion and habits only affect labor supply to the extent that they affect consumption. When consumption rises, labor falls, all else equal. Since positive permanent technology shocks drive consumption up farther in the EZ-habit model compared to the EZ-CRRA case, labor supply will rise by less in the EZ-habit model. Following a temporary technology shock, there is little change in risk aversion, so labor supply in that case will look similar in the EZ-habit and EZ-CRRA models.

This result is not specific to the Cobb-Douglas utility specification studied here. In general, preferences consistent with balanced growth will specify labor supply as some function $H(\omega_t/C_t)$ (see King, Plosser, and Rebelo, 1988). Since $\omega_t/C_t$ is stationary with balanced growth, labor supply will be too. If $H$ is monotonically increasing, regardless of its functional form, the increase in consumption induced by a decline in risk aversion will also lower labor supply.

To see how habits affect labor supply here, figure 7 plots the response of employment to a shock to technology in the EZ-habit model versus a model with constant relative risk aversion. With EZ-CRRA we have the usual RBC result that the increase in technology increases $\omega_t/C_t$ thus raising employment. Employment then slowly falls back down to its steady state.

In the simple RBC model, it is possible to make labor supply fall following a shock by varying the parameters, but it always monotonically returns to steady state. In the EZ-habit specification, though, the response of labor supply has a hump shape. On the impact of the shock, employment barely increases at all, and it then rises slowly thereafter. This behavior actually matches the response of employment to technology shocks in the literature following Gali (1999). In particular, all of those papers, though they use different methods, and though they find different initial responses of employment to technology, find a pronounced hump shape. Basu, Fernald, and Kimball (2006) argue that this could be explained by a New Keynesian model. Figure 7 shows that variation in risk aversion could also explain that behavior.

Boldrin, Christiano, and Fisher (2001) and Jaccard (2011) note that in the RBC model with power utility and additive habits, variable labor supply undermines the ability of the RBC model to generate a volatile SDF. Intuitively, households can use labor supply to smooth consumption growth. Under power utility, the volatility of consumption growth is what determines the volatility of the SDF. Under Epstein–Zin utility, though, the ability to smooth consumption shocks does not reduce the volatility of the SDF.

\[46\] All of the parameters are identical to the main text, and $v$ is set to 0.33 as in Gourio (2010).


\[48\] Note, though, that those papers also find hump-shaped responses for output and consumption, which I do not obtain.
since the SDF loads almost purely on the permanent component of consumption, as we saw in the previous subsection.\footnote{I confirm this result numerically; the Hansen–Jagannathan bound is essentially identical with and without labor supply.} The EZ-habit model thus does not suffer from the drawback of previous habit-based models that freely variable labor supply could substantially reduce the Hansen–Jagannathan bound.

6 Conclusion

This paper presents a model of time-varying risk aversion. It simultaneously matches the basic behavior of macroeconomic and financial aggregates. The EZ-habit model gives a framework in which consumption, output, and investment growth are all realistically volatile in both the short and long-run, consumption growth is nearly a random walk, and risk premia are high and volatile.

More generally, this paper provides a general framework for modeling time-varying discount rates that can be used with other macro models. As pointed out by Cochrane (2011), asset-pricing research has recently focused on understanding variation in the price of risk over time. This paper gives a way of analyzing time-varying risk prices in the standard macro framework. I show that for the RBC model, the effect of an increase in risk aversion on consumption and investment looks similar to a decline in the household’s rate of time preference in the sense that it temporarily increases investment and reduces consumption.

An obvious next step is to study the EZ-habit preferences in a richer setting. Dew-Becker (2011) estimates a standard medium-scale DSGE model with sticky prices and wages, but with the added feature that risk aversion varies over time, as here. Complementing the results in this paper on equity pricing, Dew-Becker (2011) shows that the EZ-habit model, when augmented with a model of inflation, can match the behavior of the nominal term structure well, generating a strongly upward-sloping term structure of nominal interest rates and a volatile term premium.

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A The certainty equivalent

This section looks at the relationship between the certainty equivalents using $G^{\text{habit}}$ and $G^{TV}$. I first show that the two certainty equivalents are equal up to a second order approximation around the non-stochastic version of the model. Next, I show that in the continuous-time limit, the preferences associated with the two certainty equivalents are identical.

A.1 Second-order approximation

This section approximates the certainty equivalent $G^{-1}(E_t(G(V_{t+1})))$ where $V_{t+1} = V_t \times (1 + \sigma \varepsilon_{t+1})$ around the point $\sigma = 0$. We assume that $E_t\varepsilon_{t+1} = 0$ and $E_t\varepsilon_{t+1}^2 = 1$.

Now consider the derivative of $G^{-1}(E_t(G(V_{t+1})))$ with respect to $\sigma$,

$$
\frac{d}{d\sigma} G^{-1}(E_t(G(V_{t+1}))) = \frac{d}{d\sigma} E_t(G(V_{t+1}))
$$

(A.1)

We have

$$
\frac{d}{d\sigma} E_t(G(V_{t+1})) = \int G'(V_t(1 + \sigma \varepsilon_{t+1})) \varepsilon_{t+1} V_t dF(\varepsilon_{t+1})
$$

(A.2)

where $F$ is the cdf of $\varepsilon_{t+1}$. Evaluated at $\sigma = 0$, $\frac{d}{d\sigma} E_t(G(V_{t+1})) = 0$, and therefore $\frac{d}{d\sigma} G^{-1}(E_t(G(V_{t+1}))) = 0$. So all certainty equivalents taking this form are identical up to the first order in approximations around $\sigma$.

Next, consider the second derivative,

$$
\frac{d^2}{d\sigma^2} G^{-1}(E_t(G(V_{t+1}))) = \frac{G'(G^{-1}(E_t(G(V_{t+1}))))}{[G'(G^{-1}(E_t(G(V_{t+1}))))]^2} \frac{d^2}{d\sigma^2} E_t(G(V_{t+1})) - \left[ \frac{d}{d\sigma} E_t(G(V_{t+1})) \right] \left[ \frac{d}{d\sigma} G'(G^{-1}(E_t(G(V_{t+1})))) \right]
$$

(A.3)

Since $\frac{d}{d\sigma} E_t(G(V_{t+1}))$ is equal to zero at $\sigma = 0$, we can ignore the second term in the numerator. The second derivative of the expectation is

$$
\frac{d^2}{d\sigma^2} E_t(G(V_{t+1})) = \int G''(V_t(1 + \sigma \varepsilon_{t+1})) \varepsilon_{t+1}^2 V_t^2 dF(\varepsilon_{t+1})
$$

(A.4)

At $\sigma = 0$, $\frac{d^2}{d\sigma^2} E_t(G(V_{t+1})) \big|_{\sigma=0} = G''(V_t) V_t^2$. We also have $G'(G^{-1}(E_t(G(V_{t+1})))) \big|_{\sigma=0} = G'(V_t)$, and hence

$$
\frac{d^2}{d\sigma^2} G^{-1}(E_t(G(V_{t+1}))) \big|_{\sigma=0} = \frac{G''(V_t) V_t^2}{G'(V_t)}
$$

So any two choices of $G$, say $G_1$ and $G_2$ are equivalent up to the second order if $\frac{G''(V_t)}{G'(V_t)} = \frac{G''(V_t)}{G'_2(V_t)}$ for any $V_t$. That relationship holds for $G^{\text{habit}}$ and $G^{TV}$.

A.2 Continuous time

Duffie and Epstein (1992) show how to extend Epstein–Zin preferences to continuous time. They derive a utility function following the process

$$
dV_t = \mu_t + \sigma_t dB_t
$$

(A.5)

$$
dV_t = \left(-f(c_t, V_t) - \frac{1}{2} A(V_t) \sigma_t^2 \right) dt + \sigma_t dB_t
$$

(A.6)

for a Wiener process $dB_t$.

As in the main text, suppose the household’s certainty equivalent under discrete-time Epstein–Zin preferences is $G^{-1}(E_t(G(V_{t+1})))$. Duffie and Epstein (1992) show that the analogous choice of $A$, obtained as a limiting case as the length of time periods approaches zero, is $A(V_t) = \frac{G''(V_t)}{G'(V_t)}$. In the case where $G^{\text{power}}(V_t) = V_t^{1-\alpha}$, we have

$$
A^{\text{power}}(V_t) = \frac{G^{\text{power}}''(V_t)}{G^{\text{power}}'(V_t)} = \frac{-\alpha}{V_t}
$$

(A.7)
and for $G^{habit} = (V_t - H_t)^{1-\alpha}$

$$A^{habit}(V_t) = \frac{-\alpha}{V_t - H_t} \quad (A.8)$$

For $G^TV = V_t^{1-\alpha}$,

$$A^TV(V_t) = \frac{-\alpha}{V_t} \quad (A.9)$$

So $G^TV$ and $G^{habit}$ are identical if $\alpha_t = \alpha \frac{V_t}{V_t - H_t}$, which is what is used in the text.

