Inefficient provision of inside money

Oliver Hart

Harvard University & NBER

and

Luigi Zingales*

University of Chicago, NBER & CEPR

May 2011

(First Version March 2011)

Abstract

We study the role of the private sector in creating inside money. We show that, even in the absence of asymmetric information or an agency problem, the private provision of inside money is inefficient. The reason is that inside money affects prices and the welfare of others, and creators do not internalize this. This distortion is present even if we introduce lending and government money. In fact, we show that lending is complementary to, rather than a substitute for, inside and outside money. To eliminate this inefficiency the government has to restrict the creation of money by the private sector; it cannot just crowd it out with outside money.

* We would like to thank David Abrams, Itay Goldstein, Jonathan Klick, Andrei Shleifer, Jeremy Stein, and participants at the ILE/Wharton finance seminar and the NYU law and economic seminar for helpful comments. Oliver Hart gratefully acknowledges financial support from the U.S. National Science Foundation through the National Bureau of Economic Research. Luigi Zingales gratefully acknowledges financial support from the Center for Research in Security Prices (CRSP), the Stigler Center, and the Initiative on Global Markets at the University of Chicago.
1. Introduction

Historically, one of the primary functions of the banking sector has been to create means of payment (“money like” liabilities). Banks started as money changers; then they added a safe deposit box facility; finally they extended credit, first in the form of actual coins, then as promissory notes. Soon these promissory notes started to exceed the value of the coins deposited, creating what is called “inside money”. Yet, most models of banking focus on other aspects (risk sharing, monitoring, etc.). Less attention has been dedicated to the role of banks in creating money and how this interacts with the government’s role in this area. Does a competitive banking sector generate the socially optimal amount of the means of payment?

In this paper we show that the answer is no, even if we abstract from any moral hazard and asymmetry of information. In particular, we analyze the general equilibrium effects that the availability of money has on prices and identify two pecuniary externalities: more money increases the equilibrium price of the goods that those with the money buy; but it also increases the wealth of the agents supplying these goods and so the prices of the goods they buy. A competitive bank, which ignores the externality imposed on other buyers, will generate too much money. Only a monopolist bank, mutually owned by all the agents in the economy, will be able to internalize all these externalities. But this is tantamount to a central planner. As a result, we show that the private provision of inside money is generally inefficient.

We analyze these inefficiencies arising from the transactional role played by money in a basic economy. We have two groups of agents: doctors and builders. Doctors buy building services from builders and then builders buy doctors’ services from doctors (or the other way round). Agents are also endowed with wheat. The lack of a double coincidence of wants (each builder requires a doctor different from the one he is building for) generates a need for money. Wheat is costly to carry and can easily rot, so banks arise in our model as depositary institutions, which store wheat and issue notes.

We start by studying the effect of this supply of notes on equilibrium prices and on social welfare when banks are purely passive institutions, which notify all the deposits, i.e., an agent who deposits a unit of wheat receives a note of equal value. In notifying the deposits of a doctor a bank imposes a negative externality on doctors at other banks: raising the amount of inside money increases the price of building services, which is bad for them since they consume these services. Because of this externality some wheat endowment is stored to create liquidity instead...
of being invested in profitable opportunities. This result is related to the inefficiency of commodity money derived in Sargent and Wallace (1983). But our result is more general, since it applies to any investment decision that trades off a lower return for a higher liquidity.

This distortion is present even when banks can control the notification process, as long as they act in a competitive way. A monopoly bank, instead, ends up under-producing inside money; this is the standard result that a profit maximizing monopolist restricts production. This distortion is present when we introduce lending and government money. While the introduction of government money potentially crowds out the need for inside money, there is a limit to the amount of government money that can be injected in the system. This limit is driven by the ability of the government to tax.

We find that the introduction of government money does not crowd out the need for bank lending. In fact, it is complementary to bank lending. Money is not neutral in our model: an injection of government money has a positive effect on the level of economic activity and on welfare. Lending multiplies the effect that an injection of money has on the economy.

There exists a large literature on the role played by banks and the need for bank regulation (see, for example, Dewatripont and Tirole (1994)). One branch of this literature, starting with Diamond (1984) and continuing with Holmstrom and Tirole (1997), focuses on the asset side of banks: their role in monitoring loans. Another branch of this literature, starting with Diamond and Dybvig (1983), focuses on the liability side of banks: the ability of banks to provide risk sharing in the face of liquidity needs (Diamond and Dybvig (1983) and Allen and Gale (1998) and (2007)), or to reduce future adverse selection (Gorton and Pennacchi (1990)). Finally, there are some papers, such as Diamond and Rajan (2001), which try to integrate the two sides, showing how demand deposits are critical in making credible the ability of lenders to extract a repayment for their loans.

Holmstrom and Tirole (1998) and (2011) focus on moral hazard on the side of suppliers of liquidity, rather than on the side of users. They show that, in the presence of aggregate uncertainty, the state power to tax future income creates liquidity for the corporate sector, improving its ability to invest.

In all these papers the source of friction is either some informational asymmetry or some agency problem (or both). While these problems are important, they are not the only ones relevant for banks. One important reason why banks are unique is that they issue liabilities that
are used as a means of payment. Our goal is to analyze the implications of this role. For this reason, we abstract from all the other frictions and focus on the pecuniary externalities in the creation of inside money. A general theory of banking would bring all these frictions together.

A new strand of the banking literature, which deals with the transactional role of deposits, introduces behavioral features. This strand derives the uniqueness of banks from the misperception by depositors that their claim are safe (Gennaioli et al. (2010), Rotemberg (2010)) or from the banks’ ability to arbitrage irrationally exuberant markets and rationally priced deposits (Shleifer and Vishny (2010)). We do not introduce behavioral aspects here.

Our result that the creation of inside money is excessive is similar to Stein (2011). In his model, however, it is assumed that agents have a discontinuous demand for a riskless claim (money), while we do not make such an assumption. Similarly, his inefficiency arises from an assumed friction in the financial markets (that patient investors cannot raise additional money), while ours arises endogenously in the model. Stein’s model, however, is much richer in terms of implications for monetary policy. In this respect, the two models can be seen as complementary.

There is also a huge literature on money. Much of this literature is concerned with the role money plays in a general equilibrium model (e.g., Hahn (1965)). To create such a role, one needs to introduce explicitly an exchange process in the standard Arrow-Debreu model, dispensing with the traditional Walrasian auctioneer. Ostroy and Starr (1990) provide an excellent survey of attempts in this direction. As far as we can tell, none of these attempts analyze the pecuniary externality we identify in our paper. The role money plays in our model (i.e., addressing the lack of double coincidence of wants) is similar to Kyotaki and Wright (1989). Their focus, however, is on what goods can become money and how. Our focus is to what extent private banks can provide the efficient quantity of medium of exchange. Another large slice of this literature on money analyzes the role of inside money on monetary policy, as in Brunnermaier and Sannikov (2010), Diamond and Rajan (2006), and Kashyap and Stein (2004). Our model is silent on this.