For all three choices of the certainty equivalent $G$, we can use the standard choice for $f, f(c_t, V_t) = \frac{\beta}{1-\rho} \frac{c_t - \rho V_t}{V_t^{1-\rho}}$. $\rho$ then determines the elasticity of intertemporal substitution, while $A$ determines risk aversion.

**B Derivation of the SDF**

We can obtain the stochastic discount factor (SDF) by calculating the intertemporal marginal rate of substitution. We calculate two derivatives. First,

$$\frac{\partial V_t}{\partial C_t} = V_t^\rho (1 - \beta) C_t^{-\rho} \quad (B.1)$$

Next, we differentiate $V_t$ with respect to $C_{t+1}(w)$, where $w$ denotes one state of the world, and $\pi_w$ is the probability of that state,

$$\frac{\partial V_t}{\partial C_{t+1}(w)} = \pi_w V_t^\rho \beta R_t^{-\rho} G_t^{(-1)\rho}(E_t[G_t(V_{t+1}(w))]) G_t'(V_{t+1}(w)) V_{t+1}^\rho(w) (1 - \beta) C_{t+1}^{-\rho}(w) \quad (B.2)$$

where $G_t'$ is the derivative of $G_t$ and $G_t^{(-1)\rho}$ the derivative of $G_t^{-\rho}$. $R_t \equiv G^{-1}(E_t G_t(V_{t+1}))$. The subscripts on $G_t$ refer to the fact that $G_t$ depends on the potentially-time-varying parameter $\alpha_t$. The assumption that $\alpha_t$ is exogenous to the household is necessary for this formula for the derivative to be correct (in the same way that external habits lead to a more tractable formula for the SDF than do internal habits).

The SDF can be derived from a consumer’s first order conditions for optimization as $M_{t+1}(w) = \frac{1}{\pi_w} \frac{\partial V_t/\partial C_{t+1}(w)}{\partial V_t/\partial C_t}$. We then have

$$M_{t+1}(w) = \beta \frac{G_t'(V_{t+1}(w)) V_{t+1}^\rho(w) C_t^{-\rho}}{R_t^{\rho-\alpha_t} G_t(R_t)} \quad (B.3)$$

where the last line follows from the fact that $G_t^{(-1)\rho}(x) = 1/G_t'(x)$.

In the case of $G_t(V) = V^{1-\alpha}$, the SDF becomes

$$M_{t+1} = \beta \frac{V_{t+1}^{\rho-\alpha_t}(w) C_t^{-\rho}}{R_t^{\rho-\alpha_t}} \quad (B.4)$$

**B.1 Substituting in an asset return**

Consider an asset that pays $C_t$ as its dividend. We guess that its cum-dividend price is $W_t = V_t^{1-\rho} C_t^\rho (1 - \exp(-\beta))^{-1}$. This guess can be confirmed by simply inserting it into the household’s Euler equation.

The return on the consumption claim is

$$R_{w,t+1} = \frac{W_{t+1}}{W_t - C_t} = \frac{V_{t+1}^{1-\rho}}{\exp(-\beta) R_t (V_{t+1})^{1-\rho}} \left(\frac{C_{t+1}}{C_t}\right)^\rho \quad (B.5)$$

Which yields

$$\frac{V_{t+1}^{\rho-\alpha_t}(w)}{R_t^{\rho-\alpha_t}} = (R_{w,t+1} \exp(-\beta))^{\frac{\rho-\alpha_t}{1-\rho}} \left(\frac{C_{t+1}}{C_t}\right)^{-\rho \frac{\rho-\alpha_t}{1-\rho}} \quad (B.6)$$
We can then insert this into the SDF to yield

\[ M_{t+1} = \exp(-\beta)^{1-\alpha_t} \left( \frac{C_{t+1}}{C_t} \right)^{-\rho^{1-\alpha_t}} R_{w,t+1}^{1-\alpha_t} \] (B.7)

C The log-linear model with production

C.1 Steady state

In the nonstochastic steady state, the interest rate earned by all assets, \( r \), is equal to

\[ r = \beta + \rho \mu \] (C.1)

Standard manipulations show that the steady-state ratio of capital to technology is then

\[ K = \left( \frac{\exp(\beta + \rho \mu) - 1 + \delta}{\gamma} \right)^{1/(\gamma-1)} \] (C.2)

We can obtain the steady-state consumption-output ratio by using the budget constraint,

\[ \bar{C} = \bar{Y} + (1 - \delta) \bar{K} - \exp(\mu) \bar{K} \] (C.3)

\[ \frac{1 - \bar{C}}{\bar{Y}} = -(1 - \delta - \exp(\mu)) \frac{\bar{K}}{\bar{Y}} \] (C.4)

C.2 The budget constraint

The approximation I use for the budget constraint is identical to Campbell (1994). The budget constraint is\[ K_{t+1} = A_t K_t^{\gamma-1} - C_t + (1 - \delta) K_t. \] I look for a log-linear approximation taking the form \( k_{t+1} = \lambda_0 + \lambda_a k_t + \lambda_a a_t + \lambda_c c_t, \) where the \( \lambda \) terms are coefficients from the approximation.

The budget constraint can be rewritten as

\[ \log(\exp(\Delta k_{t+1}) - (1 - \delta)] = y_t - k_t + \log(1 - \exp(c_t - y_t)) \] (C.5)

Taking a log-linear approximation to the left-hand side around the point \( \Delta k_{t+1} = \mu, \) we have

\[ \log(\exp(\Delta k_{t+1}) - (1 - \delta)] \approx \log(\exp(\mu) - (1 - \delta)] + \frac{\exp(\mu)}{\exp(\mu) - (1 - \delta)} (\Delta k_{t+1} - \mu) \] (C.6)

To approximate the right-hand side of (C.5), we approximate \( \log(1 - \exp(c_t - y_t)) \) around the steady state \( cy, \)

\[ \log(1 - \exp(c_t - y_t)) \approx \log(1 - \exp(cy)) + \frac{-\exp(cy)}{1 - \exp(cy)} (c_t - y_t - cy) \] (C.7)

This implies

\[ \log(\exp(\mu) - (1 - \delta)] + \frac{\exp(\mu)}{\exp(\mu) - (1 - \delta)} (\Delta k_{t+1} - \mu) \approx \log(1 - C/Y) + y_t - k_t + \frac{-\exp(cy)}{1 - \exp(cy)} (c_t - y_t - cy) \] (C.8)

Now we can find the coefficients in the linear approximation to the budget constraint. The constant term is

\[ \lambda_0 = \frac{\exp(\mu) - (1 - \delta]}{\exp(\mu)} \left( \log(1 - C/Y) - \left( \frac{-\exp(cy)}{1 - \exp(cy)} \right) cy \right) \] (C.9)
The coefficients on $k, a,$ and $c,$ are then

$$\lambda_k = \frac{\exp(\mu) - (1 - \delta)}{\exp(\mu)} \left[ \gamma - 1 - \gamma \left( -\exp(cy) \right) \right] + 1 \quad (C.10)$$

$$\lambda_a = \frac{\exp(\mu) - (1 - \delta)}{\exp(\mu)} (1 - \gamma) - (1 - \gamma) \frac{\exp(\mu) - (1 - \delta)}{\exp(\mu)} \left( -\exp(cy) \right) \quad (C.11)$$

$$\lambda_c = \frac{\exp(\mu) - (1 - \delta)}{\exp(\mu)} \left( -\exp(cy) \right) \quad (C.12)$$

Now note that $\lambda_k + \lambda_a + \lambda_c = 1,$ So we have

$$k_{t+1} = \lambda_0 + \lambda_k k_t + \lambda_a a_t + (1 - \lambda_k - \lambda_a) c_t \quad (C.13)$$

### C.3 Capital return

To approximate the return on capital, we say

$$r_{k,t+1} = \log \left( \gamma \exp ((1 - \gamma) (a_{t+1} - k_{t+1})) + 1 - \delta \right) \quad (C.14)$$

$$\approx \log \left( \gamma \exp ((\gamma - 1) \tilde{k}) + 1 - \delta \right) + \frac{(\gamma - 1) \gamma \exp ((\gamma - 1) \tilde{k}) (k_{t+1} - a_{t+1} - \tilde{k})}{\gamma \exp ((\gamma - 1) \tilde{k}) + 1 - \delta} \quad (C.15)$$

$$r_{k,t+1} \approx r + r_{kk} (\tilde{k}_{t+1} - \tilde{k}) \quad (C.16)$$

where $r_{kk} \equiv (\gamma - 1) (\exp(r) - 1 + \delta) / \exp(r)$

### C.4 Risk aversion

I guess that the innovation to the value function van be written as $\kappa_v \xi_{t+1},$ so that

$$\alpha_{t+1} = \phi a_t + (1 - \phi) \tilde{\alpha} + \lambda \kappa_v \xi_{t+1} \quad (C.17)$$

I confirm this guess below.

### C.5 Consumption dynamics

Writing $\tilde{k}_t \equiv k_t - a_t$ and $\tilde{c}_t \equiv c_t - a_t,$ we have

$$\tilde{k}_{t+1} = \lambda_0 + \lambda_k \tilde{k}_t + \lambda_c \tilde{c}_t - \sigma_{a \xi_{a,t+1}} - \mu \quad (C.18)$$

Now we guess that the consumption function is $\tilde{c}_t = \eta_{c0} + \eta_{ck} \tilde{k}_t + \eta_{ca} a_t$ (note here that I use $\lambda$ terms for the budget constraint, which are terms depending only on the underlying parameters of the model; the $\eta$ terms are coefficients from the optimal consumption rule). Then we have

$$\tilde{k}_{t+1} = \lambda_0 + \lambda_k \tilde{k}_t + \lambda_c \left( \eta_{c0} + \eta_{ck} \tilde{k}_t + \eta_{ca} a_t \right) - \sigma_{a \xi_{a,t+1}} - \mu \quad (C.19)$$

$$\eta_{k0} = \lambda_0 + \lambda_c \eta_{c0} - \mu$$

$$\eta_{kk} = \lambda_k + \lambda_c \eta_{ck} \quad \eta_{ka} = \lambda_c \eta_{ca} \quad (C.20)$$

This equation specifies the dynamics of capital conditional on the underlying parameters of the model and the two unknown coefficients determining the dynamics of consumption.

For consumption growth, we say $\Delta c_{t+1} = \eta_{d0} + \eta_{dk} \tilde{k}_t + \eta_{da} a_t + \kappa_d \xi_{t+1},$ where

$$\eta_{d0} \equiv \eta_{ck} \eta_{k0} - \eta_{ca} (\phi_\alpha - 1) \tilde{\alpha} + \mu \quad \eta_{dk} \equiv \eta_{ck} (\eta_{kk} - 1)$$

$$\kappa_d \equiv \sigma_a (1 - \eta_{ck}) + \eta_{ca} \lambda \kappa_v \quad \eta_{da} \equiv \eta_{ck} \eta_{ka} + \eta_{ca} (\phi_\alpha - 1)$$
The remainder of the appendix confirms that our guesses for the form of the consumption and value functions are correct.

C.6 Wealth return

In the presence of balanced growth, the long-run response of consumption to an innovation of \( \sigma_a \varepsilon_{t+1} \) to technology must be exactly \( \sigma_a \varepsilon_{t+1} \). This is equivalent to saying that

\[
\Delta E_{t+1} \sum_{j=0}^{\infty} \Delta c_{t+j+1} = \sigma_a \varepsilon_{t+1}
\]  
(C.21)

In the case where \( \theta \) approaches 1 (the steady-state dividend/price ratio approaches zero) or the consumption response only takes one period, \( \Delta E_{t+1} \sum_{j=0}^{\infty} \Delta c_{t+j+1} = \Delta E_{t+1} \sum_{j=0}^{\infty} \theta^j \Delta c_{t+j+1} \). We therefore have the approximation,

\[
r_{w,t+1} = E_t r_{w,t+1} + \sigma_a \varepsilon_{t+1} - \Delta E_{t+1} \sum_{j=1}^{\infty} \theta^j r_{w,t+j+1}
\]  
(C.22)

This extra approximation is not strictly necessary, and the model is straightforward to solve without it. However, it substantially simplifies many of the formulas and makes them more transparent. The results reported below on the accuracy of the log-linear solution apply to the solution using this approximation.

Now note that

\[
\Delta E_{t+1} \sum_{j=1}^{\infty} \theta^j r_{w,t+j+1} = \Delta E_{t+1} \sum_{j=1}^{\infty} \theta^j (\alpha_{t+j} \eta_{we} + \rho E_{t+j} \Delta c_{t+j})
\]  
(C.23)

\[
= \eta_{we} \frac{\theta}{1-\theta \phi} \sigma_{aa} + \rho (\eta_{ck} \sigma_a - \eta_{ca} \sigma_{aa})
\]  
(C.24)

The second term follows from the approximation \( \Delta E_{t+1} \sum_{j=1}^{\infty} \theta^j \rho E_{t+j} \Delta c_{t+j} \approx \Delta E_{t+1} \sum_{j=1}^{\infty} \rho E_{t+j} \Delta c_{t+j} \). The right hand side of this equation is simply \( \rho \) multiplied by the total amount of consumption growth expected following period \( t+1 \). Since we know that in the long run, the consumption-technology ratio is stationary, we just need to know how much consumption declines relative to technology at period \( t+1 \). That’s going to be exactly \( \eta_{ck} \sigma_a \varepsilon_{t+1} - \eta_{ca} \sigma_{aa} \varepsilon_{t+1} \) (since capital falls by \( \sigma_a \) and \( \alpha \) falls by \( -\sigma_{aa} \)).

We then have

\[
\kappa_r = \sigma_a - \eta_{we} \frac{\theta}{1-\theta \phi} \sigma_{aa} - \rho (\eta_{ck} \sigma_a - \eta_{ca} \sigma_{aa})
\]  
(C.25)

The return is thus

\[
r_{t+1} = \eta_{we} \rho E_t \Delta c_{t+1} + \eta_{we} \alpha_t + \sigma_a \varepsilon_{t+1} + \left( -\eta_{we} \frac{\theta}{1-\theta \phi} \sigma_{aa} + \frac{\theta}{1-\theta \phi} \eta_{dk} \sigma_a \right) \varepsilon_{t+1}
\]  
(C.26)

C.7 The Euler equation for wealth

The asset pricing equation gives us

\[
1 = E_t \left[ \exp \left( \beta \frac{1-\alpha}{1-\rho} \left( C_{t+1} \frac{1}{C_t} \right) - \frac{1-\alpha}{1-\rho} R_{w,t+1} \right) \right]
\]  
(C.27)
The log of the term inside the expectation is
\[
\log \left[ \exp \left( \frac{(1-\alpha_t)}{1-\rho} \left( \frac{C_{t+1}}{C_t} \right) \frac{(1-\alpha_t)}{1-\rho} R_{w,t+1} \right) \right] = -\frac{1-\alpha_t}{1-\rho} \beta - \frac{1-\alpha_t}{1-\rho} \Delta c_{t+1}
\]
\[
+ \frac{1-\alpha_t}{1-\rho} \left( \eta_{w0} + \rho E_t \Delta c_{t+1} + \eta_{wa} \alpha_t + \kappa_r \varepsilon_{t+1} \right)
\]
\[
= -\frac{1-\alpha_t}{1-\rho} \beta - \frac{1-\alpha_t}{1-\rho} \kappa_d \varepsilon_{t+1} + \frac{1-\alpha_t}{1-\rho} \left( \eta_{w0} + \eta_{wa} \alpha_t + \kappa_r \varepsilon_{t+1} \right)
\]
\[
(C.28)
\]
Now taking the log of the expectation and dividing by \(\frac{1-\alpha_t}{1-\rho}\) gives
\[
0 = -\beta + \eta_{w0} + \eta_{wa} \alpha_t + \frac{1-\alpha_t}{2} \left( -\rho \kappa_d + \kappa_r \right)^2
\]
\[
(C.30)
\]
which implies
\[
\eta_{w0} = \beta - \frac{1}{2} \left( -\rho \kappa_d + \kappa_r \right)^2
\]
\[
(C.31)
\]
\[
\eta_{wa} = \frac{1}{2} \left( -\rho \kappa_d + \kappa_r \right)^2
\]
\[
(C.32)
\]
\[
\kappa_r = \sigma_a - \eta_{wa} \frac{\theta}{1 - \theta \sigma_a} - \rho \left( \eta_{ck} \sigma_a - \eta_{ca} \sigma_a \right)
\]
\[
(C.33)
\]
C.8 The Euler equation for capital
The stochastic discount factor follows
\[
m_{t+1} = -\beta - \rho E \Delta c_{t+1} - \rho \frac{1-\alpha_t}{1-\rho} \kappa_d \varepsilon_{t+1} + \rho - \rho \frac{1-\alpha_t}{1-\rho} \left( \frac{1}{2} \left( -\rho \kappa_d + \kappa_r \right)^2 + \kappa_r \varepsilon_{t+1} \right)
\]
\[
(C.34)
\]
For the capital return we have
\[
1 = E_t \left[ \exp \left( m_{t+1} + r + r_{kk} \left( \eta_{k0} + \eta_{kk} \bar{k}_t + \eta_{ka} \alpha_t - \sigma_a \varepsilon_{a,t+1} - \bar{k} \right) \right) \right]
\]
\[
(C.35)
\]
which implies
\[
0 = -\beta - \rho E \Delta c_{t+1} + \frac{1}{2} \left( -\rho \kappa_d + \kappa_r \right)^2
\]
\[
+ r + r_{kk} \left( \eta_{k0} + \eta_{kk} \bar{k}_t + \eta_{ka} \alpha_t - \bar{k} \right)
\]
\[
+ \frac{1}{2} \left( r_{kk} \sigma_a + \kappa_r \right)^2 - \left( r_{kk} \sigma_a + \kappa_r \right) \left( \frac{1-\alpha_t}{1-\rho} \left( -\rho \kappa_d + \kappa_r \right) \right)
\]
\[
(C.36)
\]
Note that all of the nonlinearities disappear (i.e. the \(\alpha_t^2\) terms), and this equation is linear in the state variables. We can thus solve through the method of undetermined coefficients as usual.