Our approach resembles that of Kiyotaki and Moore (1997) and (2002). Like us, they rely on some limited pledgeability of future income. Their main focus, however, is on the multiplier/contagion effect that the failure of one intermediary can have on the overall system. Our paper, instead, is concerned with the pecuniary externality in the creation of inside money.
Our approach is also close to that of Mattesini et al (2009). They study banking using the tools of mechanism design. They consider an economy with two groups of consumers who want to trade with each other. As in our model, there is a timing problem: the first group has to buy from the second group before they have sold their own output. Mattesini et al (2009) analyze how claims on deposits with third parties are a better means of exchange than claims on individual wealth. They study the social optimum but not the market equilibrium. In contrast, we take the superiority of third-party deposits as given, and study the divergence between the market equilibrium and the social optimum.

The rest of the paper proceeds as follows. In Section 2 we lay out the framework and describe the Walrasian equilibrium. In Section 3 we analyze the effect of the introduction of banks as storage facilities. In Section 4 we introduce the possibility of bank lending. In Section 5 we study the interaction between private money and government money. In Section 6 we consider some extensions. Conclusions follow.

2. The Framework
We consider an economy that lasts three dates:

1 ----------------------------------------------------2-----------------------------------------------3

There are two types of agents in equal numbers: doctors and builders. The doctors want to consume building services and the builders want to consume doctor services. Doctors and builders can consume wheat at any date and there is no discounting. Both doctors and builders have an endowment of wheat at date 1 equal to \(0 < e < 1\).

We write agents’ utilities as:

Doctors: \(U_d = x_d + b_d - \frac{1}{2}l_d^2\)

Builders: \(U_b = x_b + d_b - \frac{1}{2}l_b^2\)

where \(x_i\) is the sum of the quantities of wheat consumed by individual \(i = d, b\) at each date; \(b_d\) is the quantity of building services consumed by the doctors; \(l_d\) is the labor supplied by the doctors; \(d_b\) is the quantity of doctor services consumed by the builders; and \(l_b\) is the labor
supplied by the builders. We assume constant returns to scale: one unit of builder labor yields one unit of building services and one unit of doctor labor yields one unit of doctor services. We normalize the price of wheat to be 1 at all dates. Let \( p_b \) and \( p_d \) be the price respectively of building and doctor services.

In words, doctors and builders have a constant marginal utility of wheat, a constant utility of the service provided by the other group of agents, and a quadratic disutility of labor.

At date 1 a coin is flipped: if it comes up heads the doctors will buy building services from the builders at date 2 and the builders will buy doctor services from the doctors at date 3; if tails, it’s the other way around. (The outcome of the coin flip is not verifiable and so agents cannot write contracts contingent on this.) After the coin is flipped, each agent can invest part of his wheat endowment in a risky project \( \bar{R} \), whose date 3 expected return is \( \bar{R} > 1 \). This investment opportunity is unique to the agent.

At dates 2 and 3 the market meets and the doctors and builders trade in the order determined at date 1. Throughout the paper we consider the realization of the coin toss where doctors buy at date 2 and builders at date 3; the reverse case is completely symmetrical.

We assume there are many doctors and many builders, and so the prices for both services are determined competitively. It is crucial for our analysis that there is no double coincidence of wants: the builder a doctor buys from requires the doctor services of another doctor: receiving the doctor services of this doctor would be useless (one can imagine that builders and doctors have symmetric but different skills).

2.1 A benchmark: the Walrasian equilibrium

In an ideal world the doctors could pledge to pay the builders out of income from supplying doctor services that they will earn later at date 3 and from the return from their risky project. This is the assumption made in classic Walrasian theory and it is easy to compute the Walrasian equilibrium.

Let’s start by analyzing the case where both the doctors and the builders invest all their endowment in the project and its return is perfectly certain. Then, they solve the following maximization problem:

\[
\text{(2.1) } \quad \text{Max } x_d + b_d - \frac{1}{2} I_d^2
\]
S.T. \[ x_d + p_d b_d \leq p_d l_d + e\bar{R} \].

The solution is

\[(2.2)\]

\[ l_d = p_d \quad \text{if} \quad p_d \geq 1 \]

\[ = \frac{p_d}{p_b} \quad \text{if} \quad p_d < 1 \]

\[ b_d = 0 \quad \text{if} \quad p_b > 1 \]

\[ b_d = \frac{p_d^2}{p_b} + \frac{e\bar{R}}{p_b} \quad \text{if} \quad p_b < 1 \]

\[ 0 \leq b_d \leq \frac{p_d^2}{p_b} + \frac{e\bar{R}}{p_b} \quad \text{if} \quad p_b = 1 \]

The intuition is that, if \( p_b > 1 \), doctors prefer wheat to building services, if \( p_b = 1 \) they are indifferent, and, if \( p_b < 1 \), they prefer building services. The marginal utility of wealth for doctors is 1 if \( p_b \geq 1 \) and \( \frac{1}{p_b} \) if \( p_b < 1 \), and this affects their labor supply decision. If \( p_b \geq 1 \), doctors choose labor supply to maximize \( p_d l_d - \frac{1}{2} l_d^2 \); and, if \( p_b < 1 \), they maximize \( \frac{p_d}{p_b} l_d - \frac{1}{2} l_d^2 \).

Similarly, the builders solve:

\[(2.3)\]

\[ \text{Max } x_b + d_b - \frac{1}{2} l_b^2 \]

S.T. \[ x_b + p_d d_b \leq p_d l_b + e\bar{R} \].

The solution is

\[(2.4)\]

\[ l_b = p_b \quad \text{if} \quad p_d \geq 1 \]

\[ = \frac{p_b}{p_d} \quad \text{if} \quad p_d < 1 \]

\[ d_b = 0 \quad \text{if} \quad p_d > 1 \]

\[ d_b = \frac{p_d^2}{p_b^2} + \frac{e\bar{R}}{p_d} \quad \text{if} \quad p_d < 1 \]
\[ 0 \leq d_b \leq \frac{p_b^2}{p_a^2} + \frac{eR}{p_d} \quad \text{if } p_d = 1 \]

Again the marginal utility of wealth for builders is 1 if \( p_d \geq 1 \) and \( \frac{1}{p_d} \) if \( p_d < 1 \).

For markets to clear we must have

(2.5) \hspace{1cm} b_d = l_b

(2.6) \hspace{1cm} d_b = l_d

On the one hand, (2.5) and (2.6) cannot be satisfied if either \( p_b > 1 \) or \( p_d > 1 \) (demand will be less than supply for building/doctor services, respectively). On the other hand, we cannot have both \( p_b < 1 \) and \( p_d < 1 \) because then the demand for wheat would be zero, while the supply is \( eR \) ((2.5) and (2.6) imply that the wheat market clears). Hence, either \( p_b < 1 \) and \( p_d = 1 \), or \( p_b = 1 \), \( p_d < 1 \), or \( p_b = p_d = 1 \). It is easily seen that the first case is inconsistent with (2.5) and the second with (2.6).