For the coefficients on capital,
\[
0 = -\rho \left( \lambda_k \eta_{ck} + \lambda_c \eta_{ck}^2 - \eta_{ck} \right) + r_{kk} \left( \lambda_k + \lambda_c \eta_{ck} \right)
\]
\[
(C.37)
\]
This is quadratic in \(\eta_{kk}\), and we have
\[
\eta_{ck} = \frac{\rho (1 - \lambda_k) + r_{kk} \lambda_c \pm \sqrt{(\rho (1 - \lambda_k) + r_{kk} \lambda_c)^2 + 4 \rho \lambda_c r_{kk} \lambda_k}}{2 \rho \lambda_c}
\]
\[
(C.38)
\]
Now note that $\lambda_c < 0$, $\lambda_k > 0$, and $r_{kk} < 0$. This implies that

$$
\sqrt{\left(\rho (1 - \lambda_k) + r_{kk} \lambda_c\right)^2 + 4 \rho \lambda_c r_{kk} \lambda_k} > \rho (1 - \lambda_k) + r_{kk} \lambda_c 
$$

(C.39)

and hence $\eta_{ck}$ has a positive and a negative root. The root where $\eta_{ck} < 0$ violates the transversality condition (high capital implies low consumption), so we choose the root with $\eta_{ck} > 0$. Note that the formula for $\eta_{ck}$ does not involve $\bar{\alpha}$ or $\sigma_{aa}$, which confirms remark 1.

For the coefficients on $\alpha_t$, we have

$$
0 = -\rho \eta_{da} - \frac{1}{2} \left(\frac{-\rho \kappa_d + \kappa_r}{1 - \rho}\right)^2 + r_{kk} \eta_{ka} + (r_{kk} \sigma_a + \kappa_r) \frac{(-\rho \kappa_d + \kappa_r)}{1 - \rho} 
$$

(C.40)

### C.9 Other parameters

To solve for $(-\rho \kappa_d + \kappa_r)$, simply combine the equations for $\eta_{wa}$ and $\kappa_r$, yielding

$$
-\rho \kappa_d + \kappa_r = -1 + \sqrt{1 + 2 \frac{\theta}{1 - \theta} \sigma_{aa} \sigma_a} \frac{1 - \rho}{1 - \rho} \frac{\theta}{1 - \theta} \sigma_{aa} 
$$

(C.41)

We choose the root for this equation that has the property that it approaches zero as $\sigma_a$ approaches zero. That is, we know that when the shocks have zero variance, all assets have the same return, and so $\eta_{aa} = 0$.

### C.10 Excess returns and the risk-free rate (result 2)

To calculate excess returns, we can simply calculate the covariance of the wealth return with the SDF. The Sharpe ratio of the wealth portfolio is

$$
\frac{E_t r_{w,t+1} - r_{f,t+1} + \frac{1}{2} \kappa_r^2}{\kappa_r} = \frac{\alpha_t - \rho (-\rho \kappa_d + \kappa_r) + \rho \kappa_d}{1 - \rho} \frac{\alpha_t - \rho (-\rho \kappa_d + \kappa_r) + \rho \kappa_d}{1 - \rho} 
$$

(C.42)

$$
= \frac{\alpha_t - \rho (-\rho \kappa_d + \kappa_r) + \rho \kappa_d}{1 - \rho} 
$$

(C.43)

This also immediately gives a formula for the risk-free rate

$$
\frac{E_t r_{w,t+1} - r_{f,t+1} + \frac{1}{2} \kappa_r^2}{\kappa_r} = \frac{\alpha_t - \rho (-\rho \kappa_d + \kappa_r) + \rho \kappa_d \kappa_r}{1 - \rho} \frac{\alpha_t - \rho (-\rho \kappa_d + \kappa_r) + \rho \kappa_d \kappa_r}{1 - \rho} 
$$

(C.44)

$$
\kappa_r^2 = \frac{\alpha_t - \rho (-\rho \kappa_d + \kappa_r) + \rho \kappa_d \kappa_r}{1 - \rho} \frac{\alpha_t - \rho (-\rho \kappa_d + \kappa_r) + \rho \kappa_d \kappa_r}{1 - \rho} 
$$

(C.45)

### C.11 The wealth-consumption ratio

The Campbell–Shiller approximation for the wealth-consumption ratio is

$$
w_t - c_t = \frac{z}{1 - \theta} + E_t \sum_{j=0}^{\infty} \theta^j (\Delta c_{t+j+1} - r_{t+j+1}) 
$$

(C.46)

for a constant $z$ depending on the average consumption-wealth ratio (i.e. related to $\theta$), $z = -\log \theta - (1 - \theta) \log \left(\frac{1}{\theta} - 1\right)$. Now

$$
E_t \sum_{j=0}^{\infty} \theta^j \Delta c_{t+j+1} = -\eta_{ck} (k_t - \bar{k}) - \eta_{ca} (\alpha_t - \bar{\alpha}) 
$$

(C.47)
under the approximation $\theta = 1$ from above, and
\[
E_t \sum_{j=0}^{\infty} \theta^j r_{w,t+j+1} = E_t \sum_{j=0}^{\infty} \theta^j (\alpha_{t+j} \eta_{wa} + \rho E_{t+j} \Delta c_{t+j+1})
\]
(C.48)
So
\[
w_t - c_t = \frac{z}{1-\theta} + (1-\rho) (\eta_{ck} (k_t - \bar{k}) - \eta_{ca} (\alpha_t - \bar{\alpha})) - \eta_{wa} \frac{\theta}{1-\theta} (\alpha_t - \bar{\alpha})
\]
(C.49)

### C.12 The value function and risk aversion (result 4)

At any time, household value is
\[
W_t = V_t^{1-\rho} C_t^{\rho} / (1 - \exp (-\beta)) \quad (C.50)
\]
\[
v_t = \frac{(w_t - c_t)}{1-\rho} + c_t + \frac{\log (1 - \exp (-\beta))}{1-\rho} \quad (C.51)
\]

The innovation to the value function, $v_{t+1} - E_t v_{t+1}$, is equal to the sum of the innovations to $\frac{(w_{t+1} - c_{t+1})}{1-\rho}$ and $c_{t+1}$, which are

\[
\Delta E_{t+1} \frac{(w_{t+1} - c_{t+1})}{1-\rho} + \Delta E_{t+1} c_{t+1} = \sigma_a \varepsilon_{t+1} - \frac{\eta_{wa}}{1-\rho} \frac{\theta}{1-\theta \phi} \sigma_{aa} \varepsilon_{t+1}
\]
(C.52)

\[
\kappa_v = \sigma_a \varepsilon_{t+1} - \frac{\theta}{1-\theta \phi} \frac{1}{2} \left( \frac{1}{1-\rho} \right)^2 \left( -\rho \kappa_d + \kappa_r \right)^2 \sigma_{aa} \varepsilon_{t+1}
\]
(C.53)