We are left with \( p_b = p_d = 1 \). It is immediate that (2.2)-(2.6) hold if \( b_d = l_b = d_b = l_d = 1 \). Given this result, it follows that it is optimal for the agents to invest all their endowment in the project. Hence, we have established

**Proposition 1:** The unique Walrasian equilibrium satisfies \( p_b = p_d = b_d = l_b = d_b = l_d = 1 \), and \( f_d = f_b = 0 \). The utilities of the doctors and builders are \( U_d = eR + \frac{1}{2} \), \( U_b = eR + \frac{1}{2} \), respectively, and total welfare (social surplus) equals \( W \equiv U_d + U_b = 2Re + 1 \).

Note that the Walrasian allocation and prices are independent of the initial endowment \( e \) (except for the doctors’ consumption of wheat, which varies one to one with \( e \)). It follows from this that our analysis is unchanged if the project return is uncertain rather than certain—all that matters is the expectation of \( R \). Finally, the Walrasian equilibrium achieves maximal social surplus, which is to be expected given the first theorem of welfare economics and the symmetry of the parties.
3. Introducing Banks

We now suppose that parties cannot pledge their future labor income or wheat income—the returns from these activities can be diverted or hidden. We also assume that it is impossible for individuals to carry wheat around with them when they trade; it is too cumbersome or the wheat would rot or be stolen. In the absence of any further assumption the model now becomes trivial. Each agent would invest and eat his wheat and no trade between doctors and builders would occur. We would have \( U_d = eR \) and \( U_b = eR \).

However, we now introduce storage facilities. These storage facilities are perfectly secure in the sense that wheat deposited at a facility at date 1 will remain there and be intact at the end of date 3.

Storage per se does not change anything since there is no advantage to doctors from storing wheat rather than consuming it right away. However, let us suppose that claims can be issued on the wheat deposited in a storage facility. In particular, if a doctor deposits \( f \) units of wheat he will receive \( f \) notes, where each note is a claim on a unit of wheat at the end of date 3; he can then use these notes to pay builders. The builders in turn can use these notes to pay doctors. At the end of date 3 the holders of the notes can go to the storage facility and redeem them for an equal number of units of wheat. We call these storage facilities banks. At the moment, these are completely passive institutions, which just store and issue notes on a one-for-one basis.

3.1 Individual optimization

Let’s consider first the doctors. At date 1, a doctor invests \( e - f_d \) units of wheat. He also receives \( f_d \) in notes from his bank. At date 2 he uses these notes to purchase building services. At date 3, he will choose \( l_d \) to maximize \( p_d l_d - \frac{1}{2} l_d^2 \), i.e., set \( l_d = p_d \). This yields revenue \( p_d^2 \) in the form of notes, which he redeems for wheat at the end of date 3 (it is too late to buy more building services); in addition he incurs an effort cost of \( \frac{1}{2} p_d^2 \). Finally, he will also receive the payoff from his investment.

Therefore, a doctor’s utility when he has to buy first is
\[
\frac{f_d}{p_b} + \frac{1}{2} p_d^2 + \bar{R}(e - f_d)
\]

Each doctor chooses \( f_d \) to maximize (3.1), taking prices as given. The first order condition is given by

\[
\frac{1}{p_b} = \bar{R}
\]

Similarly, a builder’s utility is given by

\[
\frac{f_b}{p_d} + \frac{1}{2} \left( \frac{p_b}{p_d} \right)^2 + \bar{R}(e - f_b)
\]

We shall see shortly that builders will be a corner, so their first order condition is

\[
\frac{1}{p_d} \leq \bar{R}
\]

3.2 Market Equilibrium

We solve for the equilibrium under the conjecture that \( f_b = 0 \). Since only \( f_d > 0 \), we will drop the subscript and set \( f_d = f \). We also conjecture that \( p_d \) and \( p_b \) are less than one. In due course we will show that all these conjectures are correct.

We work backwards, starting with the market for doctors at date 3. After the doctors have bought building services, the builders find themselves with a quantity \( f \) of wheat. Hence, their demand for doctor services will be given by \( \frac{f}{p_d} \). The supply for doctor services is given by

\[
l_d = p_d,
\]

as in (2.2), since any income received by doctors is spent on wheat (it is too late to consume more building services). Hence, market clearing requires
Similarly, the market-clearing condition in the building services market is
\[
\frac{f}{p_d} = \frac{p_b}{p_d}.
\]

Combining this with (3.5) yields
\[
p_d = f^2, \quad p_b = f^3.
\]

Substituting the equilibrium price into (3.2) we have:
\[
\hat{f} = \frac{1}{4} \frac{1}{R^3}
\]
where we assume that \(e\) is larger than \(\hat{f}\), so the solution is not at the corner. Since \(R > 1\), \(f < 1\), which implies \(p_d > p_b\). Hence (3.3) holds and the builders will be at a corner solution with \(f_b = 0\), as initially conjectured.

3.3 Social Optimum
Recall that ex ante it is not known who will buy first: doctors or builders. Thus the expected utility of each group is
\[
\frac{1}{2} U_d + \frac{1}{2} U_b.
\]

The social optimum is obtained by maximizing \(U_d + U_b\) taking into account the effect of \(f\) on prices. That is, the planner maximizes
\[
\frac{1}{4} f^4 + \frac{1}{2} f + \frac{1}{2} f^2 + (e - f)R + eR
\]
The first order condition is
Comparing the right hand side of (3.9) and (3.12) we have

**Proposition 3:** If \( \bar{R} > 1 \), the market equilibrium leads to an excessive amount of inside money \( f \).

**Proof:** Since the l.h.s. of both (3.9) and (3.12) are decreasing in \( f \), to prove that the solution of (3.9) is large than the solution of (3.12) we have to show that

\[
\frac{3}{4} f^{-\frac{3}{4}} + \frac{1}{2} + \frac{1}{4} f^{-\frac{1}{2}} = \bar{R}.
\]

This can be rewritten as

\[
\frac{1}{2} (f^{-\frac{3}{4}} - 1) + \frac{1}{4} (f^{-\frac{3}{4}} - f^{-\frac{1}{2}}) > 0,
\]

which is true as long as \( f < 1 \). When \( f=1 \), the inequality becomes an equality.

The intuition is the following. Suppose the doctors buy first. Then, the creation of more means of payments imposes a positive pecuniary externality on the builders (who see the price of their building services go up) and a negative pecuniary externality on the other doctors (who see the price of what they are buying go up). The social planner takes into consideration these externalities, while the market does not. In standard models the effect of these pecuniary externalities is second order and thus does not create a divergence between social and private optimality. Here, however, while the positive externality on other builders is second order, the negative pecuniary externality on other doctors is first order, since the doctors are liquidity constrained.

Notice that if \( \bar{R} = 1 \), the social and private solutions do not differ: they are both \( f = e \). There is still a divergence between private and social incentives, but this divergence is infra marginal. The reason is that both solutions are at a corner, since there is no opportunity cost in the use of wheat as money.

### 3.4 Optimal Regulation

In this simple economy, it is straightforward for the government to restore the first best. Let \( f^* \) be the solution of (3.12). Then, the government can require every agent to deposit an amount equal to \( f^* \). In fact, it is enough for the government to impose the requirement that no agent can deposit more than \( f^* \).
In practice, this simple solution is not easily implementable. People will differ in their wealth and the informational requirements to impose this restriction and enforce it are considerable. One solution is for the government to impose a cap on deposits on the entire banking system and then let the banking system enforce the individual caps. An even simpler way is for the government to impose a tax on deposits.

The optimal tax $\tau$ can be derived by comparing the left hand side of (3.9) and (3.12):

$$\tau = f^{r/\frac{3}{4}} - \frac{1}{4} f^{r/\frac{3}{4}} - \frac{1}{2} f^{r/\frac{1}{2}} - \frac{1}{4} R - \frac{1}{2} \cdot \frac{1}{4} R^2$$

This explicit tax on deposit is tantamount to Regulation Q, introduced with the Glass–Steagall Act and eliminated by 1986, which prohibited banks from paying interest on demand deposits (Gilbert, 1986). We are not trying to justify Regulation Q or say that it was introduced with this purpose, but simply to show that the type of regulation implied by the model is not so far-fetched.

4. Bank Lending

Let’s now consider the case where banks have some ability to seize payments that go through the banking sector. One possibility is that all payments take place through check transfers and that the bank is able to seize salaries before these are cashed for consumption. In other words, the bank can ensure that someone who defaults has zero consumption.

Since we consider the case where doctors go first, we have that only doctors want to borrow. In fact, builders (who buy second) will have no advantage from borrowing. The bank, knowing that they will get no extra revenues at the end of date 3, will insist in being repaid before they endorse their payment to the doctors. But then builders would have to repay their debt before they buy doctors’ services, making their borrowing useless.

Let $\beta$ be the amount of borrowing done by each doctor. By borrowing an amount $\beta$ an individual doctor can consume $\frac{\beta}{p_b}$ of building services, which is attractive if he has to pay back only $\beta$ (given $p_b < 1$). Of course, a bank needs to make sure that it will be repaid. At date 3, a doctor exerts $\frac{1}{2} p_d^2$ of effort, receiving in exchange $p_d^2$ in terms of payment. His net utility is
Thus, he cannot borrow more than \( \frac{1}{2} p_d^2 \); if he did he would prefer not to work at date 3, default, and consume nothing.

A doctor’s utility is now given by

\[
(4.1) \quad \frac{f + \beta}{p_b} + \frac{1}{2} p_d^2 - \beta + (e - f)\bar{R}
\]

Notice that given \( p_b < 1 \), doctor’s utility is increasing in \( \beta \), thus a doctor will borrow up to the constraint.

The equilibrium in the market for doctors’ services will be given by

\[
\frac{f + \beta}{p_d} = p_d
\]

and the equilibrium in the market for building services will be given by

\[
\frac{f + \beta}{p_b} = \frac{p_b}{p_d}.
\]

Since doctors will borrow as much as possible we will have

\[
\beta = \frac{1}{2} p_d^2 = \frac{1}{2} (f + \beta)
\]

or

\[
(4.2) \quad \beta = f
\]

Hence,

\[
p_d = (2f)^{\frac{1}{2}} \quad \text{and} \quad p_b = (2f)^{\frac{3}{4}}.
\]

From (3.2) the FOC in a competitive market equals to

\[
(4.3) \quad (2f)^{\frac{3}{4}} = \bar{R}
\]

or

\[
\hat{f} = \frac{1}{2} \left( \frac{1}{\bar{R}^{\frac{3}{4}}} \right) = \frac{1}{2} \hat{f}
\]
where $\hat{f}$ is the solution of (3.9) when there was no borrowing and $\hat{f}$ is the solution of (4.3) when there is borrowing. Thus, the presence of lending cuts in half the amount of wheat that is notified, reducing the welfare losses from notification.

It is easy to see that $U_d(\hat{f}) > U_d(\hat{f})$, since

$$U_d(\hat{f}) = \frac{\hat{f} + \beta}{p_b} + \frac{1}{2} p_d^2 + (e - \hat{f}) \bar{R} - \hat{f}$$

or

$$(2\hat{f})^{\frac{1}{2}} + (e - \hat{f}) \bar{R} = (\hat{f})^{\frac{1}{2}} + (e - \frac{1}{2} \hat{f}) \bar{R} > (\hat{f})^{\frac{1}{2}} + (e - \hat{f}) \bar{R} = U_d(\hat{f}).$$

Similarly, we have that

$$U_b(\hat{f}) = \frac{1}{2} (\frac{p_b}{p})^2 + e\bar{R} = \frac{1}{2} (2\hat{f})^{\frac{1}{2}} + e\bar{R} = \frac{1}{2} \hat{f}^{\frac{1}{2}} + e\bar{R} = U_b(\hat{f}).$$

The central planner solution is obtained by maximizing the sum of the utility of the doctor and the builder, i.e., the expected utility of each:

$$W = \frac{\hat{f} + \beta}{p_b} + \frac{1}{2} p_d^2 + (e - \hat{f}) \bar{R} - \beta + \frac{1}{2} (\frac{p_b}{p})^2 + e\bar{R}$$

Substituting the value of $p_d$ and $p_b$, we obtain

$$W = (2\hat{f})^{\frac{1}{3}} + (e - \hat{f}) \bar{R} + \frac{1}{2} (2\hat{f})^{\frac{1}{3}}$$

Thus, the FOC is

$$(4.4) \quad \frac{1}{2} (2\hat{f})^{\frac{3}{4}} + \frac{1}{2} (2\hat{f})^{\frac{1}{2}} = \bar{R}$$

By comparing (4.3) and (4.4) we have

Proposition 4: If $\bar{R} > 1$, the market equilibrium leads to an excessive amount of inside money even in the presence of lending. Lending, however, increases welfare.
Proof: Since the l.h.s. of both (4.3) and (4.4) are decreasing in \( f \), to prove that the solution of (4.3) is larger than the solution of (4.4) we have to show that \((2f)^{-\frac{3}{4}} > \frac{1}{2}(2f)^{-\frac{3}{4}} + \frac{1}{2}(2f)^{-\frac{1}{2}}\), or \((2f)^{-\frac{3}{4}} > (2f)^{-\frac{1}{2}}\), which is always true since \(2f < 1\).