Using the formula from above that defines $-\rho \kappa_d + \kappa_r$, we have $\kappa_v = \frac{(\rho \kappa_d + \kappa_r)}{1-\rho}$ and

\[
\sigma_{aa} = \lambda \kappa_v = \lambda \frac{-1 + \sqrt{1 + 2 \frac{\theta}{1-\theta \phi} \sigma_{aa} \sigma_a}}{1-\theta \phi \sigma_{aa}}
\]
(C.54)

### C.13 Affine bond pricing (result 5)

\[
m_{t+1} = -\beta - \rho D c_{t+1} + \frac{1 - \alpha_t}{1-\rho} \kappa_d \varepsilon_{t+1} + \frac{\rho - \alpha_t}{1-\rho} \left( \frac{1}{2} \frac{1-\alpha_t}{1-\rho} \left( -\rho \kappa_d + \kappa_r \right)^2 + \kappa_r \varepsilon_{t+1} \right)
\]
(C.55)

\[
\text{var} (m_{t+1}) = \left( \frac{1 - \alpha_t}{1-\rho} (-\rho \kappa_d + \kappa_r) - \kappa_r \right)^2 \sigma^2
\]
(C.56)

\[
= \left[ \left( \frac{1 - \alpha_t}{1-\rho} \right)^2 (-\rho \kappa_d + \kappa_r)^2 + \kappa_r^2 - \frac{2}{1-\rho} \frac{1 - \alpha_t}{1-\rho} (-\rho \kappa_d + \kappa_r) \kappa_r \right] \sigma^2
\]
(C.57)

And hence
\[
r_{f,t+1} = - \left( E_t m_{t+1} + \frac{1}{2} \text{var} (m_{t+1}) \right)
\]
(C.58)
\[
= - \left( -\beta - \rho \left( \eta_{d0} + \eta_{dk} \bar{k}_t + \eta_{da} \alpha_t \right) + \frac{1}{2} \frac{1 - \alpha_t}{1-\rho} \left( -\rho \kappa_d + \kappa_r \right)^2 + \frac{1}{2} \kappa_r^2 - \frac{1 - \alpha_t}{1-\rho} (-\rho \kappa_d + \kappa_r) \kappa_r \right)
\]
(C.59)

It is straightforward to show that
\[
m_{t+1} = - r_{f,t+1} + \left( \frac{1 - \alpha_t}{1-\rho} (-\rho \kappa_d + \kappa_r) - \kappa_r \right) \varepsilon_{t+1} - \frac{1}{2} \left( \frac{1 - \alpha_t}{1-\rho} (-\rho \kappa_d + \kappa_r) + \kappa_r \right)^2 \sigma^2
\]
(C.60)
\[
= - r_{f,t+1} - \frac{1}{2} (\omega_0 + \omega_1 \alpha_t)^2 \sigma^2 + (\omega_0 + \omega_1 \alpha_t) \varepsilon_{t+1}
\]
(C.61)
So the SDF takes the essentially affine form with
\[ \omega_0 = \frac{-(dn + \kappa_r)}{1-p} - \kappa_r \quad \omega_1 = \frac{-(dn + \kappa_r)}{1-p} \]

C.14 Accuracy of the approximation

Table A1 reports simple statistics summarizing the relationship between the projection solution and the log-linear approximation to the model. The first column lists the mean difference between the solutions, the second column the standard deviation of the gap, and the third column the standard deviation of the gap scaled by the standard deviation of the variable in the projection solution. I report deviations for log capital, log consumption growth, the coefficient of relative risk aversion, and the Sharpe ratio of the wealth portfolio. For the simulations, both models start with the same initial levels of capital and risk aversion and use the same technology shocks. I then simulate the models for 20,000 periods.

Table A1 shows that for capital and consumption growth, the log-linear approximation is nearly identical to the projection solution. The mean differences are essentially zero, and the standard deviations of the errors are both less than 3 percent of the standard deviations of the variables themselves. For risk aversion and the Sharpe ratio, the log-linear approximation is essentially identical to the projection solution on average, but the standard deviation of the differences is now roughly 12 percent of the standard deviation of the variables themselves.

An alternative method of checking the accuracy of the approximation is to look at Euler equation errors. Figure A1 plots histograms of the log10 Euler equation errors, \( \log_{10} E_t^M M_{t+1} + 1 \) under the projection solution and the log-linear solution at each date in the simulation.

D Details of return forecasting

D.1 The method from Lettau and Ludvigson (2001)

If consumption and wealth are cointegrated, then we have the relationship
\[ c_t = \zeta w_t + \xi_t \] (D.1)
where \( \zeta \) is a parameter, and \( \xi_t \) is a mean-zero, stationary, and not necessarily i.i.d. error term. If we observed wealth, \( \zeta \) and \( \xi_t \) could be directly estimated. We do not observe wealth, though, especially the human component. Lettau and Ludvigson (2001) therefore use the approximation
\[ w_t = \omega a_t + (1 - \omega) h u_t \] (D.2)
where \( a_t \) is asset wealth and \( h u_t \) human wealth. This equation simply says that log aggregate wealth is equal to the sum of log asset and human wealth. Since the level of aggregate wealth is equal to the sum of the levels of asset and human wealth, the approximation is valid as long as the shares of asset and human wealth in aggregate wealth are stationary not not too variable. The fact that labor’s share of income has been stationary in the post-war US data makes this assumption reasonable.

Finally, we assume that labor income, \( y_t \), can be viewed as the dividend from human wealth and that the dividend/price ratio for human wealth is stationary. That is,
\[ y_t = g + h u_t - \mu_t \] (D.3)
where \( g \) is a parameter and \( \mu_t \) is a mean-zero stationary \( b_{z,1} \) term. This implies that
\[ w_t = \omega a_t + (1 - \omega) y_t + (1 - \omega) g + \mu_t \] (D.4)
\[ c_t = \zeta_0 a_t + \zeta (1 - \omega) y_t + \zeta (1 - \omega) g + \zeta \mu_t + \xi_t \] (D.5)
since \( \xi_t + \zeta \mu_t \) is mean-zero and stationary, regardless of any correlation between \( \xi_t \) and \( \mu_t \), the variables \( c_t, a_t, \) and \( y_t \) are jointly cointegrated. The parameters \( \zeta, \omega, \) and \( g \) can be estimated through standard methods for cointegrated models. As Lettau and Ludvigson point out, the estimation is of these parameters is superconsistent, converging
I follow Lettau and Ludvigson in referring to the cointegrating residual, \( \zeta \mu_t + \xi_t = c_t - \zeta \omega_t - \zeta (1 - \omega) y_t - \zeta (1 - \omega) y_t \) as \( \text{me} \), and I refer to \( \omega_t + (1 - \omega) \times y_t \) as \( \text{me}_\omega \). \( \text{me} \) is an estimate of total wealth derived from data on consumption, asset wealth, and labor income, taking advantage of an assumed cointegrating relationship between the three variables. I estimate the parameters using standard maximum likelihood methods.

### D.2 Sensitivity analysis for return forecasting

The results in section 4.2 depend on choices for two parameters – the EIS and the persistence of risk aversion. Tables A3 and A4 report the ratio of the \( R^2 \) for excess value to \( \text{me} \) for 1, 5, 10, and 20-quarter returns across a variety of choices for the EIS and the persistence of risk aversion.

Table A2 varies the EIS between 0.75 and 10. The numbers in bold represent points where \( \text{me} \) outperforms \( \hat{\alpha} \). When the EIS is greater than 1, \( \text{me} \) only ever outperforms at the 1-quarter horizon, and then only if the EIS is set to 10. With an EIS less than 1, though, \( \text{me} \) always has a \( R^2 \) substantially larger that of \( \hat{\alpha} \). Moreover, the sign on \( \hat{\alpha} \) in the return regressions flips. Intuitively, this is because in the construction of \( \hat{v} \), when the EIS is less than 1, the weight on aggregate wealth is negative. The theory would predict that high risk aversion is associated with low returns, but with the EIS less than 1, \( \hat{\alpha} \) and future returns are actually positively correlated.