Lending, thus, does not resolve the tension between private and social objectives. Nevertheless, lending does improve welfare since it increases the volume of trade without sacrificing the higher return of the alternative investment. Interestingly, however, lending does not substitute for the notification of wheat, it only complements it. To see why, look at equation (4.2). The amount of feasible lending is directly related to the amount of notes present in the system. The reason is that lending faces a repayment constraint. With no notes in the system, the only reason doctors would work at date 3 would be to repay the loan. This would imply that the amount of loan is equal to the amount of revenue they expect to receive by selling their doctor’s services. If the revenue is equal to the debt, however, it is not in the interest of the doctors to work, given that they have to exert costly effort. Hence, the doctor will default. To have a functioning lending market, we need a minimum amount of deposits.

Finally, it is important to emphasize that we have only scratched the surface of borrowing. If we introduce uncertainty in the proceeds from trade, then borrowing will be risky. (In principle it could be contingent.) Some loans may not be repaid, which might cause some banks not to be able to honor their claims. This may lead to contagion effects, as consumers cannot redeem claims and in turn default, leading other banks to default. (Contagion effects are analyzed in Kiyotaki and Moore (1997), (2002).) The analysis becomes much more complex, and richer, and we hope to explore the consequences in future work.

5. Interaction between private and public money

So far we have ignored any role of the government in providing liquidity. In this finite horizon economy, to introduce government money we need to specify why it is accepted. Following a long tradition (e.g., Cochrane (1998)), we assume that government money is valuable because one can pay taxes with it. Thus, each agent receives an amount \( m \) of government money at date 1 and will have to pay some taxes at date 3. We assume that the government has one source of tax revenue: it can impose a mill tax on those who turn wheat into flour.
Assume that each agent can obtain $\lambda$ units of flour at date 3 at the cost of $\frac{1}{2}c\lambda^2$ units of wheat. One unit of flour yields one unit of utility. An agent’s utility is now:

Doctors: $U_d = x_d + b_d - \frac{1}{2}l_d^2 + (1 - t)\lambda_d - \frac{1}{2}c\lambda_d^2$

Builders: $U_b = x_b + d_b - \frac{1}{2}l_b^2 + (1 - t)\lambda_b - \frac{1}{2}c\lambda_b^2$

where $t$ is the tax rate on flour.

The assumption is that this transformation from wheat into flour can be financed out of the return from the risky investment. We assume that this return is always high enough so that agents are not at a corner solution, and so $\lambda_d, \lambda_b$ satisfy the first order condition

(5.1) $\lambda_d = \lambda_b = \frac{1-t}{c}$.

Hence, the budget balance for the government implies

(5.2) $m = \frac{t(1-t)}{c}$.

5.1 No private borrowing

For simplicity let’s start with the case where there is no private borrowing. Consider the market for doctors. The number of notes in the hands of builders will be $f+m$ (from trading with doctors) plus $m$ of their own. Thus, market equilibrium is given by

(5.3) $\frac{f+2m}{p_d} = p_d$

On the other hand, in the market for builders the number of notes available to doctors is $f+m$, so the market equilibrium is given by

(5.4) $\frac{f+m}{p_b} = \frac{p_b}{p_d}$

Solving (5.3) and (5.4) yields

(5.5) $p_d = (f + 2m)^{\frac{1}{2}}$ and $p_b = (f + m)^{\frac{1}{2}}(f + 2m)^{\frac{1}{2}}$.

We can write the utilities of doctors and builders as
where we use (5.1) to substitute for $\lambda_d$, $\lambda_b$.

The competitive equilibrium is characterized by the first order condition of (5.6) with respect to $f$, where prices are taken as given:

\begin{equation}
\frac{1}{p_b} = \bar{R}.
\end{equation}

In contrast, the planner maximizes $W = U_d + U_b$ taking into account the effects of $f$ and $m$ on prices. In other words, the planner maximizes

\begin{equation}
W = \frac{(f + m)^{\frac{1}{2}}}{(f + 2m)^{\frac{1}{2}}} + \frac{1}{2} f + m + \frac{1}{c} (1-t)^2 - f\bar{R} + \frac{1}{2} \frac{f + m}{(f + 2m)^{\frac{1}{2}}} + \frac{m}{(f + 2m)^{\frac{1}{2}}} + 2e\bar{R}.
\end{equation}

Several questions are worth asking. First, starting at the competitive equilibrium, does the planner want to introduce outside money, i.e. $m > 0$, given that the market will adjust to a new competitive equilibrium? The answer seems to be ambiguous, perhaps because the market provides too much of its own money (sets $f$ too high).

A second question is: given that the planner can regulate $f$, does she want to set $m > 0$? Here the answer is affirmative. To see this, set $f$ at the regulatory optimum, i.e., maximize $W$ with respect to $f$ when $m = 0$ (this is equivalent to maximizing (3.11)). Now consider a small change in $m$ (or equivalently in $t$). By the envelope theorem,

\begin{equation}
\frac{dW}{dm} \bigg|_{m=0} = \frac{\partial W}{\partial m} \bigg|_{m=0} = \frac{f^{\frac{1}{2}}}{f^{\frac{1}{2}}} - \frac{f^{\frac{1}{2}}}{4} \frac{1}{2} f^{\frac{3}{2}} - \frac{2(1-t)}{c} \frac{dt}{dm} + \frac{1}{2} \frac{f^{\frac{1}{2}}}{f^{\frac{1}{2}}} - \frac{f^{\frac{1}{2}}}{2} \frac{f^{\frac{3}{2}}}{f} + \frac{f^{\frac{1}{2}}}{f}
\end{equation}

From (5.2), $\frac{dt}{dm} = \frac{c}{1-2t} = c$ at $m = 0$, and so

\begin{equation}
\frac{dW}{dm} \bigg|_{m=0} = -1 + \frac{1}{f^{\frac{1}{2}}} > 0
\end{equation}
since (5.5) and (5.8) imply \( f < 1 \) when \( m = 0 \) (given \( R > 1 \)). It follows that it is always better for the planner to set \( m > 0 \) than \( m = 0 \).

This result can also be interpreted as a proof of the non-neutrality of money: welfare goes up when \( m \) goes up. To be fair, our model is unable to distinguish between monetary policy and fiscal policy, since the helicopter drop of money could easily be replaced by a government purchase of goods financed with debt (government money and debt are equivalent in our model). If we were to take that interpretation, though, we could have an example in which fiscal policy is not neutral even in the presence of Ricardian equivalence.

A third question is: given that the regulator can choose \( m \) (and \( t \)) optimally, does she want to set \( f = 0 \), i.e., does outside money crowd out inside money? Here the answer is: it depends.