Table A3 presents \( R^2 \) ratios for the same set of regressions, but now varying the persistence of risk aversion. Across a fairly wide range of autocorrelations, \( \hat{\alpha} \) outperforms \( \text{cay} \) at most horizons. The best performance is found with an annual autocorrelation of 0.9, which corresponds to \( \phi = 0.974 \). Even with an autocorrelation as low as 0.65 (\( \phi = 0.9 \)), though, \( \hat{\alpha} \) performs nearly as well as \( \text{cay} \). As with the EIS, the place where \( \text{cay} \) is most likely to outperform is with 1-quarter returns. Table A4 lists \( R^2 \)s for \( \text{cay} \), PE, and \( \hat{\alpha} \) for pre and post-1980 samples.

### D.3 Out-of-sample forecasting regressions

An alternative to the in-sample regressions studied in the main text is out-of-sample tests of forecasting power. I consider the mean squared forecast ratio \( b_{z,1} \) (MSFE) based tests from analyzed in Clark and McCracken (2001, 2005) and Clark and West (2007).

Suppose we want to test whether a single variable, \( x_t \), forecasts stock returns, \( r_t \), against the null that \( r_t \) is i.i.d. (the methods used here apply to any null model that is nested; i.e. they are appropriate for asking whether \( x_t \) has marginal forecasting power when added to some other model). The forecast horizon can be any length. Therefore, denote \( r_{t,t+j} = \sum_{\tau=t}^{t+j} r_\tau \).

We compare the residuals from the null model, \( e_{1t} = r_{t,t+j} - \beta_{0,t} \) (for an estimated constant mean \( \beta_{0,t} \) using data prior to date \( t \)) to the residuals from the alternative model, \( e_{2t,t+j} = r_{t,t+j} - \beta_{0,t} - \beta_{1,t} x_{t+j-1} \) (where \( \beta_{1,t} \) is a constant regression coefficient estimates on the data from \( \tau = 0 \) to \( \tau = t-1 \)). The samples for the regressions are begun after the first 20 percent of the sample.

The measure of the difference in MSFE is

\[
\mathcal{f}_{t,t+j} = e_{1t,t+j}^2 - e_{2t,t+j}^2 + (e_{1t,t+j} - e_{2t,t+j})^2
\]

Under the null, the MSFE for the \( e_1 \) model tends to be smaller than the MSFE for the \( e_2 \) model because the \( e_2 \) model has added noise due to the extraneous predictor. Intuitively, model \( e_1 \) correctly imposes the constraint that \( \beta_1 = 0 \) under the null. The term \( (e_{1t,t+j} - e_{2t,t+j})^2 \) is essentially a correction for this effect.

When the forecast horizon is more than a single observation, \( \mathcal{f}_{t,t+j} \) is serially correlated. To correct for this, we divide by a consistent estimate of its long-run variance (spectral density at frequency zero). Following Clark and West (2007), I use the Newey–West measure with a lag window of \( 1.5 \times j \). Denote this measure of the long-run variance as \( S(\mathcal{f}_{t,t+j}) \). The long-run variance corrects for the fact that the forecast \( b_{z,1} \) from overlapping samples will be serially correlated. Clark and McCracken tabulate the critical values of the statistic \( (T - j) \sum_{\tau=1}^{T-j} \mathcal{f}_{t,t+j} / S(\mathcal{f}_{t,t+j}) \).

In the main text, \( \hat{\alpha} \) is calculated using full-sample information. In particular, we need to calculate the cointegrating relationship between consumption, labor income, and financial wealth. We also need to know the average growth rate of value. For the out-of-sample forecasts, all of those parameters are estimated using only backward-looking information. The only possible source of look-ahead bias here would be data revisions.
The top panel of figure A2 plots the values of the statistics using $\hat{\alpha}_t$ as the predictor against a null of a constant expected equity for horizons from 1 to 20 quarters. We can easily reject the null at the 5 percent level at all horizons and at the 1% level for 2–13 quarter horizons.

D.3.1 Bootstrapping

A major concern with predictive regressions is that asymptotic distribution theory is often a poor guide to small-sample behavior. A simple way to deal with that concern is to use a bootstrap to construct confidence intervals for the test statistics. I construct bootstrap samples in the following way. I select bootstrap samples of stock returns and growth rates of consumption, asset wealth, and labor income. I then construct level series for consumption, wealth, and income, and calculate $\hat{\alpha}$ using purely backward-looking information as above. Finally, I construct the test statistic from above for each bootstrapped sample at each horizon from 1 to 20 quarters. I bootstrap 10,000 samples of data. The top panel of figure A2 plots the 95th and 99th percentiles of the bootstrapped test statistics, and the out-of-sample forecasting power is still significant at the 5 percent level.

D.3.2 $\hat{\alpha}$ versus $cay$

We can also use the out-of-sample test to ask whether estimated risk aversion forecasts stock returns better than $cay$. The null model is now one where stock returns depend on a constant and the lagged value of $cay$, and the encompassing alternative adds the lagged value of $\hat{\alpha}$. The bottom panel of figure A2 plots the test statistics. At every horizon, we can reject the null that $\hat{\alpha}$ does not improve the forecast using $cay$ at the 5 percent level, and we can reject the null at the 1 percent level at every horizon longer than 1 quarter.

Figure A2 also plots the statistic for a test of whether $cay$ has any marginal predictive power above that of $\hat{\alpha}$. At horizons shorter than 8 quarters, we cannot reject the null that it does not. At longer horizons, though, there is evidence that both variables contain important information for forecasting stock returns.
### Table 1. Calibration

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Target</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\gamma$</td>
<td>0.33</td>
<td>Capital income share</td>
</tr>
<tr>
<td>$\beta$</td>
<td>0.9975</td>
<td>2% annual real risk-free rate</td>
</tr>
<tr>
<td>$\delta$</td>
<td>0.02</td>
<td>8% annual depreciation (BEA data)</td>
</tr>
<tr>
<td>$\mu$</td>
<td>0.005</td>
<td>2% annual output growth</td>
</tr>
<tr>
<td>$\varphi$</td>
<td>0.94</td>
<td>Persistence of price/dividend ratio</td>
</tr>
<tr>
<td>$\rho$</td>
<td>0.67</td>
<td>A priori (see text)</td>
</tr>
<tr>
<td>$\sigma_a$</td>
<td>0.0088</td>
<td>Long-run standard deviation of consumption growth</td>
</tr>
<tr>
<td>$\text{mean}(\alpha_t)$</td>
<td>14</td>
<td>Mean Sharpe ratio (0.32 annualized)</td>
</tr>
<tr>
<td>$\text{stdev}(\alpha_t)$</td>
<td>6.2</td>
<td>Stock return predictability</td>
</tr>
<tr>
<td>$\sigma_x$</td>
<td>0.012</td>
<td>Variance of output growth</td>
</tr>
<tr>
<td>$\varphi_x$</td>
<td>0.9</td>
<td>Variance of output growth</td>
</tr>
</tbody>
</table>

Note: Parameters used for the structural models. In table 2, the CRRA model uses with $\text{stdev}(\alpha)=0$; the benchmark EZ-habit model (column 3) sets $\sigma_x=0$.

### Table 2. Comparison of preference specifications

<table>
<thead>
<tr>
<th>Model:</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Real moments:</td>
<td>Data</td>
<td>EZ-CRRA</td>
<td>EZ-habit</td>
<td>Dual-shock</td>
</tr>
<tr>
<td>1 Long-run SD(dC,dY,dI) (%)</td>
<td>0.88</td>
<td>0.88</td>
<td>0.88</td>
<td>0.88</td>
</tr>
<tr>
<td>2 StdDev(dY) (%)</td>
<td>0.99</td>
<td>0.59</td>
<td>0.59</td>
<td>1.03</td>
</tr>
<tr>
<td>3 StdDev(dC) (%)</td>
<td>0.46</td>
<td>0.28</td>
<td>0.47</td>
<td>0.56</td>
</tr>
<tr>
<td>4 StdDev(dI) (%)</td>
<td>2.65</td>
<td>1.11</td>
<td>0.83</td>
<td>2.37</td>
</tr>
<tr>
<td>5 corr(dC(t),Rf(tv1))</td>
<td>-0.09</td>
<td>0.28</td>
<td>0.07</td>
<td>0.08</td>
</tr>
<tr>
<td>Financial moments:</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>6 Mean SR (annualized)</td>
<td>0.32</td>
<td>0.22</td>
<td>0.32</td>
<td>0.32</td>
</tr>
<tr>
<td>7 Std. dev. SR</td>
<td>0.22</td>
<td>0.12</td>
<td>0.22</td>
<td>0.22</td>
</tr>
<tr>
<td>8 p-value SR std. dev.</td>
<td>N/A</td>
<td>0.16</td>
<td>0.50</td>
<td>0.42</td>
</tr>
<tr>
<td>9 Mean Rw (annualized %)</td>
<td>6.78</td>
<td>1.04</td>
<td>4.17</td>
<td>4.15</td>
</tr>
<tr>
<td>10 StdDev(Rw) (annualized %)</td>
<td>21.19</td>
<td>4.71</td>
<td>13.30</td>
<td>12.98</td>
</tr>
<tr>
<td>11 Mean Rf (annualized %)</td>
<td>0.91</td>
<td>2.20</td>
<td>2.04</td>
<td>1.94</td>
</tr>
<tr>
<td>12 StdDev(Rf) (annualized %)</td>
<td>1.16</td>
<td>0.21</td>
<td>0.25</td>
<td>0.26</td>
</tr>
</tbody>
</table>