Again, taking advantage of the envelope theorem, one can look at

\[
\left. \frac{dW}{df} \right|_{f=0}
\]

From this it is clear that, on the one hand, if \( \bar{R} \) is very large, the socially optimal \( f \) is indeed zero. On the other hand, if \( c \) is very large, then the maximum possible value of \( m \) (given by \( \frac{1}{4c} \)) is very small.\(^1\) In other words, a pure outside money economy will achieve very little trade, and inside money by itself can do better. Thus, the optimal \( f \) is bigger than zero.

5.2 Case with private borrowing

We now show that our result that some outside money is optimal \((m > 0)\) generalizes to the case of borrowing. In the presence of private borrowing (5.3) and (5.4) are replaced by

\[
(5.12) \quad \frac{f + \beta + 2m}{p_d} = p_d
\]

and the equilibrium in the market for building services will be given by

\[
(5.13) \quad \frac{f + \beta + m}{p_b} = \frac{p_b}{p_d},
\]

and hence

\[
(5.14) \quad p_d = (f + \beta + 2m)^{\frac{1}{2}} \quad \text{and} \quad p_b = (f + \beta + m)^{\frac{1}{2}} (f + \beta + 2m)^{\frac{1}{4}}.
\]

---

\(^1\) This maximum value of \( m \) can be obtained from maximizing (5.2) with respect to \( t \).
We know that $\beta = \frac{1}{2} p_d^2$, from which it follows that

\begin{equation}
\beta = f + 2m
\end{equation}

Hence, total welfare is given by

\begin{equation}
W = \frac{f + \beta + m}{p_b} + \frac{1}{2} p_d^2 - \beta + (e-f) \bar{R} + \frac{1}{2c} (1-t)^2 + \frac{m}{p_d} + \frac{1}{2} (\frac{p_b}{p_d})^2 + e \bar{R} + \frac{1}{2c} (1-t)^2 =
\end{equation}

\begin{equation}
= \frac{(2f+3m)^\frac{1}{2}}{c} + \frac{1}{2} (1-t)^2 - f \bar{R} + \frac{1}{2} \frac{2f+3m}{(2f+4m)^\frac{1}{2}} + \frac{m}{(4f+4m)^\frac{1}{2}} + 2e \bar{R}
\end{equation}

The planner will maximize (5.16) with respect to $f$ and $m$ (given that $\beta$ adjusts to satisfy (5.15)).

Using the envelope theorem we have

\begin{equation}
\left. \frac{dW}{dm} \right|_{m=0} = \left. \frac{\partial W}{\partial m} \right|_{m=0} = \frac{1}{2} (2f)^{-\frac{1}{2}} - 2 + \frac{3}{2} (2f)^{-\frac{1}{2}} > 0
\end{equation}

if $f < \frac{1}{2}$, which we know to be the case from (4.3). Hence, it is optimal to set $m > 0$.

Interestingly, the introduction of government money does not eliminate the role for bank lending. In fact, it is complementary to bank’s lending, as shown by equation (5.15). The reason is similar to the one highlighted in Section 4: with no notes in the system, the only reason why doctors would work at date 3 would be to repay the loan. This would imply that the amount of loan is equal to the amount of revenue they expect to receive by selling their doctor’s services. If the revenue is equal to the debt, however, it is not in the interest of the doctors to work, because they have to exert costly effort. Hence, the doctor will default. To have a functioning lending market, thus, we need either inside or outside money. Lending multiplies the effect from the injection of money.

6 Extensions

6.1 Non competitive banks

Thus far we have treated banks as purely passive institutions. In this subsection we explore how the results change once we allow banks to have a strategy. To keep the analysis simple we assume that the builders, who buy second, have no initial endowment $e$. 
Let us assume that there is a fixed number of banks, \( \frac{1}{\alpha} \), where \( 0 < \alpha < 1 \). Each bank serves a fraction \( \alpha \) of the doctors. For simplicity we assume that each bank is a monopolist with respect to its constituency of doctors; however, we doubt that much would change if we allowed several banks to compete for the same constituency of doctors.

Note that the case \( \alpha = 0 \) can be interpreted as the (limiting) situation where every doctor can set up his own bank.

We will distinguish between mutual (or cooperative) banks and banks with outside owners. We start with the former.

**Mutual Banks**

We suppose that each bank offers the doctors in its constituency the following service: a doctor can deposit an amount of wheat up to \( \sigma \) and receive notes (or checks) equal to \( \sigma \). Hence, \( \sigma \) is a policy instrument of the bank, which is the same for all customers; moreover, we assume that the bank can announce and commit to it. \(^2\)

If \( \sigma \geq \hat{\sigma} \) obtained in (3.9), then each doctor will set \( f = \hat{\sigma} \) and the bank policy would be irrelevant. Hence, we focus on the possibility of a \( \sigma < \hat{\sigma} \), where each doctor will deposit the full amount allowed \( \sigma \).

Consider a single bank’s choice of \( \sigma \) given that the bank serves a fraction \( \alpha \) of the population of doctors and that the average choice of other banks is \( \bar{\sigma} \). We know from Section 2.2 that even if every doctor deposits all his endowment \( e \) and \( \sigma = 1 \) for all banks, then \( p_b < 1 \) and \( p_d < 1 \). A fortiori this must be true when \( \sigma \leq e \). Thus we can focus on situations where \( p_b < 1 \) and \( p_d < 1 \).

The doctor’s utility will be given by

\[
\frac{\sigma}{p_b} + \frac{1}{2} p_d^2 + (e - \sigma)\bar{R}
\]

\[(6.1)\]

---

\(^2\) One can imagine more complicated arrangements between banks and customers, e.g., banks could put limits on how much each customer can deposit. For simplicity, however, we restrict attention to the above contracts.
The mutual bank chooses $\sigma$ to maximize the utility of a representative member, given by (6.1), taking into account the effect of the bank’s choice of $\sigma$ on prices, $p_b$ and $p_d$.