Note: Column 2 gives results under Epstein–Zin preferences with constant relative risk aversion, column 3 uses EZ-habit preferences and random-walk technology, and column 4 EZ-habit preferences with the dual-shock specification. All models are calibrated as in table 1. All variables are measured using quarterly values. dI is investment growth, dY output growth, and dC consumption growth. Rf is the risk-free rate (measured empirically as the nominal 3-month yield minus an inflation forecast), and Rw is the annualized return on a levered consumption claim (with a leverage ratio of 2.74). The long-run SD is the square root of the spectral density at frequency zero multiplied by $2\pi$. SR is the annualized Sharpe ratio; the standard deviation of the Sharpe ratio is measured by the standard deviation of the fitted values in forecasts of one-quarter-ahead returns in 228-quarter simulated samples divided by the unconditional standard deviation of returns. Row 7 reports the median of the standard deviations of the Sharpe ratio in the simulated samples. Row 8 reports the fraction of simulated samples in which the standard deviation of the Sharpe ratio is as large as in the data.
Table 3. Proportion of simulated samples that match empirical statistics

<table>
<thead>
<tr>
<th>Forecasting R^2</th>
<th>Model:</th>
<th>EZ-CRRA</th>
<th>EZ-habit</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Predictor:</td>
<td>cay</td>
<td>P/D</td>
</tr>
<tr>
<td>Horizon (quarters)</td>
<td>1</td>
<td><strong>0.00</strong></td>
<td>0.14</td>
</tr>
<tr>
<td>4</td>
<td><strong>0.01</strong></td>
<td>0.13</td>
<td>0.11</td>
</tr>
<tr>
<td>8</td>
<td><strong>0.01</strong></td>
<td>0.13</td>
<td>0.12</td>
</tr>
<tr>
<td>12</td>
<td><strong>0.02</strong></td>
<td>0.16</td>
<td>0.17</td>
</tr>
<tr>
<td>16</td>
<td>0.04</td>
<td>0.20</td>
<td>0.26</td>
</tr>
<tr>
<td>20</td>
<td>0.10</td>
<td>0.25</td>
<td>0.45</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>KVB t-statistics</th>
<th>Model:</th>
<th>EZ-CRRA</th>
<th>EZ-habit</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Predictor:</td>
<td>cay</td>
<td>P/D</td>
</tr>
<tr>
<td>Horizon (quarters)</td>
<td>1</td>
<td>0.10</td>
<td>0.05</td>
</tr>
<tr>
<td>4</td>
<td><strong>0.04</strong></td>
<td>0.11</td>
<td>0.17</td>
</tr>
<tr>
<td>8</td>
<td><strong>0.02</strong></td>
<td>0.11</td>
<td>0.08</td>
</tr>
<tr>
<td>12</td>
<td><strong>0.03</strong></td>
<td>0.15</td>
<td>0.12</td>
</tr>
<tr>
<td>16</td>
<td>0.06</td>
<td>0.14</td>
<td>0.22</td>
</tr>
<tr>
<td>20</td>
<td>0.06</td>
<td>0.20</td>
<td>0.19</td>
</tr>
</tbody>
</table>

Note: The top panel reports, for each predictor, the proportion of simulated 228-quarter samples in which the R^2 in a return-forecasting regression is at least as large as observed in the data and where the predictive relationship has the correct sign. The bottom panel reports the proportion of simulated samples that generate Kiefer-Vogelsang-Bunzel t-statistics for each variable that are as large as in the data and have the same sign. The EZ-habit model is the benchmark (single-shock) model. cay is the consumption/wealth ratio from Lettau and Ludvigson (2001). P/D is the price/dividend ratio from CRSP. All simulated regressions use as the predictor the wealth/consumption ratio (equivalently, the price/dividend ratio on a claim to aggregate consumption) and the dependent variable is the excess return on a consumption claim. The regressions are run at horizons listed in the left-hand column. Bold numbers are less than 0.05, bold italics less than 0.01.
Table 4. Various return predictors

<table>
<thead>
<tr>
<th></th>
<th>Data</th>
<th>EZ-CRRA</th>
<th>EZ-habit</th>
<th>Dual-shock</th>
</tr>
</thead>
<tbody>
<tr>
<td>Five-year excess stock return correlations:</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\alpha_t$</td>
<td>0.69</td>
<td>0.05</td>
<td>0.31</td>
<td>0.27</td>
</tr>
<tr>
<td>Real interest rate</td>
<td>0.09</td>
<td>-0.05</td>
<td>-0.30</td>
<td>-0.23</td>
</tr>
<tr>
<td>RREL</td>
<td>-0.09</td>
<td>-0.03</td>
<td>-0.22</td>
<td>-0.17</td>
</tr>
<tr>
<td>Term spread</td>
<td>0.20</td>
<td>0.05</td>
<td>0.29</td>
<td>0.10</td>
</tr>
<tr>
<td>P/D</td>
<td>-0.39</td>
<td>-0.05</td>
<td>-0.30</td>
<td>-0.23</td>
</tr>
</tbody>
</table>

Term spread summary statistics:

<table>
<thead>
<tr>
<th></th>
<th>Mean</th>
<th>Std. dev.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>1.43</td>
<td>0.11</td>
</tr>
<tr>
<td>Std. dev.</td>
<td>0.11</td>
<td>0.07</td>
</tr>
</tbody>
</table>

Note: The bottom two rows are the mean and standard deviation of the spread between the yield on a ten-year equivalent bond and a one-quarter riskless bond (measured in the data using nominal Treasuries). The remaining rows give population correlations between various variables and five-year stock returns. For the simulations, population values are reported. RREL is the gap between the risk-free rate and its four-quarter moving average; P/D is the price/dividend ratio. In the simulations, P/D is measured as the wealth/consumption ratio. $\bar{\alpha}$ is the value of risk aversion in the model, and estimated in the data as in section 4.

Table 5. Regressions estimating the elasticity of intertemporal substitution

<table>
<thead>
<tr>
<th>Model:</th>
<th>Data</th>
<th>EZ-CRRA</th>
<th>EZ-habit</th>
<th>Dual-shock</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 Population, infeasible ($E_t[\Delta c_{t+1}]$)</td>
<td>N/A</td>
<td>1.50</td>
<td>0.64</td>
<td>0.78</td>
</tr>
<tr>
<td>2 Population</td>
<td>N/A</td>
<td>1.50</td>
<td>0.56</td>
<td>0.71</td>
</tr>
<tr>
<td>3 Small sample</td>
<td>0.14</td>
<td>1.16</td>
<td>0.03</td>
<td>0.35</td>
</tr>
<tr>
<td>4 [2.5%, 97.5%]</td>
<td>N/A</td>
<td>[0.03, 1.79]</td>
<td>[-1.98, 1.02]</td>
<td>[-1.32, 1.34]</td>
</tr>
<tr>
<td>5 Population, RRA control</td>
<td>N/A</td>
<td>N/A</td>
<td>1.50</td>
<td>1.50</td>
</tr>
<tr>
<td>6 Small sample, RRA control</td>
<td>0.18</td>
<td>N/A</td>
<td>0.07</td>
<td>0.94</td>
</tr>
<tr>
<td>7 [2.5%, 97.5%]</td>
<td>N/A</td>
<td>N/A</td>
<td>[-3.08, 3.11]</td>
<td>[-1.07, 2.78]</td>
</tr>
</tbody>
</table>

Note: Values reported are the coefficient from regressions of consumption growth or expected consumption growth on the risk-free rate. The dependent variable in row 1 is expected consumption growth (computed numerically in the simulations); all other rows use actual consumption growth. The small-sample regressions are based on 228 quarters of data, and median coefficient estimates are reported; 2.5 and 97.5 percentiles are reported in brackets. The RRA control is actual risk aversion in the simulations and estimated risk aversion (section 4) in the empirical regressions.
Figure 1. Simulated and empirical R²s