Let us consider this price effect. Given $\sigma$ the total value of notes in circulation will be $\alpha \sigma + (1 - \alpha) \bar{\sigma}$; the first term represents the contribution of this bank and the second term the contribution of the other banks. Since doctors use all their notes on building services, the demand for building services is

\[(6.2)\quad \frac{\alpha \sigma + (1 - \alpha) \bar{\sigma}}{p_b},\]

while the supply is, as in (2.4), $\frac{p_b}{p_d}$. (Builders will spend all the proceeds of their building services on doctor services.) Equating these yields

\[(6.3)\quad p_b^2 = (\alpha \sigma + (1 - \alpha) \bar{\sigma}) p_d.\]

In the market for doctors, demand is

\[(6.4)\quad \frac{\alpha \sigma + (1 - \alpha) \bar{\sigma}}{p_d},\]

since the builders use all the notes received from doctors to buy doctor services; and supply is $p_d$. Combining this with (6.3) yields

\[(6.5)\quad p_b = (\alpha \sigma + (1 - \alpha) \bar{\sigma})^{\frac{3}{2}},\]

\[p_d = (\alpha \sigma + (1 - \alpha) \bar{\sigma})^{\frac{1}{2}}.\]

Substituting into (6.2), we see that the utility of a representative doctor at the bank choosing $\sigma$ is

\[(6.6)\quad \frac{\sigma}{(\alpha \sigma + (1 - \alpha) \bar{\sigma})^{\frac{3}{2}}} + \frac{1}{2} (\alpha \sigma + (1 - \alpha) \bar{\sigma}) + (e - \sigma) \bar{R}.\]

We study a Nash equilibrium in which each bank chooses $\sigma$ to maximize (6.6), taking $\bar{\sigma}$ as given. Let $\gamma = \alpha \sigma + (1 - \alpha) \bar{\sigma}$ and $z = (1 - \alpha) \bar{\sigma}$. Then, maximizing (6.6) is equivalent to maximizing
(6.7) \[ y^3 - z \frac{1}{y^3} - e^y (1 - \frac{1}{2} \alpha)(y - z) \]

with respect to \( y \). It is easy to see that (6.7) is strictly concave in \( y \). Thus, there is a unique maximizer \( y \) and hence a unique maximizer \( \sigma \) of (6.6), given \( \hat{\sigma} \).

Moreover, the optimal \( y \) is strictly increasing in \( z \). It follows that, if two different banks choose different values of \( \sigma \) in equilibrium, i.e., they face different values of \( z \), then they will choose different values of \( y \). But \( y \) equals the average value of \( \sigma \) over all banks, and must therefore be the same for each bank. It follows that the equilibrium \( \sigma \) is the same for all banks, i.e., any Nash equilibrium is symmetric.

Differentiating (6.6) and setting \( \hat{\sigma} = \hat{\sigma} \), we may conclude that the equilibrium level of \( \sigma \), if \( 0 < \sigma < 1 \), satisfies

(6.8) \[ \sigma^3 + \frac{1}{2} \alpha [1 - \frac{3}{2} \sigma^3] = \bar{R}, \]

Comparing (6.8) with (3.12) we have

Proposition 5. In a competitive market (\( \alpha \) close to 0) banks choose too high a level of deposits with respect to what it is socially efficient. In a monopolistic market (\( \alpha = 1 \)) banks choose too low a level of deposits with respect to what it is socially efficient.

Proof: For \( \alpha = 0 \) (6.8) becomes (3.9) and the same result applies. For \( \alpha = 1 \) the l.h.s. (6.8) becomes \( \sigma^3 + \frac{1}{2} \sigma^3 = \bar{R} \). Since both (6.8) and (3.12) are decreasing in their arguments, to prove that the solution of (6.8) is smaller than the solution of (3.9) we have to show that

\[
\frac{1}{4} f^{-\frac{3}{2}} + \frac{1}{2} f^{-\frac{1}{2}} + \frac{1}{4} f^{-\frac{3}{4}} > \frac{3}{2} + \frac{3}{4} f^{-\frac{3}{4}}, \text{ or } \frac{1}{4} f^{-\frac{3}{2}} > 0, \text{ which is always true.}
\]

The intuition for the competitive result is as before, the intuition for the monopolist one, it rather simple. Large mutual banks restrict \( \sigma \), i.e., issue too few notes, to lower the price of building services; this helps their members since their members consume these services. In doing this, however, large banks ignore the positive externality they impose on builders, who gain from high prices since this allows them to buy more doctor services. Small banks choose a high \( \sigma \) because their impact on prices is limited.
Outside Owned Banks

Suppose now that banks are owned by outsiders rather than the doctors themselves. Each bank approaches individual doctors in its constituency with the following proposal: “You pay a fee $F$ and in return you can decide how much to store with us, $0 \leq \sigma \leq e$, and we will give you notes equal to $\sigma$.” Here we again assume that each bank announces and commits to a particular level of $\sigma$, $0 \leq \sigma \leq e$.

Each doctor always has the option to turn down the bank’s offer. If he does so, he will invest his wheat $e$, and consume no building services. However, he can supply his labor in the market for doctors in the second half of period 2, receiving $\frac{1}{2} p_d^2$ in net terms. Thus, his utility is

$$\frac{1}{2} p_d^2 + eR.$$  \hspace{1cm} (6.9)

In contrast, if the doctor accepts the bank’s offer, his utility will be

$$\frac{\sigma}{p_b} + \frac{1}{2} p_d^2 + (e - \sigma)R - F.$$  \hspace{1cm} (6.10)

A bank’s profits are given by $F$ and hence a profit maximizing bank will choose $F$ so that (6.9) and (6.10) are equal. If an individual bank chooses $\sigma$, while the average choice of other banks is $\bar{\sigma}$, $p_b$ and $p_d$ are given by (6.6), and so

$$F = \frac{\sigma}{p_b} + \frac{1}{2} p_d^2 + (e - \sigma)R - eR - \frac{1}{2} p_d^2.$$  \hspace{1cm} (6.10)

$$= \frac{\sigma}{(\alpha \sigma + (1 - \alpha)\bar{\sigma})^2} - eR.$$  \hspace{1cm} (6.10)

Consider the Nash equilibrium in which each bank maximizes (6.10), taking $\hat{\sigma}$ as given. As before, it is easy to show that the Nash equilibrium is unique and symmetric. Setting the derivative of (6.10) with respect to $\sigma$ equal to zero, substituting $\sigma = \bar{\sigma}$ yields

$$\sigma^3 - \alpha \frac{3}{4} \sigma^2 = \bar{R}.$$  \hspace{1cm} (6.11)

Comparing (6.11) and (6.8), we see that a profit-maximizing bank chooses a lower level of $\sigma$ than does a similarly competitive mutual bank. The intuition is that the profit-maximizing bank ignores the fact that a higher $\sigma$ increases $p_d$, which increases the income doctors obtain...
from supplying doctor services. The point is that this increase is also enjoyed by doctors who do not contract with the bank and so the bank cannot charge for it.

Note, however, that if $\alpha$ is small the solution to (6.11) is still an excessive creation of inside money, while if $\alpha = 1$, the private market delivers too little inside money.

6.2 Banks investing in risky projects
We have assumed that the risky projects can only be undertaken by the agents and not by the banks, because they require some unique human skills that the agents have. What will happen if the banks can undertake risky projects too? One could imagine that each doctor deposits all his endowment in the bank in exchange for shares in the bank’s assets (in proportion to his fraction of the total funds deposited). These shares could then be used for trading. The doctors will endorse these shares to the builders in exchange for building services and the builders will further endorse them to doctors when they buy their services.

In analyzing this case the two key assumptions are when the uncertainty about the risky projects gets resolved and what the risk-return characteristics of the projects undertaken by the banks are. If we take the extreme assumption that all the uncertainty about the risky projects get resolved after the last round of trading at time 3, then the risk characteristics of the projects are irrelevant. For all practical purposes the value of claims backed by risky projects, rather than the wheat, will be valued at the expected value of the risky projects and for a bank investing in the risky projects strictly dominates storing wheat. Hence, if the return of the banks’ risky projects is the same as those of the individual agents, we achieve the second best, with all the wheat deposited and invested in risky projects. If $e > \frac{1}{2}$ and there is the possibility of borrowing, then we achieve also the first best.

If we maintain the same assumption about the resolution of uncertainty, but the return of the risky projects available to banks is inferior to the return available to individuals, then we have the same model as in Section 3 where the storage at the bank will be replaced by the investment in risky projects and the previous $R$ is replaced by the difference between the return on private investments and the return of banks’ investment.

The more interesting (and realistic) case, though, is when some uncertainty gets resolved before the two rounds of trading, so that the value of the notes backed by risky investments
change. While in this economy individual agents are risk neutral, the transactional role played by money makes them risk-averse vis-à-vis fluctuations in the value of notes.

To see this point clearly, let’s consider an extreme example in which the risky project has two possible realizations: \(2R\) and zero with equal probability and uncertainty gets fully resolved before the first round of trading. We want to show that a certain claim with value \(R\) leads to a higher welfare than a claim with expected value \(R\):

\[
W(\tilde{R} = R) > \frac{1}{2} W(\tilde{R} = 2R) + \frac{1}{2} W(\tilde{R} = 0).
\]

If \(Re < 1\), even when all the wheat is invested in risky project and used as a collateral, the economy will be working below its Walrasian potential. Thus, from equation (3.11):

\[
W(\tilde{R} = R) = (Re)^{\frac{1}{2}} + \frac{1}{2} Re + \frac{1}{2} (Re)^{-\frac{1}{2}} + Re
\]

By contrast, if \(2Re > 1\), when all the wheat is invested in risky project and used as a collateral and the risky project turns out to be successful, the economy will be working at its Walrasian potential. Thus,

\[
W(\tilde{R} = 2R) = 1 + 2(2Re),
\]

while if the realization of the risky project is zero, the welfare level will be zero. Comparing the two we have that

\[
(Re)^{\frac{1}{2}} + \frac{1}{2} Re + \frac{1}{2} (Re)^{-\frac{1}{2}} + Re > \frac{1}{2} + 2eR
\]

for \(1 \leq eR \leq 2\), since the function \(x^{\frac{1}{2}} + \frac{1}{2} x^{-\frac{1}{2}} - \frac{1}{2} \) is positive at \(x = 1\) and increasing in \(x\) in the range \(1 \leq x \leq 2\).

Agents dislike the uncertainty about the value of their claims because if the value of their claims drops below 1, the economy operates below full potential, while if the value of the claims operate above 1, no extra benefits are generated.

Let’s now consider the case where banks can choose between a risky project and a safe project and this choice is perfectly contractible, thus there is no moral hazard involved. As above, we assume that the uncertainty is resolved before the first round of trading. Finally, we
assume that the risky projects are perfectly correlated and they have the following payoff per dollar (constant return to scale):

\[ (6.12) \quad \tilde{R} = \frac{R}{\varepsilon} \quad \text{with probability } \varepsilon \]

\[ = 0 \quad \text{with probability } 1 - \varepsilon \]

The expected return of the investment is \( R \) which exceeds 1. We will focus on the limiting case where \( \varepsilon \to 0 \) and the investment becomes infinitely risky.

In this set up the individuals do not have any advantage vis-à-vis the banks and since they can perfectly contract on the mix of safe deposits and risky investments, it is easy to show that they deposit all their money in the banks. Thus, the crucial variable is the proportion \( \mu \) of deposits that a typical bank invests in the riskless asset (storage), where \( 1 - \mu \) is invested in the risky one.

Since trading will take place with banks’ shares, the equilibrium prices will differ according to the state that is realized before the beginning of trading. Denote prices in the bad state by \( p_b, p_d \) and in the good state by \( p_b^g, p_d^g \).

With probability \( 1 - \varepsilon \) the risky investment will yield zero and the value of the doctor’s claim will be \( \mu e \). By contrast, with probability \( \varepsilon \) the risky investment will yield \( \frac{R}{\varepsilon} \) and the value of the doctor’s claim will be

\[ (6.13) \quad g = \mu e + (1 - \mu)\frac{R}{\varepsilon} \]

A doctor’s expected utility is

\[ (6.14) \quad (1 - \varepsilon)[\frac{\mu e}{p_b} + \frac{1}{2} p_d^2] + \varepsilon[\frac{g}{p_d^g} + \frac{1}{2} p_d^{g2}] \]

From the analysis of Sections 3 and 4,

\[ p_b = (\mu e)\frac{3}{2} \quad p_d = (\mu e)^{\frac{1}{2}} \]

\[ p_b^g = \min \left\{ \frac{3}{g^4}, 1 \right\} \quad p_d^g = \min \left\{ \frac{1}{g^2}, 1 \right\} \]
If we keep $\mu < 1$ constant and let $\varepsilon \rightarrow 0$, $p_b^* \converges to one and (6.14) becomes
\[
\frac{\mu e}{p_b} + \frac{1}{2} p_d^2 + (1 - \mu) e R
\]
which is identical to (6.1) with $\mu e = \sigma$. Hence, all the solutions will be the same as in Section 6.1. In particular, for $\varepsilon \rightarrow 0$ the welfare function will become
\[
W \equiv U_d + U_b = (\mu e)^\frac{1}{2} + \frac{1}{2} (\mu e)^\frac{1}{2} + \frac{1}{2} \mu e + (1 - \mu) e R + e R
\]
which is identical to (3.11) with $\mu e = f$. Thus we have

**Proposition 6.** When uncertainty is resolved before trading takes place, the model of this section with trading in bank shares is equivalent to the model of Section 3 with trading in fixed claims on deposits.

7. **Conclusions**

We have built a simple framework to analyze the general equilibrium implications of the creation of inside money by banks. To isolate these effects in this paper we consider either fully-backed money or lines of credit secured by a certain income stream. In so doing we avoid any risk of banks’ default. This risk is clearly a very important problem, which we plan to address in future work.

We show that the competitive equilibrium leads to the creation of an excessive amount of inside money. This distortion is present even if we introduce lending and government money. The source of this distortion is a pecuniary externality that arises from the creation of money. More money increases the equilibrium price of goods purchased by the agents who are liquidity constrained (think for example of the effect of the relaxed credit on the price of U.S. houses). This pecuniary externality has welfare effects, because the buyers are liquidity constrained. The government can remove this distortion by imposing a tax on deposits.
References