Note: R²s from univariate regressions of stock returns on various predictors. The forecast horizon is reported in quarters. Data for $c_{ay}$ is obtained from Sydney Ludvigson's website; Price/dividend data comes from CRSP; the Campbell–Cochrane excess consumption ratio is computed using their parameter values and consumption data from the BEA. The gray lines give the mean and 95th percentiles in the simulation of the EZ-habit model.
Figure 2. Consumption predictability

Autocorrelation of consumption growth

Empirical 95% confidence interval
Empirical estimate
Model-implied values

Correlation of long-horizon consumption growth with W/C

Simulated 95%
Empirical estimate
Simulated mean
Simulated 5%

Forecast horizon (quarters)

Note: The top panel reports the empirical autocorrelation function for consumption. The gray shaded region is the 95% confidence interval using Newey–West standard errors with a lag window of 12 quarters. The bottom panel reports the correlation of consumption growth between periods t and t+x with the wealth-consumption ratio ($c_{ay}$) at date t, where x is reported in quarters on the horizontal axis. The solid lines give the mean and 5th and 95th percentiles of the same correlation in simulated 228-quarter samples.
Figure 3. Impulse response functions

Note: Impulse responses for the EZ-CRRA and EZ-habits models. The shock is a positive unit-standard-deviation increase in technology. The dotted lines are for EZ-CRRA, solid are for EZ-habit. All functions are reported as fractions of the variables’ means except for the risk-free rate, for which the response is in annualized percentage points. Value is lifetime utility; the Sharpe ratio is for an asset that pays aggregate consumption as its dividend.
Figure 4. Value, raw and linearly detrended

Note: Household value (lifetime utility) is measured using data on wealth and consumption from Sydney Ludvigson's website. The thin line is the absolute level of value (left-hand axis); the thick line is value linearly detrended. Both variables are measured in logs. Grey bars are NBER-recorded recessions.
Figure 5. Estimated risk aversion and 5-year excess stock returns

Note: Excess stock returns are for the CRSP value-weighted index minus the risk-free rate, from Ken French's website. Returns are forward-looking five-year averages. Risk aversion is estimated from data on aggregate wealth and consumption and is normalized to have zero mean and unit variance.
Figure 6. Stock returns and estimates of risk aversion from productivity growth

Note: Total factor productivity is the quarterly Solow residual obtained from John Fernald's website. Labor productivity is output per hour in the non-farm private business sector from the BLS. Risk aversion is an AR(1) with innovations equal to the (negative) innovations to the Beveridge–Nelson trend in productivity.
Figure 7. Response of employment to a technology shock

Note: Response of labor supply to a unit-standard-deviation permanent increase in the level of technology.
Table A1. Comparison of results from simulations of projection and log-linear model solutions

<table>
<thead>
<tr>
<th>Differences</th>
<th>Mean</th>
<th>Std. dev.</th>
<th>Scaled std. dev.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Capital</td>
<td>-8.72E-04</td>
<td>0.00073</td>
<td>0.020</td>
</tr>
<tr>
<td>Cons. Growth</td>
<td>3.16E-08</td>
<td>0.0013</td>
<td>0.028</td>
</tr>
<tr>
<td>RRA</td>
<td>1.11E-02</td>
<td>0.75344</td>
<td>0.116</td>
</tr>
<tr>
<td>Sharpe Ratio</td>
<td>-2.15E-03</td>
<td>0.00892</td>
<td>0.123</td>
</tr>
</tbody>
</table>

Note: Comparison of the projection and log-linear solutions. The two simulations use the same shocks but different policy functions. The first column is the mean difference between the simulations, the second column the standard deviation, and the third column the standard deviation of the difference scaled by the standard of the variable in the projection solution. RRA is relative risk aversion.

Table A2: Relative R2s for varying EIS

<table>
<thead>
<tr>
<th>Span</th>
<th>EIS=0.1</th>
<th>0.25</th>
<th>0.75</th>
<th>1.25</th>
<th>1.5</th>
<th>2</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 quarter</td>
<td>0.39</td>
<td>0.26</td>
<td>0.29</td>
<td>1.07</td>
<td>1.10</td>
<td>1.08</td>
<td>0.96</td>
</tr>
<tr>
<td>5 quarters</td>
<td>0.44</td>
<td>0.28</td>
<td>0.29</td>
<td>1.16</td>
<td>1.21</td>
<td>1.21</td>
<td>1.09</td>
</tr>
<tr>
<td>10 quarters</td>
<td>0.61</td>
<td>0.42</td>
<td>0.31</td>
<td>1.41</td>
<td>1.49</td>
<td>1.49</td>
<td>1.37</td>
</tr>
<tr>
<td>20 quarters</td>
<td>1.28</td>
<td>1.02</td>
<td>0.21</td>
<td>1.92</td>
<td>2.08</td>
<td>2.15</td>
<td>2.09</td>
</tr>
</tbody>
</table>

Note: This table lists the ratio of the R2 for a univariate regression of long-horizon returns on estimated risk aversion to the R2 for $cay$. Values less than 1 are in bold. The span in quarters is listed in the left hand column. The top row gives the EIS. The EIS is used to calculate household value and risk aversion.

Table A3. Relative R2s for varying persistence of risk aversion

<table>
<thead>
<tr>
<th>Span</th>
<th>Autocorr.=0.95</th>
<th>0.9</th>
<th>0.85</th>
<th>0.8</th>
<th>0.75</th>
<th>0.7</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 quarter</td>
<td>0.86</td>
<td>1.27</td>
<td>1.10</td>
<td>0.90</td>
<td>0.75</td>
<td>0.64</td>
</tr>
<tr>
<td>5 quarters</td>
<td>1.03</td>
<td>1.44</td>
<td>1.21</td>
<td>0.97</td>
<td>0.78</td>
<td>0.64</td>
</tr>
<tr>
<td>10 quarters</td>
<td>1.17</td>
<td>1.73</td>
<td>1.49</td>
<td>1.20</td>
<td>0.99</td>
<td>0.82</td>
</tr>
<tr>
<td>20 quarters</td>
<td>1.24</td>
<td>2.24</td>
<td>2.08</td>
<td>1.76</td>
<td>1.48</td>
<td>1.27</td>
</tr>
</tbody>
</table>

Note: This table lists the ratio of the R2 for a univariate regression of long-horizon returns on estimated risk aversion to the R2 for $cay$. Values less than 1 are in bold. The span in quarters is listed in the left hand column. The top row gives the annual autocorrelation of risk aversion.

Table A4. R2s from pre and post-1980 univariate return forecasting regressions

<table>
<thead>
<tr>
<th>Span</th>
<th>Estim. RRA</th>
<th>P/D</th>
<th>post-1980</th>
<th>Estim. RRA</th>
<th>P/D</th>
</tr>
</thead>
<tbody>
<tr>
<td>1q</td>
<td>0.10</td>
<td>0.03</td>
<td>1q</td>
<td>0.03</td>
<td>0.03</td>
</tr>
<tr>
<td>5q</td>
<td>0.28</td>
<td>0.25</td>
<td>5q</td>
<td>0.18</td>
<td>0.15</td>
</tr>
<tr>
<td>10q</td>
<td>0.27</td>
<td>0.16</td>
<td>10q</td>
<td>0.48</td>
<td>0.36</td>
</tr>
<tr>
<td>20q</td>
<td>0.38</td>
<td>0.13</td>
<td>20q</td>
<td>0.56</td>
<td>0.33</td>
</tr>
</tbody>
</table>

Note: R2s from univariate regressions of long-horizon stock returns on estimated risk aversion, $cay$, and the price/dividend ratio. The highest value for each horizon and sample is listed in bold.
Figure A1. $\log_{10}$ Euler equation error densities

Note: Densities of Euler equation errors under the two solution methods. The log errors are defined as $\log_{10}(|E[M_{t+1} R_{k,t+1}] - 1|)$. Densities are estimated using a kernel smoother on simulated data. In both cases, the model used is the benchmark single-shock model with EZ-habit preferences and constant labor supply.
Estimated risk aversion depends on the cointegrating model used to estimate $cay$. The top panel tests whether estimated risk aversion has marginal forecasting power against a null of a constant-mean model for returns. The cointegrating vector is reestimated in each period using only backward-looking information. The bottom panel tests adding estimated risk aversion to a null model including a constant and $cay$ and vice versa.

Note: Out-of-sample test statistics from Clark and McCracken (2001, 2005) based on the reduction in out-of-sample RMSE.